Three-loop Matching Coefficient of the Vector Current

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in collaboration with

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ICHEP 2012, Melbourne

Outline

Motivation

The matching coefficient of the vector current comprises an important building block for

- Measurement of the top-quark mass at a future linear collider

$$\Delta M_t < 100\,\mathrm{MeV}$$

  Measurements of the top-quark mass at hadron colliders limited by systematic errors

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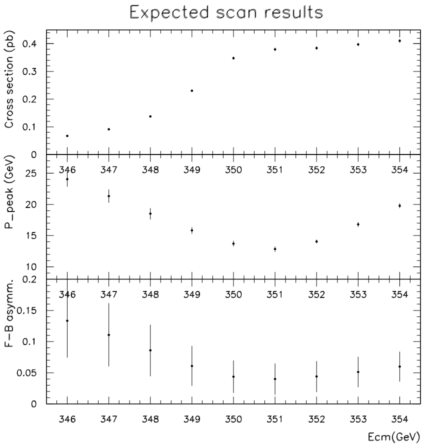
- Measurement of the bottom-quark mass from Υ sum rules

Introduction

The top-quark mass can be obtained from a scan of the top anti-top threshold at a linear collider [Martinez, Miquel '02]



$M_t = 175\,\mathrm{GeV}$

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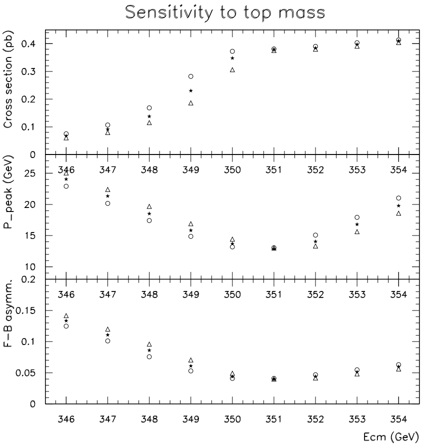

$M_t = 175\,\mathrm{GeV} \pm 200\,\mathrm{MeV}$

Threshold expansion $\leftrightarrow$ effective field theories

QCD

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$\downarrow$

integrate out hard modes

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integrate out soft/potential modes

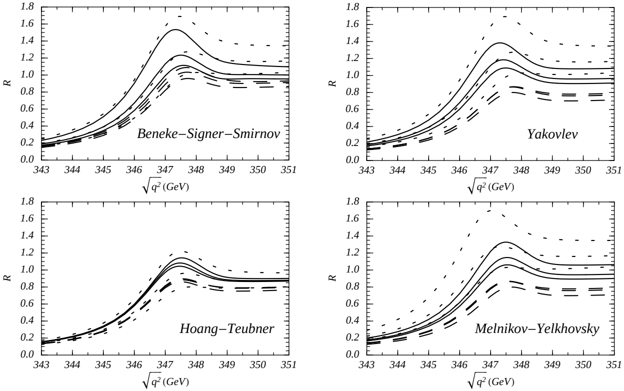
$\downarrow$

potential NRQCD (PNRQCD)

Theoretical Status

two-loop: calculation done by several groups

[Hoang,Teubner; Melnikov,Yelkhovsky; Yakovlev; Beneke, Signer, Smirnov; Nagano, Ota, Sumino; Penin, Pivovarov]



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three-loop:

- matching coefficient c_v
  - n_l ✓ [PM,Piclum,Seidel,Steinhauser]
  - n_f ✓ [PM,Piclum,Seidel,Steinhauser]
  - $n_f^0 \to$ this talk
- heavy-quark potential a_3 ✓
  [Smirnov,Smirnov,Steinhauser; Anzai,Kiyo,Sumino]
- potential contributions (✓) [Beneke,Kiyo,Schuller]
- ultrasoft contributions ✓ [Beneke,Kiyo,Penin]

Definition

QCD vector current	NRQCD vector current
$j_v^\mu = \bar{Q}\gamma^\mu Q$	$\tilde{j}_v^k = \phi^\dagger \sigma^k \chi$

$$j_v^k = c_v \tilde{j}_v^k + \mathcal{O}\left(\frac{1}{M^2}\right)$$

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c_v can be extracted by calculating vertex corrections involving j_v and $\tilde{j}_v$

$$Z_2 \Gamma_v = c_v \tilde{Z}_2 \tilde{Z}_v^{-1} \tilde{\Gamma}_v + \cdots$$

Details

full and effective theory contain the same soft, ultra-soft and potential contributions $\Rightarrow$ sufficient to calculate vertex functions at threshold

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Setup of the Calculation

- Feynman diagrams generated using `QGRAF`
- mapped onto 78 topologies using `Q2E/EXP`
- Feynman integrals reduced to master integrals with `CRUSHER`
- master integrals in different topologies have to be identified
- $\mathcal{O}(100)$ master integrals calculated analytically/numerically using various techniques
- numerical errors added in quadrature

Calculation of Master Integrals

- some simple (propagator-type) master integrals known analytically
- others can be calculated precisely using Mellin-Barnes methods
- difficult (vertex-type) integrals calculated numerically using `FIESTA` (Feynman Integral Evaluation by a Sector decomposiTion Approach) [Smirnov,Tentyukov]

