IR-Improved Operator Product Expansions in non-Abelian Gauge Theories

B.F.L. Ward
Baylor University

OUTLINE:
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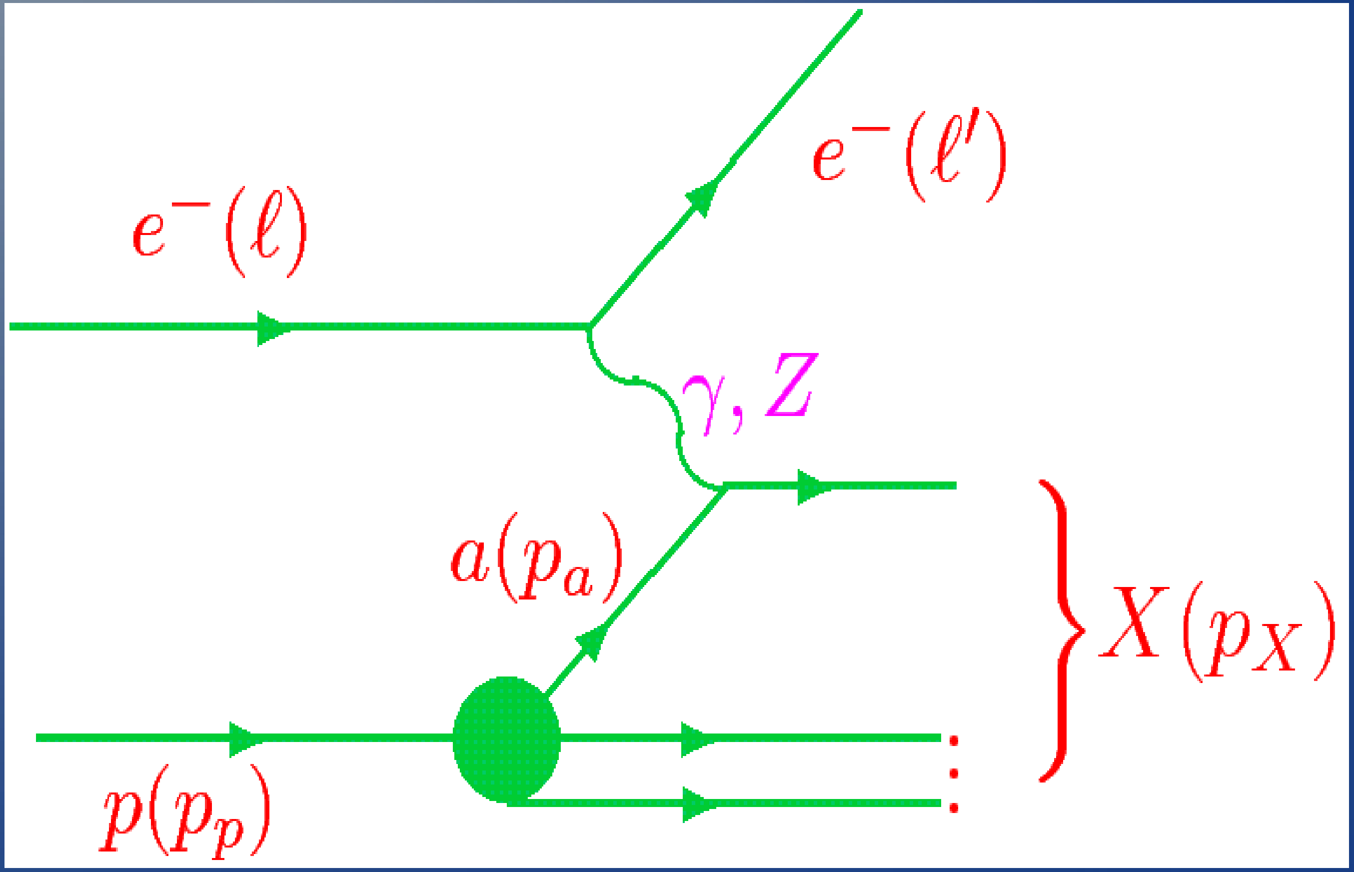
Introduction

