The Simplest Neutrino Mass Matrix Revisited

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TriBimaximal Mixing (2002)

Defined by 3 Symmetries:

$$U_{MNS} = \begin{matrix} \\ e \\ \mu \\ \tau \end{matrix} \begin{matrix} \nu_1 & \nu_2 & \nu_3 \\ \left(\begin{matrix} \frac{2}{\sqrt{6}} \\ -\frac{1}{\sqrt{6}} \\ -\frac{1}{\sqrt{6}} \end{matrix} \right. & \begin{matrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{matrix} & \left. \begin{matrix} 0 \\ \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{matrix} \right) \end{matrix}$$

CP Symmetry (row e)

$\mu - \tau$ *Symmetry* (rows μ and τ)

Democracy (column ν_2)

TriBimaximal Mixing (2002)

Defined by 3 Symmetries:

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\begin{array}{cc} & \begin{array}{ccc} \nu_1 & \nu_2 & \nu_3 \end{array} \\
U_{MNS} = \begin{array}{c} e \\ \mu \\ \tau \end{array} & \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix}
\end{array}
$$

CP Symmetry (row e)

$\mu - \tau$ *Symmetry* (rows μ, τ)

Democracy (column ν_2)

Symmetries reflected in neutrino mass matrix in the flavour basis:

$$
M_\nu^2 \equiv M_\nu M_\nu^\dagger = \begin{array}{c} \\ \nu_e \\ \nu_\mu \\ \nu_\tau \end{array} \begin{array}{c} \begin{array}{ccc} \nu_e & \nu_\mu & \nu_\tau \end{array} \\ \begin{pmatrix} c+a+b & b & b \\ b & c+b & a+b \\ b & a+b & c+b \end{pmatrix} \end{array}
$$

CP Symm. $\Rightarrow$ parameters Real; $\mu - \tau$ Symm. is Manifest;

Democracy $\Rightarrow$ rows and cols sum to same value - Magic matrix.

But Tribimaximal Mixing is Excluded

Daya Bay and RENO experiments find

$$\theta_{13} \simeq 9^\circ$$

But Tribimaximal Mixing is Excluded

Daya Bay and RENO experiments find

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$\Rightarrow$ TBM is excluded!

Triχmaximal Mixing (2002): Relax *CP* Symmetry

PFH and W.G. Scott, PLB535 (2002) 163.

Retains two symmetries:

$$U_{MNS} = \begin{pmatrix} \sqrt{\frac{2}{3}}\cos\chi & \frac{1}{\sqrt{3}} & i\sqrt{\frac{2}{3}}\sin\chi \\ -\frac{\cos\chi}{\sqrt{6}} + i\frac{\sin\chi}{\sqrt{2}} & \frac{1}{\sqrt{3}} & \frac{\cos\chi}{\sqrt{2}} - i\frac{\sin\chi}{\sqrt{6}} \\ -\frac{\cos\chi}{\sqrt{6}} - i\frac{\sin\chi}{\sqrt{2}} & \frac{1}{\sqrt{3}} & -\frac{\cos\chi}{\sqrt{2}} - i\frac{\sin\chi}{\sqrt{6}} \end{pmatrix}$$

CP Violated (first row)

$\mu - \tau$ *Symmetry* (second and third rows)

Democracy (middle column)

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CP Violated

$\mu - \tau$ *Symmetry*

Democracy

Allows b param. to become complex $(b + id) \Rightarrow$ 3 masses and $\theta_{13} \neq 0$ arbitrary.

$$M_\nu^2 \equiv M_\nu M_\nu^\dagger = \begin{matrix} & \nu_e & \nu_\mu & \nu_\tau \\ \nu_e & c+a+b & b+id & b-id \\ \nu_\mu & b-id & c+b & a+b+id \\ \nu_\tau & b+id & a+b-id & c+b \end{matrix}$$

$\mu - \tau$ Symm. still Manifest;

Democracy $\Rightarrow$ rows and cols still sum to same value.

