B-physics from lattice QCD...with a twist

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and in progress

ETMCb group

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G. C. Rossi, F. Sanfilippo, A.S., S. Simula, C. Tarantino
Use relativistic quarks (Wilson twisted mass fermions)

For each observable $b$-mass point is reached through interpolation $\frac{1}{\mu_h}$ from $c$-mass region to the static point

c-mass region computation is reliable

the $b$-mass point is related to its $c$-mass counterpart by a chain of suitable, HQET-inspired ratios at successive values of the heavy-mass

Ratios show a smooth chiral and continuum limit

Static limit value of ratios is exactly known
Wilson twisted mass

Family of lattice actions parametrized by $\omega$

$\omega = 0 \Rightarrow$ Wilson fermions

$\omega = \pi/2 \Rightarrow$ Wilson twisted mass at maximal twist

- Infrared cutoff to the spectrum of Wilson operator
  
- No exceptional configurations in the quenched model

- Exact bound on the smallest eigenvalues

- Automatic $O(a)$ improvement on physical correlators at $\omega = \pi/2$

- The freedom in the choice of $\omega$ for different quark flavours allows simplifications in the renormalization pattern

- No renormalization needed for decay constants (like with overlap)

- Multiplicative renormalization for bag parameters


R. Frezzotti, G.C. Rossi: 2004

C. Pena, S. Sint, A. Vladikas: 2004

Ratio method: example

HQET expansion

\[ M_{hl} = \mu_h + \Lambda - \frac{\lambda_1 + 3\lambda_2}{2} \frac{1}{\mu_h} + O\left(\frac{1}{(\mu_h)^2}\right) \]

\[ y(\mu_h, \lambda_{\text{pole}}) = \frac{1}{\lambda_{\text{pole}}} \frac{M_{hl}(\mu_h)}{M_{hl}(\mu_h/\lambda_{\text{pole}})} \]

\[ y(\mu_h, \lambda) = 1 + \frac{\eta_1}{\mu_h} + \frac{\eta_2}{\mu_h^2} \]

\[ y = 1 - \Lambda \frac{\lambda_{\text{pole}} - 1}{\mu_h} + \left[ \frac{\lambda_1 + 3\lambda_2}{2} \right] \left( \lambda_{\text{pole}} + 1 \right) + \Lambda^2 \lambda_{\text{pole}} \left( \frac{\lambda_{\text{pole}} - 1}{(\mu_h)^2} \right) \]
Simulation points

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>$a_{\mu \ell}$</th>
<th>$a_{\mu s}$</th>
<th>$a_{\mu h}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.80</td>
<td>0.0080, 0.0110</td>
<td>0.0175, 0.0194</td>
<td>0.1982, 0.2331, 0.2742, 0.3225, 0.3793, 0.4461</td>
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<td>0.0213</td>
<td>0.5246, 0.6170, 0.7257, 0.8536, 1.004</td>
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<tr>
<td>3.90</td>
<td>0.0030, 0.0040</td>
<td>0.0159, 0.0177</td>
<td>0.1828, 0.2150, 0.2529, 0.2974, 0.3498</td>
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<tr>
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<td>0.0064, 0.0085, 0.0100</td>
<td>0.0195</td>
<td>0.4114, 0.4839, 0.5691, 0.6694, 0.7873, 0.9260</td>
</tr>
<tr>
<td>4.05</td>
<td>0.0030, 0.0060</td>
<td>0.0139, 0.0154</td>
<td>0.1572, 0.1849, 0.2175, 0.2558, 0.3008</td>
</tr>
<tr>
<td></td>
<td>0.0080</td>
<td>0.0169</td>
<td>0.3538, 0.4162, 0.4895, 0.5757, 0.6771, 0.7960</td>
</tr>
<tr>
<td>4.20</td>
<td>0.0020, 0.0065</td>
<td>0.0116, 0.0129</td>
<td>0.13315, 0.1566, 0.1842, 0.2166, 0.2548</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.0142</td>
<td>0.2997, 0.3525, 0.4145, 0.4876, 0.5734, 0.6745</td>
</tr>
</tbody>
</table>

$0.098 \lesssim a \lesssim 0.054$ fm  $m_s/6 \lesssim \bar{\mu}_l \lesssim m_s/2$  $m_c \lesssim \bar{\mu}_h \lesssim 3$ $m_c$