- PRECISION QCD FOR LHC
- $\Delta\sigma = \Delta\sigma^{expt} \oplus \Delta\sigma^{th} \leqslant 1\%$
- NEED $\Delta\sigma^{th} \leqslant 0.67\%$
- NEW ERA: QCD X QED
- EXACT AMPLITUDE RESUMMATION THEORY
- RESUM LARGE IR EFFECTS EXACTLY
- REVISIT OPE IN THIS CONTEXT

Review of OPE

- We use Bjorken's Problem here:



- DIS on the proton: x, Q^2, $\nu = qp_p/m_p$ as usual.

Imaginary Part of Forward Compton Amplitude

$$W_{\alpha\beta}^{EM}(p_p,q)=\frac{1}{2\pi}\int d^4y\, e^{iqy}\langle p|[J_\beta^{EM}(y),J_\alpha^{EM}(0)]|p\rangle$$

$$=(-g_{\alpha\beta}+q_\alpha q_\beta/q^2)W_1(\nu,q^2)$$

$$+\frac{1}{m_p^2}(p_p-qqp_p/q^2)_\alpha(p_p-qqp_p/q^2)_\beta W_2(\nu,q^2),$$

$$J_\alpha^{EM}(y)\equiv J_\alpha(y)\Leftrightarrow hadronic\ \ electromagnetic\ \ current,$$

$$W_{1,2}\Leftrightarrow usual\ \ deep\ \ inelastic\ \ structure\ \ functions$$

$$Bjorken\ \ scaling\ \ by\ \ SLAC-MIT\, expts:\ x=\frac{Q^2}{2m_p\nu},\ \lim_{B_j}\ \equiv\ \lim_{Q^2\to\infty}\ \ |_{\text{x fixed}}$$

$$Q^2\simeq 1_+\ GeV^2,\ precocious\ \ scaling-$$

$$\lim_{Bj}m_pW_1(\nu,q^2)=F_1(x),\ \ \lim_{Bj}\nu W_2(\nu,q^2)=F_2(x)$$

Rapidly Oscillating Phase⇔Short Distance⇔OPE

- In $\lim_{Bj}$ (Gross, Wilczek, Politzer,....)

$$J_\beta(y) J_\alpha(0) = \frac{1}{2} g_{\beta\alpha} \left(\frac{\partial}{\partial y}\right)^2 \frac{1}{y^2 - i\epsilon y_0} \sum_{n=0}^{\infty} \sum_j C_{j,1}^{(n)}(y^2 - i\epsilon y_0) O^j_{\mu_1 \cdots \mu_n}(0) y^{\mu_1} \cdots y^{\mu_n}$$

$$+ \frac{1}{y^2 - i\epsilon y_0} \sum_{n=0}^{\infty} \sum_j C_{j,2}^{(n)}(y^2 - i\epsilon y_0) O^j_{\beta\alpha\mu_1 \cdots \mu_n}(0) y^{\mu_1} \cdots y^{\mu_n} + \cdots,$$

- Here, ε ↓ 0, $O^j_{\mu1\ldots\mu n}(y)$, twist = dim - spin =2,
- $C^{(n)}_{j,k}$ dimensionless.

Standard Methods

- $$\langle p | O^{j}_{\mu_1\ldots\mu_n}(0) | p \rangle |_{\text{spin averaged}} = i^n \frac{1}{m_p} p_{p\mu_1} \ldots p_{p\mu_n} M^n{}_j + \cdots$$

$$\Rightarrow$$

$$\int_0^1 dx\, x^n F_1(x, q^2) = \sum_j \bar{C}^{(n+1)}_{j,1}(q^2) M_j^{n+1},$$

$$\int_0^1 dx\, x^n F_2(x, q^2) = \sum_j \bar{C}^{(n)}_{j,2}(q^2) M_j^{n+2}, \quad \text{where}$$

$$\bar{C}^{(n)}_{j,k}(q^2) = \frac{1}{2} i (q^2)^{n+1} \left(\frac{-\partial}{\partial q^2} \right)^n \int d^4 y\, e^{iqy} \frac{C^{(n)}_{j,k}(y^2)}{y^2 - i \epsilon y_0}, \quad \text{and}$$

$$\left[\left(\mu \frac{\partial}{\partial \mu} + \beta(g) \frac{\partial}{\partial g} \right) \delta_{ij} - \gamma^{(n)}_{ij}(g) \right] C^{(n)}_{j,k} = 0$$

- (the Callan-Symanzik eqn.)

