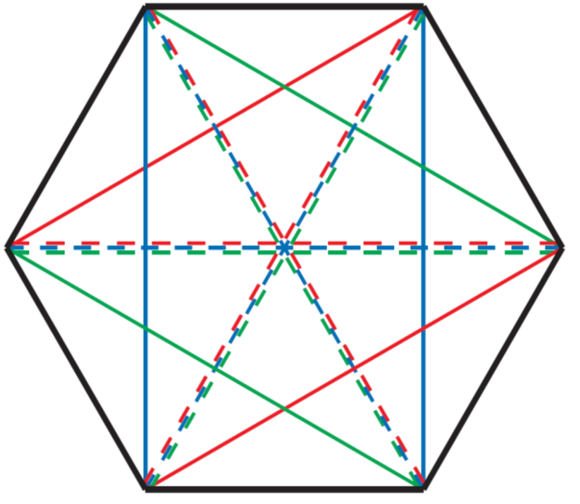
Scattering in Planar N=4 Super-Yang-Mills Theory and the Multi-Regge-Limit



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with C. Duhr and J. Pennington,

arXiv:1207.0186

ICHEP Melbourne, Australia July 5, 2012

Scattering amplitudes in planar N=4 Super-Yang-Mills

- Planar (large N_c) N=4 SYM is a 4-dimensional analog of QCD, (potentially) solvable to all orders in $\lambda = g^2 N_c$
- It can teach us what types of mathematical structures will enter multi-loop QCD amplitudes
- Its amplitudes have remarkable hidden symmetries
- In strong-coupling, large λ limit, AdS/CFT duality maps the problem into weakly-coupled gravity/semi-classical strings moving on $AdS_5 \times S^5$

Remarkable, related structures recently unveiled in planar N=4 SYM scattering

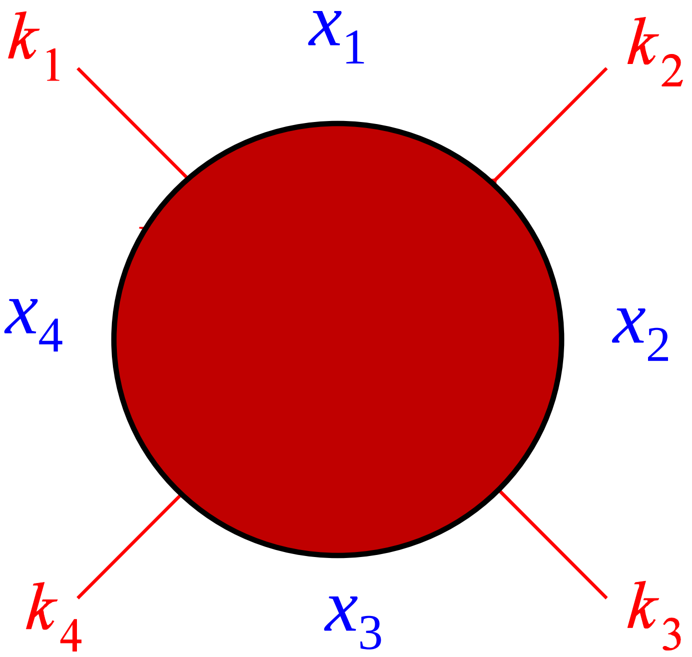
- Exact exponentiation of 4 & 5 gluon amplitudes
- Dual (super)conformal invariance
- Amplitudes equivalent to Wilson loops
- Strong coupling → minimal area “soap bubbles”

Can these structures be used to solve exactly for all planar N=4 SYM amplitudes?
What is the first nontrivial case to solve?