Computations on HLRN (Germany) & CINECA (Italy)
Lattice implementation

\[ y(\bar{\mu}_h^{(n)}, \lambda; \bar{\mu}_l, \alpha) \equiv \frac{M_{hl}(\bar{\mu}_h^{(n)}; \bar{\mu}_l, a)}{M_{hl}(\bar{\mu}_h^{(n-1)}; \bar{\mu}_l, a)} \cdot \frac{\bar{\mu}_h^{(n-1)}}{\bar{\mu}_h^{(n)}} \cdot \frac{\rho(\bar{\mu}_h^{(n-1)}, \mu^*)}{\rho(\bar{\mu}_h^{(n)}, \mu^*)} = \]

\[ = \lambda^{-1} \frac{M_{hl}(\bar{\mu}_h^{(n)}; \bar{\mu}_l, a)}{M_{hl}(\bar{\mu}_h^{(n)}/\lambda; \bar{\mu}_l, a)} \cdot \frac{\rho(\bar{\mu}_h^{(n)}/\lambda, \mu^*)}{\rho(\bar{\mu}_h^{(n)}, \mu^*)}, \quad n = 2, \ldots, N \]

\[ \frac{\bar{\mu}_h^{(n)}}{\bar{\mu}_h^{(n-1)}} = \lambda \quad \mu_h = \rho(\bar{\mu}_h, \mu^*)\bar{\mu}_h(\mu^*) \quad \bar{\mu}_h \leftarrow \text{MS scheme} \]

\[ \rho(\bar{\mu}_h, \mu^*) \leadsto N^3\text{LO} \]
Continuum-chiral limit

- Triggering input: PS meson mass at the charm region --> affected by tolerable cutoff effects
- Ratios have small cutoff effects
Evolution determination

Fit ansatz valid at NP level

\[ y(\bar{\mu}_h) = 1 + \frac{\eta_1}{\bar{\mu}_h} + \frac{\eta_2}{\bar{\mu}_h^2} \]

Strong cancellation of perturbative factors

- Resolve recursion

\[ y(\bar{\mu}_h^{(2)}) y(\bar{\mu}_h^{(3)}) \cdots y(\bar{\mu}_h^{(K+1)}) = \lambda^{-K} \frac{M_{hu/d}(\bar{\mu}_h^{(K+1)})}{M_{hu/d}(\bar{\mu}_h^{(1)})} \cdot \left[ \frac{\rho(\bar{\mu}_h^{(1)}, \mu^*)}{\rho(\bar{\mu}_h^{(K+1)}, \mu^*)} \right] \]

- Adjust \((\lambda, \bar{\mu}_h^{(1)})\) such that \(M_{hu/d}(\bar{\mu}_h^{(K+1)}) \equiv M_B^{\text{expt}}\) for \(K\) integer

- Our calculation

\[ \lambda = 1.1784, \quad \bar{\mu}_h^{(1)} = 1.14 \text{ GeV}(\overline{MS}, 2 \text{ GeV}) \equiv \bar{\mu}_b = \lambda^K \bar{\mu}_h^{(1)} (K = 9) \]
y(\bar{\mu}_h) = 1 + \frac{\eta_1}{\bar{\mu}_h} + \frac{\eta_2}{\bar{\mu}_h^2}

\bar{\Lambda} = 0.39(11) \text{ GeV}, \quad \lambda_1 = -0.19(10) \text{ GeV}^2, \quad \lambda_2 = 0.12(2) \text{ GeV}^2
b-quark mass summary

Perfect agreement using Mhs as observable

\[ m_b^{\overline{MS}}(m_b)\mid_{N_f=2} = 4.35(12) \text{ GeV} \quad (NLL & LL) \]

\[ m_b^{\overline{MS}}(m_b)\mid_{N_f=2} = 4.32(12) \text{ GeV} \quad (TL) \]

\[ m_b(m_b, \overline{MS})\mid_{N_f=2} = 4.29(14) \text{ GeV} \]

- stat. \(\rightarrow\) 0.2 %
- ZP + scale setting 2.1–2.8%
- y-ratios negligible
**Ratio method: \( f_{B_s} \)**

\[
\Phi_{hs}(\bar{\mu}_h, \mu^*_b) = \left[ C_A^{\text{stat}}(\mu^*_b, \bar{\mu}_h) \right]^{-1} \cdot \Phi_{hs}^{\text{QCD}}(\bar{\mu}_h) \quad C_A^{\text{stat}}(\mu^*_b, \bar{\mu}_h) \sim N^2LO
\]

