

$|V_{ub}|$ determination in Lattice QCD

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Couplings of flavor-changing *weak interactions*:

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

processes with $b \rightarrow u$ transitions

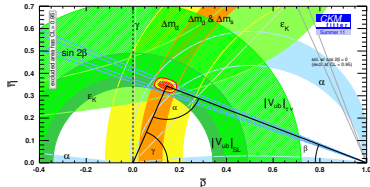
- **Inclusive** $B \rightarrow X_u \ell \nu$
heavy quark and α_S expansion
- **Exclusive Semileptonic** $B \rightarrow \pi \ell \nu$
hadronic formfactor $f_+(q^2)$
- **Exclusive Leptonic** $B \rightarrow \tau \nu$
hadronic decay constant f_B

Lattice input

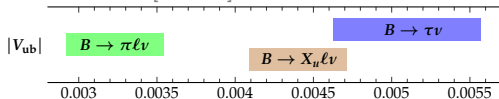
V_{ub} puzzle

+

$(\mathcal{B}(B \rightarrow \tau \nu), \sin(2\beta))$ discrepancy



Summer 2012: [PDG'12]



Compute B -physics in $N_f = 2$ QCD, keeping others **systematic errors** under control:

- f_B this talk, preliminary (but nearly finished)
- f_{B_s} this talk, preliminary
- $f_+(q^2)$ next year, show progress in this talk

Treatment of light quarks

- Volume effects exponentially suppressed:

$$Lm_\pi \geq 4.0$$

- Light quark mass chiral extrapolation:

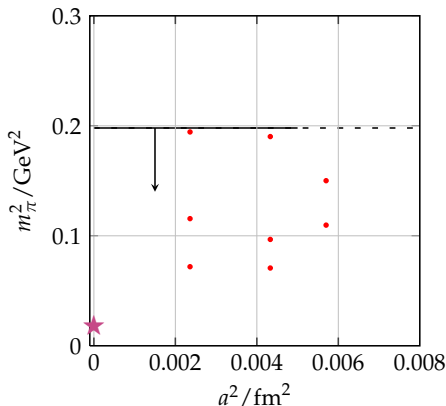
$$(250 \lesssim m_\pi \lesssim 400 - 450) \text{ MeV}$$

- Discretization effects:

- 3 lattice spacings a

$$(0.048, 0.065, 0.075 < 0.1) \text{ fm}$$

- NP renormalization
- NPly $O(a)$ improved Wilson fermions



7 simulations fulfill our current criteria

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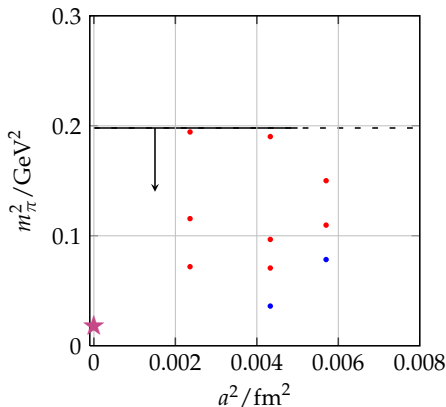
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7 simulations fulfill our current criteria

CLS
based



+ 2 more by end of this year

Treatment of b quark

$m_b \gg \Lambda_{\text{QCD}}$: b treated in HQET

- expansion in $1/m_b$

$$\mathcal{L}_{\text{HQET}}(x) = \bar{\psi}_h(x) D_0 \psi_h(x) - \omega_{\text{kin}} \bar{\psi}_h(x) \mathbf{D}^2 \psi_h(x) - \omega_{\text{spin}} \bar{\psi}_h(x) \boldsymbol{\sigma} \cdot \mathbf{B} \psi_h(x)$$

