PARAMETRIZING THE NEUTRINO SECTOR OF THE SEESAW EXTENSION IN TAU DECAYS

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1 Introduction

The Standard Model includes neutrinos as massless particles, but neutrino oscillations showed, that neutrinos are not massless. A simple extension of adding gauge singlet fermions to the particle spectrum allows normal Yukawa mass terms for neutrinos. The smallness of the neutrino masses can be well understood within the seesaw mechanism (type I). After spontaneous symmetry breaking of the Standard Model gauge group one obtains a $(n_L+n_R)\times(n_L+n_R)$ Majorana mass matrix M_ν for neutrinos. The mixing between the n_R "right-handed" singlet fermions and the neutral parts of the n_L lepton doublets gives masses for the neutrinos which are of the size expected from neutrino oscillations.

The diagonalization of the mass matrix gives rise to a split spectrum consisting of heavy and light states of neutrinos given by $U^T M_\nu U = \mathrm{diag}(m^{light}_{n_L}, m^{heavy}_{n_R})$. For the case $n_R=1$ we diagonalize M_ν with a rotation matrix determined by two angles, two masses, and Majorana phases. For the case $n_R=2$ we diagonalize the mass matrix with a unitary matrix determined by complex parameters, four masses, and Majorana phases. In both cases we take $n_L=3$.

We calculate the one-loop radiative corrections to the mass parameters which produce mass terms for the neutral leptons. In both cases we numerically analyse light neutrino masses as functions of the heavy neutrinos masses. Parameters of the model are varied to find light neutrino masses that are compatible with experimental data of solar $\Delta m^2_\odot$ and atmospheric $\Delta m^2_{\rm atm}$ neutrino oscillations for normal and inverted hierarchy.

2 Description Of The Model

2.1 The Tree Level

The mass terms for the neutrinos can be written in a compact form as a mass term with an $(n_L+n_R)\times(n_L+n_R)$ symmetric mass matrix

$$M_\nu = \begin{pmatrix} 0 & M_D^T \\ M_D & \hat{M}_R \end{pmatrix}, \tag{1}$$

where M_D is $n_L\times n_R$ Dirac neutrino mass matrix, while the hat indicates that $\hat{M}_R$ is a diagonal matrix. M_ν can be diagonalized as

$$U^T M_\nu U = \hat{m} = \mathrm{diag}\left(m_1, m_2, \ldots, m_{n_L+n_R}\right), \tag{2}$$

where the m_i are real and non-negative. In order to implement the seesaw mechanism [1, 2] we assume that the elements of M_D are of order m_D and those of M_R are of order m_R, with $m_D \ll m_R$. Then, the neutrino masses m_i with $i=1,2,\ldots,n_L$ are of order m_D^2/m_R, while those with $i=n_L+1,\ldots,n_L+n_R$ are of order m_R. For calculation of corrections it is useful to decompose the $(n_L+n_R)\times(n_L+n_R)$ unitary matrix U as $U=(U_L\ U_R^*)^{\rm T}$ [3, 4], where the submatrix U_L is $n_L\times(n_L+n_R)$ and the submatrix U_R is $n_R\times(n_L+n_R)$.

2.2 One-loop Corrections

In the standard seesaw, one-loop corrections to the mass matrix, i.e. the self energies, are determined by the neutrino interactions with the Z boson, the neutral Goldstone boson G^0, and the Higgs boson h^0 [5]. Each diagram contains a divergent piece but when summing up the three contributions the result turns out to be finite [6].

Once the one-loop corrections are taken into account the neutral fermion mass matrix is given by

$$M_\nu^{(1)} = \begin{pmatrix} \delta M_L & M_D^T+\delta M_D^T \\ M_D+\delta M_D & \hat{M}_R+\delta M_R \end{pmatrix} \approx \begin{pmatrix} \delta M_L & M_D^T \\ M_D & \hat{M}_R \end{pmatrix} \tag{3}$$

where the $0_{3\times3}$ matrix appearing at tree level (1) is replaced by the contribution δM_L. This correction is a symmetric matrix, it has the largest influence as compared to other corrections.