The Simplest Neutrino Mass Matrix (2004)

PFH and W.G. Scott - PLB 594 (2004) 324.

Motivation was to relate the two small observables in ν oscillations to each other.

Set $b = 0 \Rightarrow$ special case of Triχmaximal mixing.

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Motivation was to relate the two small observables in ν oscillations to each other.

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We are back to 3 parameters:

$$M_\nu^2 \equiv M_\nu M_\nu^\dagger = \begin{array}{c} \\ \nu_e \\ \nu_\mu \\ \nu_\tau \end{array} \begin{array}{c} \begin{array}{ccc} \nu_e & \nu_\mu & \nu_\tau \end{array} \\ \begin{pmatrix} c+a & id & -id \\ -id & c & a+id \\ id & a-id & c \end{pmatrix} \end{array}$$

Simplest Neutrino Mixing Prediction

$\mu - \tau$ and Democracy symmetries still hold

$\Rightarrow$ Mixing as in TriχM, with $\theta_{13} \neq 0$.

Single small parameter links θ_{13} with mass hierarchy parameter, predicts:

$$\sin\theta_{13} \simeq \sqrt{\frac{2}{3}\frac{\Delta m^2_{sol}}{\Delta m^2_{atm}}}$$

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$$\begin{array}{llll} ie. \ \sin^2 2\theta_{13} & \simeq & 0.086^{+0.003}_{-0.006} & \text{(Predicted)} \\ cf. \ \sin^2 2\theta_{13} & = & 0.093 \pm 0.010 & \text{(Daya Bay \& RENO \& D-CHOOZ)} \end{array}$$

Problems with Simplest

Despite startling success of the Simplest neutrino mass matrix, has a number of significant shortcomings:

- Is only a texture without a theory behind it.
- Deals with M_ν^2.
    - Great for phenomenology, since mass-squared differences measured in ν-osc.
    - But need M_ν in Lagrangian and square-root of matrix not unique.

Moving Simplest to M_ν

- Will use symmetries to guide us:
- Try an M_ν which has Democracy and $\mu - \tau$ symmetry - symmetries survive squaring: $M_\nu \to M_\nu^2$.
- But $b = 0$ is not a *symmetry*!
  - Try either an M_ν with $b = 0$ (then M_ν^2 does not have this successful texture).
  - Or find an M_ν which when squared gives $b = 0$?

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    - Try either an M_ν with $b = 0$ (then M_ν^2 does not have this successful texture).
    - Or find an M_ν which when squared gives $b = 0$?
- If neutrinos are Dirac particles, M_ν has no constraints other than our symmetries - can be chosen hermitian.
- If Majorana, M_ν should be complex-symmetric.

Either way, Simplest texture in M_ν relates θ_{13} to $\frac{(m_2 - m_1)}{(m_3 - m_1)}$, not to $\frac{\Delta m^2_{sol}}{\Delta m^2_{atm}}$! - Function of m_1.

Towards a Theory of Simplest Mixing

Have proved a simple theorem:

Define P as the $\mu - \tau$ permutation matrix, $P = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$.

Given a 3×3 matrix, M, which is Hermitian and $\mu - \tau$ symmetric, the matrix MP will be complex symmetric and $\mu - \tau$ symmetric. Democracy also survives this transformation.

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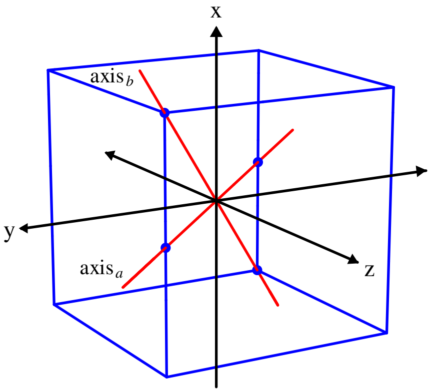
Thus, starting with M_H in Simplest form, we construct a theory with Majorana neutrinos leading to a neutrino mass matrix of the complex-symmetric, Simplest form:

$$M_H P = M_S = a\begin{pmatrix}1&-ik&ik\\-ik&1+ik&0\\ik&0&1-ik\end{pmatrix}+c\begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix}$$

with a, $k=\frac{d}{a}$ and c real.

