Probing Flavor Transition Mechanisms of Astrophysical Neutrinos

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Outline

- Flavor transition
  - Q-representation
- Observables
  - at low and high energies
- Discrimination between models
  - Statistical analysis

Probability Transition Matrix

- "Astrophysical": $\Delta m_{ij}^2 \frac{L}{E} >> 1$
- Oscillation only

$$P_{\alpha\beta} = \sum_i |U_{\alpha i}|^2 |U_{\beta i}|^2$$

- Interplay with decay

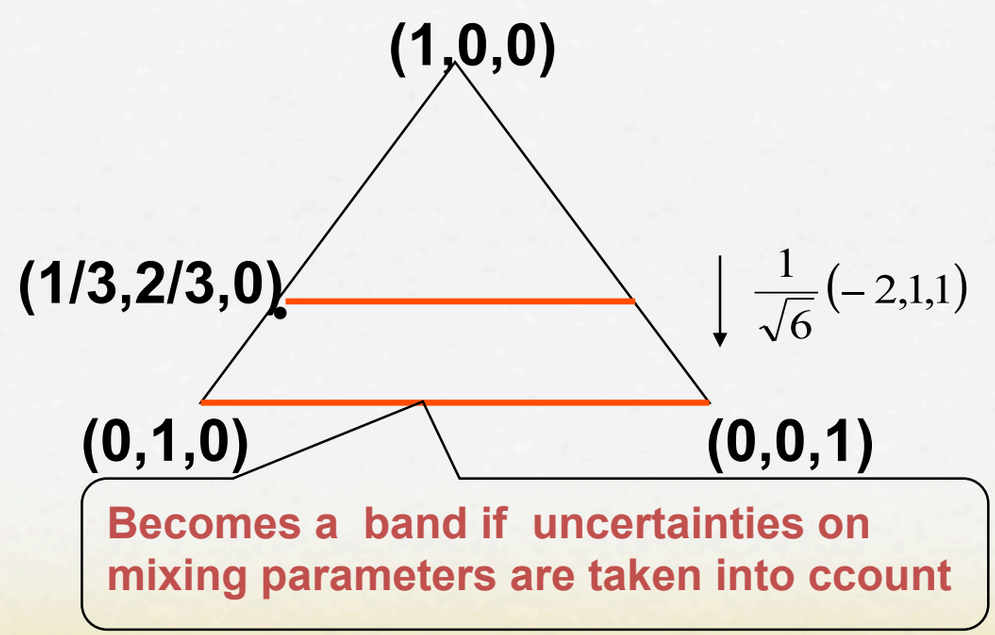
$$P_{\alpha\beta} = \sum_{f\ stable} \left(|U_{\alpha f}|^2 + \sum_{i\ unstable} |U_{\alpha i}|^2 \, \mathrm{Br}_{i \to f} \right) |U_{\beta f}|^2$$

Tri-Bimaximal Matrix

- When $\theta_{23}=45°$, $\theta_{13}=0°$, P matrix is singular!
- Results: Degeneracy in ν_μ and ν_τ ϕ $\phi_0(\nu_\mu)+\phi_0(\nu_\tau)=$ constant
- Need $\theta_{23}\neq 45°$ or $\theta_{13}\neq 0$ to break degeneracy

$$P=\frac{1}{8}\begin{pmatrix} 8-4\omega & 2\omega & 2\omega \\ 2\omega & 4-\omega & 4-\omega \\ 2\omega & 4-\omega & 4-\omega \end{pmatrix},$$

where $\omega=\sin^2 2\theta_{12}$



Q-representation

Tri-bimaximal matrix

$$P^{\mathrm{TBM}} = \frac{1}{18}\begin{pmatrix} 10 & 4 & 4 \\ 4 & 7 & 7 \\ 4 & 7 & 7 \end{pmatrix}$$

Eigenvectors of TBM:

$$V_1 = (1,\ 1,\ 1)$$
$$V_2 = (0,\ -1,\ 1)$$
$$V_3 = (2,\ -1,\ -1)$$

$$A = \begin{pmatrix} 1 & 0 & 2 \\ 1 & -1 & -1 \\ 1 & 1 & -1 \end{pmatrix}.$$

$$\varphi = P\varphi_0$$

$$\varphi_0 = (\varphi_{0,e},\ \varphi_{0,\mu},\ \varphi_{0,\tau}) = \frac{1}{3}V_1 + rV_2 + sV_3,$$

$$\varphi = \kappa V_1 + \rho V_2 + \lambda V_3$$

$$\Longrightarrow\ Q \equiv A^{-1}PA$$

Transition Mechanisms

$$Q = \begin{pmatrix} Q_{11} & Q_{12} & Q_{13} \\ Q_{21} & Q_{22} & Q_{23} \\ Q_{31} & Q_{32} & Q_{33} \end{pmatrix}$$

$$\rho = \phi(\nu_\tau) - \phi(\nu_\mu),\ \lambda = \phi(\nu_e)/3 - (\phi(\nu_\mu) + \phi(\nu_\tau))/6$$

