Galactic Dark Matter in the Phantom Dark Energy Background

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Abstract

We study the possibility that the galactic dark matter exists in the phantom field responsible for the dark energy. The statically and spherically exact solution for this kind of the galaxy system with a supermassive black hole at its center is obtained. The solution of the metric functions is satisfied with $g_{tt} = -g_{rr}^{-1}$. In a galaxy, the background of the phantom field, which is spatially inhomogeneous, has an exponential potential. The absorption cross section of the low-energy S-wave excitations, arising from the phantom dark energy background, into the central black hole is shown to be the horizontal area of the central black hole. The accretion of the phantom energy is companied with the decrease of the black hole mass, which is estimated to be much less than a solar mass in the lifetime of the Universe. Using a simple model with the cold dark matters very weakly coupled to the excited phantom particles, we show that these two densities can be stable in the galaxy.

I. Introduction

The recent experimental data have shown that the current Universe is undergoing a phase of accelerated expansion. Considering the universe filled with a barotropic perfect fluid which corresponds to the dark energy component, its equation of state w$< -1/3$ is required for cosmic acceleration, where w$= p/\rho$ with ρ and p being the density and pressure, respectively.

Observations related to the CMB and LSS support that the universe is very close to spatially flat geometry. At the present time our universe is dominated by dark energy with the fraction$\sim$72%. The most accessible component of the universe is baryonic matter which amounts to only 4.6%. The main remaining part that is non-baryonic and non-luminous is believed to be the so-called dark matter responsible for $\sim$23%. The WIMPs are considered one of the main candidates for by its gravitational effect on a spiral galaxy's rotation curve, dominates the mass of a galaxy and can stretch up to be larger than 50 kpc from the center of a galaxy. In this paper, we are interested in the static solution of the Einstein equations that can describe the dark matter halo with the existences of the supermassive adopt the standard assumption that the dark matter halo consists of the WIMPs with w$= 0$. We find that an exact solution of the metric exists for describing this galactic halo scenario when we take the metric functions to be satisfied with $g_{tt} = -g_{rr}^{-1}$. To study the stability of the space-time structure for the galaxy.

II. The Exact Solution in the Static Limit

We consider the real phantom field minimally coupled to gravity

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{2\kappa^2} + \frac{1}{2}g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi - V(\Phi) + \mathcal{L}_m + \mathcal{L}_I\right],$$

where $\kappa^2 = 8\pi G$ is the reduce Planck mass, $V(\Phi)$ is phantom field potential, $\mathcal{L}_m$accounts for the massive dark matter in the galaxy, and $\mathcal{L}_I$ describes possible interactions between the phantom field and dark matter. We employ the static spherically symmetric metric$ds^2 = -e^\nu dt^2 + e^\lambda dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$ with adding the ansatz $\lambda = -\nu$. Thus for the static situation, the Einstein equations read

$$g^{ii}R_{ii} - \frac{1}{2}R = e^\nu\left(\frac{1}{r^2} + \frac{\nu'}{r}\right) - \frac{1}{r^2} = k^2 T^i{}_i,$$

$$g^{jj}R_{jj} - \frac{1}{2}R = \frac{e^\nu}{2}\left(\nu'' + \nu'^2 + \frac{2\nu'}{r}\right) = k^2 T^j{}_j,$$

where $i = t, r$,$j = \theta, \phi$ and the energy-momentum tensor corresponds to the massive dark matters in the background of the phantom field

$$T^t{}_t = -\rho = -\rho_{ph} - \rho_{DM} = \frac{1}{2}e^\nu\Phi'^2 - V(\Phi) - \rho_{DM},$$

$$T^r{}_r = p_r = p_{r,ph} = -\frac{1}{2}e^\nu\Phi'^2 - V(\Phi)\,,$$

$$T^j{}_j = p_j = p_{j,ph} = \frac{1}{2}e^\nu\Phi'^2 - V(\Phi)\,,$$

and a *prime* denotes the differentiation with respect to r.