Usual definitions:

$$\beta(g) = \mu \frac{\partial g}{\partial \mu},$$

$$\gamma_{ij}^{(n)}(g) = \left(Z_O^{-1} \mu \frac{\partial}{\partial \mu} Z_O \right)_{ij} \Big|_{g_0, \text{ regularization fixed}},$$

$$O_i^{(n)} \equiv O_{i,R}^{(n)} = \sum_j O_{j,\text{bare}}^{(n)} \left(Z_O^{-1} \right)_{ji}$$

Asymptotic Bjorken limit $\Leftrightarrow$ Operators with the smallest eigenvalue for anomalous dimension matrix $\gamma_{ij}^{(n)}$

IR-Improved OPE

- To get $\gamma^{(n)}_{ij}$, we study DIS from partons:

$$W^F_{\alpha\beta}(p_F,q)=\frac{1}{2\pi}\int d^4y\, e^{iqy}\langle p_F|[J_\beta(y),J_\alpha(0)]|p_F\rangle$$

$$=(2\pi)^3\sum_X \delta(q+p_F-p_X)\langle p_F|J_\beta(0)|p_X\rangle\langle p_X|J_\alpha(0)|p_F\rangle$$

- We apply exact, amplitude-based resummation theory

$$d\hat{\sigma}_{\exp}=e^{\mathrm{SUM}_{IR}(QCED)}\sum_{n,m=0}^{\infty}\frac{1}{n!m!}\int\frac{d^3p_2}{p_2^0}\frac{d^3q_2}{q_2^0}\prod_{j_1=1}^{n}\frac{d^3k_{j_1}}{k_{j_1}}\prod_{j_2=1}^{m}\frac{d^3k'_{j_2}}{k'_{j_2}}$$

$$\times\int\frac{d^4y}{(2\pi)^4}e^{iy\cdot(p_1+q_1-p_2-q_2-\sum k_{j_1}-\sum k'_{j_2})+D_{QCED}}\;\tilde{\bar{\beta}}_{n,m}(k_1,\cdots,k_n;k'_1,\cdots,k'_m)$$

Observations:

- EXACT – Compare:

  Sterman-Catani-Trentadue Threshold Resummation

  As for any f(z),

$$\left|\int_0^1 dz\, z^{n-1} f(z)\right| \le \left(\frac{1}{n}\right) max\left|f(z)\right|,$$

  drop non-singular contributions to cross section at z→1

- SCET:

  drop O(λ) terms, λ = √(Λ/Q),

  Λ∿ .3 GeV, Q∿100 GeV ⇒ λ≃5.5%

- These methods give approximations to our $\bar{\beta}_{n,m}$

To apply the "exact" theory to the OPE, observe

$$\langle p_X | J_\alpha(0) | p_F \rangle = e^{\alpha_s B_{\mathrm{QCD}}} \langle p_X | J_\alpha(0) | p_F \rangle_{\text{IRI-virt}} \quad \Rightarrow$$

$$W^F_{\alpha\beta}(p_F, q) = (2\pi)^3 \sum_X \delta(q + p_F - p_X) e^{2\alpha_s \Re B_{\mathrm{QCD}}} \ {}_{\text{IRI-virt}}\langle p_F | J_\beta(0) | p_X \rangle \langle p_X | J_\alpha(0) | p_F \rangle_{\text{IRI-virt}} \quad \Rightarrow$$

$$_{\text{IRI-virt}}\langle p_F | J_\beta(0) | p_X \rangle \langle p_X | J_\alpha(0) | p_F \rangle_{\text{IRI-virt}}$$