Flux-conservation ➔ $(Q_{11}, Q_{12}, Q_{13}) = (1,0,0)$

ν_μ - ν_τ symmetry ➔ $(Q_{21}, Q_{22}, Q_{23}) \approx (0,0,0)$
$(Q_{12}, Q_{22}, Q_{32}) \approx (0,0,0)$

$\Rightarrow$ Q_{31} and Q_{33} classify possible flavor transition models

Observables R & S

	$<10^{16}$eV	$>10^{16}$eV
shower	ν_e, ν_τ	ν_e
track	ν_μ	ν_μ, ν_τ

R: ratio between showers and tracks
S: ratio between two flavors

For energy $<10^{16}$eV

$R=\phi(\nu_\mu)/(\phi(\nu_e)+\phi(\nu_\tau))$
$S=\phi(\nu_e)/\phi(\nu_\tau)$

For energy $>10^{16}$eV

$R=\phi(\nu_e)/(\phi(\nu_\mu)+\phi(\nu_\tau))$
$S=\phi(\nu_\mu)/\phi(\nu_\tau)$

Classifying Mechanisms

Q_{31}, Q_{32} and Q_{33} are determined by the fraction of ϕ_e.
The pion (1/3, 2/3, 0) and damped muon (1, 0, 0) sources are plausible to probe the Q's.

At low energies, IceCube regime,
R plus ν_μ-ν_τ symmetry probe Q_{31} and Q_{33}.
At high energies, ARA regime,
R alone probes Q_{31} and Q_{33},
via $f_{12}=Q_{31}-Q_{32}$ and $f_{23}=Q_{31}+Q_{33}$.

Decay Scenario

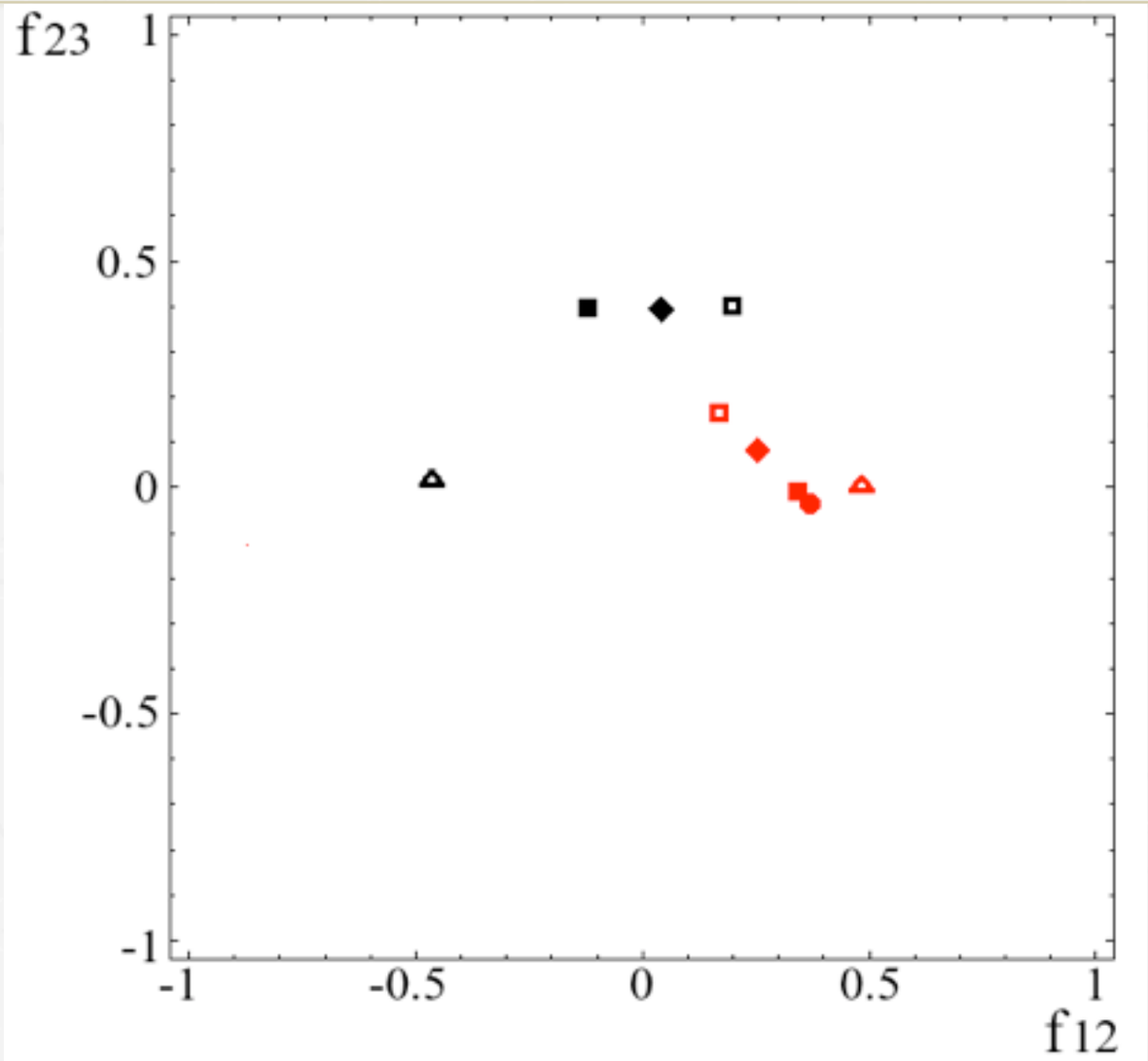
	One stable state			One unstable state		
Scenario	$\cancel{3}\cancel{2}1$	$\cancel{3}2\cancel{1}$	$3\cancel{2}\cancel{1}$	$\cancel{3}21$	$3\cancel{2}1$	$32\cancel{1}$
Branching ratio	Br_{31}=a Br_{21}=b	Br_{32}=a	Br_{ij}=0 all i, j	Br_{32}=a, Br_{31}=b	Br_{21}=a	Br_{ij}=0 all i, j

