Large lepton mixing angles from a 4+1-dimensional $SU(5) \times A_4$ domain-wall braneworld model

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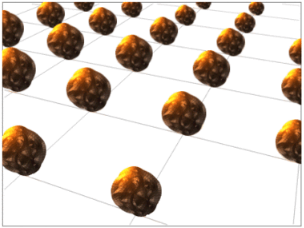
Theoretical Particle Physics
ICHEP 2012

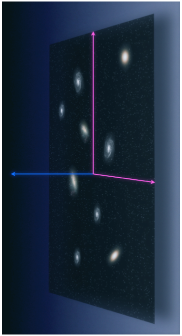
July 7, 2012

THE UNIVERSITY OF MELBOURNE

Extra Dimensions



String Theory



Braneworld

Motivations

Why domain walls?

- It is a bottom-up approach, contrasting with the top-down string-theory philosophy.
- It uses field theory only, including for the origin of the brane.
- All spatial dimensions on equal footing in the action.
- Davies, George and Volkas proposed a 4+1D domain-wall braneworld model with an $SU(5)$ gauge group broken to the Standard Model on the domain wall. It was shown that this model could generate charged fermion spectra, light neutrinos and quark mixing naturally using the split fermion mechanism which arises in the model.

Motivations

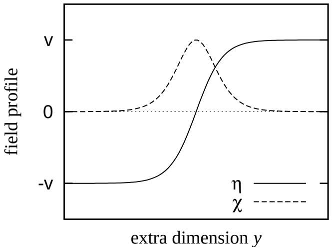
Why add an A_4 flavour symmetry?

- It was shown that neutrino mixing could not be accounted for in the same model with the desired regime to generate all other fermion mass spectra.
- Flavour symmetries are a popular approach. Perhaps impose an additional A_4 symmetry?
- Scalars can also be split in our model; solution to the vacuum alignment problem?

The background DW

- To set up the domain-wall kink, we need a singlet scalar field η with a $\mathbb{Z}_2$-symmetric Higgs potential with two distinct, degenerate minima.
- In order to confine gauge bosons on the domain wall, we invoke the Dvali-Shifman mechanism which relies on non-perturbative confinement dynamics in 4+1D. To facilitate this mechanism, we need to embed the Standard Model into a GUT gauge group G, and we need a field which has a vacuum expectation value that tends to zero out to infinity in the bulk but attains a non-zero value on the wall in order to break the G to the SM.
- Make the minimal choice, $G = SU(5)$ and add an adjoint scalar field $\chi \sim 24$

The background DW



After writing the $\mathbb{Z}_2$-symmetric Higgs potential for η and χ, arrange the global minima

$$\langle\eta\rangle = \pm v, \ \langle\chi\rangle = 0,$$

use them as boundary conditions, solve the Euler-Lagrange equations to get, e.g.

$$\eta(y) = v \tanh(ky),$$

$$\chi_1(y) = A \,\mathrm{sech}(ky).$$

This simple analytical solution holds on a certain parameter slice. Off that slice, similar solutions exist but must be obtained numerically.

Choosing our A_4 representations: Fermions

- Need to assign fermions to representations $(R_{SU(5)}, R_{A_4})$
- $\Psi_5 \sim (5^*, 1) \quad \Psi'_5 \sim (5^*, 1') \quad \Psi''_5 \sim (5^*, 1'')$
- $\Psi^i_{10} \sim (10, 1) \quad \text{for i} = 1, 2, 3$
- To generate the desired mixing patterns, we place the right-handed neutrinos into an A_4-triplet, $N \sim (1, 3)$.

Fermion charge assignment and localisation

Next, we Yukawa couple the fermions to η and χ:

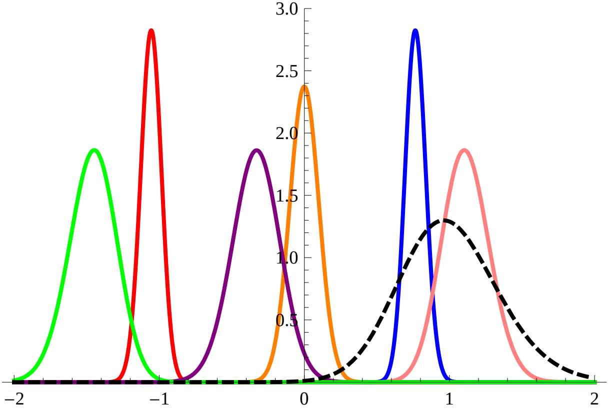
$$\begin{aligned} Y_{DW} &= h_{5\eta}\overline{\Psi^R_5}\Psi^R_5\eta + h_{5\chi}\overline{\Psi^R_5}\chi^T\Psi^R_5 \\ &+ h^{ij}_{10\eta}\mathrm{Tr}(\overline{\Psi^i}_{10}\Psi^j_{10})\eta - 2h^{ij}_{10\chi}\mathrm{Tr}(\overline{\Psi^i}_{10}\chi\Psi^j_{10}) \\ &+ h_{1\eta}(\overline{N}N)_1\eta. \end{aligned}$$

The background fields you use in the 5d Dirac Eq. are:

$$b_{nY}(y) \equiv h_{n\eta}\eta(y) + \sqrt{\frac{3}{5}}\frac{Y}{2}\,h_{n\chi}\chi_1(y).$$

SM components of different hypercharge Y couple to different linear combinations of $\eta(y)$ and $\chi_1(y)$. Fermions are *split*, but not arbitrarily.