Setting $\nu = \ln(1-U)$, we obtain $r^2U'' + 2\epsilon rU' + 2(\epsilon - 1)U = 0$ where we have set $T^\theta{}_\theta = T^\phi{}_\phi = T^t{}_t(1-\epsilon)$ with ϵ being a constant. Thus, the metric give by

$$ds^2 = -\left[1 - \frac{r_s}{r} + \frac{r^{2(1-\epsilon)}}{r_\epsilon}\right]dt^2 + \left[1 - \frac{r_s}{r} + \frac{r^{2(1-\epsilon)}}{r_\epsilon}\right]^{-1}dr^2 + r^2 d\Omega^2, for\ \epsilon \neq \frac{3}{2}\,,$$

or

$$ds^2 = -\left[1 - \frac{r_s}{r} + \frac{a}{r}\ln\left(\frac{r}{|a|}\right)\right]dt^2 + \left[1 - \frac{r_s}{r} + \frac{a}{r}\ln\left(\frac{r}{|a|}\right)\right]^{-1}dr^2 + r^2 d\Omega^2, for\ \epsilon = \frac{3}{2}\,,$$

where r_s, r_ϵ and $a \neq 0$ are the integration constants.

We discuss the obtained exact solutions in some typical limits. First, for $\epsilon = 1$ with $r_\epsilon \to \infty$, it gives the Schwarzschild metric, and r_s is the Schwarzschild radius. Second, for $\epsilon = 2$, the solution is the Reissner-Nordstrom metric, where $r_\epsilon^{-1} = GQ^2$ and Q is the charge of the black hole. Third, for $\epsilon = 0$, it gives the Schwarzschild-de Sitter/anti-de Sitter solutions which are equivalent to the replacement $r_\epsilon \equiv -\frac{3}{\Lambda}$, with Λ being the positive/negative cosmology constants.

In this paper, we are interested in the metric that can describe the motions of stars in the galaxy. This corresponds to $\epsilon \to 1$, but $\neq 1$. In this case, if neglecting the normal matter effects, $r_s \sim 10^{-7}$pc can stand for the Schwarzschild radius of supermassive black hole which is at the center of the galaxy. The rotational stars with radius $r_{halo} > r \gg r_s$ in a spiral galaxy, where $r_{halo} \gtrsim 50$ kpc denotes the radius of a typical halo in a galaxy, are moving in circular orbits with *nearly* constant tangential speed v which roughly ranges from 10^{-4} to 10^{-3}. Thus, the particle motions in this case, we get the classical phantom field background $\Phi_b(r)$ and potential

$$\Phi_b(r) - \Phi_\infty \approx \frac{v m_{pl}}{\sqrt{4\pi}}\ln\left(\frac{r}{\tilde{r}_0^2}\right),$$

$$V(\Phi_b) \approx \frac{v^2 m_{pl}^2}{8\pi\tilde{r}_0^2}e^{-\frac{\sqrt{16\pi}(\Phi_b - \Phi_\infty)}{v m_{pl}}},$$

during the distances $r_{halo} > r \gg r_s$, where $m_{pl} = G^{-1/2}$ is the Planck mass, Φ_b is the classical phantom field background and the constant $\tilde{r}_0$ is roughly $\gtrsim$6 Mpc, the intergalactic distance. As for $r \gtrsim \tilde{r}_0$, the space becomes flat, the phantom field $\Phi_b = \Phi_\infty$ is spatially uniform.

III. The Absorption of the Phantom Field by the Supermassive Black Hole

To study the stability of the space-time structure for the galaxy, we compute the accretion rate of excited phantom particles into the black hole. We will estimate the absorption probability, absorption cross section, and the accretion rate of the phantom excitation wave into the black hole.

