# Model independent determination of the axial mass parameter in CCQE neutrino nucleon scattering

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based on work with Bhubanjyoti Bhattachary and Gil Paz, 1008.4619, PRD 84, 073006 (2011)

# Take home messages

- the axial mass parameter, or axial charge radius, is a fundamental property of the nucleon, and a phenomenologically important quantity

- we can employ model-independent constraints when extracting this quantity

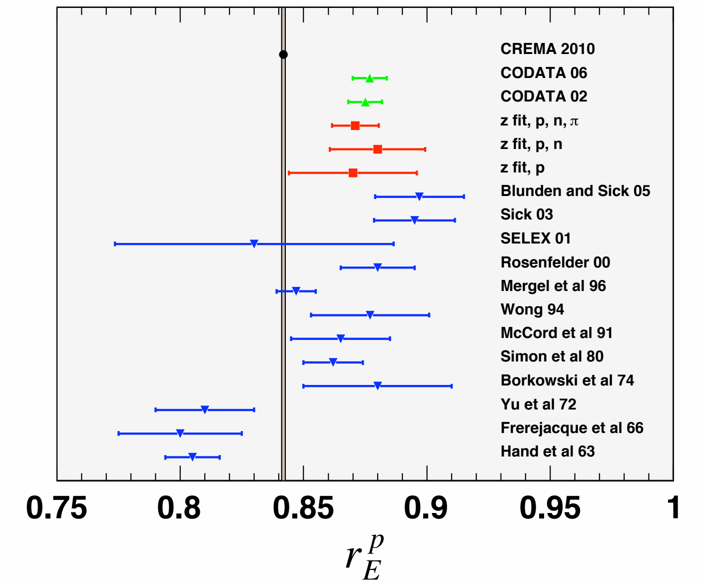
- there is no “axial mass anomaly” without unjustified model dependent assumptions on nucleon level form factors

## Outline of the talk

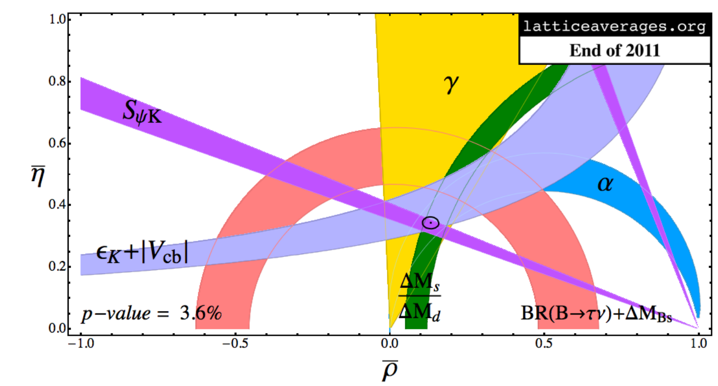
- analyticity and scattering amplitudes
- analyticity and the charge form factor of the nucleon
- model independent determination of $m_A$ in charged-current quasielastic scattering

# Motivations

- Proton (charge) radius problem: ~5σ discrepancy between electron scattering/electron hydrogen and muonic hydrogen determinations
  - most mundane resolution may be ~5σ shift in Rydberg *(less mundane resolutions proposed)*

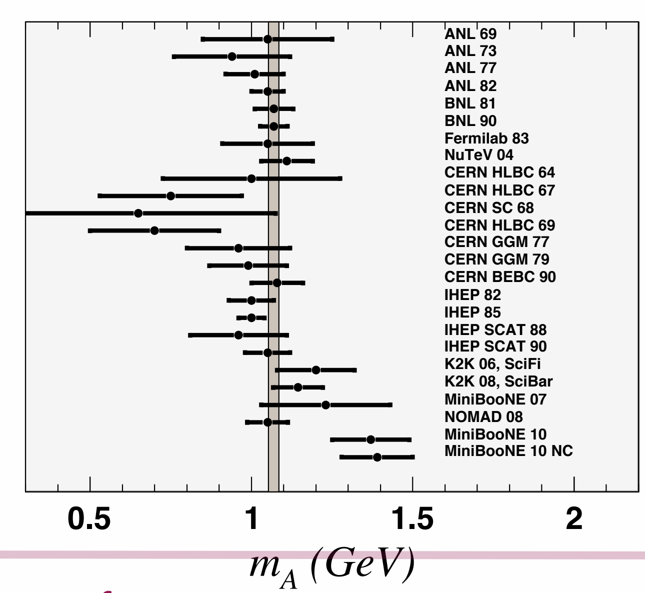



- CKM phenomenology *Tensions in semileptonic $|V_{ub}|$ from $b \to u l \nu$, $R(B \to D^{(*)} \tau\nu \,/\, B \to D^{(*)} e\nu)$*
  - need extrapolate between different kinematic regions connecting experiment and lattice QCD.
  - need model independent implementation/validation of lattice results



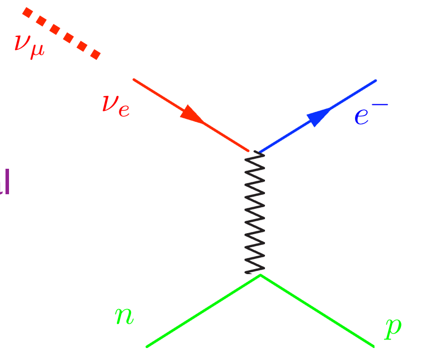

*[Laiho, Lunghi, van de Water 2012]*

- Axial radius of proton: determines strength of signal process (CCQE) in e.g. $\nu_e$ appearance oscillation searches
  - CCQE important in itself
  - moreover, CCQE studies constrain nuclear parameters that feed into other background estimations

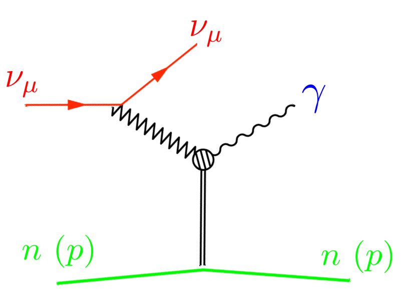


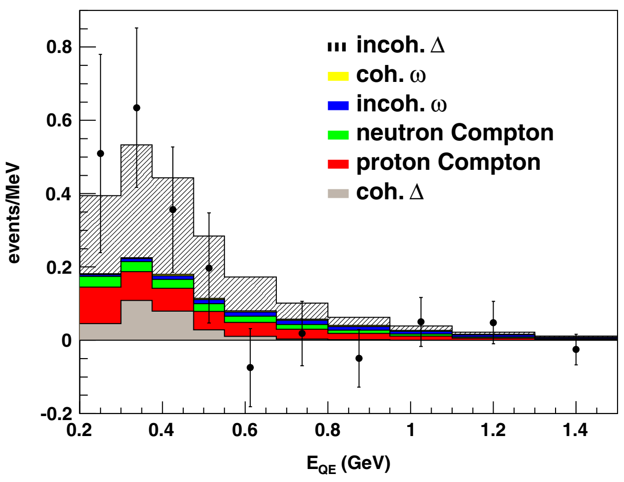
# *sterile neutrinos or SM background?*

$\nu_e \to e$ signal




$\nu_\mu \to \nu_\mu \gamma$ background

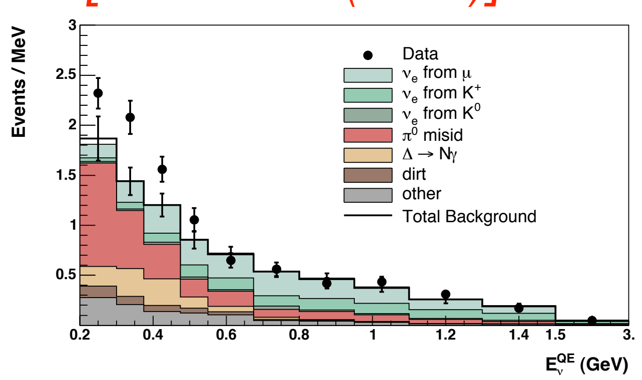


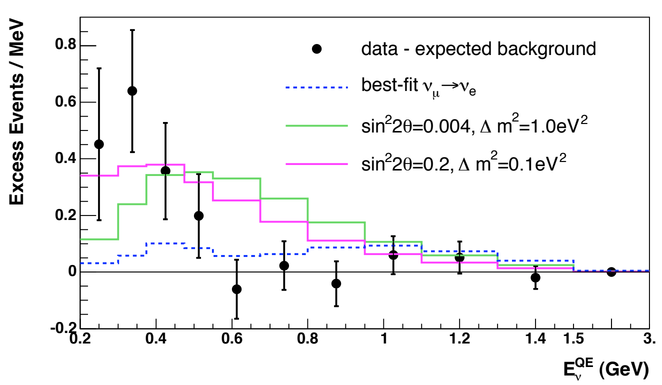




Model for single photon processes

*[Harvey, Hill, Hill 2007, RJH 2009]*

*[MiniBooNE (2009)]*

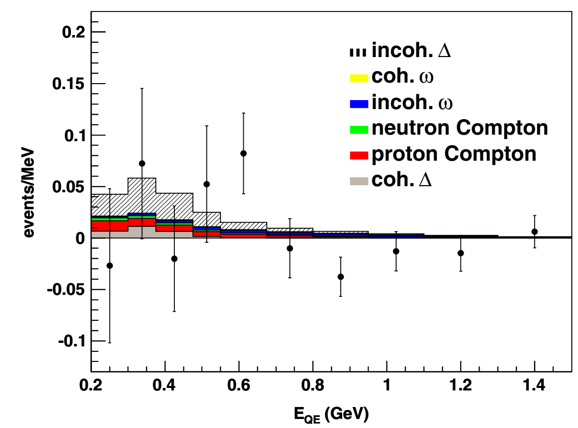


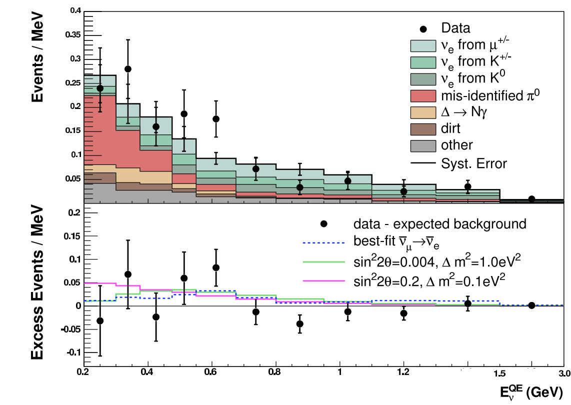




Need quantitative understanding of nucleon level amplitudes for input to nuclear modeling