Neglecting the sub-dominant pieces in (3), one-loop corrections to the neutrino masses originate via the relation

$$\delta M_L = U_L^*\Sigma_L^S(p^2)U_L^\dagger = U_L^*\Sigma_L^S(0)U_L^\dagger\,, \tag{4}$$

where we evaluate the one-loop neutrino self-energy $\Sigma_L^S(p^2)$ at zero external momentum squared. After contractions of similar terms the final expression for the finite one-loop correction is given by

$$\delta M_L = \delta M_L^{(Z)} + \delta M_L^{(h^0)}, \tag{5}$$

where

$$\delta M_L^{(Z)} = \frac{3g^2}{64\pi^2 m_W^2} U_L^*\hat{m}^3\left(\frac{\hat{m}^2}{m_Z^2}-\mathbb{1}\right)^{-1}\ln\left(\frac{\hat{m}^2}{m_Z^2}\right)U_L^\dagger\,; \tag{6}$$

$$\delta M_L^{(h^0)} = \frac{g^2}{64\pi^2 m_W^2} U_L^*\hat{m}^3\left(\frac{\hat{m}^2}{m_{h^0}^2}-\mathbb{1}\right)^{-1}\ln\left(\frac{\hat{m}^2}{m_{h^0}^2}\right)U_L^\dagger\,. \tag{7}$$

3 Case $\mathrm{n_R}=1$

First we consider the minimal extension of the standard model adding only one right-handed field ν_R to three left-handed fields contained in ν_L.

We use the parametrization of $M_D = m_D\vec{a}^T$ with $|\vec{a}|=1$. Diagonalization of the symmetric mass matrix M_ν (1) in block form is:

$$U^T M_\nu U = U^T\begin{pmatrix} 0 & m_D\vec{a} \\ m_D\vec{a}^T & \hat{M}_R \end{pmatrix}U = \begin{pmatrix} \hat{M}_l & 0 \\ 0 & \hat{M}_h \end{pmatrix}. \tag{8}$$

The non zero masses in $\hat{M}_l$ and $\hat{M}_h$ are determined analytically by finding eigenvalues of the hermitian matrix $M_\nu M_\nu^\dagger$. These eigenvalues are the squares of the masses of the neutrinos $\hat{M}_l = \mathrm{diag}(0,0,m_l)$ and $\hat{M}_h = m_h$. Solutions $m_D^2 = m_h m_l$ and $m_R^2 = (m_h - m_l)^2 \sim m_h^2$ correspond to the seesaw mechanism.

Working at tree level, we can construct the diagonalization matrix U from two diagonal matrices of phases and three rotation matrices $U_{\rm tree} = \hat{U}_\phi(\phi_i)U_{12}(\alpha_1)U_{23}(\alpha_2)U_{34}(\beta)\hat{U}_i$, where the angle β is determined by the masses m_l and m_h. The values of ϕ_i and α_i can be chosen to cover variations in M_D.

Diagonalization of the mass matrix after calculation of one-loop corrections is performed with a unitary matrix $U_{\rm loop} = U_{\rm egv}\hat{U}_\varphi(\varphi_1,\varphi_2,\varphi_3)$, where $U_{\rm egv}$ is an eigenmatrix of $M_\nu^{(1)}M_\nu^{(1)\dagger}$ and $\hat{U}_\varphi$ is a phase matrix. These radiative corrections give mass to the second lightest neutrino. The third lightest neutrino remains massless.

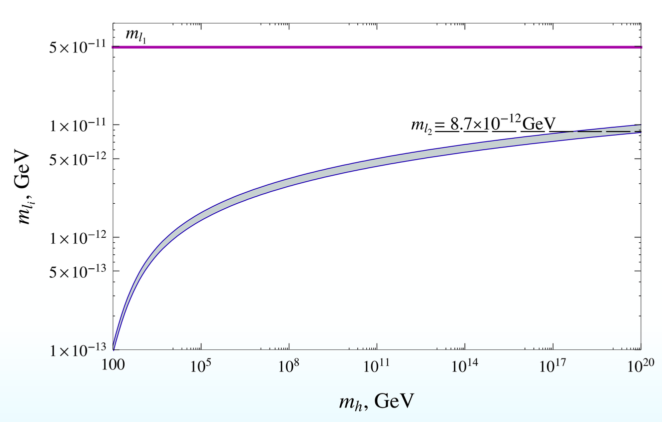

Figure 1: Calculated masses of two light neutrinos as a function of the heavy neutrino mass m_h. The mass of the third light neutrino is zero, when $n_R=1$. Solid lines show the boundaries of allowed neutrino mass ranges when the model parameters are constrained by the experimental data on neutrino oscillations. Due to the scale, the band of the allowed m_{l_1} values appears as a line. The dashed line indicates the estimated experimental mass of the second lightest neutrino for the case $n_R=1$.

