Systematic uncertainties in NLO+PS matching

Marek Schönherr

Institute for Particle Physics Phenomenology

ICHEP, 05/07/2012
Melbourne, Australia

arXiv:1111.1220, arXiv:1201.5882
JHEP04(2011)024, JHEP08(2011)123*

* in collaboration with S. Höche, F. Krauss, F. Siegert

Introduction

Importance of matching NLO calculations with parton showers

- exclusive hadronic final states ("particle" level NLO predicitons)
- observable independent combination of fixed order and resummation
- problem double counting: both NLO and PS are approximations to higher order corrections

Two methods appeared in the literature: MC@NLO and POWHEG

- two sides of one medal
- differ in choices of division of resummation and fixed-order part

Uncertainties of NLO+PS matching

- usual μ_R and μ_F variation as in NLO calculations
- also μ_Q-variation as in analytic resummations

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General NLO+PS matching

$$\langle O\rangle^{\mathsf{NLO}} = \int \mathrm{d}\Phi_B\ \bar{\mathrm{B}}^{(\mathsf{A})}(\Phi_B) \left[\Delta^{(\mathsf{A})}(t_0,\mu_Q^2)\,O(\Phi_B) + \sum_i \int_{t_0}^{\mu_Q^2} \mathrm{d}\Phi_1\, \frac{\mathrm{D}_i^{(\mathsf{A})}(\Phi_B,\Phi_1)}{\mathrm{B}(\Phi_B)}\,\Delta^{(\mathsf{A})}(t,\mu_Q^2)\,O(\Phi_R)\right] + \int \mathrm{d}\Phi_R \left[\mathrm{R}(\Phi_R) - \sum_i \mathrm{D}_i^{(\mathsf{A})}(\Phi_R)\right] O(\Phi_R)$$

- NLO calculation with subtraction terms $\mathrm{I}^{(\mathsf{A})} = \sum_i \int \mathrm{d}\Phi_1\, \mathrm{D}_i^{(\mathsf{S})}$

Frixione, Kunszt, Signer Nucl.Phys.B467(1996)399-442
Catani, Seymour Nucl.Phys.B485(1997)291-419

- introduce second set of subtraction functions $\mathrm{D}_i^{(\mathsf{A})}$

$$\bar{\mathrm{B}}^{(\mathsf{A})} = \mathrm{B} + \tilde{\mathrm{V}} + \mathrm{I}^{(\mathsf{A})} + \sum_i \int \mathrm{d}\Phi_1 \left[\mathrm{D}_i^{(\mathsf{A})}\Theta(\mu_Q^2 - t) - \mathrm{D}_i^{(\mathsf{S})}\right]$$

- NLO+PS matching methods differ in choices for $\mathrm{D}_i^{(\mathsf{A})}$ and μ_Q

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$$\Delta^{(\mathsf{A})}(t_0,t_1) = \exp\left[-\int_{t_0}^{t_1} \mathrm{d}\Phi_1\,\frac{\mathrm{D}_i^{(\mathsf{A})}}{\mathrm{B}}\right]$$

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$$+ \int \mathrm{d}\Phi_R \left[\mathrm{R}(\Phi_R) - \sum_i \mathrm{D}_i^{(\mathsf{A})}(\Phi_R)\right] O(\Phi_R) \qquad \mathbb{H}$$

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POWHEG & MC@NLO (traditional scheme)

POWHEG: Nason JHEP11(2004)040, Frixione et.al. JHEP11(2007)070

- exponentiation kernel $\mathrm{D}_i^{(\mathsf{A})} = \rho_i \cdot \mathrm{R}$ with $\rho_i = \mathrm{D}_i^{(\mathsf{S})} / \sum_i \mathrm{D}_i^{(\mathsf{S})}$
  $\to$ each $\rho_i \cdot \mathrm{R}$ contains only one divergence structure as defined by $\mathrm{D}_i^{(\mathsf{S})}$
- no $\mathbb{H}$-events, resummation scale μ_Q^2 at kinematic limit $\frac{1}{2}\, s_{\mathsf{had}}$
- exponentiation of R through matrix element corrected parton shower
- uncontrolled exponentiation of non-logarithmic terms

MC@NLO (trad. scheme): Frixione, Webber JHEP06(2002)029

- exponentiation kernel $\mathrm{D}_i^{(\mathsf{A})} = \mathrm{B} \cdot \mathcal{K}_i$ with $\mathcal{K}_i$ parton shower kernels
- resummation scale $\mu_Q^2 = t_{\mathsf{max}}$ parton shower starting scale
- in general, $\mathrm{D}_i^{(\mathsf{A})}$ only leading colour approximation
  NLO accuracy depends crucially on correctness of IR-limit

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MC@NLO – $\mathrm{D}_i^{(\mathsf{A})} = \mathrm{D}_i^{(\mathsf{S})}$ scheme