\[
\Phi_{hs}(\bar{\mu}_h, \mu^*_b) = \frac{(f_{hs \sqrt{M_{hs}}})^{\text{QCD}}}{C_A^{\text{stat}}(\bar{\mu}_h, \mu^*_b)} = \Phi_0(\mu^*_b) \left( 1 + \frac{\Phi_1(\mu^*_b)}{\bar{\mu}_h} + \frac{\Phi_2(\mu^*_b)}{(\bar{\mu}_h)^2} \right) + O \left( \frac{1}{(\bar{\mu}_h)^3} \right)
\]

\[
z(\bar{\mu}_h, \lambda; \bar{\mu}_l, a) \equiv \lambda^{1/2} \frac{f_{hl}(\bar{\mu}_h, \bar{\mu}_l, a)}{f_{hl}(\bar{\mu}_h/\lambda, \bar{\mu}_l, a)} \cdot \frac{C_A^{\text{stat}}(\mu^*_b, \bar{\mu}_h/\lambda)}{C_A^{\text{stat}}(\mu^*_b, \bar{\mu}_h)} \cdot \frac{[\rho(\bar{\mu}_h, \mu^*_b)]^{1/2}}{[\rho(\bar{\mu}_h/\lambda, \mu^*_b)]^{1/2}}
\]

\[
z_s(\bar{\mu}_h, \lambda; \bar{\mu}_l, \bar{\mu}_s, a) \equiv \lambda^{1/2} \frac{f_{hs}(\bar{\mu}_h, \bar{\mu}_l, \bar{\mu}_s, a)}{f_{hs}(\bar{\mu}_h/\lambda, \bar{\mu}_l, \bar{\mu}_s, a)} \cdot \frac{C_A^{\text{stat}}(\mu^*_b, \bar{\mu}_h/\lambda)}{C_A^{\text{stat}}(\mu^*_b, \bar{\mu}_h)} \cdot \frac{[\rho(\bar{\mu}_h, \mu^*_b)]^{1/2}}{[\rho(\bar{\mu}_h/\lambda, \mu^*_b)]^{1/2}}
\]

\[
\zeta(\bar{\mu}_h, \lambda; \bar{\mu}_l, \bar{\mu}_s, a) \equiv \frac{z_s}{z} = \frac{f_{hs}(\bar{\mu}_h, \bar{\mu}_l, \bar{\mu}_s, a)}{f_{hs}(\bar{\mu}_h/\lambda, \bar{\mu}_l, \bar{\mu}_s, a)} \cdot \frac{f_{hl}(\bar{\mu}_h/\lambda, \bar{\mu}_l, \bar{\mu}_s, a)}{f_{hl}(\bar{\mu}_h, \bar{\mu}_l, \bar{\mu}_s, a)}
\]
Continuum-chiral extrapolation

- Ratios have small discretization effects 2%-3%
- Triggering pseudoscalar decay constant affected by tolerable cutoff effects
Decay constant ratio

\[
\tilde{z}_s(\bar{\mu}_h) = 1 + \frac{\zeta(1)}{\bar{\mu}_h} + \frac{\zeta(1)}{\bar{\mu}_h^2}
\]

- stat. $\to 0.2\%$
- ZP + scale setting 2.1-2.8%
- \(z\)-ratios negligible

\[
\tilde{z}_s(\bar{\mu}_h) = \frac{\chi^{K/2}}{f_{hs}(\bar{\mu}_h^{(1)})} \cdot \left[ \frac{C_A^{\text{stat}}(\bar{\mu}_h^{(1)}, \mu^*)}{C_A^{\text{stat}}(\bar{\mu}_h^{(K+1)}, \mu^*)} \right] \cdot \left[ \frac{\rho(\bar{\mu}_h^{(K+1)}, \mu^*)}{\rho(\bar{\mu}_h^{(1)}, \mu^*)} \right]^{1/2}
\]

\[
\bar{\mu}_h^{(K+1)} \equiv m_b \implies f_{hs}(\bar{\mu}_h^{(K+1)}) = f_{B_s}
\]

\[
f_{B_s} = 232(6) \text{ MeV}
\]
$f_{B_s}/f_B$

$\frac{f_{hs}}{f_{hl}} = A \left[ 1 - \frac{3(1 + 3\hat{g}^2)}{4} \frac{2B_0\bar{\mu}_l}{(4\pi f_0)^2} \log \left( \frac{2B_0\bar{\mu}_l}{(4\pi f_0)^2} \right) + B\bar{\mu}_l + C\alpha^2 \right]$  