$$= \tilde{S}_{\mathrm{QCD}}(k_1) \cdots \tilde{S}_{\mathrm{QCD}}(k_n) \ {}_{\text{IRI-virt}}\langle p_F | J_\beta(0) | p_{X'} \rangle \langle p_{X'} | J_\alpha(0) | p_F \rangle_{\text{IRI-virt}} + \cdots +$$

$$_{\text{IRI-virt\&real}}\langle p_F | J_\beta(0) | p_{X'}, k_1, \cdots, k_n \rangle \langle p_{X'}, k_1, \cdots, k_n | J_\alpha(0) | p_F \rangle_{\text{IRI-virt\&real}}$$

$$\Rightarrow$$

The result

$$W^F_{\beta\alpha}(p_F,q)=(2\pi)^3\sum_X \delta(q+p_F-p_X)e^{2\alpha_s \Re B_{QCD}}\ \Big[\tilde{S}_{QCD}(k_1)\cdots\tilde{S}_{QCD}(k_n)$$

$$_{\text{IRI-virt}}\langle p_F|J_\beta(0)|p_{X'}\rangle\langle p_{X'}|J_\alpha(0)|p_F\rangle_{\text{IRI-virt}}+\cdots$$

$$+\ _{\text{IRI-virt\&real}}\langle p_F|J_\beta(0)|p_{X'},k_1,\cdots,k_n\rangle\langle p_{X'},k_1,\cdots,k_n|J_\alpha(0)|p_F\rangle_{\text{IRI-virt\&real}}\Big]$$

$$=\frac{1}{2\pi}\int d^4y\sum_{X'}\sum_n\frac{1}{n!}\int\prod_{j=1}^n\frac{d^3k_j}{k_j^0}e^{\text{SUM}_{\text{IR}}(\text{QCD})}e^{iy(q+p_F-p_{X'}-\sum_j k_j)+D_{\text{QCD}}}$$

$$_{\text{IRI-virt\&real}}\langle p_F|J_\beta(0)|p_{X'},k_1,\cdots,k_n\rangle\langle p_{X'},k_1,\cdots,k_n|J_\alpha(0)|p_F\rangle_{\text{IRI-virt\&real}}$$

$$=\frac{1}{2\pi}\int d^4y\,e^{iqy}e^{\text{SUM}_{\text{IR}}(\text{QCD})+D_{\text{QCD}}}\ _{\text{IRI-virt\&real}}\langle p_F|[J_\beta(y),J_\alpha(0)]|p_F\rangle_{\text{IRI-virt\&real}},$$

$$\text{SUM}_{\text{IR}}(\text{QCD})=2\alpha_s\Re B_{\text{QCD}}+2\alpha_s\tilde{B}_{\text{QCD}}(\text{Kmax}),\qquad 2\alpha_s\tilde{B}_{\text{QCD}}(\text{Kmax})=\int^{\leq \text{Kmax}}\frac{d^3k}{k^0}\tilde{S}_{\text{QCD}}(k),$$

$$D_{\text{QCD}}=\frac{\int d^3k}{k}\tilde{S}_{\text{QCD}}(k)\left[e^{-iy\cdot k}-\theta(\text{Kmax}-k)\right]$$

We use

$$W_{\beta\alpha} = \sum_a \int_0^1 \frac{dx}{x} F_a(x) W^a_{\beta\alpha}$$

to get

$$\int_0^1 dx\, x^n F_1(x, q^2) = \sum_j \tilde{\tilde{C}}^{(n+1)}_{j,1}(q^2)\, \tilde{M}^{n+1}_j,$$

$$\int_0^1 dx\, x^n F_2(x, q^2) = \sum_j \tilde{\tilde{C}}^{(n)}_{j,2}(q^2)\, \tilde{M}^{n+2}_j,$$

where

$$\tilde{\tilde{C}}^{(n)}_{j,k}(q^2) = \frac{1}{2} i (q^2)^{n+1} \left(\frac{-\partial}{\partial q^2} \right)^n \int d^4 y\, e^{iqy + \mathrm{SUM}_{\mathrm{IR}}(\mathrm{QCD}) + D_{\mathrm{QCD}}} \frac{\tilde{C}^{(n)}_{j,k}(y^2)}{y^2 - i\epsilon y_0}$$

and

$$\langle p | \tilde{O}^j_{\mu_1 \cdots \mu_n}(0) | p \rangle |_{\text{spin averaged}} \equiv$$

$$_{\text{IRI-virt\&real}}\langle p | O^j_{\mu_1 \cdots \mu_n}(0) | p \rangle_{\text{IRI-virt\&real}} |_{\text{spin averaged}} = i^n \frac{1}{m_p} p_{p\mu_1} \cdots p_{p\mu_n} \tilde{M}^n_j + \cdots$$