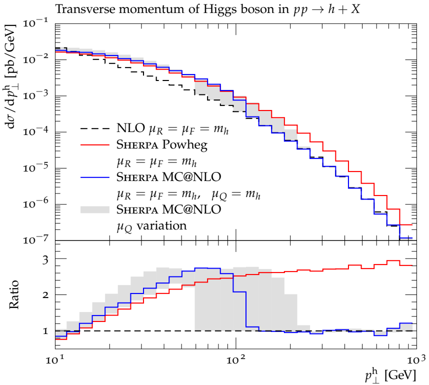
Special choices: SH, FK, MS, FS arXiv:1111.1220

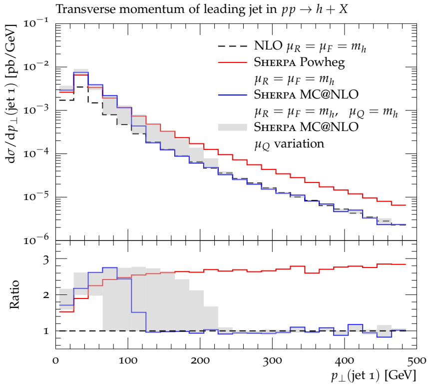
- exponentiation kernel $\mathrm{D}_i^{(\mathsf{A})} = \mathrm{D}_i^{(\mathsf{S})}$
- μ_Q as parton shower starting scale left as a free parameter
  $\rightarrow$ its uncertainty can be assessed systematically

Consequences:

- simplification of $\bar{\mathrm{B}}^{(\mathsf{A})}$-integral $\rightarrow$ integrate Θ-function numerically
- resummation as in parton shower Schumann, Krauss JHEP03(2008)038
  but resummation kernels preserve full-colour and spin dependence
- implementation in SHERPA framework process independent
  $\rightarrow$ all pieces are generated automatically,
  only V needs to be interfaced from dedicated OLE
  BLACKHAT, GOSAM, MCFM, etc.
- trivially NLO and NLL correct

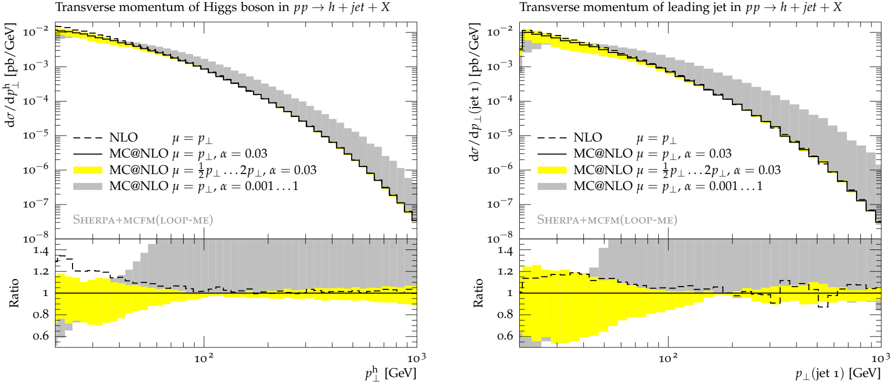
Resummation scale variation in $\mathrm{pp} \to \mathrm{h} + \mathrm{X}$





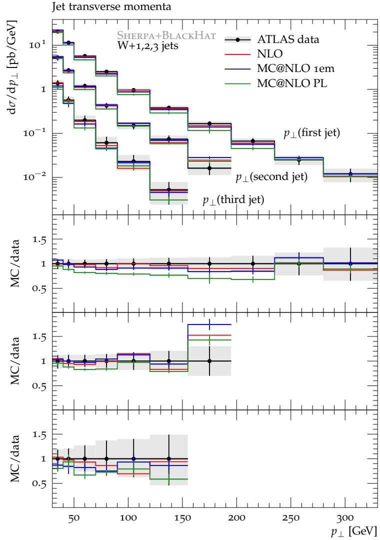
- difference in rate of real emission at $t = \mu_Q \pm \epsilon$
- strong dependence on radiative phase space, $\mu_Q = [\frac{1}{2}, 2]m_h$
- large local K-factors $\Rightarrow$ large uncertainties

Resummation scale variation in $\mathrm{pp} \to \mathrm{h} + \mathbf{jet} + \mathrm{X}$



- increased parton multiplicity worsens problems
- large dependence on exponentiated phase space, $\mu_Q \in (0, \frac{1}{4}\sqrt{s_\text{had}}]$
- unphysical results for $\mu_Q \to \frac{1}{4}\sqrt{s_\text{had}}$

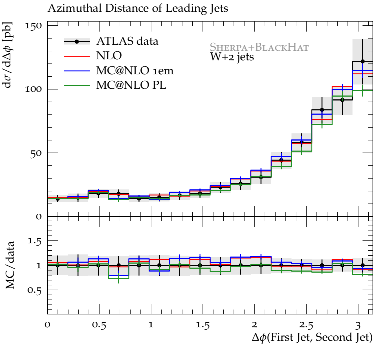
Results in pp → W + n jet + X



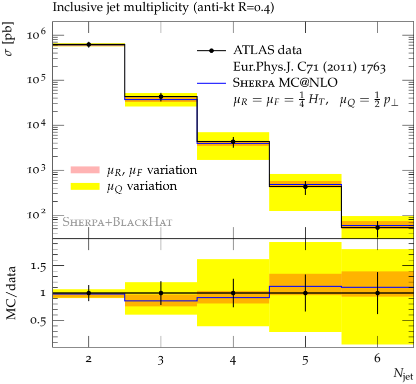
$W + 1, 2, 3$ jets at LHC (ATLAS data)