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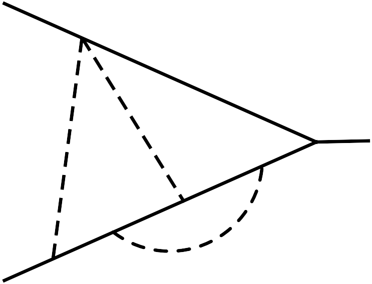

$$= \mathcal{N}\left(+ \frac{0.411236(3)}{\epsilon^2} + \frac{3.4860(1)}{\epsilon} + 34.520(2) + 339.68(4)\epsilon + \mathcal{O}(\epsilon^2) \right)$$

Checks

- Renormalization constant $\tilde{Z}_v$ of the NRQCD current can be reproduced
  - $\tilde{Z}_v$ analytically known, $1/\epsilon$ part numerically small
  - agreement within the error estimate at the percent level

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- Gauge independence: terms linear in ξ vanish after renormalization
- Change basis of master integrals and compare

Phenomenology

Residue of the QCD two-point function

$$\Pi(q^2) \xrightarrow{E \to E_n} \frac{N_c}{2m_Q} \frac{Z_n}{E_n - (E + i0)}$$

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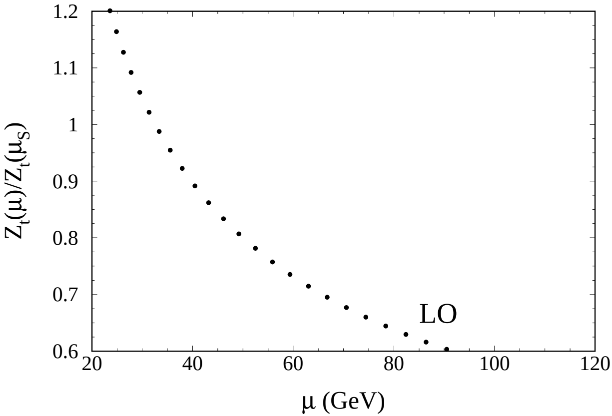

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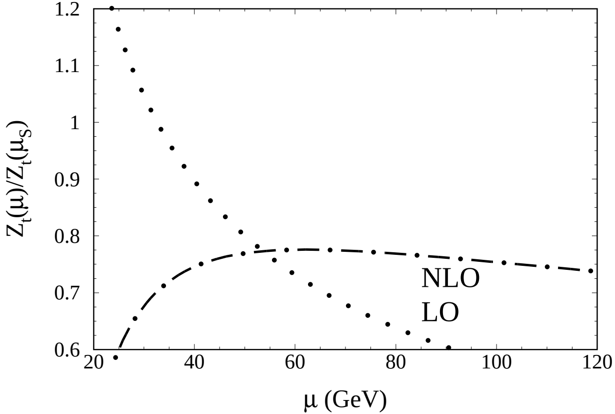

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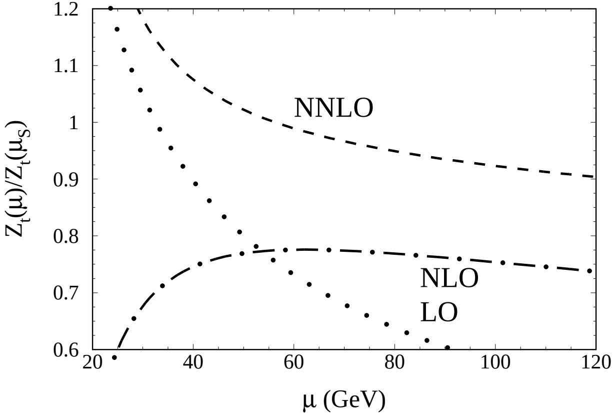

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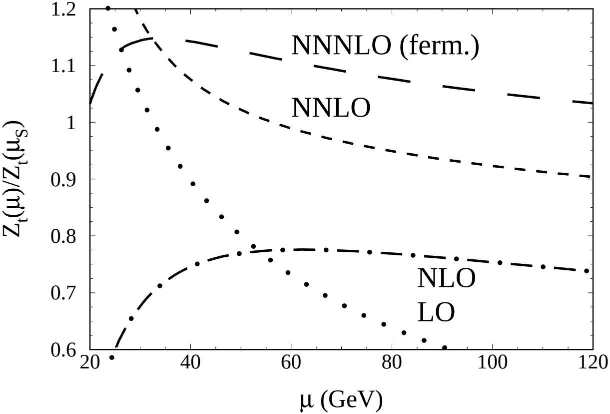

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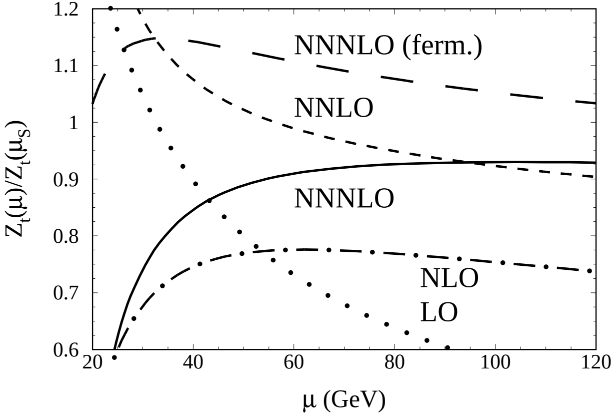

Conclusion

- Calculated the final (?) missing piece for the complete NNNLO theory prediction for $e^+e^- \to t\bar{t}$ at threshold at a future linear collider.
- Numerical result with an error in the permille range.
- Sizable corrections, but matching coefficient alone not physical
- Full analysis and final checks of other building blocks still missing.