Still have Callan-Symanzik Eqn:

$$\left[\left(\mu\frac{\partial}{\partial\mu}+\beta(g)\frac{\partial}{\partial g}\right)\delta_{ij}-\tilde{\gamma}_{ij}^{(n)}(g)\right]\tilde{\tilde{C}}_{j,k}^{(n)}=0$$

for new matrix $\tilde{\gamma}_{ij}^{(n)}(g)$

We follow Curci, Furmanski and Petronzio(NPB175(1980)27):

For the NS operator $\;{}^{N}O^{F,b}(y)=\frac{1}{2}i^{N-1}S\,\bar{\psi}(y)\gamma_{\mu_1}\nabla_{\mu_2}\cdots\nabla_{\mu_N}\lambda^{b}\psi(y)-\text{trace terms},$

where $\nabla_{\mu}=\partial_{\mu}+ig\tau^{a}A_{\mu}^{a},$ S denotes symmetrization

$$\langle p|\;{}^{N}O^{F,b}(y)|p\rangle=\;{}^{F,b}O^{N}(\alpha_s,\epsilon)p_{\mu_1}\cdots p_{\mu_N}-\text{trace terms},\qquad {}^{F,b}O^{N}(\alpha_s,\epsilon)\equiv M_{F,b}^{N}\Rightarrow$$

$${}^{F,b}O^{N}(\alpha_s,\epsilon,p^2/\mu^2)=Z_O^{-1}(\alpha_s,\frac{1}{\epsilon})\;{}^{F,b}O_{\text{bare}}^{N}((\alpha_s)(\mu^2/p^2)^{\epsilon},\epsilon)$$

Regulate collinear div. with $p^2 \neq 0$, $d=4-\varepsilon \Rightarrow$

- Use Δ, with $\Delta^2 = 0$,

$$ {}^{F,b}O^{N}(\alpha_s,\epsilon) = \langle p | \, {}^{N}O^{F,b}_{\mu_1\cdots\mu_N}(y) | p \rangle \Delta^{\mu_1}\cdots\Delta^{\mu_N} / (\Delta p)^N $$

Set $\Delta = n$, $x_{Bj} = np/np_p$, $x = nk/np$, $nA^a = 0 \Rightarrow$

$$ {}^{F,b}O^{N}(\alpha_s,\epsilon) = \int_{-1}^{1} dx\, x^{N-1}\ {}^{F,b}O(x,\alpha_s,\epsilon) $$

where

$$ {}^{F,b}O(x,\alpha_s,\epsilon) = Z_F[\delta(x-1) + x\frac{\int d^d k}{(2\pi)^d}\delta(x - \frac{kn}{pn})\left[\frac{\mathbf{n}}{4\mathrm{kn}}T(p,k)\,\mathbf{p}\right] $$

$T(p,k)$ = fully connected 4-pt fn., Z_F = field renorm., and

[**b** B denotes $b_{\alpha\alpha'} B^{\alpha\alpha'}{}_{\beta\beta'}$, etc.