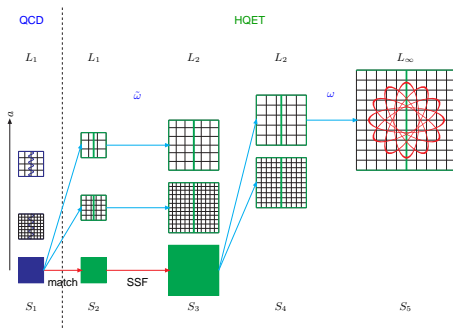
- renormalizable at every order in $1/m_b$
- restrict to processes such that $p \ll m_b$
- power divergences in $a^{-1} \Rightarrow$ **need NP renormalization** [Maiani, Martinelli, Sachrajda 92]

$$\ln(a^{3/2} f_B \sqrt{m_B/2}) = \ln(Z_A^{\text{HQET}}) + \ln(a^{3/2} p^{\text{stat}}) + b_A^{\text{stat}} am_q \\ + \omega_{\text{kin}} p^{\text{kin}} + \omega_{\text{spin}} p^{\text{spin}} + c_A^{(1)} p^{A(1)}$$

$$A_{0,R}^{\text{HQET}} = Z_A^{\text{HQET}} [A_0^{\text{stat}} + c_A^{(1)} A_0^{(1)}],$$

$$A_0^{\text{stat}} = \bar{\psi}_1 \gamma_0 \gamma_5 \psi_h,$$

$$A_0^{(1)} = \bar{\psi}_1 \gamma_5 \gamma_i \frac{1}{2} (\nabla_i^{\text{S}} - \overleftarrow{\nabla}_i^{\text{S}}) \psi_h$$



- match QCD and HQET at $a^{-1} \gg M_b$ (small volume, however $z = LM_b \gg 1$)

determine NP: $m_{\text{bare}}, Z_A^{\text{HQET}}, c_A^{(1)}, \omega_{\text{kin}}, \omega_{\text{spin}}$

- step scale to large volumes
- ⇒ determine M_b dependence of large volume observables

Example: the b-quark's mass

- 1 (left) extrapolate to physical point ($m_{PS} \rightarrow m_\pi$)

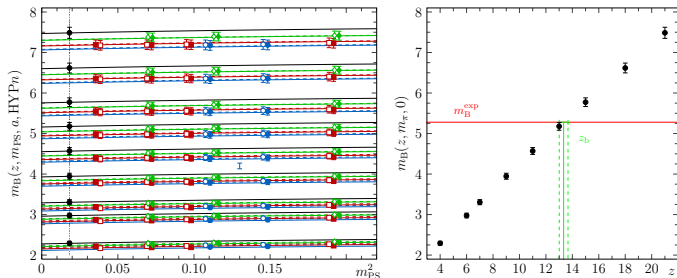
$$m_B(z, m_{PS}, a, \text{HYPn}) = B(z) + C m_{PS}^2 - \frac{3\hat{g}^2}{16\pi f_\pi^2} m_{PS}^3 + D_{\text{HYPn}} a^2$$

$$\hat{g} = 0.51(2) \text{ [Bulava et al. 10]}$$

- 2 (right) Interpolate $m_B(z)$ to get M_b :

$$m_B(z, m_\pi, a)|_{z=z_b} \equiv m_B^{\text{exp}} = 5279.5 \text{ MeV}$$

- 3 we get $z_b = 13.34(33)(13)_z$ or equivalently $\bar{m}_b(\bar{m}_b) = 4.22(10)(4)_z \text{ GeV}$.



[Della Morte et al. 07]

f_B computation on the lattice

Typically on the lattice one computes:

$$c(t) = \sum_{\vec{x}, \vec{y}} \langle P_{hl}(x) P_{hl}(y) \rangle \xrightarrow{T \rightarrow \infty, t \rightarrow \infty} f_B^2 m_B e^{-M_B t} (1 + O(e^{-\Delta_{1,0} t})) \quad (t = x_0 - y_0)$$

At finite t **excited states** contamination:

- $\sum_{\vec{y}}$ to reduce noise: stochastically (full time dilution)
- apply GEVP to n interpolating fields $P_{hl}^k, k = 1, \dots, n$

$$P_{hl}^k = \bar{\psi}_1^{(k)} \gamma_0 \gamma_5 \psi_h \quad \psi_1^{(k)}(x) = \left(1 + \kappa_G a^2 \Delta\right)^{R_k} \psi_1(x)$$

$$\Rightarrow O(e^{-\Delta_{1,0} t}) \rightarrow O(e^{-\Delta_{n,0} t})$$

- include in plateau average points such:

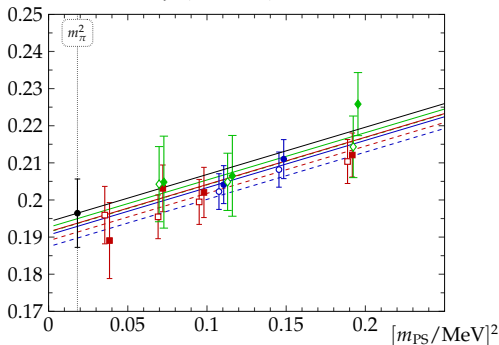
$$O(e^{-\Delta_{n,0} t}) < 1/3 \times \delta c_{stat}$$

The B-meson decay constant f_B

chiral and continuum extrapolation using HM χ PT: $f_B \equiv \lim_{(m_{PS}, a) \rightarrow (m_\pi, 0)} f_B(m_{PS}, a)$

$$f_B(m_{PS}^2, a^2) = b + cm_{PS}^2 + da^2 \quad (\text{linear})$$

$f_B(z_b, m_{PS}, a) / \text{GeV}$



■ no mixed term $(am_{PS}^2)^2 \lesssim 0.02$

$$f_B = 196(9)_{stat}$$

linear

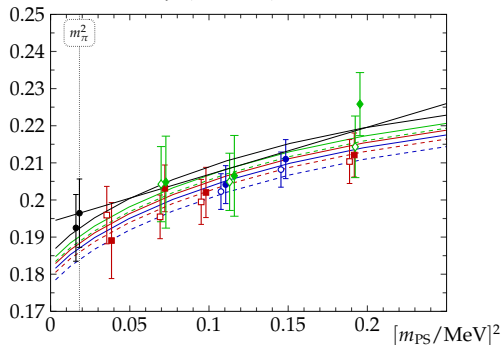
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$$f_B(m_{PS}^2, a^2) = b' \left[1 - \frac{3}{4} \frac{1+3\hat{g}^2}{(4\pi f_{\pi})^2} m_{PS}^2 \ln(m_{PS}^2) \right] + c' m_{PS}^2 + d' a^2 \quad (\text{NLO})$$

$f_B(z_b, m_{PS}, a) / \text{GeV}$



- no mixed term $(am_{PS}^2)^2 \lesssim 0.02$

$$f_B = 196(9)_{stat}$$

linear

$$f_B = 192(9)_{stat}$$

NLO

$$f_{\pi} = f_{\pi}^{\text{exp}}, \hat{g} = 0.51(2)$$

[Bulava et al. 10]

Our estimate of V_{ub}

$$\mathcal{B}_{\text{SM}}(B \rightarrow \tau\nu) = f_B^2 |V_{ub}|^2 \frac{G_F^2 m_B \tau_B}{8\pi} m_\tau^2 \left[1 - \frac{m_\tau^2}{m_B^2} \right]^2$$

using PDG values and

$$f_B = 194(9)_{\text{stat}}(4)_\chi \text{MeV}, \quad \mathcal{B}_{\text{SM}}(B \rightarrow \tau\nu)_{\text{PDG}} = (1.65(34)) \times 10^{-5}$$