Fermion localisation



Choosing our A_4 representations: Scalars

- To generate the requisite Yukawa interactions in the charged fermion sector, with mixing at tree level, we introduce:

$$\Phi \sim (5^*, 1) \quad \Phi' \sim (5^*, 1') \quad \Phi'' \sim (5^*, 1'')$$

- To generate Dirac masses for the neutrinos, we introduce:

$$\rho \sim (5^*, 3)$$

- To generate the desired off-diagonal Majorana mass terms to generate approximate tribimaximal mixing, we introduce:

$$\varphi \sim (1, 3)$$

Scalar localisation and EW symmetry breaking

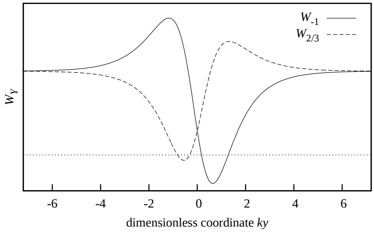
The quintet scalars contain electroweak and coloured scalar components Φ_w^R and Φ_c^R. Yukawa couple them to fermions in the usual way.

You do a mode decomposition, and are interested in the lowest modes:

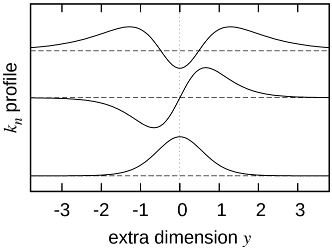
$$\Phi_{w,c}(x,y) = p_{w,c}(y)\phi_{w,c}(x)$$

You write the Higgs potential, plug the above into the Euler-Lagrange Eqs., get effective Schrödinger Eqs. for the profiles $p(y)$.

Scalar localisation and EW symmetry breaking



Scalar trapping potentials



Generic scalar modes

The p_w well is deeper (due to parameter region chosen) and gets a negative evalue m_w^2 triggering spontaneous EW symmetry breaking.

Scalar localisation and EW symmetry breaking

- Localisation potential for φ similar, but always with a potential minimum about $y = 0$ (hence φ always localised there).
- As with the electroweak components of the quintets, we can choose parameters such that φ attains a tachyonic mass on the wall.

EW Yukawa Lagrangian

- $$Y_{EW} = h^i_{-}\overline{(\Psi_5)^C}\Psi^i_{10}\Phi + h'^i_{-}\overline{(\Psi'_5)^C}\Psi^i_{10}\Phi'' + h''^i_{-}\overline{(\Psi''_5)^C}\Psi^i_{10}\Phi'$$

$$+ h^{ij}_{+}\epsilon^{\alpha\beta\gamma\kappa\delta}\overline{(\Psi^i_{10})^C_{\alpha\beta}}\Psi^j_{10}(\Phi_\delta)^*$$

$$+ h_\rho\overline{\Psi_5}(\rho N)_1 + h'_\rho\overline{\Psi'_5}(\rho N)_{1'} + h''_\rho\overline{\Psi''_5}(\rho N)_{1''}$$

$$+ M(\overline{N}N^C)_1 + h_\varphi\left[(\overline{N}N^C)_{3s}.\varphi\right]_1 + h.c.$$

Fermion Mass Fitting

- Generically, in this model, the masses of the localised fermions in the effective theory on the wall are of the form

$$m = h\langle H\rangle \int f_L(y) f_R(y) p_w(y) dy$$

- We can generate the correct charged fermion masses which has a spread of roughly 12 orders of magnitude with a set of (non-dimensionalised) domain wall background and electroweak Yukawas ranging from roughly 100 to about 800. This occurs since the splitting of the fermions lead to exponential suppressions from the overlap integrals.
- $$\Theta_{12}^{CKM} = 13.0^\circ, \quad \Theta_{13}^{CKM} = 0.201^\circ, \quad \Theta_{23}^{CKM} = 2.39^\circ.$$
- $$\Theta_{12}^{eL} = 7.32 \times 10^{-2\,\circ}, \quad \Theta_{13}^{eL} = 4.15 \times 10^{-3\,\circ}, \quad \Theta_{23}^{eL} = 0.925^\circ.$$

  This will prove to be important for generating the correct lepton mixing patterns.