We consider its S-state, which is the excitation from the background:$\delta\Phi = \Phi(t,r) - \Phi_b(r)$. The phantom potential is assumed to be around the local maximum,

$$V(\Phi) \approx \frac{v^2 m_{pl}^2}{8\pi\tilde{r}_0^2}e^{-\frac{\sqrt{16\pi}(F(\Phi) - \Phi_\infty)}{v m_{pl}}},$$

where $F(\Phi) \simeq \Phi_b + \left(\frac{F_{\Phi\Phi}(\Phi_b)}{2}\right)\delta\Phi^2$. The local potential is around the locally stable point, $V_\Phi(\Phi_b) = 0$. Here $F_{\Phi\Phi} \equiv \frac{d^2F}{d\Phi^2}$ and $V_\Phi \equiv \frac{dV}{d\Phi}$. The wave for the phantom excitation is satisfied with the Klein-Gordon equation

$$-\frac{1}{f}\partial_0^2\delta\Phi + \frac{1}{r^2}\partial_r(r^2 f\partial_r\delta\Phi) = \left(m_\Phi^2 + \lambda\langle\Psi^2\rangle\right)\delta\Phi \equiv m_{eff}^2\delta\Phi,$$

where $-g_{tt} \equiv f = e^\nu, m_\Phi^2 = V(\Phi_b)\frac{\sqrt{16\pi}}{v m_{pl}}F_{\Phi\Phi}(\Phi_b)$ is the mass squared of the excited phantom field $\delta\Phi$, λ is dimensionless coupling constant describing the interaction between the excited phantom particle and the dark matter in form of $\mathcal{L}_I = \frac{\lambda}{2}\delta\Phi^2$.

The excited phantom wave can be variable separated in form of $\delta\Phi(t,r) = \mathcal{R}[\phi(r)e^{-i\omega t}]$.

We get the absorption probability of a spherical S-wave is $\Gamma \approx 4d^2\omega^2 r_s^2$, where $d \simeq \left[1 - \frac{m_{eff}^2}{\omega^2}\right]^{1/2}$, with $\omega r_s \ll 1$. The absorption cross section of the S-wave component is

$$\sigma_{abs} = \frac{\#\ of\ particles\ absorbed\ by\ the\ area\ of\ spherical\ surface\ per\ unit\ time}{\#\ of\ incident\ particles\ crossing\ unit\ area\ per\ unit\ time}$$

$$= \frac{\left|\frac{1}{2kr}\right|^2 4\pi r^2\Gamma}{1} = 4\pi r^2\frac{d^2\omega^2}{k^2} = 4\pi r_s^2,$$

where $k = d\omega$, r is the area of the spherical surface, and $4\pi r_s^2$ is exactly the area of the Schwarzschild horizon. The decrease rate of the black hole can be estimated to be $\frac{dM_{BH}}{dt} \lesssim -10^{-21}M_\odot yr^{-1}$,where we have used $M_{BH} \approx 10^6 M_\odot$. Therefore, the decrease of the black hole mass is much less than a solar mass in the lifetime of the Universe.

IV. Stability

In this section, we consider the classical evolution of a simple system for which a non-relativistic scalar dark matter couples to the excitation of the excitation of the phantom field in a galaxy. We will show that this system can be stable for a weak coupling.

The relevant Lagrangian $\mathcal{L}$ is

$$\mathcal{L} = -\frac{1}{2}g^{\mu\nu}\partial_\mu\Psi\partial_\nu\Psi - \frac{1}{2}M_\Psi^2\Psi^2 + \frac{1}{2}g^{\mu\nu}\partial_\mu\delta\Phi\partial_\nu\delta\Phi + \frac{1}{2}m_\Phi^2\delta\Phi^2 + \frac{1}{2}\lambda\Psi^2\delta\Phi^2,$$