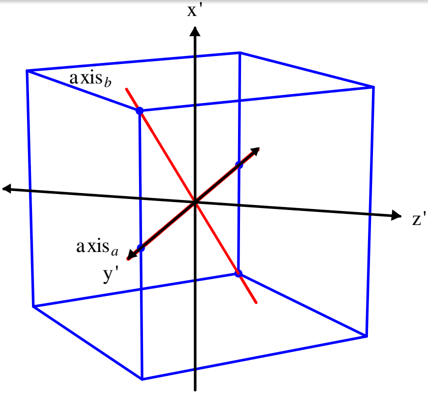
The S4 Group



- The group of permutations of 4 objects.
- Also the symmetry group of the cube.
- Used several times in models to generate TBM-like mixings.
- Normally, a triplet of neutrino states, $(\nu_e, \nu_\mu, \nu_\tau)$ is used, aligned with the x, y and z axes shown.

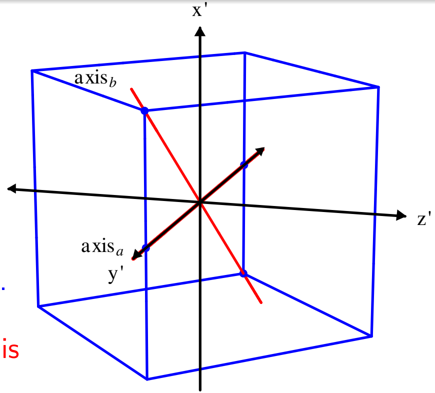
The $\mu - \tau$-rotated Basis

- Here, rotate the coordinate axes by $\pi/4$ about x-axis.
  The $(\nu_e, \nu_\mu, \nu_\tau)$ states are now defined to be parallel to the new x, y' and z' axes shown.



The $\mu - \tau$-rotated Basis

- Here, rotate the coordinate axes by $\pi/4$ about x-axis. The $(\nu_e, \nu_\mu, \nu_\tau)$ states are now defined to be parallel to the new x, y' and z' axes shown.
- This rotation of the basis states is the main innovation here.



- Leads to the $\mu - \tau$ symmetry of the neutrino mass matrix which is so important to the phenomenology of Triχmaximal and Simplest neutrino mixing.

Tensor Product

The tensor product decomposition of two χ_3's is:

$$\chi_3' \times \chi_3' = \chi_1 + \chi_2 + \chi_3 + \chi_3'$$

where we have:

$$\chi_1 = \frac{1}{\sqrt{3}}(\nu_e.\nu_e + \nu_\mu.\nu_\mu + \nu_\tau.\nu_\tau), \quad \chi_2 = \begin{pmatrix} -\sqrt{\frac{2}{3}}\nu_e.\nu_e + \frac{1}{\sqrt{6}}\nu_\mu.\nu_\mu + \frac{1}{\sqrt{6}}\nu_\tau.\nu_\tau \\ \frac{1}{\sqrt{2}}(\nu_\mu.\nu_\tau + \nu_\tau.\nu_\mu) \end{pmatrix}$$

$$\chi_3 = \begin{pmatrix} \frac{1}{\sqrt{2}}(\nu_\mu.\nu_\mu - \nu_\tau.\nu_\tau) \\ \frac{1}{\sqrt{2}}(\nu_\tau.\nu_e + \nu_e.\nu_\tau) \\ \frac{1}{\sqrt{2}}(\nu_e.\nu_\mu + \nu_\mu.\nu_e) \end{pmatrix}, \quad \chi_3' = \begin{pmatrix} \frac{1}{\sqrt{2}}(\nu_\mu.\nu_\tau - \nu_\tau.\nu_\mu) \\ \frac{1}{\sqrt{2}}(\nu_\tau.\nu_e - \nu_e.\nu_\tau) \\ \frac{1}{\sqrt{2}}(\nu_e.\nu_\mu - \nu_\mu.\nu_e) \end{pmatrix} = 0.$$