Dual conformal invariance

Broadhurst (1993); Lipatov (1999); Drummond, Henn, Smirnov, Sokatchev, hep-th/0607160

Conformal symmetry acting in momentum space, on dual or sector variables x_i : $k_i = x_i - x_{i+1}$



invariance under inversion:

$$x_i^\mu \rightarrow \frac{x_i^\mu}{x_i^2}$$

$$x_{ij}^2 \rightarrow \frac{x_{ij}^2}{x_i^2 x_j^2}$$

Dual conformal constraints

- Symmetry fixes form of amplitude, up to functions of dual conformally invariant cross ratios:

$$u_{ijkl} \equiv \frac{x_{ij}^2 x_{kl}^2}{x_{ik}^2 x_{jl}^2}$$

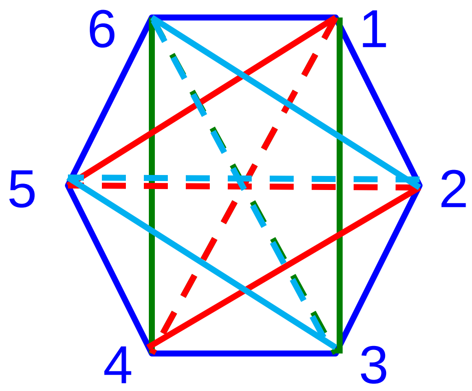
- Because $x_{i-1,i}^2 = k_i^2 = 0$ there are no such variables for n = 4,5
- Amplitude fixed to BDS ansatz:

$$\mathcal{A}_{4,5}(\epsilon; s_{ij}) = \mathcal{A}_{4,5}^{\rm BDS}(\epsilon; s_{ij})$$

For $n = 6$, precisely 3 ratios:

$$u_1 = \frac{x_{13}^2 x_{46}^2}{x_{14}^2 x_{36}^2} = \frac{s_{12} s_{45}}{s_{123} s_{345}}$$

+ 2 cyclic perm's



$$\mathcal{A}_6(\epsilon; s_{ij}) = \mathcal{A}_6^{\rm BDS}(\epsilon; s_{ij}) \exp[R_6(u_1, u_2, u_3)]$$

Formula for $R_6^{(2)}(u_1,u_2,u_3)$

- First worked out analytically from Wilson loop integrals

Del Duca, Duhr, Smirnov, 0911.5332, 1003.1702

17 pages of Goncharov polylogarithms.

- Simplified to just a few classical polylogarithms using symbology

Goncharov, Spradlin, Vergu, Volovich, 1006.5703

$$R_6^{(2)}(u_1,u_2,u_3) = \sum_{i=1}^{3}\left(L_4(x_i^+,x_i^-) - \frac{1}{2}\,\mathrm{Li}_4(1-1/u_i)\right)$$
$$-\frac{1}{8}\left(\sum_{i=1}^{3}\mathrm{Li}_2(1-1/u_i)\right)^2 + \frac{1}{24}J^4 + \frac{\pi^2}{12}J^2 + \frac{\pi^4}{72}$$

$$L_4(x^+,x^-) = \frac{1}{8!!}\log(x^+x^-)^4$$
$$+\sum_{m=0}^{3}\frac{(-1)^m}{(2m)!!}\log(x^+x^-)^m(\ell_{4-m}(x^+)+\ell_{4-m}(x^-))$$

$$\ell_n(x) = \frac{1}{2}\left(\mathrm{Li}_n(x) - (-1)^n\,\mathrm{Li}_n(1/x)\right)$$

$$J = \sum_{i=1}^{3}(\ell_1(x_i^+) - \ell_1(x_i^-))$$

$$x_i^\pm = u_i x^\pm, \qquad x^\pm = \frac{u_1+u_2+u_3-1\pm\sqrt{\Delta}}{2u_1u_2u_3} \qquad \Delta = (u_1+u_2+u_3-1)^2 - 4u_1u_2u_3$$