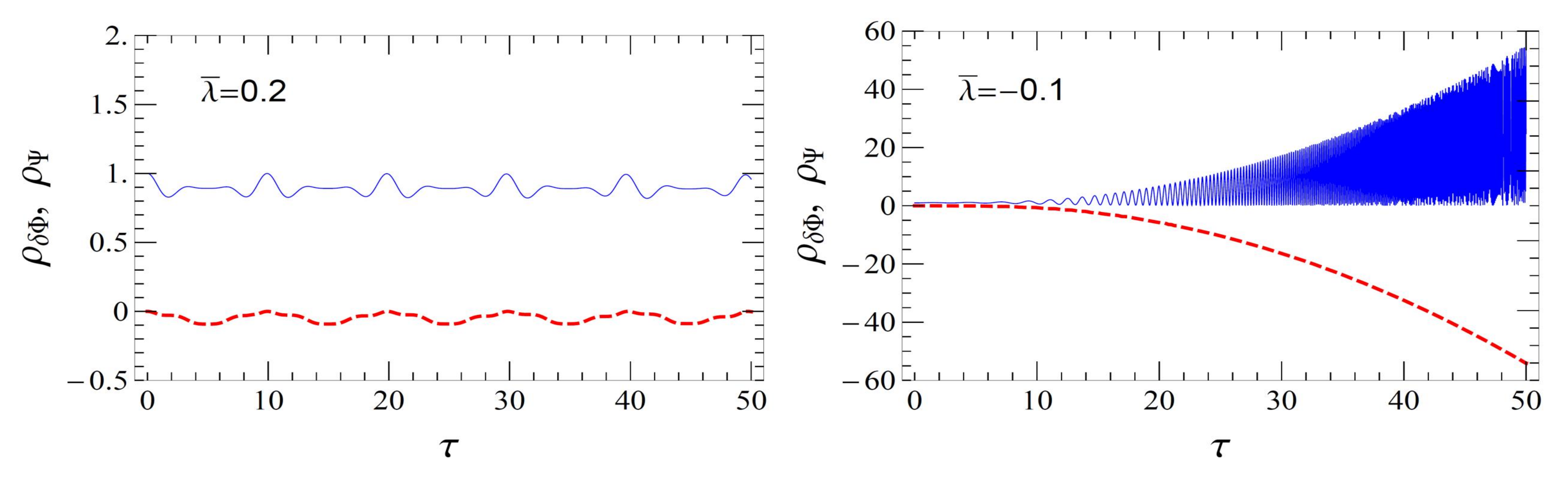

FIG. 1. Evolution of the energy densities of a coupled pair of the dark matter denoted by the solid curve and phantom excitation denoted by the dashed curve. The initial condition is $\bar{\Psi}' = \bar{\delta\Phi}' = 0$ and $\bar{\Psi} = \bar{\delta\Phi} = 1$ starting at $\tau = 0$. The energy densities are displaced in units of $m_\Psi^2 M^2/2$ and satisfied with the constraint $\rho_\Psi + \rho_{\delta\Phi} + \rho_{\Psi\delta\Phi} =$ constant.

We have used the dimensionless variable

$$\bar{\Psi} = \frac{\Psi}{M}, \qquad \bar{\delta\Phi} = \frac{\delta\Phi}{M}, \qquad \bar{\lambda} = \lambda\left(\frac{M}{m_\Psi}\right)^2, \qquad \tau = m_\Psi t,$$

We have used $m_\phi = 10^{-33}eV$ and $m_\Psi = 10^{11}eV$ as inputs, we obtain that for $0 \leq \bar{\lambda} \leq 0.37$ the densities oscillate with a stable behavior, where $\rho_\Psi \simeq v^2 m_{pl}^2/4\pi r^2$ and $\rho_{\delta\Phi} \sim 0$.

V. Summary

We have studied the possibility that the galactic dark matter exists in the phantom field. We have obtained the statically and spherically exact solution for this kind of the galaxy system with a supermassive black hole at its center. To study the stability of this system, we have computed $\sigma_{abs} \simeq 4\pi r_s^2$ and $\frac{dM_{BH}}{dt} \lesssim -10^{-21}M_\odot yr^{-1}$,so that the decrease of the black hole mass is much less than a solar mass in the lifetime of the Universe. Furthermore, we have demonstrated that the dark matter and phantom densities can be stable.