Here the product $\nu_i.\nu_j$ represents the product of the right-handed neutrino fields: $\nu_{iR}^T \epsilon \nu_{jR}$, with ϵ the antisymmetric matrix $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$.
The antisymmetric product χ_3' vanishes.

Mass Terms

We assume three types of flavons ϕ_1, ϕ_2 and ϕ_3 transforming as χ_1, χ_2 and χ_3 respectively.
Can write the invariant mass term:

$$\text{Inv} = c_1\chi_1\phi_1 + c_2\chi_2^T\phi_2 + c_3\chi_3^T\phi_3$$

with c_1, c_2 and c_3 constants.

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with c_1, c_2 and c_3 constants.

Suppose the flavons get vevs by SSB of $\langle\phi_1\rangle = 1$, $\langle\phi_2\rangle = (-\frac{1}{2}, \frac{\sqrt{3}}{2})$ and $\langle\phi_3\rangle = (1, 1, -1)$. Leads to a mass matrix:

$$M = c_1 I + \frac{c_2\sqrt{3}}{2\sqrt{2}} \begin{pmatrix} \frac{2}{3} & 0 & 0 \\ 0 & -\frac{1}{3} & 1 \\ 0 & 1 & -\frac{1}{3} \end{pmatrix} + \frac{c_3}{\sqrt{2}} \begin{pmatrix} 0 & -1 & 1 \\ -1 & 1 & 0 \\ 1 & 0 & -1 \end{pmatrix}$$

which is in the required form, assuming c_1 $(\sim a)$ and c_2 $(\sim c)$ to be real and c_3 $(\sim d = ak)$ to be imaginary.

The Model

Constructed in SM framework with additional heavy RH ν's.

Type 1 see-saw produces light Majorana neutrinos.

RH Majorana mass term of above form.

Additional $C2$ flavour symmetry to allow only singlet flavon ϕ_1 to enter Dirac mass term $\Rightarrow$ Dirac MM$\propto$ I.

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Fermion and Flavon content as follows (SM Higgs is flavour singlet):

	e_R	μ_R	τ_R	L	ν_R	ϕ_1^-	ϕ_1	ϕ_2	ϕ_3	ϕ'_{3e}	$\phi'_{3\mu}$	$\phi'_{3\tau}$
S_4	χ_1	χ_1	χ_1	χ'_3	χ'_3	χ_1	χ_1	χ_2	χ_3	χ'_3	χ'_3	χ'_3
C_2	-1	-1	-1	-1	1	-1	1	1	1	1	1	1

Table: The flavour structure of the model. L are the three left handed lepton weak isospin doublets and ν_R are the three right handed heavy neutrinos.

Mass Terms Again

Charged lepton mass term:

$$\left(y_e L^\dagger e_R \phi'_{3e} + y_\mu L^\dagger \mu_R \phi'_{3\mu} + y_\tau L^\dagger \tau_R \phi'_{3\tau}\right) \frac{H}{\Lambda} + H.C.$$

where Λ is the cut-off scale and the y_i are coupling constants. After SSB, the flavons ϕ'_{3e}, $\phi'_{3\mu}$ and $\phi'_{3\tau}$ get vev's $(1,0,0)^T$, $(0,1,0)^T$ and $(0,0,1)^T$ respectively $\rightarrow$ m_e, m_μ and m_τ.

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Dirac mass term for the neutrinos:

$$y_w L^\dagger \nu_R \frac{\phi_1^-}{\Lambda} \tilde{H} + H.C.$$

where $\tilde{H}$ is the conjugate Higgs and y_w is a coupling.