$$|V_{ub}|_{\text{lept.}} = (4.98(58)) \times 10^{-3}$$

our determination

$$|V_{ub}|_{\text{lept.}} = (5.07(54)) \times 10^{-3}$$

$$f_B = 190.6(4.6) \text{MeV} [\text{Lattice Avgs}]$$

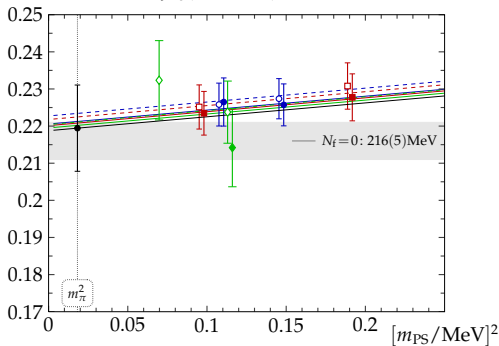
our current value significantly reduces the tension

⇒ experiment and theory have to improve further

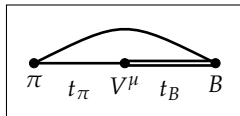
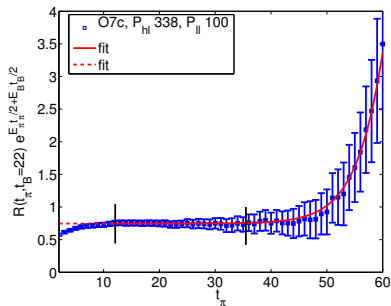
chiral and continuum extrapolation using $\text{HM}\chi\text{PT}$: $f_{B_s} \equiv \lim_{(m_{PS}, a) \rightarrow (m_{\pi}, 0)} f_{B_s}(m_{PS}, a)$

$$f_{B_s}(m_{PS}^2, a^2) = b + cm_{PS}^2 + da^2 \quad (\text{NLO})$$

$f_{B_s}(z_b, m_{PS}, a) / \text{GeV}$



- $\text{SU}(3) \text{HM}\chi\text{PT}$ not expected to work [Becirevic et al. 07]
- treat K as heavy: use $\text{SU}(2) \text{K}\chi\text{PT}$ [Roessl 99]
- $f_{B_s} = 219(12)_{\text{stat}}$ (our det.)
- $f_{B_s} = 227.6(5.0)_{\text{stat}}$ [Lattice Avgs.]

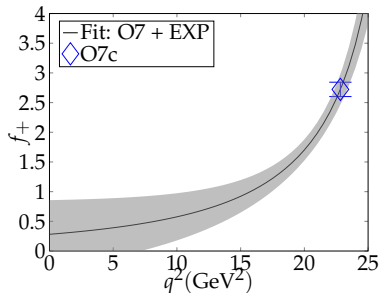


- smearing π ($R_k = 100$) and B ($R_k = 338$)
- fit finite T effects (2 parameters)
- obtain f_+ to 5% level
- $M_\pi \sim 270$ MeV, $a = 0.048$ fm

So far:

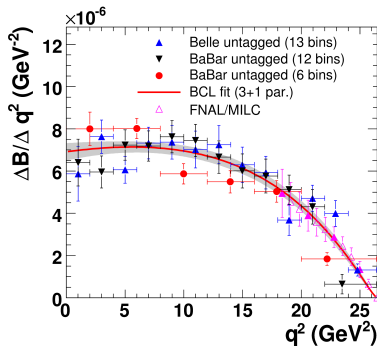
- static limit **truncation errors** $\sim E_\pi/m_B \sim 10 - 20\%$
- NP renormalization of HQET, but 3-loop matching
- light quarks $O(a)$ improved, but V^μ not improved **possibly large** $O(a\Lambda\alpha)$
- NO continuum limit, NO χ extrapolation

Expected error, next year



Using one point to $\sim 6\%$ ($q^2 \sim 22$ GeV^2):

- z-expansion (analyticity, unitarity, QCD properties, kinematics)
[Bourelly, Caprini, Lellouch 09]
 - experimental data
- $\Rightarrow V_{ub}$ to 15%



Next year:

- include all $1/m_b$ terms
- full NP renormalization and matching
- $O(a\Lambda)$ improvement; but not $O(\Lambda^2 a/m_b)$
- continuum limit, χ extrapolation