# *sterile neutrinos or SM background?*

*Excess appearing also in antineutrinos*



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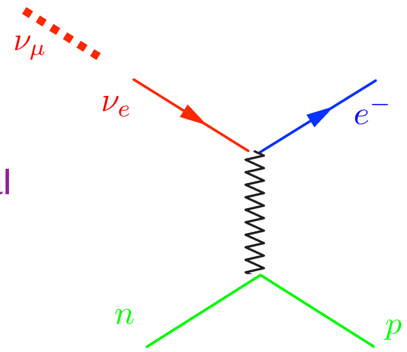
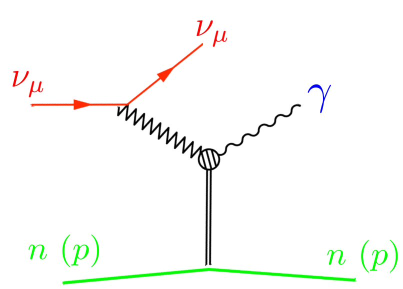
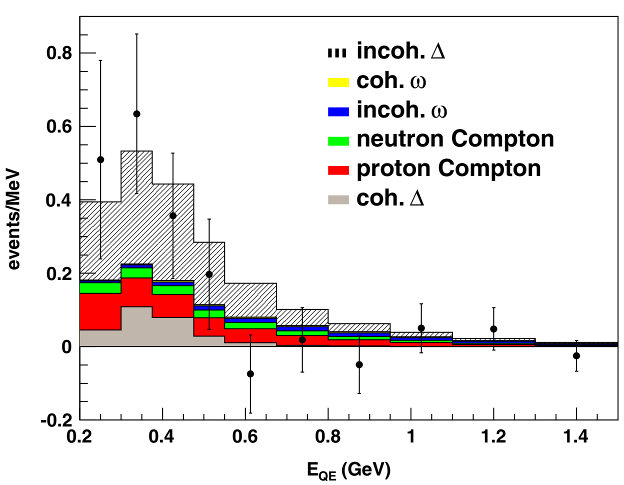


$\nu_\mu \to \nu_\mu \gamma$ background



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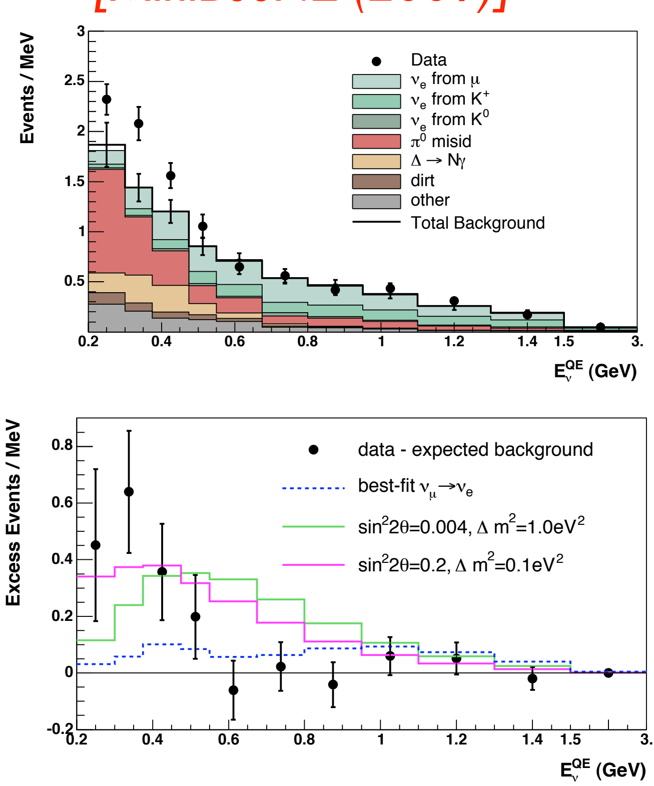


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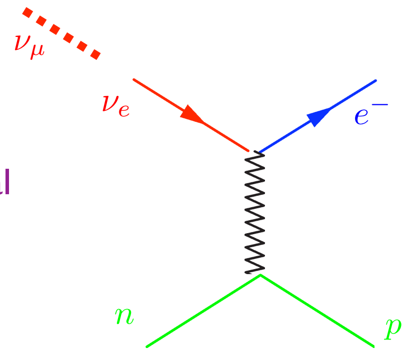
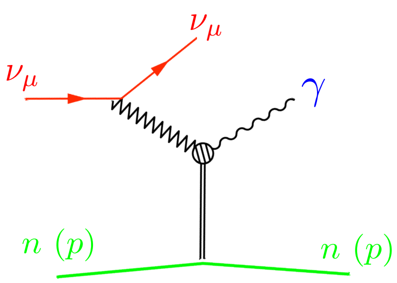
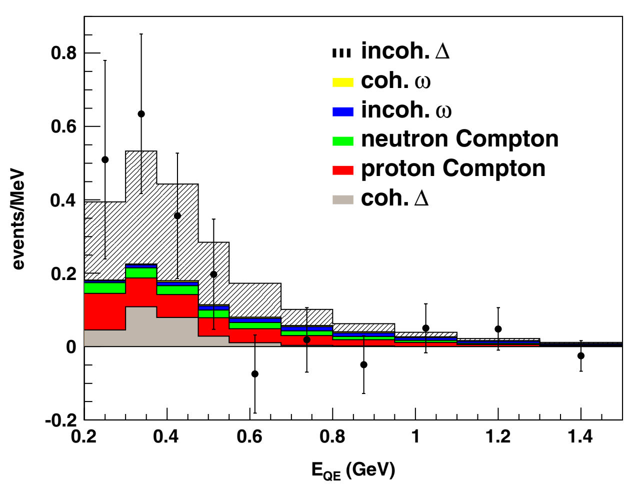



$\nu_\mu \to \nu_\mu \gamma$ background

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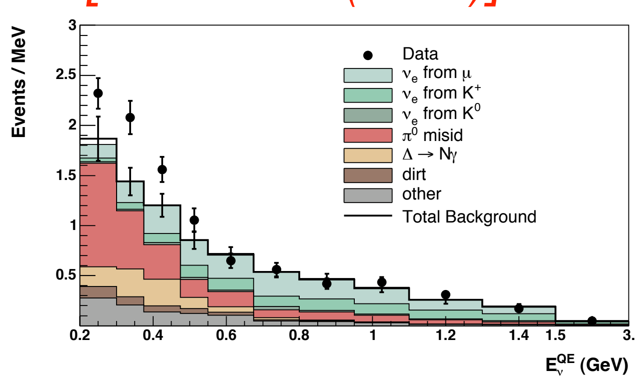
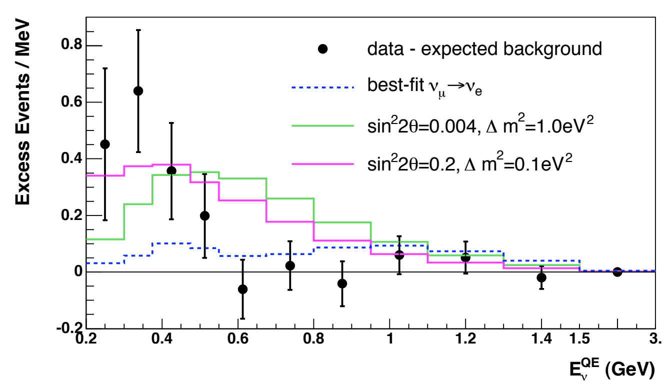
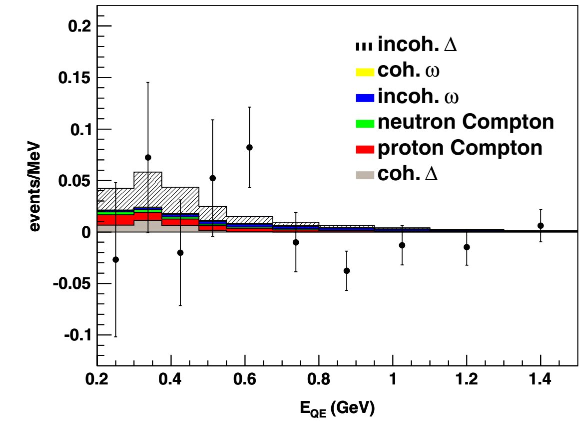
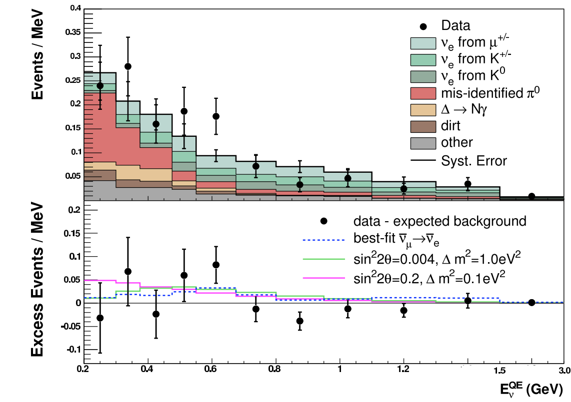
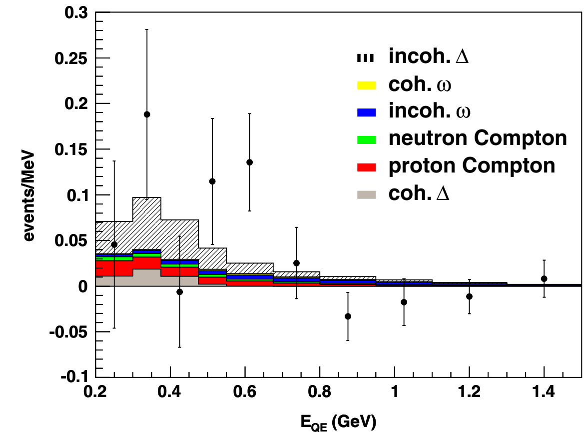


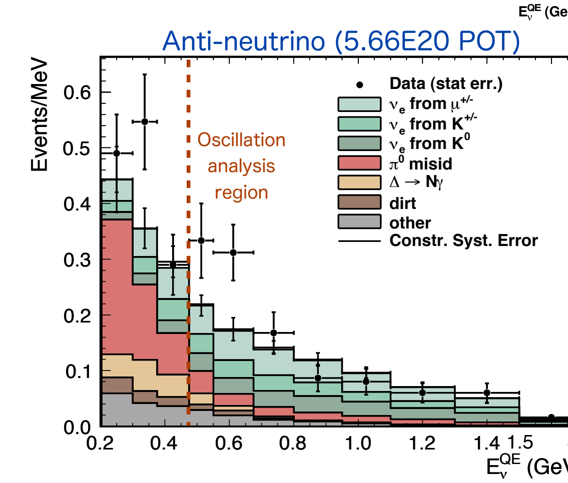




## *Excess appearing also in antineutrinos*




PRL 105, 181801 (2010)

Need quantitative understanding of nucleon level amplitudes for input to nuclear modeling

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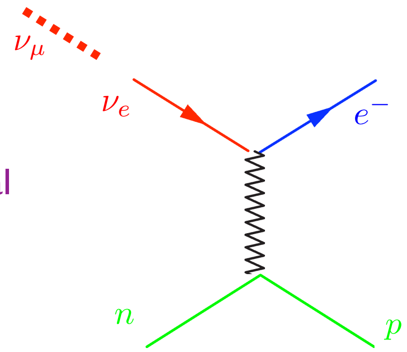


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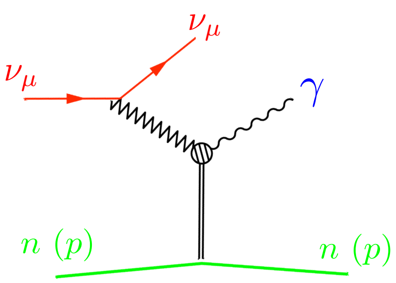