Variables for $R_6^{(L)}(u_1,u_2,u_3)$

- A "pure" transcendental function of weight $2L$:
Differentiation gives a linear combination of weight $2L$-1 pure functions, divided by rational expressions r_i.
- Here,

$$r_i \in \{u_1, u_2, u_3, 1-u_1, 1-u_2, 1-u_3, y_1, y_2, y_3\}$$

with

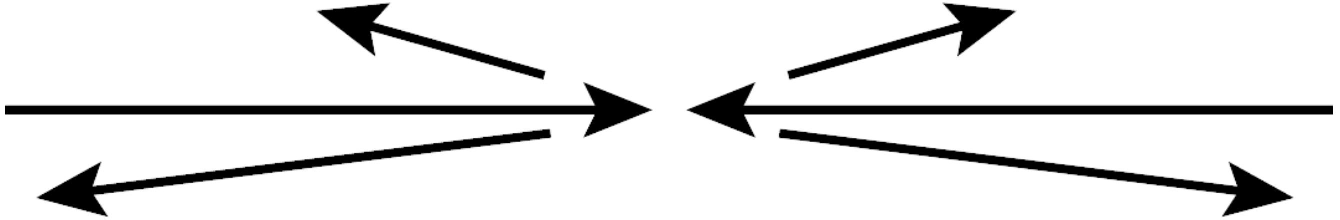
$$y_i \equiv \frac{u_i - z_+}{u_i - z_-}$$

$$\begin{aligned} z_\pm &= \frac{1}{2}\Big[-1 + u_1 + u_2 + u_3 \pm \sqrt{\Delta}\Big] \\ \Delta &= (1-u_1-u_2-u_3)^2 - 4u_1u_2u_3 \end{aligned}$$

→ Square-root complications

The multi-Regge limit

- $R_6^{(2)}(u_1,u_2,u_3)$ is pretty simple. But to go to very high loop order, we take the limit of multi-Regge kinematics (MRK): large rapidity separations between the 4 final-state gluons:

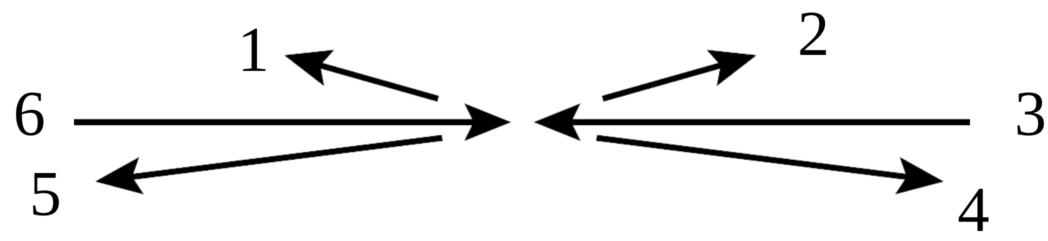


- Properties of planar N=4 SYM amplitude in this limit studied extensively already:

Bartels, Lipatov, Sabio Vera, 0802.2065, 0807.0894; Lipatov, 1008.1015; Lipatov, Prygarin, 1008.1016, 1011.2673; Bartels, Lipatov, Prygarin, 1012.3178, 1104.4709; Fadin, Lipatov, 1111.0782

Multi-Regge kinematics

$$u_1 = \frac{s_{12}^2 s_{45}^2}{s_{123}^2 s_{345}^2} \to 1$$



$$\frac{u_2}{1-u_1} \to x$$
$$\frac{u_3}{1-u_1} \to y$$

A very nice change of variables [LP, 1011.2673] is to (w, w^*) :

$$x = \frac{1}{(1+w)(1+w^*)}$$
$$y = \frac{ww^*}{(1+w)(1+w^*)}$$

$$y_1 \to 1$$
$$y_2 \to \frac{1+w^*}{1+w}$$
$$y_3 \to \frac{(1+w)w^*}{w(1+w^*)}$$

2 symmetries: conjugation $w \leftrightarrow w^*$
and inversion $w \leftrightarrow 1/w, \ w^* \leftrightarrow 1/w^*$

Physical 2→4 multi-Regge limit

- To get a nonzero result, for the physical region, one must first let $u_1 \to u_1 e^{-2\pi i}$, and extract one or two discontinuities → factors of $-2\pi i$.
- Then let $u_1 \to 1$. Bartels, Lipatov, Sabio Vera, 0802.2065, …

$$R_6^{(L)} \to (2\pi i) \sum_{n=0}^{L-1} \ln^n(1-u_1) \left[g_n^{(L)}(w,w^*) + 2\pi i\, h_n^{(L)}(w,w^*) \right]$$