Neutrino Mass Matrices

- For the localised electroweak component of ρ, ρ_w, we assign a VEV of the alignment $\langle \rho_w \rangle = (v_\rho, v_\rho, v_\rho)$
- This leads to a Dirac neutrino mass matrix of the form:

$$M_{\nu,Dirac} = \begin{pmatrix} m_\rho & m_\rho & m_\rho \\ m'_\rho & \omega m'_\rho & \omega^2 m'_\rho \\ m''_\rho & \omega^2 m''_\rho & \omega m''_\rho \end{pmatrix},$$

$$= \begin{pmatrix} \sqrt{3} m_\rho & 0 & 0 \\ 0 & \sqrt{3} m'_\rho & 0 \\ 0 & 0 & \sqrt{3} m''_\rho \end{pmatrix} .U(\omega),$$

Neutrino Mass Matrices

- For the localised mode of the A_4 triplet φ, φ_0, we assign a VEV $\langle\varphi_0\rangle = (0, v_\varphi, 0)$
- This leads to a Majorana mass matrix of the form:

$$M_{\nu,Majorana} = \begin{pmatrix} M & 0 & M_\varphi \\ 0 & M & 0 \\ M_\varphi & 0 & M \end{pmatrix},$$

- Here, $M_\varphi = h_\varphi v_\varphi \int f_N^2(y) p_{\varphi_0}(y)\, dy$

Neutrino Mass Matrices

- $M_{\nu,Majorana}$ is diagonalized by the matrix

$$P = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & -1 \\ 0 & \sqrt{2} & 0 \\ 1 & 0 & 1 \end{pmatrix} .$$

Neutrino Masses and Lepton Mixing

- Since $M_L \approx -M_{\nu,Dirac} M^{-1}_{\nu,Majorana} M^T_{\nu,Dirac}$, under the assumption that $m_\rho = m'_\rho = m''_\rho$, we get the left neutrino diagonalization matrix to be

$$\begin{aligned} V_{\nu L} &= U(\omega)^\dagger P, \\ &= \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{\omega}{\sqrt{6}} & \frac{\omega}{\sqrt{3}} & -\frac{e^{i\pi/6}}{\sqrt{2}} \\ -\frac{\omega^2}{\sqrt{6}} & \frac{\omega^2}{\sqrt{3}} & \frac{e^{5i\pi/6}}{\sqrt{2}} \end{pmatrix}. \end{aligned}$$

- We already generated electron mass textures such that $V_{eL} \sim 1$, hence we have obtained tribimaximal lepton mixing. Deviating from $V_{eL} \sim 1$ and breaking the assumption $m_\rho = m'_\rho = m''_\rho$ will lead to deviations from tribimaximal mixing.
- Left neutrino mass eigenvalues are $\frac{-3m_\rho^2}{M+M_\varphi}$, $\frac{-3m_\rho^2}{M}$, and $\frac{-3m_\rho^2}{M-M_\varphi}$.

Neutrino Masses and Lepton Mixing

- With

$$M = 2.86\ \mathrm{TeV}, \quad M_\varphi = 2.26\ \mathrm{TeV},$$

get the left neutrino eigenstate masses to be

$$m_1 = \left| \frac{-3m_\rho^2}{M + M_\varphi} \right| = 5.86 \times 10^{-3}\ \mathrm{eV},$$

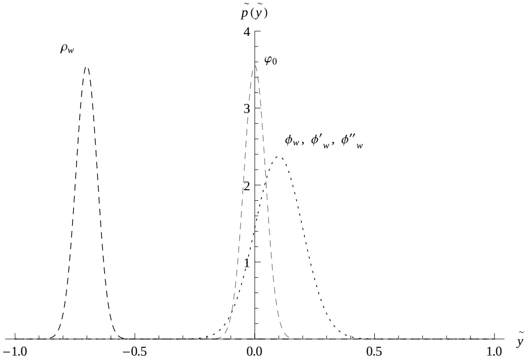
$$m_2 = \left| \frac{-3m_\rho^2}{M} \right| = 0.0105\ \mathrm{eV},$$

$$m_3 = \left| \frac{-3m_\rho^2}{M - M_\varphi} \right| = 0.0504\ \mathrm{eV},$$

Vacuum alignment

- ρ breaks $A_4 \to \mathbb{Z}_3$ and φ breaks $A_4 \to \mathbb{Z}_{\neq}$
- When cross-talking interactions switched on, troublesome interactions such as $(\rho^\dagger \rho)_{1'}(\varphi\varphi)_{1''}$ occur. The vevs of ρ and φ tend to align due to these interactions.
- It turns out we can naturally suppress these interactions by many orders of magnitude splitting the profiles of these scalars. This preserves the desired alignment up to some small corrections.

Vacuum alignment



Conclusion

- We have constructed a 4+1D $SU(5) \times A_4$ domain-wall braneworld model and localised fermions and scalars. These fermions and scalars are naturally split.
- By choosing the appropriate A_4 representations and splittings for the fermions, we were able to generate the fermion mass hierarchy, quark mixing as well as large lepton mixing angles.
- By splitting the A_4 triplet scalars we were able to solve the vacuum alignment problem.