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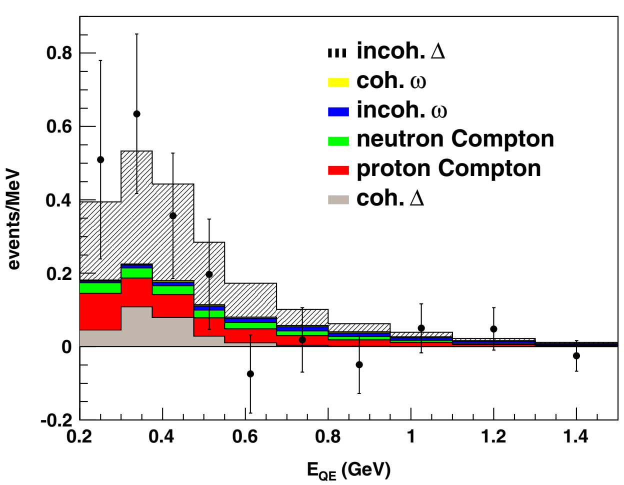


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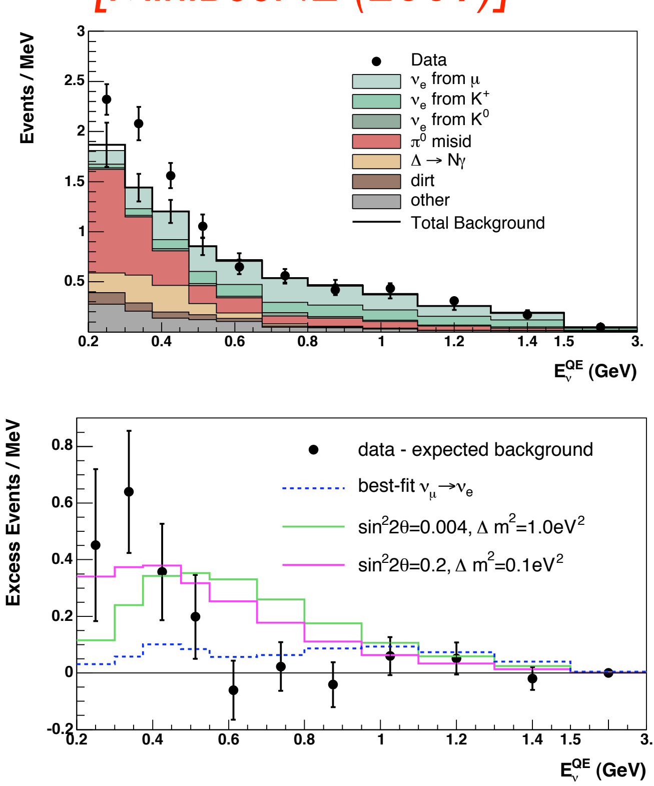
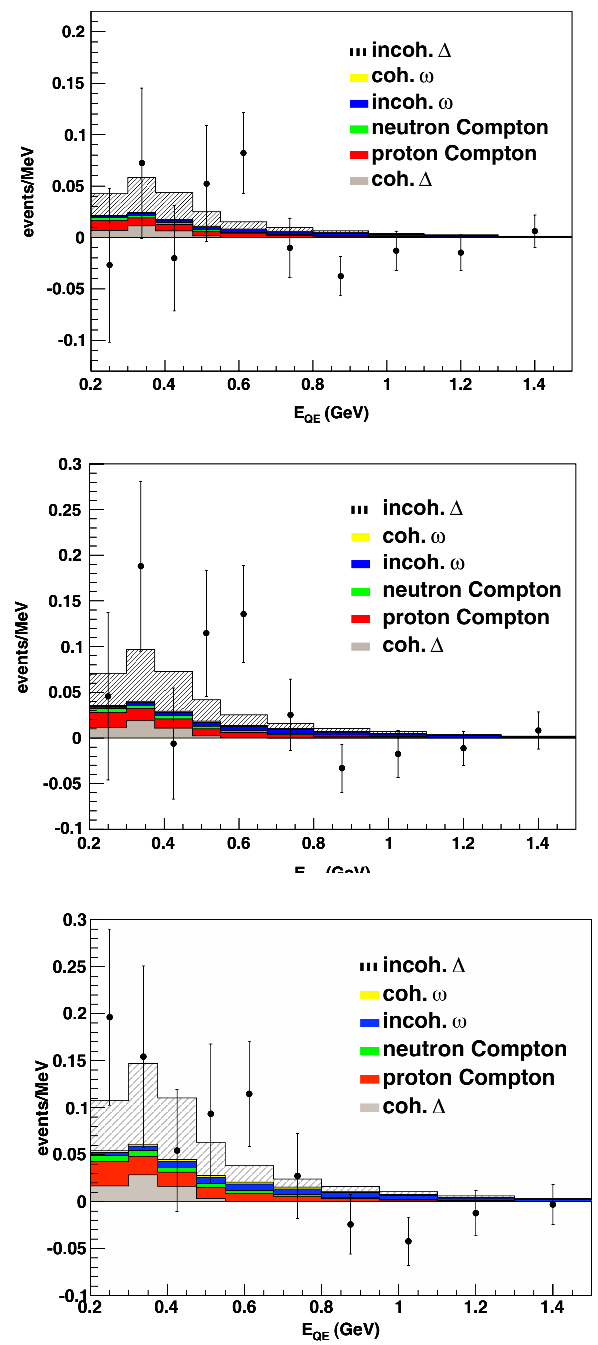
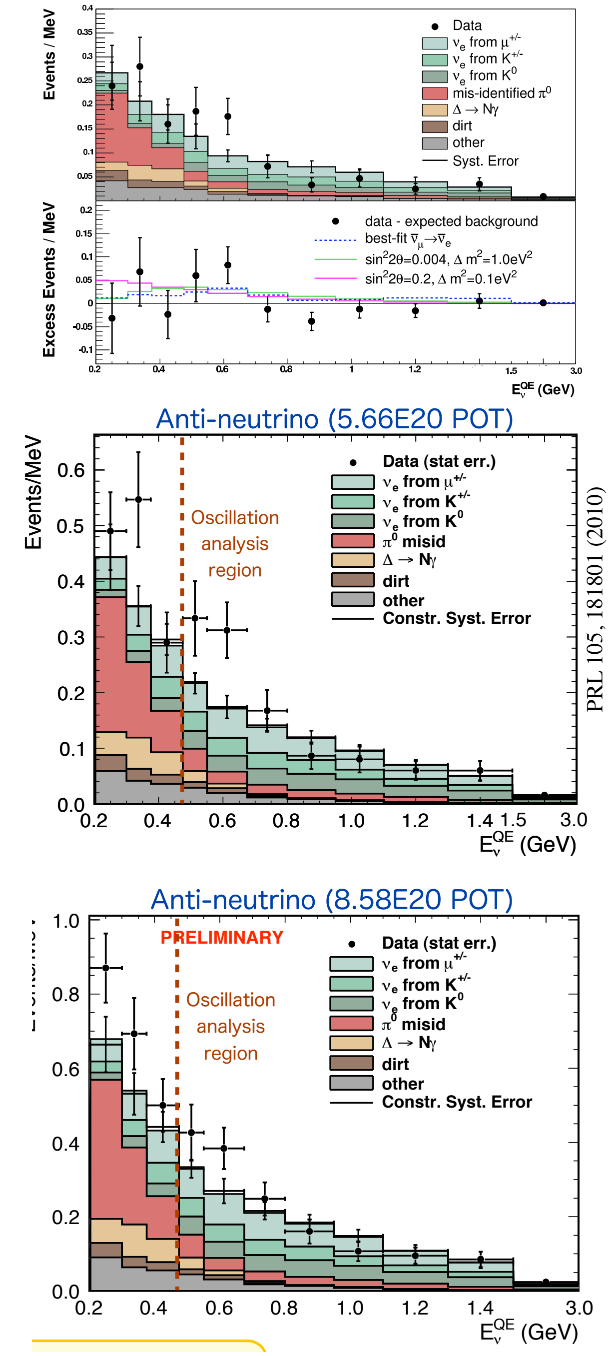


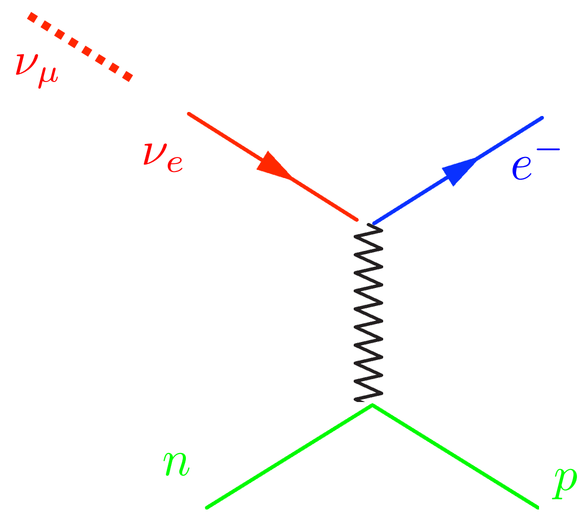




Need quantitative understanding of nucleon level amplitudes for input to nuclear modeling

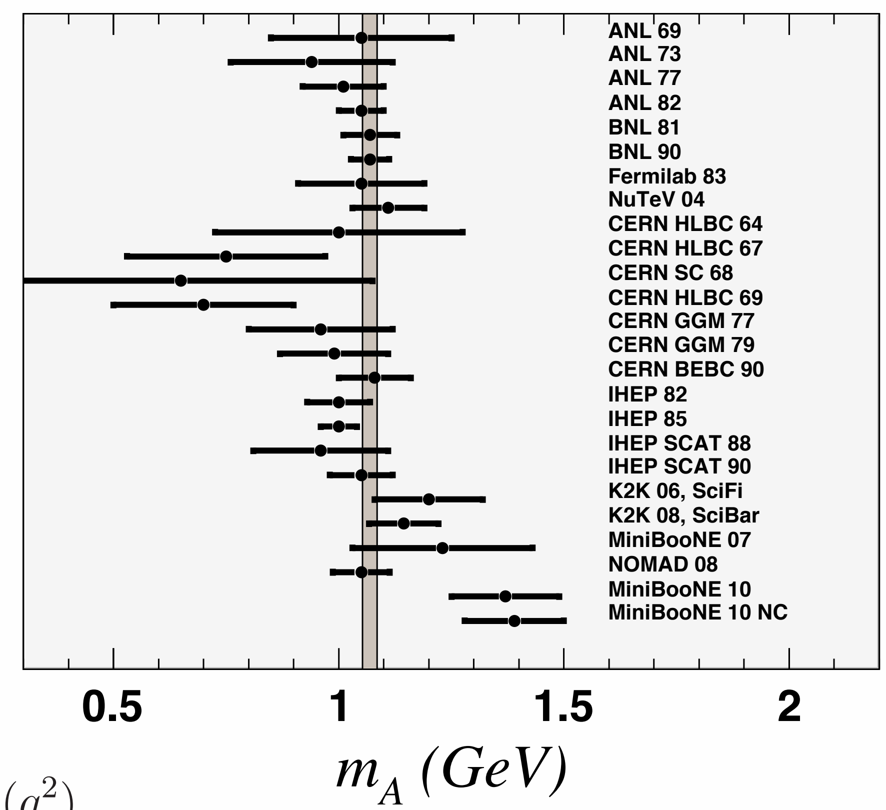
## ***Charged current quasielastic scattering (CCQE)***

In a simpler, but no less important process, find a disturbing anomaly:




pion electroproduction

*[Bernard et al 2002]*




$$F_A^{\rm dipole}(q^2) = \frac{F_A(0)}{\left[1 - q^2/(m_A^{\rm dipole})^2\right]^2}\,.$$

$$\langle p(p')|J_W^{+\mu}|n(p)\rangle \propto \bar{u}^{(p)}(p')\left\{\gamma^\mu F_1(q^2) + \frac{i}{2m_N}\sigma^{\mu\nu}q_\nu F_2(q^2) + \gamma^\mu\gamma_5 F_A(q^2) + \frac{1}{m_N}q^\mu\gamma_5 F_P(q^2)\right\}u^{(n)}(p)$$

- Discrepancies (~3 sigma) in CCQE measurements [basic signal process / flux determination at accelerator neutrino experiments ]

At the nucleon level, single-photon process is described by many Lorentz invariant amplitudes, each a function of many kinematic variables

$$\mathcal{M}^{\mu\nu}[\nu(p) + N(k) \to \gamma(q) + N(k')] = \sum_{i=1}^{12} M_i(s,t,u,p^2)\Gamma_i^{\mu\nu}$$

On top of this are nuclear effects..