Simpler pure functions

$$R_6^{(L)} \to (2\pi i) \sum_{n=0}^{L-1} \ln^n(1-u_1) \, [g_n^{(L)}(w,w^*) + 2\pi i \, h_n^{(L)}(w,w^*)]$$

weight $2L$-n-1 $2L$-n-2

$$r_i \in \{w, 1+w, w^*, 1+w^*\}$$

- Single-valued in $(w,w^*) = (-z,-\bar{z})$ plane

This is precisely the class of functions defined by Brown:
F.C.S. Brown, C. R. Acad. Sci. Paris, Ser. I 338 (2004).

SVHPLs: $\mathcal{L}_{m_1,\ldots,m_w}(z,\bar{z}) \sim \sum_{i,j} c_{i,j} H_{\vec{m}_i}(z) H_{\vec{m}_j}(\bar{z})$

H = ordinary harmonic polylogarithms

Remiddi, Vermaseren, hep-ph/9905237

MRK Master Formula

Fadin, Lipatov, 1111.0782

$$e^{R+i\pi\delta}|_{\rm MRK} = \cos\pi\omega_{ab} + i\,\frac{a}{2}\sum_{n=-\infty}^{\infty}(-1)^n\left(\frac{w}{w^*}\right)^{\frac{n}{2}}\int_{-\infty}^{+\infty}\frac{d\nu}{\nu^2+\frac{n^2}{4}}\,|w|^{2i\nu}\,\Phi_{\rm Reg}(\nu,n)$$
$$\times\exp\left[-\omega(\nu,n)\left(\log(1-u_1)+i\pi+\frac{1}{2}\log\frac{|w|^2}{|1+w|^4}\right)\right]$$

BFKL eigenvalue

MHV impact factor

$$\omega(\nu,n) = -a(E_{\nu,n} + a\,E_{\nu,n}^{(1)} + a^2\,E_{\nu,n}^{(2)} + \cdots)$$

$$\Phi_{\rm Reg}(\nu,n) = 1 + a\Phi_{\nu,n}^{(1)} + a^2\Phi_{\nu,n}^{(2)} + a^3\Phi_{\nu,n}^{(3)} + \cdots$$