TABLE I: Decay scenarios for normal mass hierarchy.

	One stable state			One unstable state		
Scenario	$\cancel{2}\cancel{1}3$	$\cancel{2}1\cancel{3}$	$2\cancel{1}\cancel{3}$	$\cancel{2}13$	$2\cancel{1}3$	$21\cancel{3}$
Branching ratio	Br_{23}=a Br_{13}=b	Br_{21}=a	Br_{ij}=0 all i, j	Br_{21}=a Br_{23}=b	Br_{13}=a	Br_{ij}=0 all i, j

TABLE II: Decay scenario for inverted mass hierarchy

Transition Models



Best fit values for transition models:
red, normal hierarchy; black, inverted hierarchy

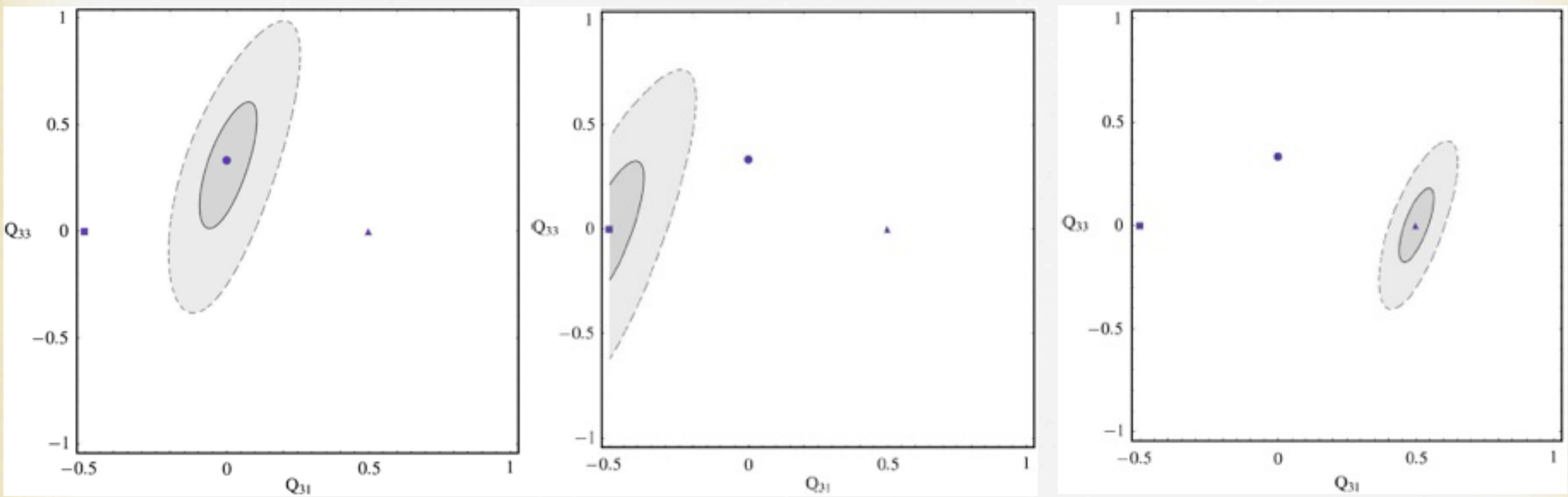
Model Identification

$$\chi^2 = \left(\frac{R_{\pi,\,\mathrm{th}} - R_{\pi,\,\mathrm{exp}}}{\sigma_{R_\pi}} \right)^2 + \left(\frac{R_{\mu,\,\mathrm{th}} - R_{\mu,\,\mathrm{exp}}}{\sigma_{R_\mu}} \right)^2$$

- Pion and damped-muon sources measured with $\sigma_R \equiv \Delta R/R = 10\%$
- Best fit: $R_{i,\mathrm{th}}$ comes from $P^{-1}\phi_0$