To gain some traction, revert to simple process: nucleon level CCQE

$$\langle p(p')|J_W^{+\mu}|n(p)\rangle \propto \bar{u}^{(p)}(p')\Big\{\gamma^\mu F_1(q^2) + \frac{i}{2m_N}\sigma^{\mu\nu}q_\nu F_2(q^2) + \gamma^\mu\boxed{\gamma_5 F_A(q^2)} + \frac{1}{m_N}q^\mu\gamma_5 F_P(q^2)\Big\}u^{(n)}(p)$$

Vector form factors well constrained (at ν level of precision) by electron scattering; pseudoscalar form factor subdominant (lepton mass suppressed), leaving axial-vector form factor

⇒ Single amplitude depending on single kinematic invariant

Should be able to make some headway here..

***Idea:***

Apply constraints of analyticity to nucleon form factor, separate

1) *nucleon* form factor model
2) *nuclear* modeling (e.g. $^{12}$C target)

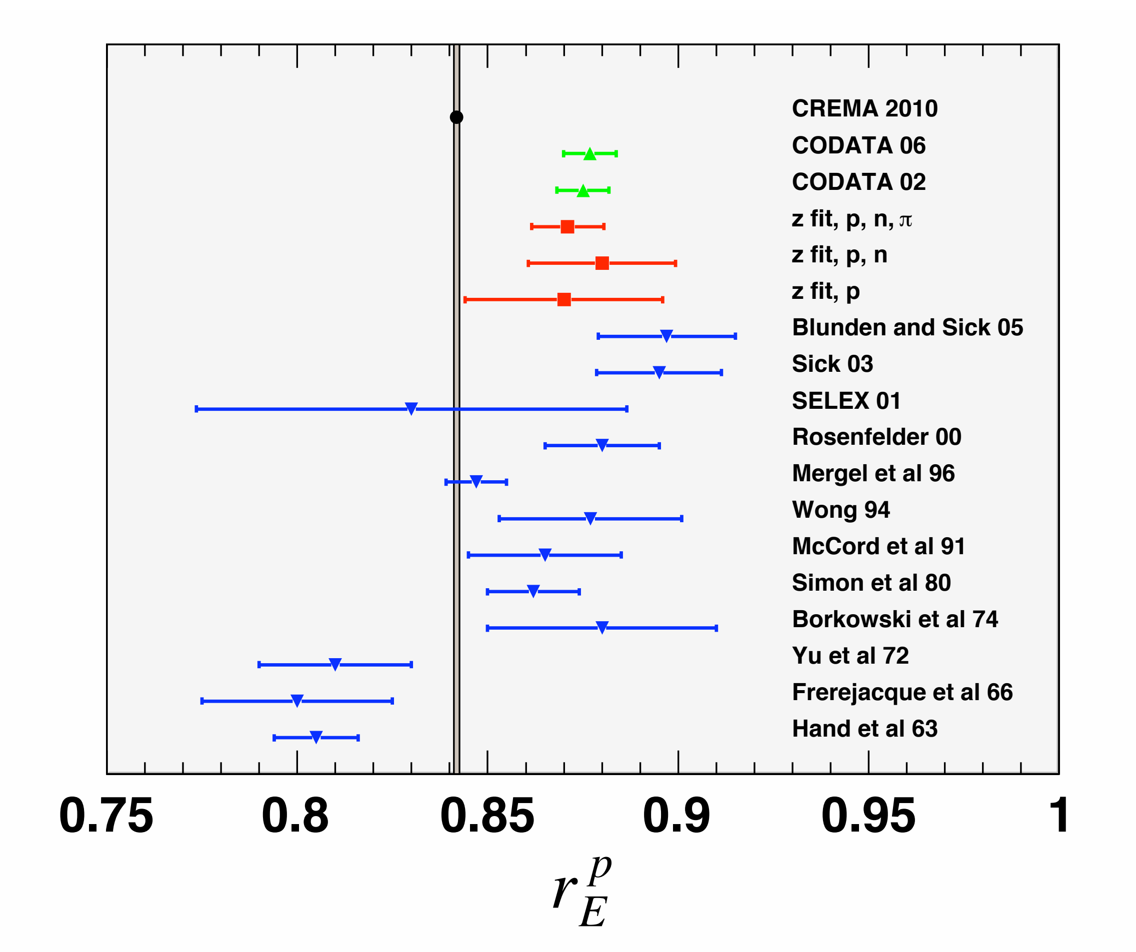
Eliminate model errors from (1). Can be used for unambiguous tests of (2) .

Apply first to better measured vector form factors

## *Similar formalism can be applied to vector form factors*

In fact, this relates to a very puzzling anomaly from atomic spectroscopy

proton radius puzzle




- inferred from muonic H *[Pohl et al, Nature 2010]*

- inferred from electronic H [CODATA 2008]

- extraction from e p, e n scattering, pi pi NN data

- previous extractions from e p scattering (as tabulated in PDG)

Let's first apply analyticity constraints to the vector form factors. Extension to axial-vector form factor will be straightforward

$$\langle k'|J^{\mu}_{\mathrm{e.m.}}|k\rangle = \bar{u}(k')\left[\gamma^{\mu}F_1(q^2) + \frac{i}{2M}F_2(q^2)\sigma^{\mu\nu}q_{\nu}\right]u(k)$$

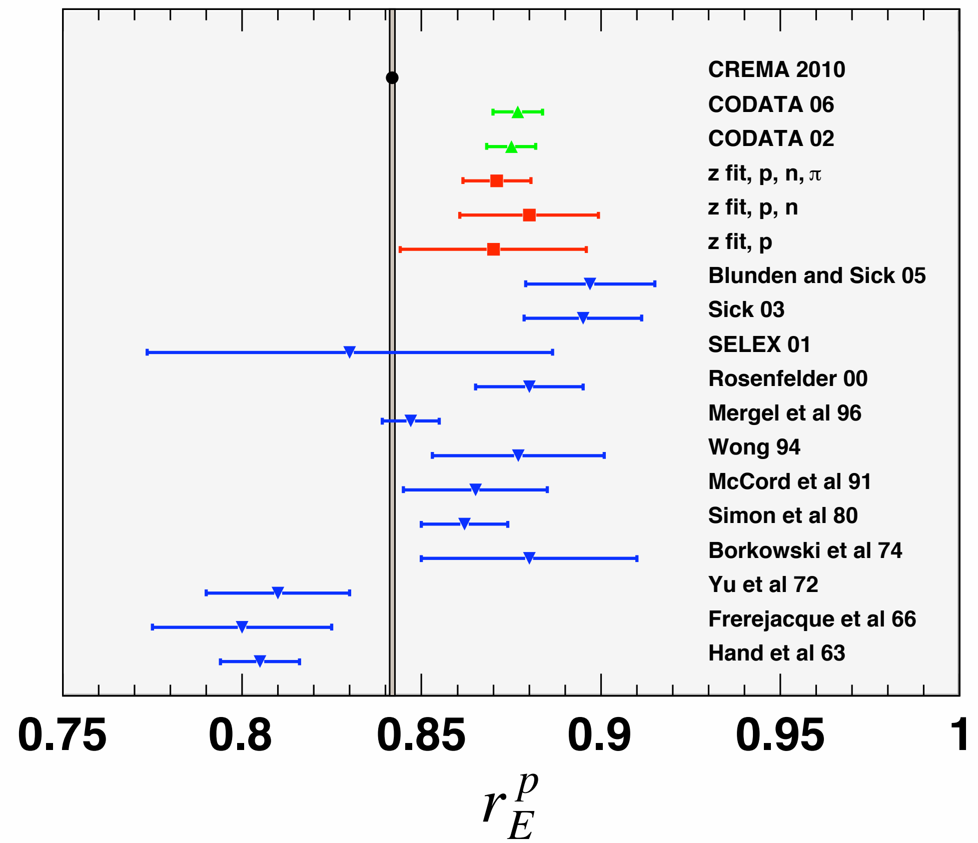
Convenient to work in terms of Sachs basis:

$$G_E = F_1 + \frac{q^2}{4M^2}F_2$$
$$G_M = F_1 + F_2$$

$$G_M(0) = \mu_p \approx 2.793$$

$$M^2 \frac{d}{dq^2} G_E \bigg|_{q^2=0} \equiv \frac{1}{6} r_E^2 = ?$$

*blue points*: fits of e p scattering data using different data sets, different functional forms of $G_E(q^2)$



## *Analyticity and form factor constraints*

What functional form to use in extrapolating to $Q^2=0$ ?

$$G_E = 1 + a_1 q^2 + a_2 q^4 + \ldots$$

*[Simon et al 1980]*

radius of convergence < 4 $m_\pi^2$

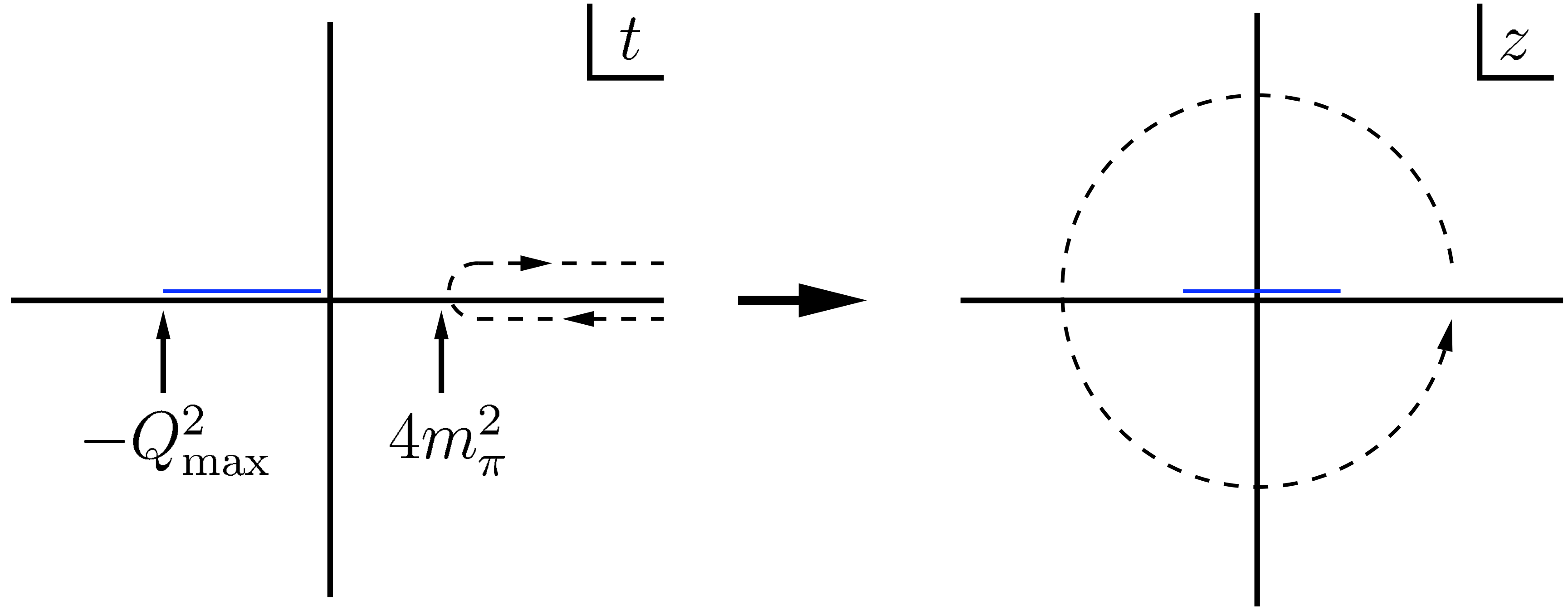
$$G_E = \frac{1}{1 + a_1 \frac{q^2}{1 + a_2 \frac{q^2}{1+\ldots}}}$$

*[Sick 2003]*

no control on parameters

*fundamental problem*: need larger $Q^2$ to increase statistics but then introduce sensitivity to more parameters (need even more statistics, ...)