LL NLL NNLL NNNLL

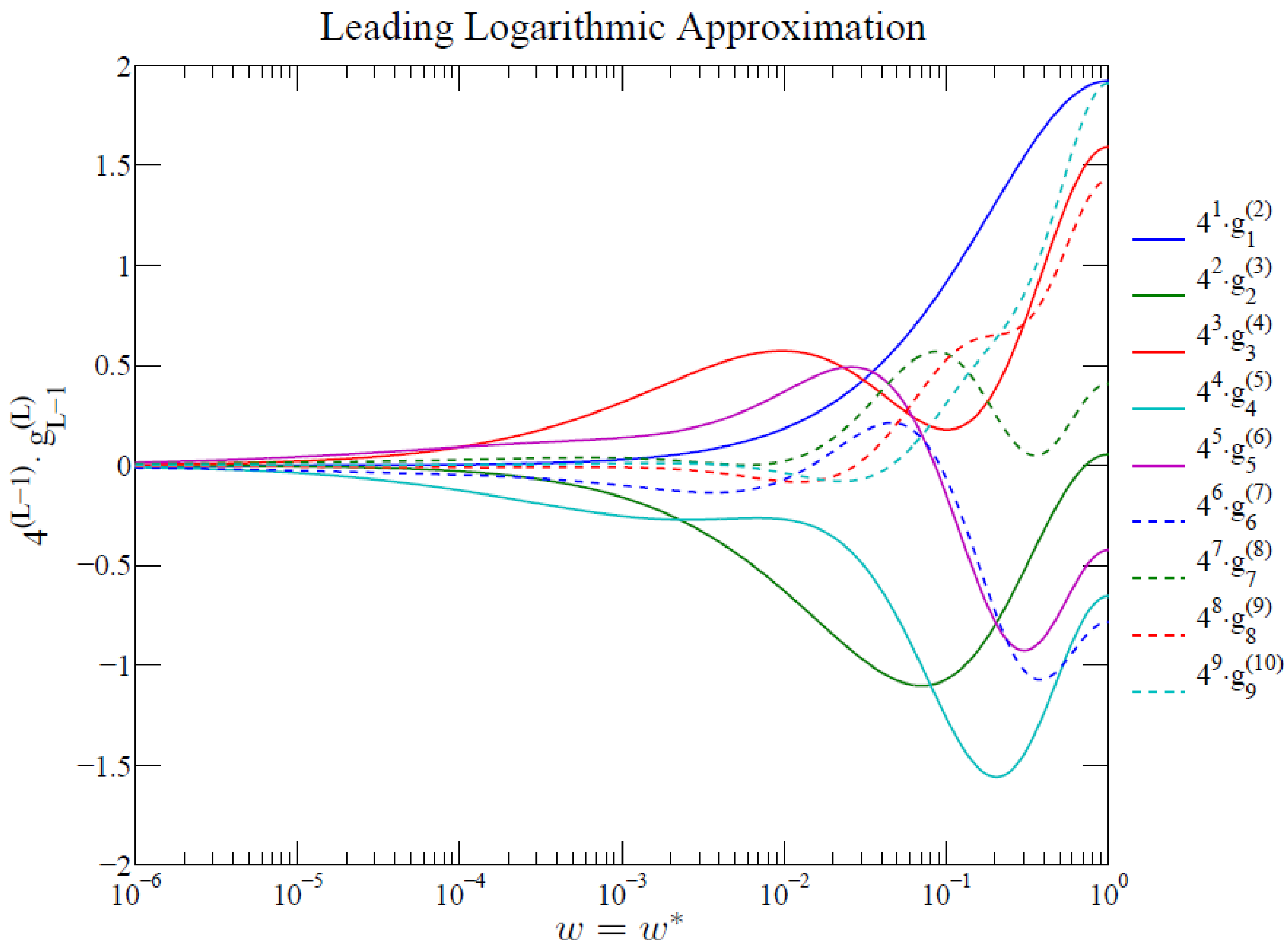
Evaluating the master formula

- We know every $g_n^{(L)}(w,w^*)$ and $h_n^{(L)}(w,w^*)$

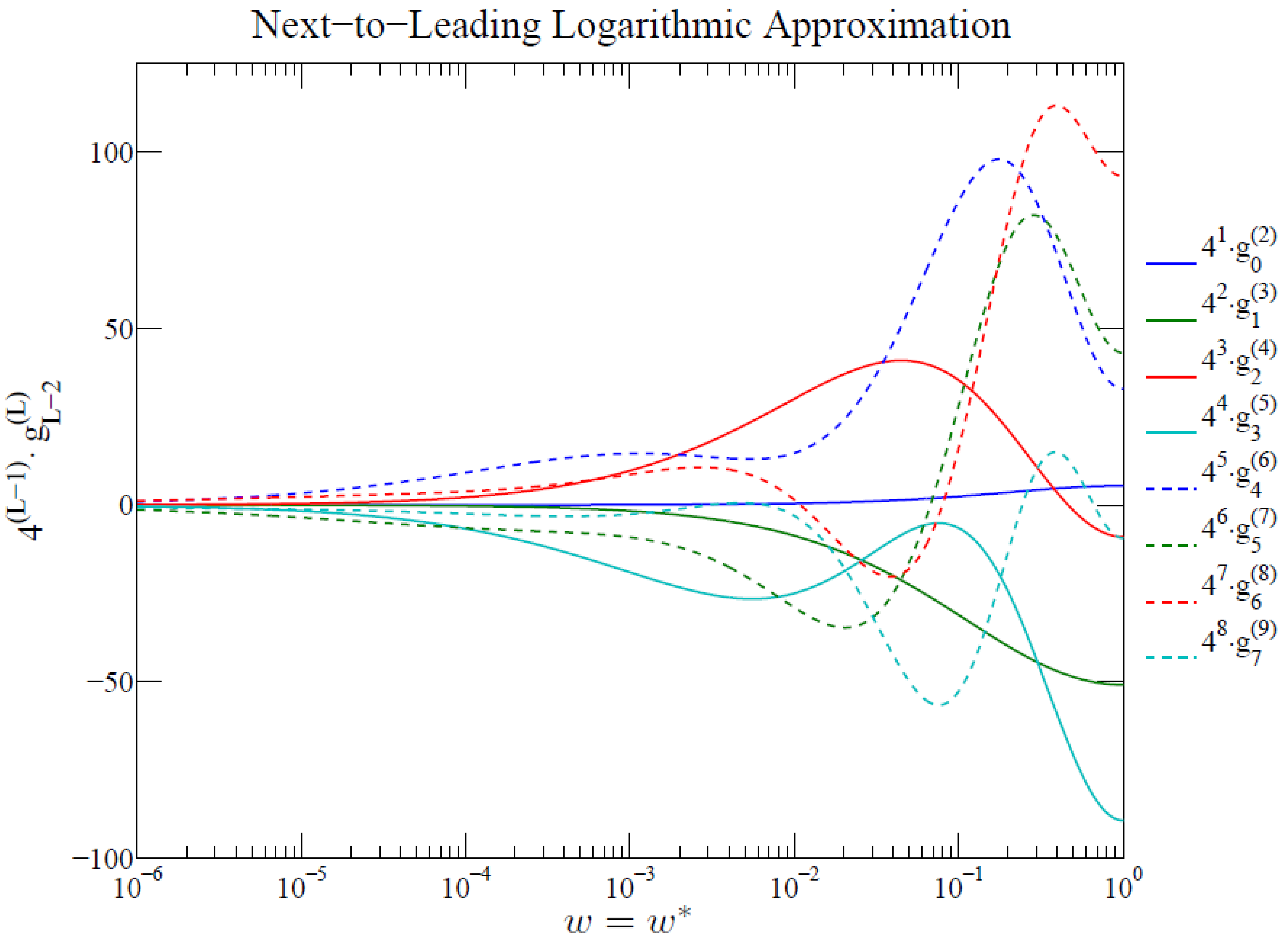
is a linear combination of a finite basis of SVHPLs.

- After evaluating the ν integral by residues, the master formula leads to a double sum.
- Truncating the double sum ←→ truncating a power series in (w,w^*) around the origin.
- Match the two series to determine the coefficients in the linear combination.
- LL and NLL data known Fadin, Lipatov 1111.0782