It is possible to estimate masses of the light neutrinos from experimental data of solar and atmospheric neutrino oscillations ($\Delta m^2_\odot = 7.59\times10^{-23}\,\mathrm{GeV}^2$, $\Delta m^2_{\rm atm} = 2.43\times10^{-21}\,\mathrm{GeV}^2$) assuming that the lightest $m_{l_3}=0$ and considering the normal ordering of the light neutrinos:

$$\begin{aligned} m_{l_1} &= 5.0\times10^{-11}\,\mathrm{GeV}, \\ m_{l_2} &= 8.7\times10^{-12}\,\mathrm{GeV}. \end{aligned} \tag{9}$$

However the numerical analysis shows that we can reach those values only for a heavy singlet with the mass near to the Plank scale, see Fig. 1.

4 Case $\mathrm{n_R}=2$

If we add two singlet fields ν_R to three left-handed fields ν_L, the radiative corrections give masses to all three light neutrinos.

Now we parametrize

$$M_D = \begin{pmatrix} m_{D_2}\vec{a}^T \\ m_{D_1}\vec{b}^T \end{pmatrix} \tag{10}$$

with $|\vec{a}|=1$ and $|\vec{b}|=1$. Diagonalizing the symmetric mass matrix M_ν (1) in block form we write:

$$U^T M_\nu U = U^T\begin{pmatrix} 0_{3\times3} & m_{D_2}\vec{a}\ \ m_{D_1}\vec{b} \\ \begin{matrix} m_{D_2}\vec{a}^T \\ m_{D_1}\vec{b}^T \end{matrix} & \hat{M}_R \end{pmatrix}U = \begin{pmatrix} \hat{M}_l & 0 \\ 0 & \hat{M}_h \end{pmatrix}. \tag{11}$$

The non zero masses in $\hat{M}_l$ and $\hat{M}_h$ are determined by the seesaw mechanism: $m_{D_i}^2 \approx m_{h_i}m_{l_i}$ and $m_{R_i}^2 \approx m_{h_i}^2$, $i=1,2$. Here we use $m_1>m_2>m_3$ ordering of masses. The third lightest neutrino is massless at tree level.

The diagonalization matrix for tree level $U_{\rm tree} = U_{12}(\alpha_1,\alpha_2)U_{\rm egv}(\beta_i)\hat{U}_\phi(\phi_i)$ is composed of a rotation matrix, an eigenmatrix of $M_\nu M_\nu^\dagger$ and a diagonal phase matrix, respectively. Diagonalization of the mass matrix including the one-loop correction is performed with a unitary matrix $U_{\rm loop} = U_{\rm egv}\hat{U}_\varphi(\varphi_i)$, where $U_{\rm egv}$ is the eigenmatrix of $M_\nu^{(1)}M_\nu^{(1)\dagger}$ and $\hat{U}_\varphi$ is a phase matrix.

In numerical calculations the model parameters as well as the derived masses of the light neutrinos are obtained in several steps. First, the diagonal mass matrix for tree level is constructed. The lightest neutrino is massless, and the masses of other two light neutrinos are estimated from experimental data on solar and atmospheric neutrino oscillations. The masses of the heavy neutrinos are input parameters. This diagonal matrix is used to constrain the parameters α_i and ϕ_i that enter the tree-level mass matrix M_ν and its diagonalization matrix $U_{\rm tree}$. Then the diagonalization matrix is used to evaluate one-loop corrections to the mass matrix. Diagonalization of the corrected mass matrix yields masses for three light neutrinos. If the calculated mass difference is compatible with the experimental neutrino mass difference, the parameter set is kept. Otherwise, another set of the parameters is generated. Figures 2-6 illustrate the obtained results. Both normal and inverted neutrino mass orderings are considered.