*analyticity:*




- extended to complex values of $t=q^2$, form factor is analytic outside cut in t plane

$$z(t, t_{\rm cut}, t_0) = \frac{\sqrt{t_{\rm cut} - t} - \sqrt{t_{\rm cut} - t_0}}{\sqrt{t_{\rm cut} - t} + \sqrt{t_{\rm cut} - t_0}}$$

$4m_\pi^2$ (isovector channel)

point mapping to z=0 (scheme choice)

- "resums" simple Taylor expansion, ensuring convergence through entire physical range

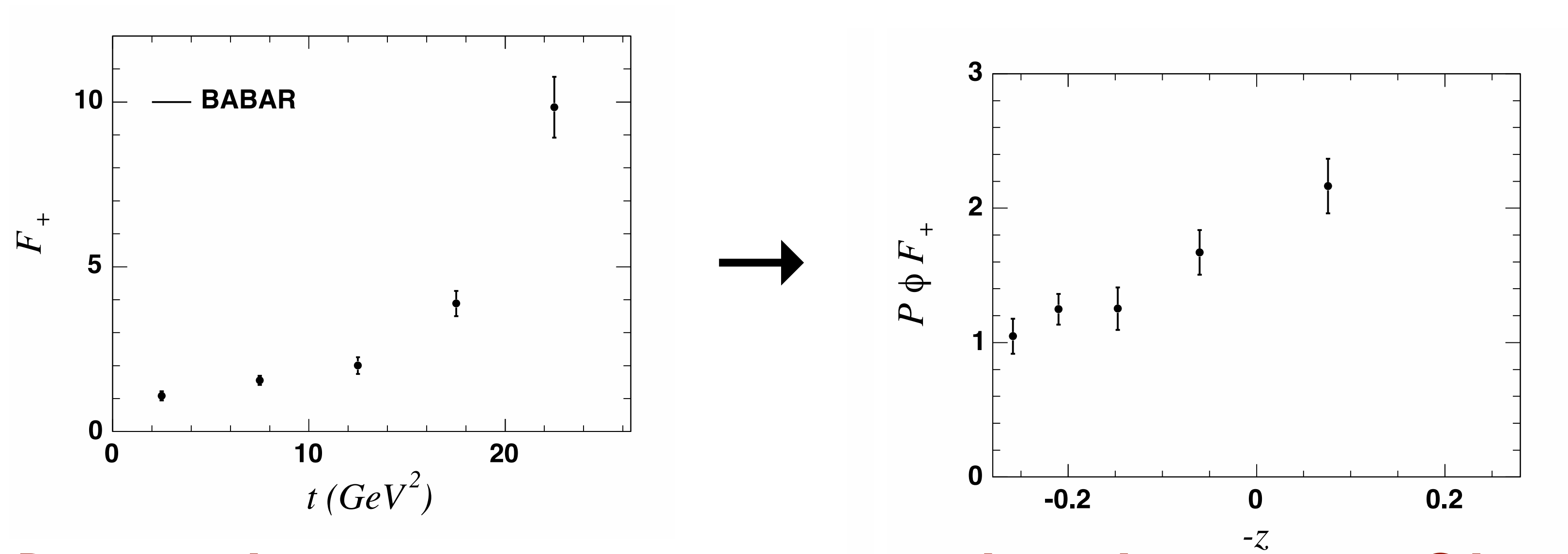
- basic idea: small expansion parameter, z, with *order unity* expansion coefficients

$$G(q^2) = \sum_{n=0}^{\infty} a_n z(q^2)^n$$

- in fact, a little better, e.g.

$$\sum_{n=0}^{\infty} a_n^2 < \infty \qquad \Rightarrow \qquad a_n \textit{ smaller} \text{ for large n}$$

The z expansion has become a standard tool for meson transitions (e.g. |Vub| determinations in B → π l ν )



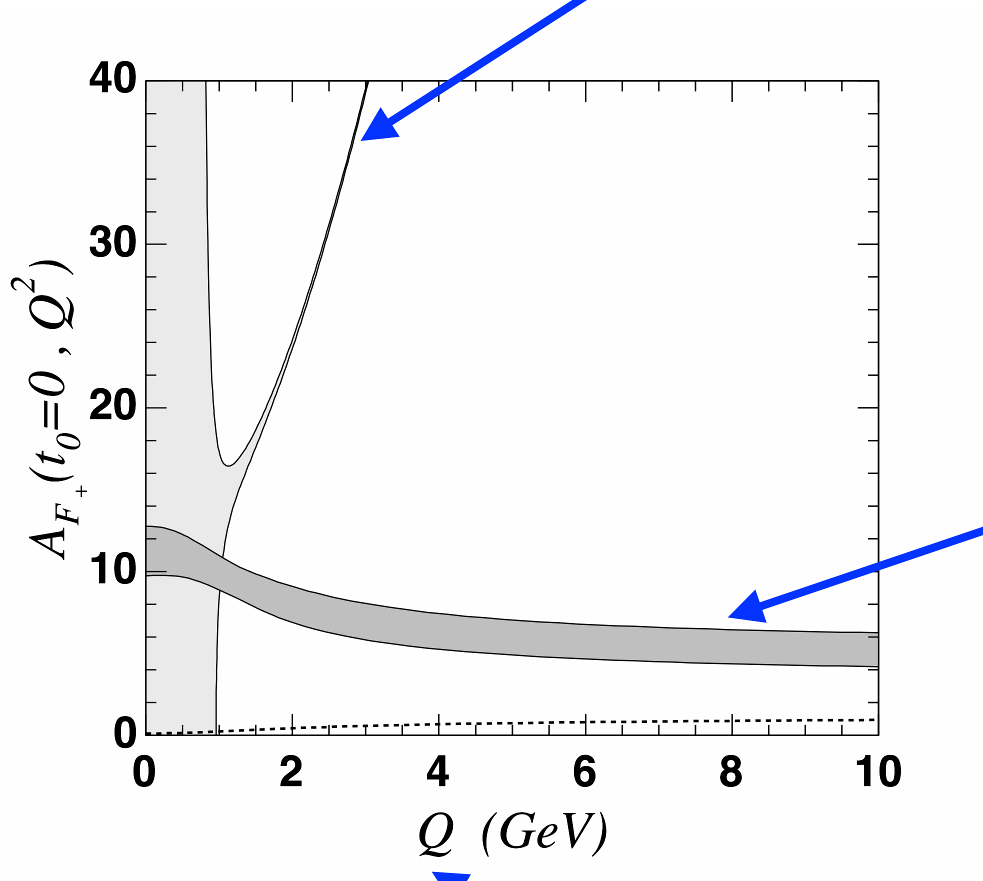

*[Bourrely et al 1981]*
*[Boyd, Grinstein, Lebed 1995]*
*[Lellouch et al 1996]*
*[Arnesen et al 2005]*
*[Becher, Hill 2006] ....*

*Bring the z expansion into the domain of baryon form factors*

For the cognoscenti, the real power of the expansion is based on observation of O(1) coefficients, not unitarity bounds. Example for K→π vector form factor, can *measure* bound

unitarity bound on A (require exclusive rate< inclusive rate)

$$A = \sqrt{\sum_k \frac{a_k^2}{a_0^2}}$$




actual size of A (measured τ→Kπν)

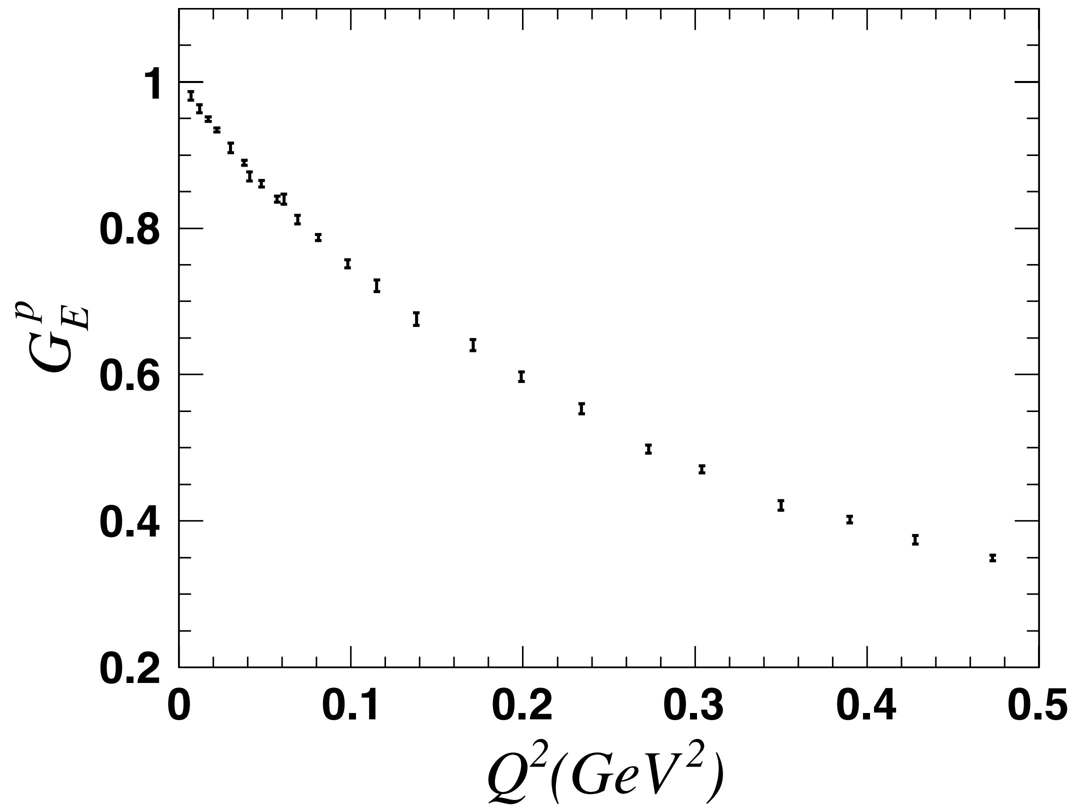
[RJH PRD 2006]