- HMCHPT and linear fits increase systematic error
- Ratios show very weak dependence on the light and heavy quark masses
- Apply recursion for the ratio
Comparison

\[ f_{B_s} = 232(6) \text{ MeV} \ (NLL \ & \ LL) \]
\[ f_{B_s} = 229(6) \text{ MeV} \ (TL) \]
\[ \frac{f_{B_s}}{f_B} = 1.17(2)(4) \Rightarrow f_B = 198(7)(7) \text{ MeV} \]

\[ f_B = 195(12) \text{ MeV} \]
\[ f_{B_s} = 232(10) \text{ MeV} \]
\[ f_{B_s}/f_B = 1.19(5) \]
\[ f_b = 194(9) \text{ MeV} \]

PDG

ETMC 2011
\[
\Delta M_q = \frac{G_F^2 M_W^2}{6\pi^2} |V_{tq} V_{tb}^*|^2 \eta_2^B S_0(x_t) M_{Bq} f_{Bq}^2 \hat{B}_{Bq}
\]

\[
\langle B_q | O | B_q \rangle_{\overline{MS}} = \frac{8}{3} f_{Bq}^2 B_{Bq}^2 (\mu) M_{Bq}^2
\]

\[
\tilde{B}_{B_{d/s}} (\bar{\mu}_h, \mu_b^*) = \left[ C(\bar{\mu}_h, \mu_b^*, \mu) \right]^{-1} B_{B_{d/s}} (\bar{\mu}_h, \mu)
\]

\[
\omega_{d/s} (\bar{\mu}_h, \lambda; \bar{\mu}_l, a) = \frac{B_{B_{d/s}} (\bar{\mu}_h, \bar{\mu}_l, a; \mu)}{B_{B_{d/s}} (\bar{\mu}_h / \lambda, \bar{\mu}_l, a; \mu)} \cdot \frac{C(\bar{\mu}_h / \lambda; \mu_b^*, \mu)}{C(\bar{\mu}_h; \mu_b^*, \mu)}
\]

@ TL and LO there is no mixing

\[
\omega(\bar{\mu}_h) = 1 + \frac{c_1}{\bar{\mu}_h}
\]

@TL and LO No renormalization or matching needed
Chiral continuum extrapolation

\[ \omega_s (\bar{\mu}_h^{(2)}) \omega_s (\bar{\mu}_h^{(3)}) \cdots \omega_s (\bar{\mu}_h^{(K+1)}) = \]
\[ \frac{B_{B_s} (\bar{\mu}_h^{(K+1)}; \bar{\mu}_b)}{B_{B_s} (\bar{\mu}_h^{(1)}; \bar{\mu})} \cdot \left[ \frac{C(\bar{\mu}_h^{(1)}; \bar{\mu}_b, \mu)}{C(\bar{\mu}_h^{(K+1)}; \bar{\mu}_b, \mu)} \right] \]

Cutoff effects well under control
Ratios interpolation

\[ B_{B_d}^{MS}(\bar{\mu}_b) = 0.86(4)(3) \]
\[ B_{B_s}^{MS}(\bar{\mu}_b) = 0.89(4)(3) \]

\[ \frac{B_{B_s}}{B_{B_d}} = 1.023(18)(2) \]

\[ \xi = \frac{f_{B_s}\sqrt{B_{B_s}}}{f_B\sqrt{B_B}} = 1.183(22)(40) \]
Conclusions

- Better interpolating operators for heavy-light 2-point functions
  => sub-percent statistical error

- Very small statistical errors --> strong check on the validity of the ratio method

- Precise determination of $m_b$, $f_B$, $f_{Bs}$ for $N_f=2$

- Final error dominated by error in the scale setting and renormalization factor

- Ratio method extended to the $B$ parameters

- First indications that ratio method is valid also in this case

- Preliminary estimate of $B$ parameters
Backup slides
Improving projection

\[ \beta = 3.80 \quad a \mu = 0.080 \quad a \mu_h = 0.5246 \sim 2.5 \, m_c \]

\[ \frac{1}{L^3} \sum_x \langle P_5(0, 0) P_5(x, x_0) \rangle \]
Improving projection

\[ \psi^S = (1 + \kappa_G a^2 \nabla^2_{\text{APE}})^{N_G} \psi^L \quad \kappa_G = 4 \quad N_G = 30 \]

\[ \alpha_{\text{APE}} = 0.5 \quad N_{\text{APE}} = 20 \]
Improving projection

\[ \beta = 3.80 \quad a \mu = 0.080 \quad a \mu_h = 0.5246 \sim 2.5 \, m_c \]

\[ \Phi_{source}(w) = w \Phi^S + (1 - w) \Phi^L \]

\[ \max [m_{eff}(w - L)] \simeq \min [m_{eff}(w - L)] \]
Checks on smearing

Earlier plateau essential for 3-point functions
Checked all the masses with GEVP