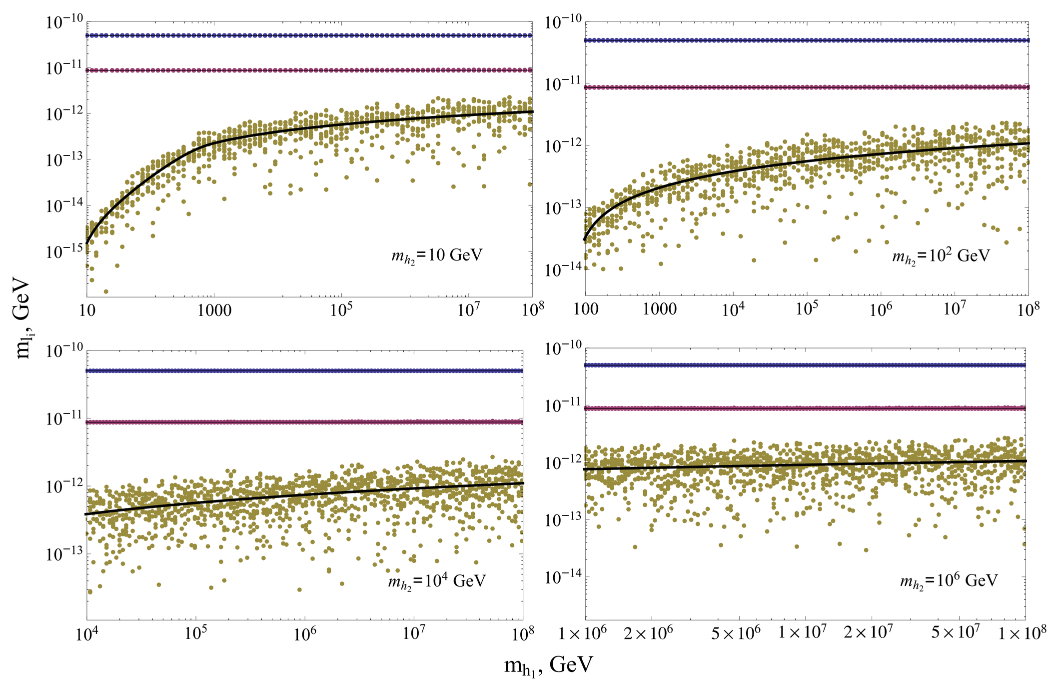

Figure 2: The masses m_{l_i} of the light neutrinos as functions of the heaviest right-handed neutrino mass m_{h_1}, for the case $n_R=2$. Normal hierarchy of the light neutrinos is assumed. The value of m_{h_2} is shown in the plots. The black solid lines indicate the mean values of the scatter data.

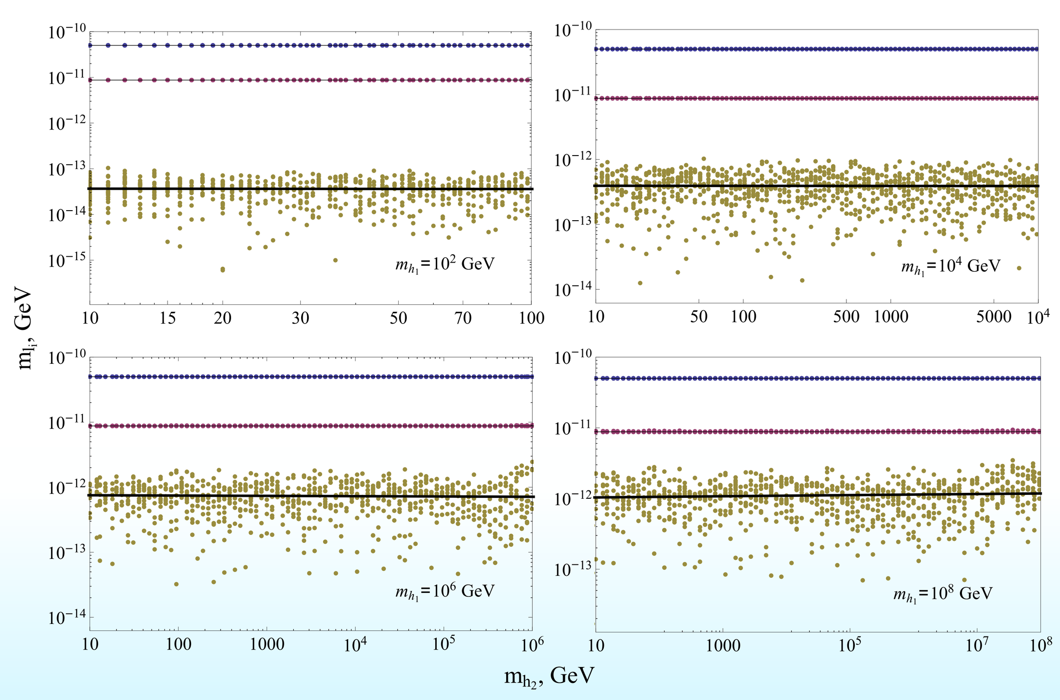

Figure 3: Same as Fig. 2, but the masses m_{l_i} of the light neutrinos are shown as functions of the lightest right-handed neutrino mass m_{h_2}. The value of m_{h_1} is shown in the plots. The black solid lines indicate the mean values of scatter data.