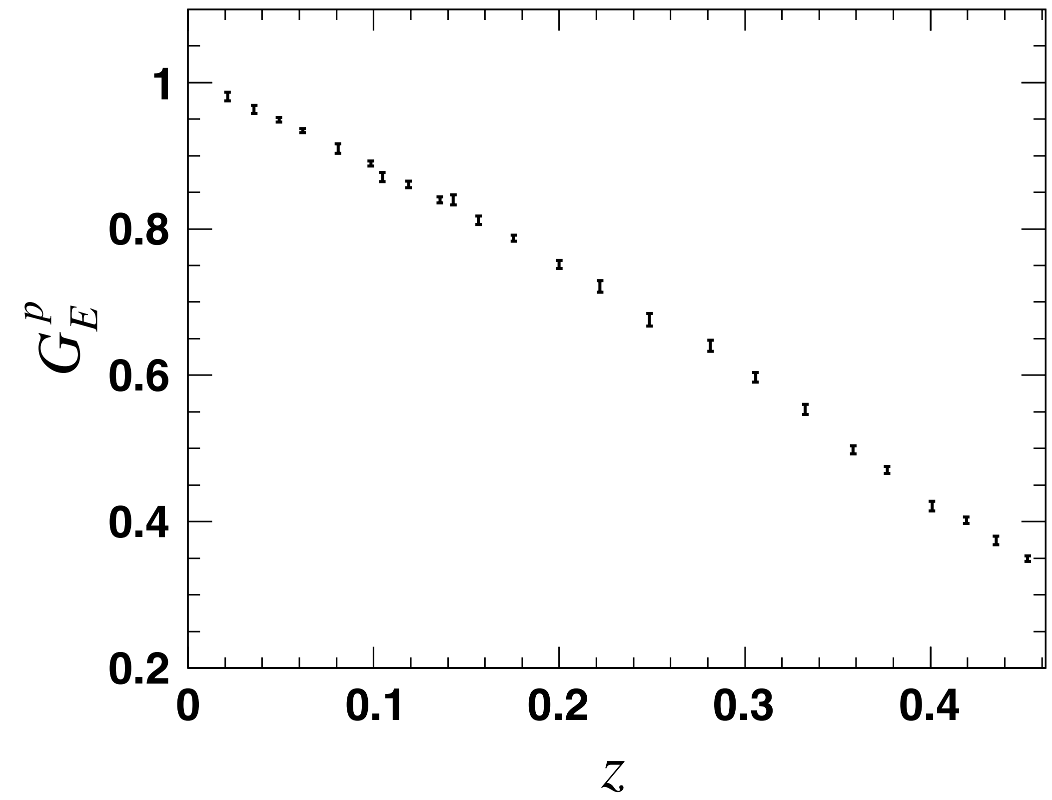
scheme choice to evaluate OPE for inclusive rate

⇒ Unitarity bound either uncertain (low Q) or overestimates bound (high Q)

For nucleon form factors, unitarity even less relevant, as dominant dispersive contribution to form factors is from states below NN threshold

- study of vector dominance models, ππ approximation to isovector form factors: expect O(1) is really order 1 (e.g. not 10)



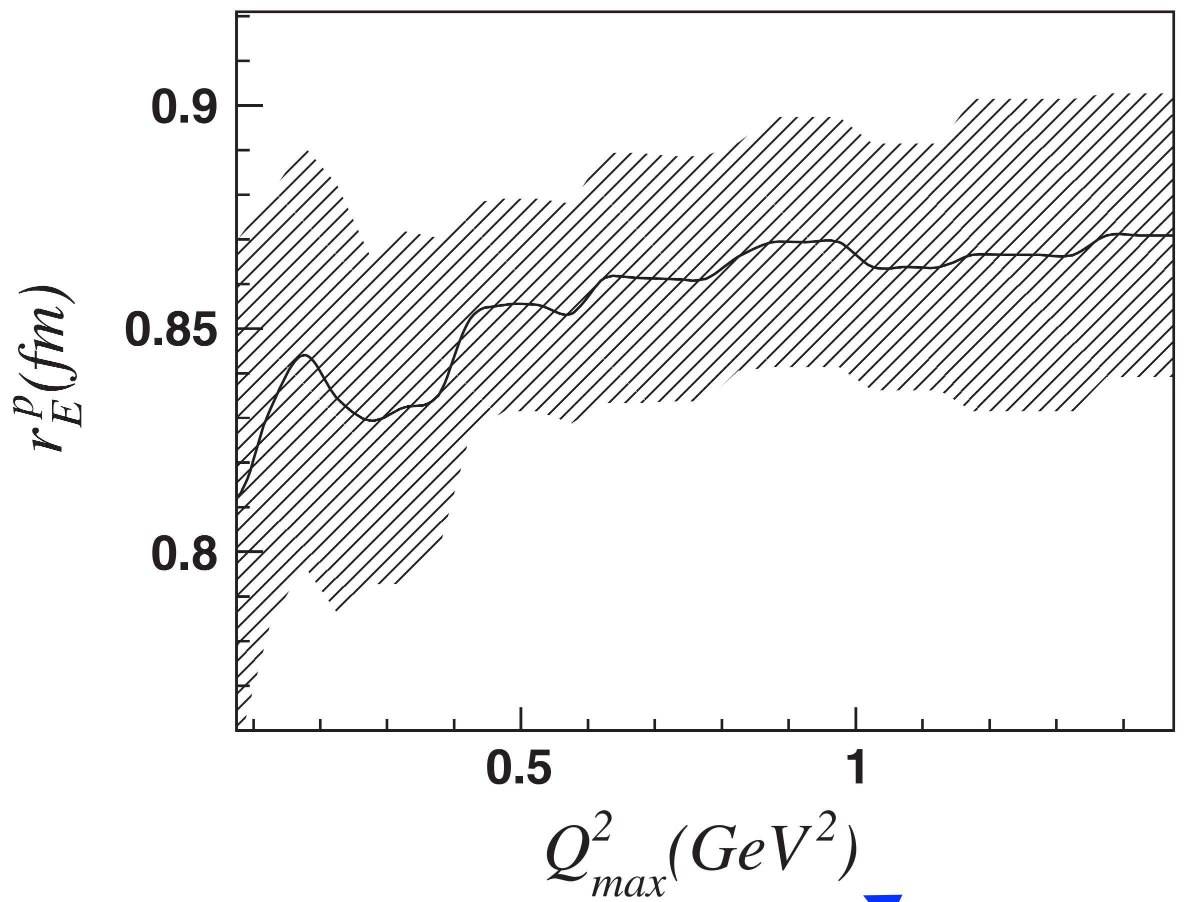




- more concretely, fits to data yield

$$a_0 \equiv 1\,, \quad a_1 = -1.01(6)\,, \quad a_2 = -1.4^{+1.1}_{-0.7}\,, \quad a_3 = 2^{+2}_{-6}$$

- to assign error, constrain coefficients <5 (conservative) or <10 (very conservative)

# results for proton charge radius:



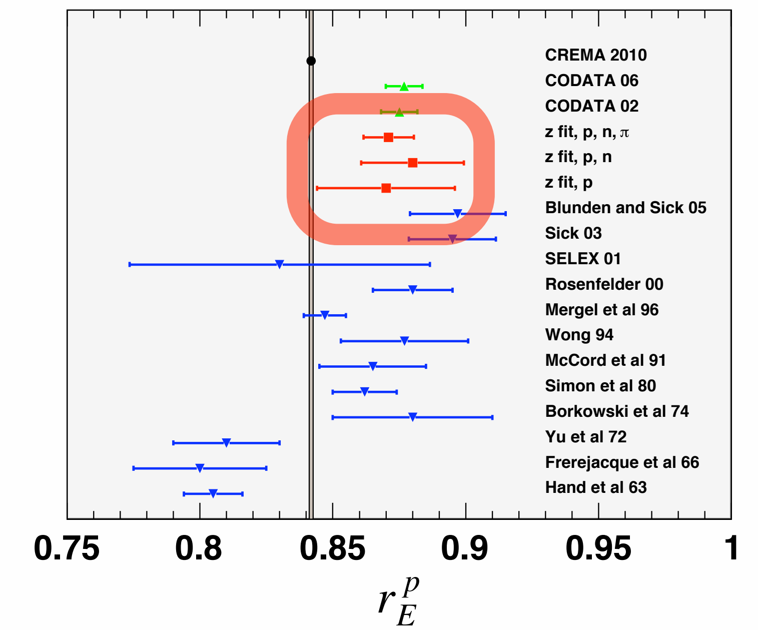

*data from Arrington et al PRC 2007*

maximum $Q^2$

$$r_E^p = 0.870 \pm 0.023 \pm 0.012\,\mathrm{fm}$$

expt

shape:
$|a|<5 \rightarrow |a|<10$




- larger $Q^2$ range: sensitive to more coefficients in expansion, but doesn't improve slope at $Q^2$=0

- can reduce error by decomposing isospin amplitudes, adding neutron scattering data, $\pi\pi \rightarrow N\tilde{N}$ data

# Investigate "scheme" dependence, finding little:

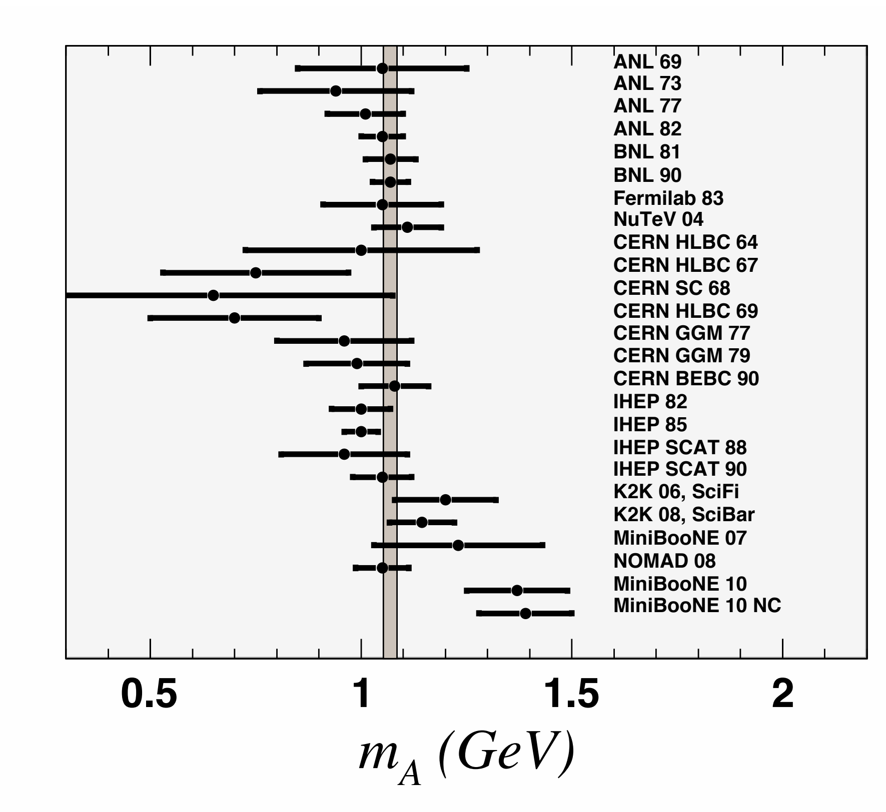
TABLE III. The rms charge radius extracted using electron-proton and electron-neutron scattering data, and different schemes presented in the text. The neutron form-factor slope is constrained using (31). A cut $Q^2_{\rm max} = 0.5$ GeV$^2$ is enforced. In the lower part of the table, the bounds on $\sum_k a_k^2$ from Table II are multiplied by 4. $\phi_{\rm VMD}$ and $\phi_{\rm OPE}$ are defined in Eqs. (22) and (23).