Analytic continuation to d=4+ε, ε>0, $p^2 \to 0$ gives (CFP)

$$Z_O^{-1}(\alpha_s,\frac{1}{\epsilon})=\int_{-1}^{1}dx\,x^{N-1}\left[\Gamma_{qq}(x,\alpha_s,\frac{1}{\epsilon})\theta(x)-\Gamma_{q\bar{q}}(-x,\alpha_s,\frac{1}{\epsilon})\theta(-x)\right]$$

$\Gamma_{qq}(\Gamma_{q\bar{q}}) \Leftrightarrow$ respective parton density for a quark(anti-quark) in a quark $\Rightarrow$ coefficients of $\frac{1}{\epsilon}$ give

$$-\gamma^{(N)}(\alpha_s)=2\int_{-1}^{1}dx\,x^{N-1}\left[P_{qq}(x,\alpha_s)\theta(x)-P_{q\bar{q}}(-x,\alpha_s)\theta(-x)\right]$$

$$=2\left[P_{qq}(N,\alpha_s)+(-1)^N P_{q\bar{q}}(N,\alpha_s)\right],$$

$F(N)=\int_0^1 dx\,x^{N-1}F(x)$, and

$P_{BA} \Leftrightarrow$ usual DGLAP-CS kernels in CFP convention,

with $\gamma^{(N)}(\alpha_s) \Leftrightarrow$ anomalous dimension of ${}^{N}O^{F,b}$

Appliication to new IR-improved anomalous dimension matrix:

We IR-improve each step –

$$\langle p|\ {}^{N}O^{F,b}(y)|p\rangle \Rightarrow \langle p|\ {}^{N}\tilde{O}^{F,b}(y)|p\rangle$$

and $${}^{F,b}O^{N}(\alpha_s,\epsilon) \Rightarrow {}^{F,b}\tilde{O}^{N}(\alpha_s,\epsilon) \ \Rightarrow$$

$${}^{F,b}\tilde{O}^{N}(\alpha_s,\epsilon,p^2/\mu^2) = Z_{\tilde{O}}^{-1}(\alpha_s,\frac{1}{\epsilon})\ {}^{F,b}\tilde{O}^{N}_{bare}((\alpha_s)(\mu^2/p^2)^{\epsilon},\epsilon) \Rightarrow$$

$${}^{F,b}\tilde{O}^{N}(\alpha_s,\epsilon) = \int_{-1}^{1} dx\, x^{N-1}\ {}^{F,b}\tilde{O}(x,\alpha_s,\epsilon) \quad \text{where}$$

$${}^{F,b}\tilde{O}(x,\alpha_s,\epsilon) = Z_F[\delta(x-1) + x\frac{\int d^d k}{(2\pi)^d}\delta(x-\frac{kn}{pn})\left[\frac{\boldsymbol{n}}{4\mathrm{kn}}\tilde{T}(p,k)\boldsymbol{p}\right]$$

$\tilde{T}(p,k)$ is IR-improved T(p,k) →

IR-improved result:

$$Z_{\tilde{Q}}^{-1}(\alpha_s,\frac{1}{\epsilon})=\int_{-1}^{1}dx\,x^{N-1}[\Gamma_{qq}^{\exp}(x,\alpha_s,\frac{1}{\epsilon})\theta(x)-\Gamma_{q\bar{q}}^{\exp}(-x,\alpha_s,\frac{1}{\epsilon})\theta(-x)]$$

where $\Gamma_{qq}^{\exp}$, $\Gamma_{q\bar{q}}^{\exp}$ are the respective IR-improved parton densities $\Rightarrow$

$$-\tilde{\gamma}^{(N)}(\alpha_s)=2\frac{\alpha_s}{2\pi}[P_{qq}^{\exp}(N,\alpha_s)+(-1)^N P_{q\bar{q}}^{\exp}(N,\alpha_s)]$$

where $P_{qq}^{\exp}$, $P_{q\bar{q}}^{\exp}\Leftrightarrow$ respective IR-improved kernels $\Rightarrow$

IR-improved one-loop identifications

$$-\tilde{\gamma}^{(N)}(\alpha_s)_{ij}=2\frac{\alpha_s}{2\pi}P_{ij}^{\exp}(N),$$

rigorous contact with new IR-improved DGLAP-CS theory

Conclusions

- IR-improved DGLAP-CS Theory Rigorously Related to Wilson's Expansion
- Realization in Herwiiri1.031 Should be Closer to Data without AD HOC Parameters: See talk later today
- Implementation in Other Parton Shower MC's in Progress: Herwig++, Sherpa, Pyhtia