SH, FK, MS, FS arXiv:1201.5882

- complexity not a problem
- speed limited by the virtual amplitude in $W + 3$jet



Results in $\mathrm{pp} \to$ dijet $+ \mathrm{X}$



$p_\perp > 60$ GeV, $p_\perp^{\mathrm{lead}} > 80$ GeV

Incl. production at LHC (ATLAS data)

Höche, MS to appear

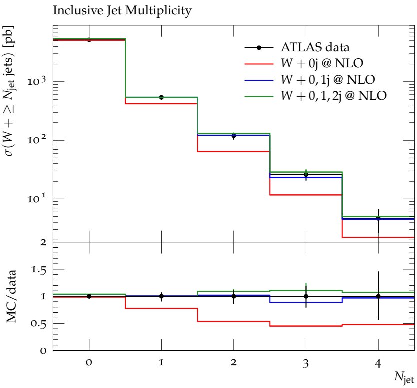
- central scale choices:
  $\mu_R = \mu_F = \frac{1}{4}\, H_T$
  $\mu_Q = \frac{1}{2}\, p_\perp$
- vary μ_F, μ_R by factor 2 independently
- vary μ_Q by factor $\sqrt{2}$
  Dasgupta, Salam JHEP08(2002)032
- μ_Q dependence larger than μ_F, μ_R dependence

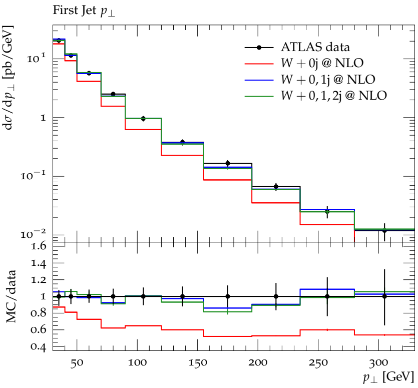
Preview: NLO merging in $pp \to W + \text{jets}$

extend CKKW merging of tree-level amplitudes to NLO

Höche, Krauss, MS, Siegert upcoming publication

talk by L. Lönnblad earlier





Conclusions

- NLO+PS is LO+(N)LL matching
  $\Rightarrow$ uncertainties are correspondingly large
- sensible values for μ_Q have to be chosen for sensible results
- higher precission can achieved by adding higher order calculations
  - (N)NLL resummation
  - NNLO corrections
  - NLO$\otimes$NLO merging with $Q_{\text{cut}} < \mu_Q^2$

SHERPA-1.4.0

http://sherpa.hepforge.org

Thank you for your attention!

Full-colour parton showering

Implemented in SHERPA – full-colour first parton shower emission

Tricky point: $\mathrm{D}_i^{(\mathsf{A})} < 0$ e.g. for subleading colour dipoles

Use modified Sudakov veto algorithm SH, FK, MS, FS arXiv:1111.1220

- Assume $f(t)$ as function to be generated, and overestimate $g(t)$
  Standard probability for *one* acceptance with n rejections

$$\frac{f(t)}{g(t)}\,g(t)\exp\left\{-\int_t^{t_1}\mathrm{d}\bar{t}\,g(\bar{t})\right\}\prod_{i=1}^{n}\left[\int_{t_{i-1}}^{t_{n+1}}\mathrm{d}t_i\left(1-\frac{f(t_i)}{g(t_i)}\right)g(t_i)\exp\left\{-\int_{t_i}^{t_{i+1}}\mathrm{d}\bar{t}\,g(\bar{t})\right\}\right]$$

- Can split weight into MC and analytic part using auxiliary function $h(t)$

$$\frac{f(t)}{g(t)}\,h(t)\exp\left\{-\int_t^{t_1}\mathrm{d}\bar{t}\,h(\bar{t})\right\}\prod_{i=1}^{n}\left[\int_{t_{i-1}}^{t_{n+1}}\mathrm{d}t_i\left(1-\frac{f(t_i)}{g(t_i)}\right)h(t_i)\exp\left\{-\int_{t_i}^{t_{i+1}}\mathrm{d}\bar{t}\,h(\bar{t})\right\}\right]$$

$$w(t,t_1,\ldots,t_n) = \frac{g(t)}{h(t)}\prod_{i=1}^{n}\frac{g(t_i)}{h(t_i)}\,\frac{h(t_i)-f(t_i)}{g(t_i)-f(t_i)}$$

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Identify $f(t)$, $g(t)$, $h(t)$:

- $f(t)$ determined by MC@NLO $\Rightarrow \mathrm{D}_i^{(\mathsf{A})}$
- $h(t)$ determined by parton shower $\Rightarrow \mathrm{D}_i^{(\mathsf{PS})}$
- $g(t)$ **can be chosen freely** $\Rightarrow$ const. $\cdot\, f$
  constraints: sign(f) = sign(g), $|f| \le |g|$