| | $k_{\rm max} = 2$ | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|
| $\phi = 1,\ t_0 = 0,\ \lvert a_k\rvert \le 10$ | $888^{+5}_{-5}$ | $865^{+11}_{-11}$ | $888^{+17}_{-22}$ | $882^{+21}_{-22}$ | $878^{+20}_{-19}$ |
| | $\chi^2 = 33.67$ | 23.65 | 21.80 | 21.13 | 20.47 |
| $\phi = 1,\ t_0 = 0,\ \lvert a_k\rvert \le 5$ | $888^{+5}_{-5}$ | $865^{+11}_{-11}$ | $881^{+10}_{-16}$ | $885^{+16}_{-21}$ | $882^{+18}_{-20}$ |
| | $\chi^2 = 33.67$ | 23.65 | 21.95 | 21.46 | 21.06 |
| $\phi = \phi_{\rm VMD},\ t_0 = 0,\ \lvert a_k\rvert \le 10$ | $865^{+6}_{-6}$ | $874^{+12}_{-13}$ | $884^{+23}_{-24}$ | $879^{+24}_{+22}$ | $877^{+22}_{-20}$ |
| | $\chi^2 = 23.26$ | 22.50 | 22.15 | 21.59 | 21.09 |
| $\phi = 1,\ t_0 = 0$ | $888^{+5}_{-5}$ | $865^{+11}_{-11}$ | $880^{+13}_{-16}$ | $882^{+14}_{-18}$ | $882^{+15}_{-18}$ |
| | $\chi^2 = 33.67$ | 23.65 | 22.07 | 21.45 | 21.18 |
| $\phi = \phi_{\rm OPE},\ t_0 = 0$ | $904^{+5}_{-5}$ | $861^{+10}_{-11}$ | $888^{+14}_{-21}$ | $883^{+20}_{-20}$ | $881^{+20}_{-19}$ |
| | $\chi^2 = 61.34$ | 24.38 | 21.62 | 20.86 | 20.51 |
| $\phi = \phi_{\rm OPE},\ t_0 = t_0^{\rm opt}(0.5\ {\rm GeV}^2)$ | $912^{+5}_{-5}$ | $869^{+9}_{-9}$ | $887^{+18}_{-19}$ | $881^{+20}_{-19}$ | $880^{+20}_{-19}$ |
| | $\chi^2 = 93.69$ | 22.54 | 21.05 | 20.32 | 20.32 |

# Recap:

- Motivated by anomalies in neutrino cross sections, we look for model-independent constraints on nucleon-level amplitudes
- We employed analyticity to describe the vector form factors

*Return to our mission of constraining the axial-vector form factor: in particular, what is $m_A$?*




First, let's *define* $m_A$ (motivated by, but not dependent on, dipole ansatz)

$$F_A(q^2) = F_A(0)\left[1 + \frac{2}{m_A^2}q^2 + \dots\right] \implies m_A \equiv \sqrt{\frac{2F_A(0)}{F_A'(0)}}$$

Slope and normalization essentially the only parameters probed (form factor almost linear in z variable)

Same techniques as for vector form factors

$$F_A(q^2) = \sum_{k=0}^{\infty} a_k z(q^2)^k$$

$$z(t, t_{\rm cut}, t_0) = \frac{\sqrt{t_{\rm cut} - t} - \sqrt{t_{\rm cut} - t_0}}{\sqrt{t_{\rm cut} - t} + \sqrt{t_{\rm cut} - t_0}}$$

$9m_\pi^2$

point mapping to z=0
(scheme choice)

## define “norms”

$$||F_A||_p = \left( \sum_k |a_k|^p \right)^{1/p}$$

$$||F_A||_2 = \left( \frac{1}{\pi} \int_{t_{\rm cut}}^{\infty} \frac{dt}{t - t_0} \sqrt{\frac{t_{\rm cut} - t_0}{t - t_{\rm cut}}} |F_A(t)|^2 \right)^{1/2}$$

$$||F_A||_\infty = \sup_k |a_k| = \lim_{p\to\infty} ||F_A||_p$$

$$a_0 = \frac{1}{\pi} \int_0^\pi d\theta \, {\rm Re}\, F_A[t(\theta) + i0] = F_A(t_0)\,,$$

$$a_{k\ge 1} = -\frac{2}{\pi} \int_0^\pi d\theta \, {\rm Im}\, F_A[t(\theta) + i0] \, \sin(k\theta) = \frac{2}{\pi} \int_{t_{\rm cut}}^\infty \frac{dt}{t - t_0} \sqrt{\frac{t_{\rm cut} - t_0}{t - t_{\rm cut}}} {\rm Im} F_A(t) \sin[k\theta(t)]$$

$$t = t_0 + \frac{2(t_{\rm cut} - t_0)}{1 - \cos\theta} \equiv t(\theta)$$

## how big is “order unity” ?

## e.g. "axial-vector dominance" ansatz

$$\mathrm{Im} F_A(t+i0) = \frac{\mathcal{N} m_{a_1}^3 \Gamma_{a_1}}{(t-m_{a_1}^2)^2 + \Gamma_{a_1}^2 m_{a_1}^2} \theta(t - t_{\rm cut})$$

## can then compute explicitly:

| | $t_0 = 0$ | $t_0 = t_0^{\rm opt}(1.0\,{\rm GeV}^2)$ |
|---|---|---|
| $\lvert\lvert F_A \rvert\rvert_2 / \lvert F_A(t_0)\rvert$ | 1.5-1.7 | 1.9-2.3 |
| $\lvert\lvert F_A \rvert\rvert_\infty / \lvert F_A(t_0)\rvert$ | 1.0-1.4 | 1.4-1.8 |

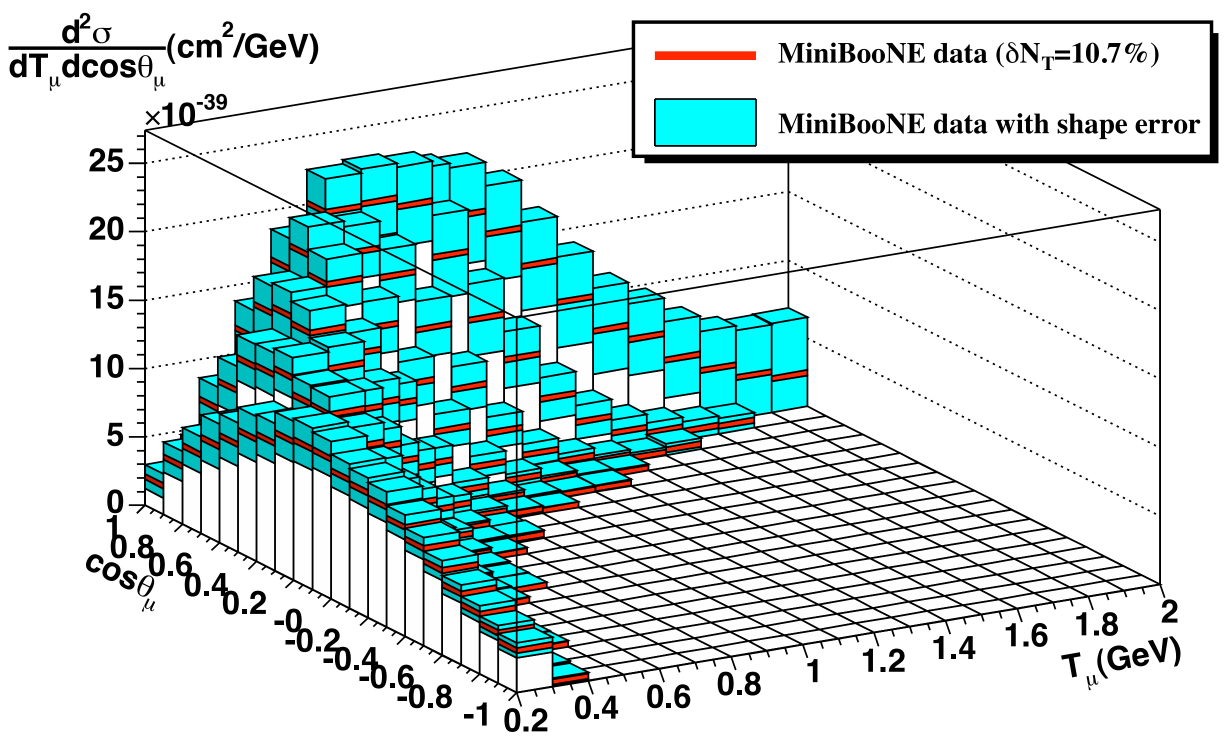
⇒ Indicates that "order unity" really means order unity

| Parameter | Value |
|---|---|
| $\lvert V_{ud}\rvert$ | 0.9742 |
| $\mu_p$ | 2.793 |
| $\mu_n$ | $-1.913$ |
| $m_\mu$ | 0.1057 GeV |
| $G_F$ | $1.166 \times 10^{-5}$ GeV$^{-2}$ |
| $m_N$ | 0.9389 GeV |
| $F_A(0)$ | $-1.269$ |
| $\epsilon_b$ | 0.025 GeV |
| $p_F$ | 0.220 GeV |

$$E_{ij} = (\delta\sigma_i)^2\delta_{ij} + (\delta N)^2\sigma_i\sigma_j$$

$$\chi^2 = \sum_{ij}(\sigma_i^{\text{expt.}} - \sigma_i^{\text{theory}})E_{ij}^{-1}(\sigma_j^{\text{expt.}} - \sigma_j^{\text{theory}})$$