- $\chi^2_{\mathrm{min}} \equiv 0$ when $R_{i,\mathrm{th}} = R_{i,\mathrm{exp}}$
- $\Delta\chi^2 = \chi^2 - \chi^2_{\mathrm{min}} = \chi^2$
- Then 1σ and 3σ regions are defined as boundaries of $\Delta\chi^2 = 2.3$ and $\Delta\chi^2 = 11.8$ respectively

Low Energy Case

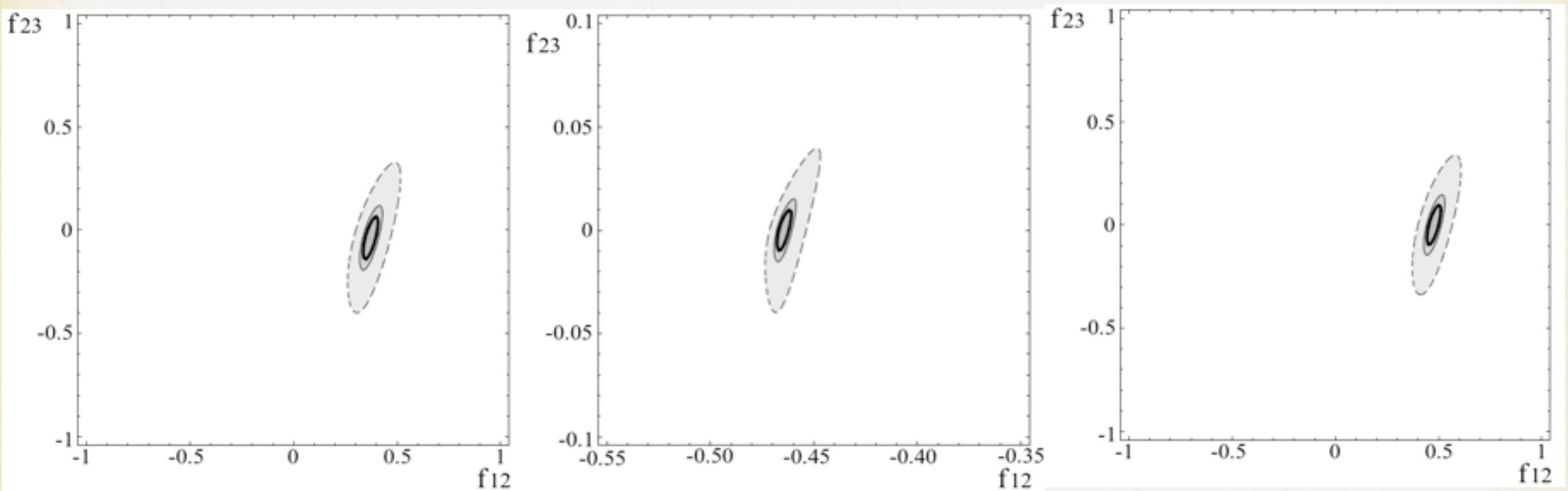
$\Delta R_\pi / R_\pi = \Delta R_\mu / R_\mu = 10\%$



from left to right: models for standard oscillation, decay with inverted and normal hierarchy.

High Energy Case

$\Delta R_\pi / R_\pi = \Delta R_\mu / R_\mu = 10\%$



from left to right: models for standard oscillation, decay with inverted and normal hierarchy, with $(f_{12}, f_{23})=(Q_{31}-Q_{32}, Q_{32}+Q_{33})$.

Summary

- Q-representation classifies flavor transition models
- χ^2 analysis to probe transition models
- Different cases at low and high energies included
- Events of $\sim O(100)$ assumed for $\Delta R/R=10\%$