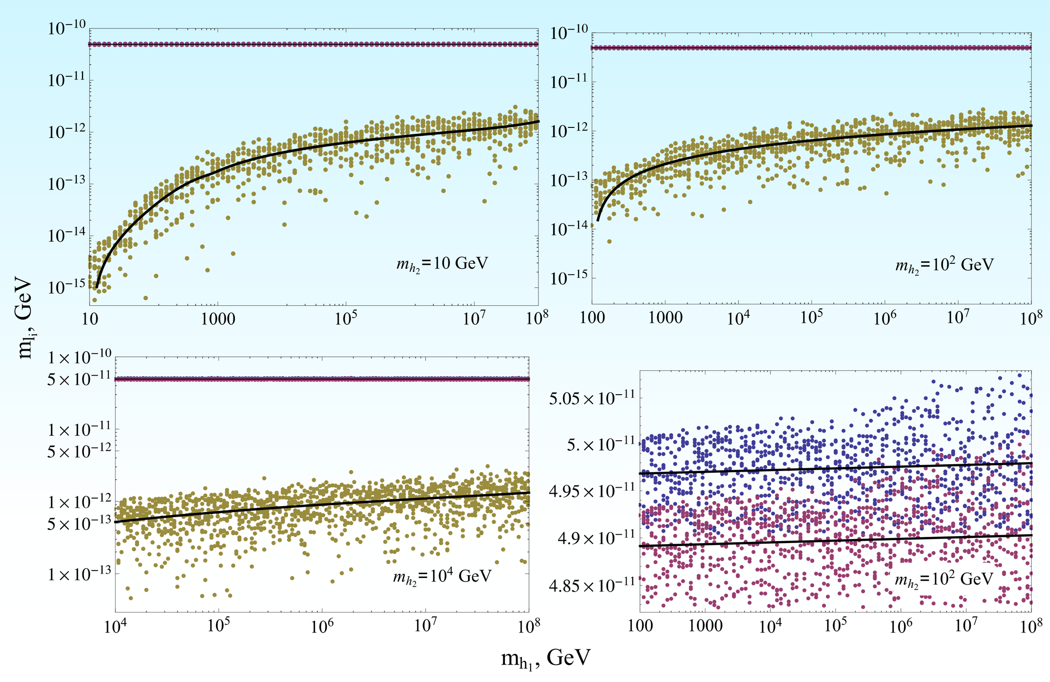

Figure 4: The masses m_{l_i} of the light neutrinos as functions of the heaviest right-handed neutrino mass m_{h_1}, for the case $n_R=2$. Inverted hierarchy of light neutrinos is assumed. The value of m_{h_2} is shown in the plots. The black solid lines indicate the mean values of the scatter data. Due to the scale, the masses m_{l_1} and m_{l_2} are very close to each other and look like one line. Their values are shown separately in the lower right plot for $m_{h_2}=10^2$.