Fit to double differential CCQE data from MiniBooNE

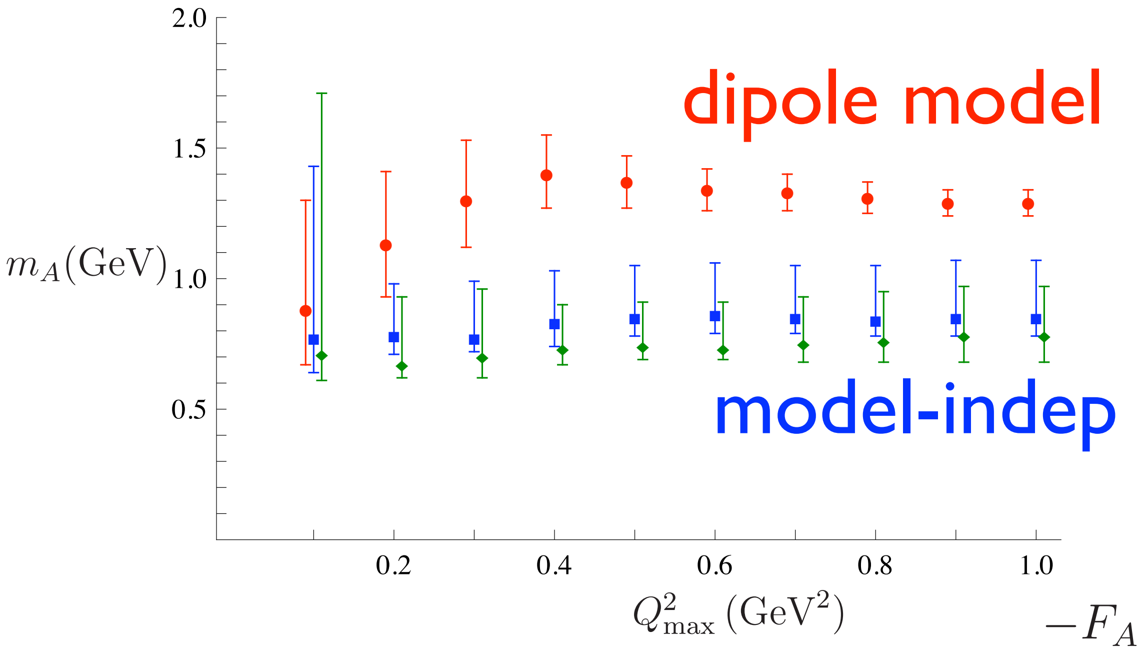


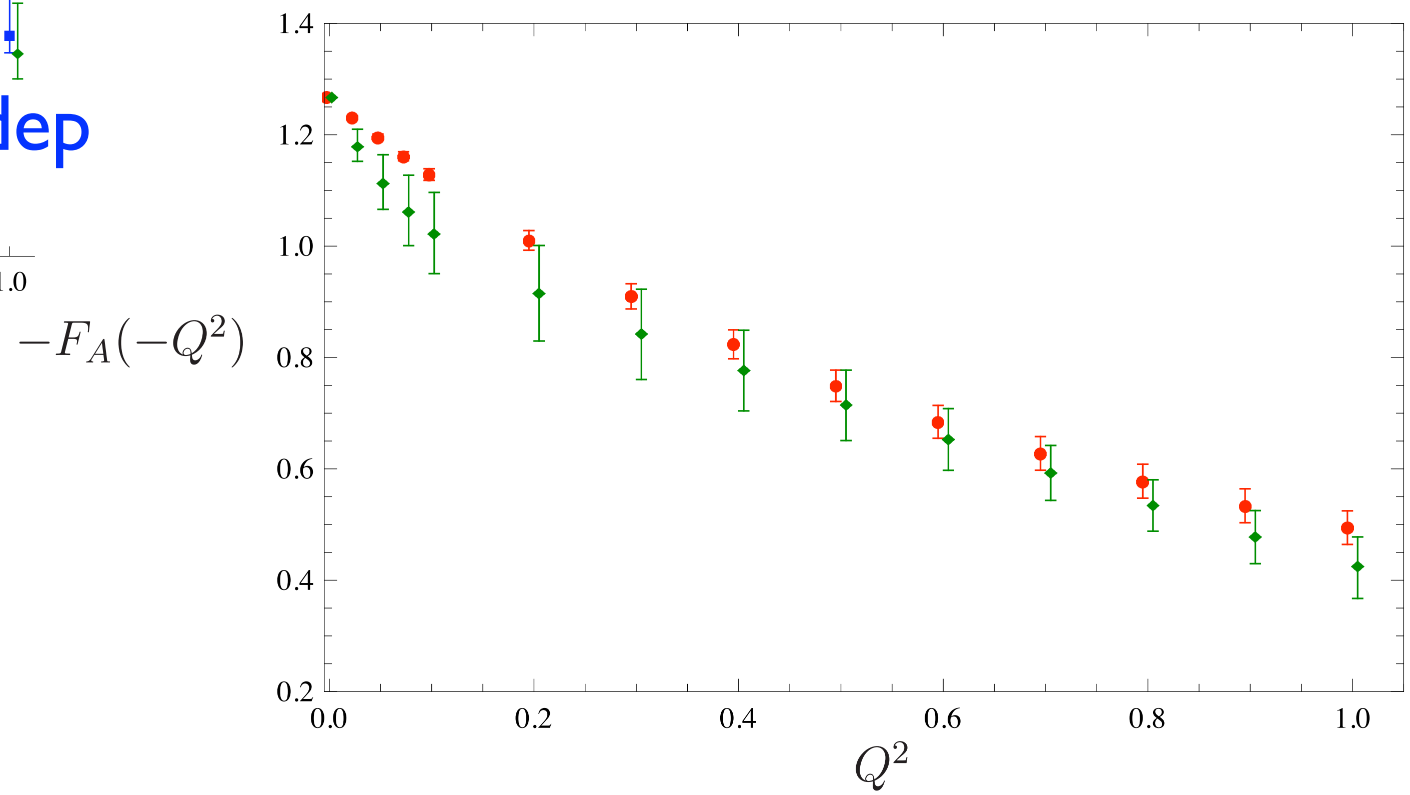

[MiniBooNE, PRD81, 092005 (2010)]

Assume Relativistic Fermi Gas nuclear model

[Smith and Moniz (1972)]

# Results for axial mass:

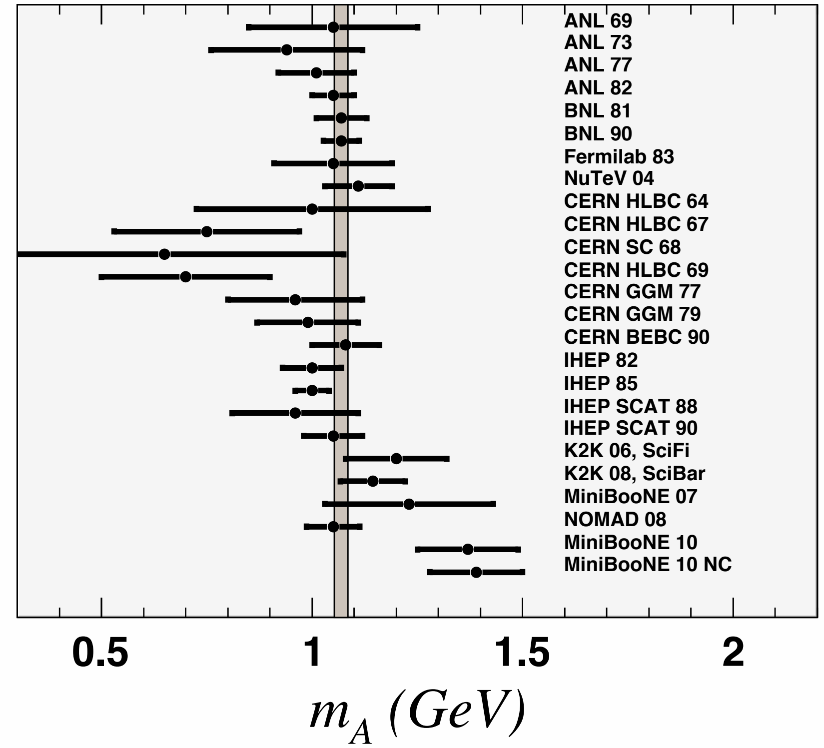



$$m_A = 0.85^{+0.22}_{-0.07} \pm 0.09\,\mathrm{GeV} \qquad \text{(neutrino scattering)}$$

$$m_A^{\rm dipole} = 1.29 \pm 0.05\,\mathrm{GeV}$$

## ***Revisit pion electroproduction***



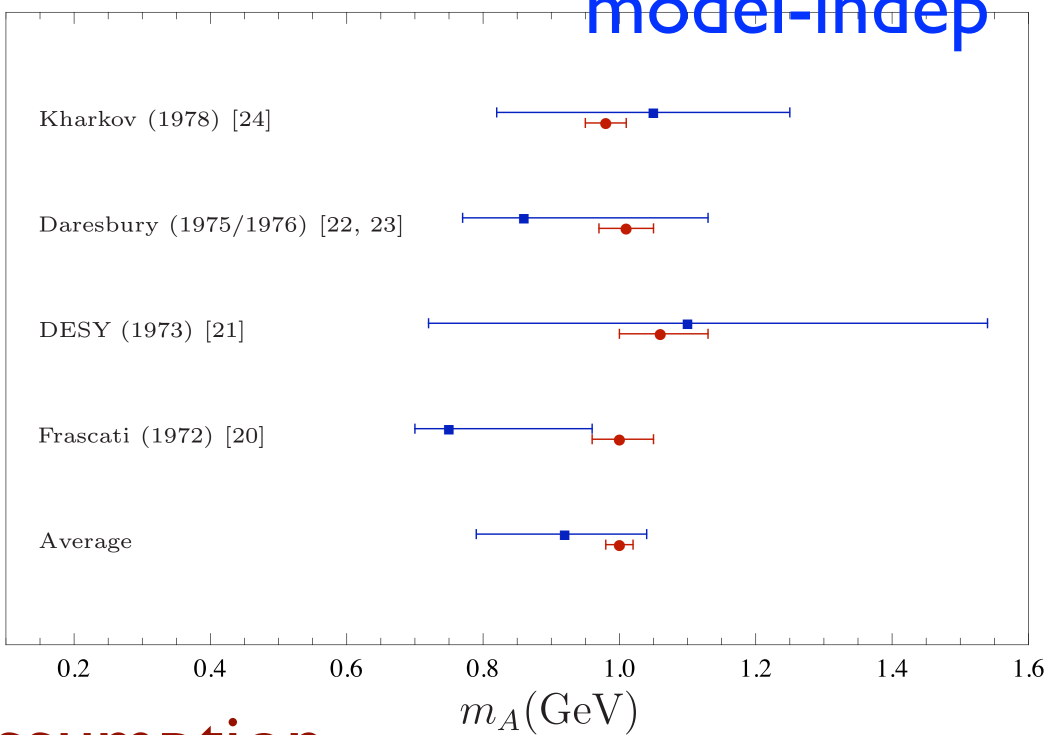

Experimental anomalies are between

a) high and low energy neutrino data

b) neutrino data and electroproduction data

dipole model

model-indep




$$m_A = 0.92^{+0.12}_{-0.13} \pm 0.08\,\mathrm{GeV} \qquad \text{(electroproduction)}$$

⇒ World average strongly affected by dipole assumption.

Errors previously underestimated

Where does that leave us?

- No model-independent basis for an axial-mass anomaly, either between low- and high-energy neutrino data, or between neutrino and electroproduction data

- Nuclear cross sections are the convolution of a) nucleon-level process and b) nuclear effects.

Given a) can make progress on b)

Constraints on nuclear model

Example: consider the binding energy parameter in RFG nuclear model

$$\sigma_{\text{nuclear}} \approx 2V \int \frac{d^3p}{(2\pi)^3} n_i(\boldsymbol{p})$$

$$\left\{ \frac{G_F^2}{16|k\cdot p|} \int \frac{d^3k'}{(2\pi)^3 2E_{\boldsymbol{k}'}} \int \frac{d^3p'}{(2\pi)^3 2E_{\boldsymbol{p}'}} (2\pi)^4 \delta^4(p-p'+q) L^{\mu\nu} H_{\mu\nu} \right\} [1 - n_f(\boldsymbol{p}')]$$

$$p^0 \to \epsilon_{\boldsymbol{p}} \equiv E_{\boldsymbol{p}} - \epsilon_b$$

$$\epsilon_b = 28 \pm 3\,\text{MeV}$$

# Summary

- enforced analyticity constraints on invariant amplitudes for nucleon level processes

- extracted fundamental nucleon parameters independent of model assumptions ($m_A$, $r_E$, ..)

- can disentangle nucleon level process from nuclear effects in a model-independent way

Should make use of model-independent information when it's available: it's a free lunch !