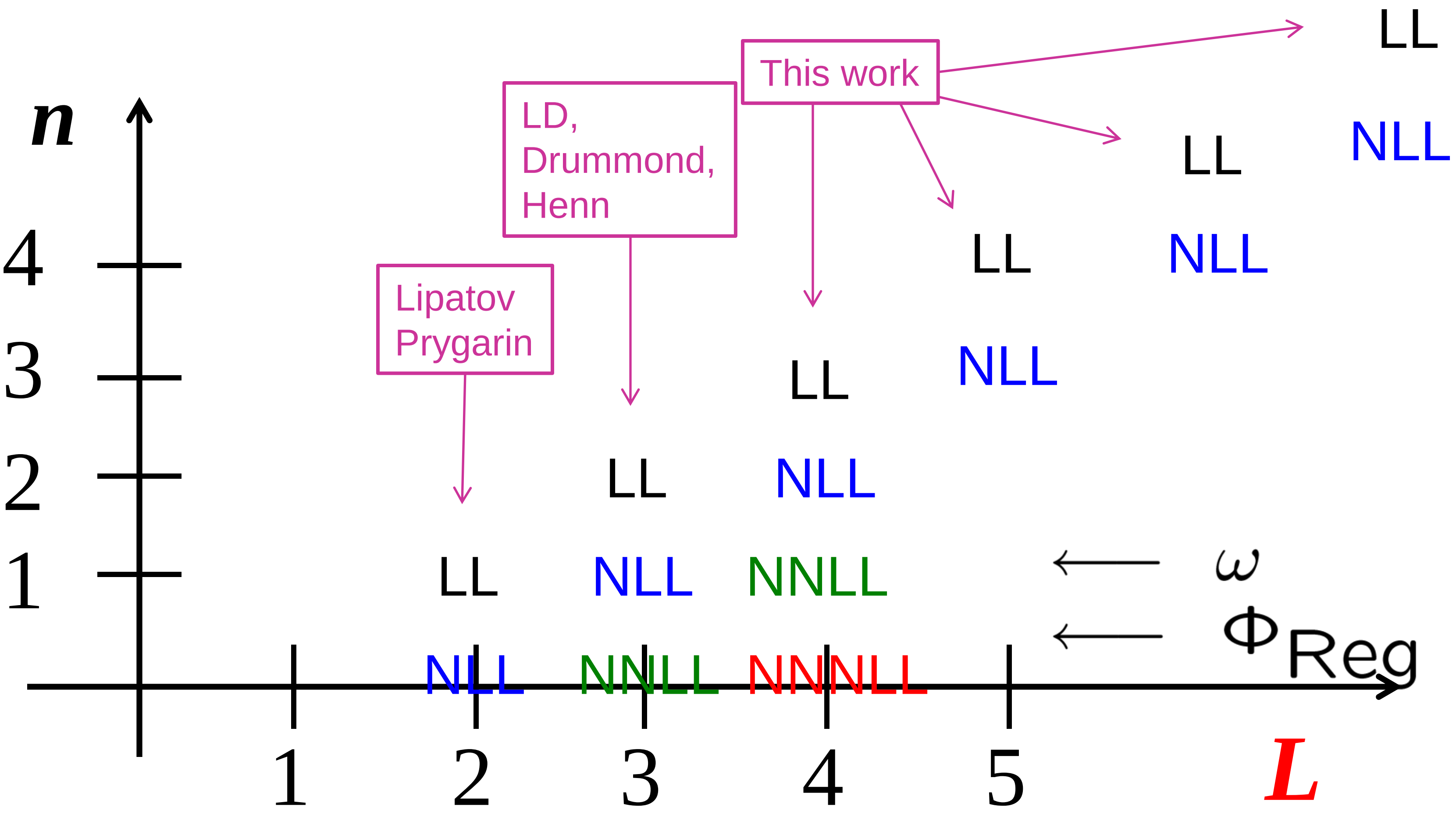
MHV LLA $g_{L\text{-}1}^{(L)}$ through 10 loops



MHV NLLA $g_{L\text{-}2}^{(L)}$ through 9 loops



LL, NLL, and beyond



(modulo some "beyond-the-symbol" constants starting at NNLL)

BFKL beyond NLL

- Using various constraints, we determined the 4-loop remainder function in MRK, up to a number of undetermined constants. In particular,

$$g_1^{(4)}(w, w^*) \qquad g_0^{(4)}(w, w^*)$$

- This let us compute the NNLL $E^{(2)}_{\nu,n}$ and the NNNLL $\Phi^{(3)}_{\nu,n}$ – once we understood the (ν,n) ←→ (w,w^*) map (a Fourier-Mellin transform) in both directions
- Only a limited set of (ν,n) functions enter: polygamma functions $\psi^{(k)}(x)$ + rational

Conclusions

- Planar N=4 SYM is a powerful laboratory for studying 4-d scattering amplitudes, thanks to dual (super)conformal invariance
- 6-gluon amplitude is first nontrivial case.
- The multi-Regge limit offers a simpler setup to try to solve first, greatly facilitated by Brown's SVHPLs
- NMHV amplitudes in MRK also naturally described by same functions   Lipatov, Prygarin, Schnitzer, 1205.0186; DDP
- Can also input fixed-order information, obtained using the "symbol", along with a key constraint from the collinear OPE, to go to NNLL, part of NNNLL
- Multi-Regge limit of 6-gluon amplitude may well be first case solved to all orders

What further secrets lurk within the hexagon?