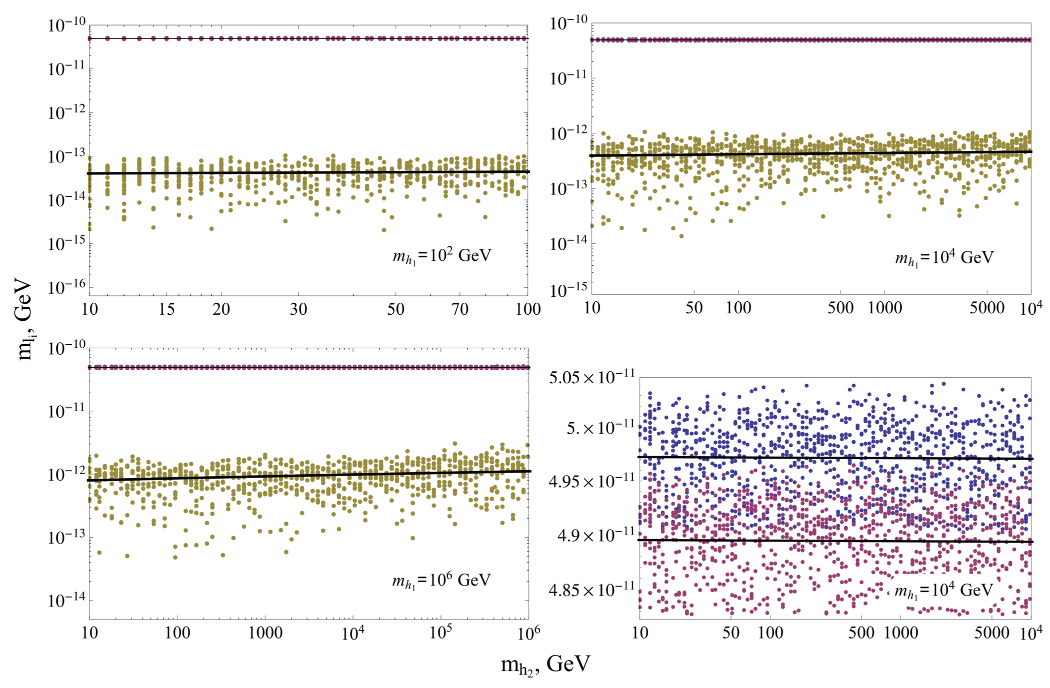

Figure 5: Same as Fig. 4, but the masses m_{l_i} of the light neutrinos are shown as functions of the lightest right-handed neutrino mass m_{h_2}. The value of m_{h_1} is shown in the plots. The black solid lines indicate the mean values of scatter data. The nearly-degenerate masses of m_{l_1} and m_{l_2} are in the lower right plot for $m_{h_1}=10^4$.

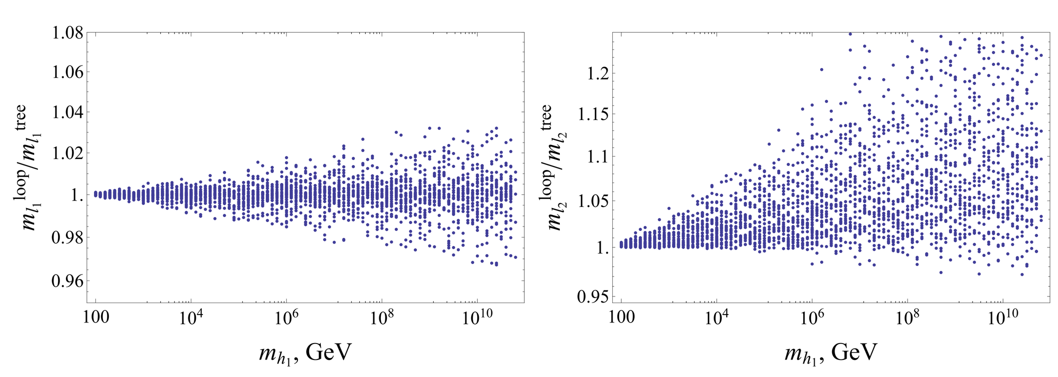

Figure 6: The ratio between the one-loop and the tree-level result for the masses of the light neutrinos ν_{L_i} $(i=1,2)$ with $m^{\rm loop} = m^{\rm tree} + \delta m^{\rm loop}$ as a function of the heaviest right-handed neutrino mass m_{h_1}, with $m_{h_2} = 100$ GeV fixed. Normal hierarchy of the light neutrino masses is assumed.

5 Conclusions

1. For the case $n_R=1$ we can match the differences of the calculated light neutrino masses to $\Delta m^2_\odot$ and $\Delta m^2_{\rm atm}$ only for a mass of the heavy singlet of order 10^{17} GeV. Only normal ordering of neutrino masses is possible.
2. In the case $n_R=2$ we obtain three non vanishing masses of light neutrinos for normal and inverted hierarchies. The numerical analysis shows that the values of light neutrino masses (especially of the lightest mass) depend on the choice of the heavy neutrinos masses. The radiative corrections generate the lightest neutrino mass and have a big impact on the second lightest neutrino mass.
3. In future we plan to apply our parametrization to study the τ polarization coming from the decay of a W boson in the data of the CMS experiment at LHC and thus determine restrictions to the parameters of the neutrino sector.

6 References

[1] M. Gell-Mann, P. Ramond, and R. Slansky, in *Supergravity, Proceedings of the Workshop, Stony Brook, New York, 1979*, edited by F. van Nieuwenhuizen and D. Freedman (North Holland, Amsterdam, 1979).

[2] J. Schechter and J. W. F. Valle, Phys. Rev. D 22 (1980) 2227.

[3] W. Grimus and H. Neufeld, Nucl. Phys. B 325 (1989) 18.

[4] W. Grimus and L. Lavoura, Phys. Rev. D 66 (2002) 014016.

[5] W. Grimus and L. Lavoura, Phys.Lett. B 546 (2002) 86.

[6] D. Aristizabal Sierra and C. E. Yaguna, JHEP 1108 (2011) 013.

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