Majorana mass term for the neutrinos:

$$\left(y_1 \chi_1 \phi_1 + y_2 \chi_2^T \phi_2 + i y_3 \chi_3^T \phi_3\right) \frac{1}{\Lambda}$$

where the χs were given above and y_1, y_2 and y_3 are coupings.

Phenomenology

Neutrino Dirac mass term is proportional to the identity

Get Simplest Mass matrix for Majorana

Get Type I See-saw mechanism and Simplest results for neutrino mixing matrix

Predictions of Model

We have: $|U_{e2}| = \frac{1}{\sqrt{3}}$; $\theta_{23} = \frac{\pi}{4}$; $\delta_{CP} = \pm\frac{\pi}{2}$; $\theta_{13} \simeq \frac{k}{\sqrt{2}}$ and

$$\frac{m_2^2 - m_1^2}{m_3^2 - m_2^2} = \frac{(\sqrt{1+3k^2} - r)^2(3k^2 + 2(-1+\sqrt{1+3k^2})r)}{(\sqrt{1+3k^2} + r)^2(-3k^2 + 2(1+\sqrt{1+3k^2})r)}$$

where $r = c/a$.

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where $r = c/a$.

For $\theta_{13} = 9^\circ \Rightarrow k = 0.22$,
get 3 real solutions for r:

- $r = 0.41$ (normal);
- $r = 14.4$(normal)
- $r = -1.04$ (inverted);

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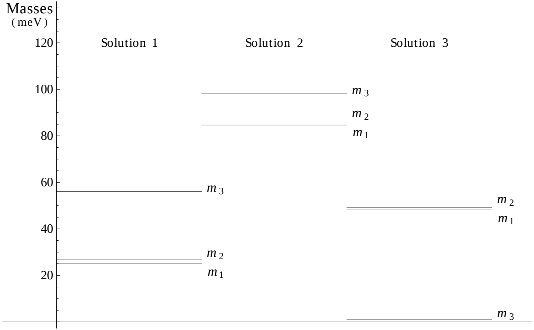
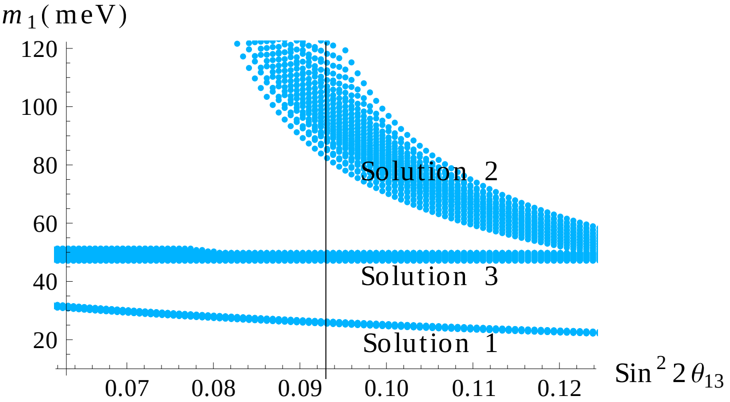


Predictions of Model

Allowing $\pm 3\sigma$ variation for θ_{13}, and Δm^2_{sol} and Δm^2_{atm} to vary within errors:



Concluding remarks

- Simplest texture for M_ν^2 (2004) give precise predictions for all neutrino mixing angles and is in good agreement with all neutrino oscillation data.
- Simplest texture for M_ν (this work) is in good agreement with all neutrino oscillation data, and correllates values of θ_{13} within experimental bounds with lightest neutrino mass
- A simple model based on $S4$ symmetry with majorana neutrinos and a seesaw mechanism can implement Simplest for M_ν, and is in agreement with all neutrino oscillation data.
- The model predicts $\delta_{CP} = \pm\frac{\pi}{2}$ and thus, when combined with experimental data, predicts large CP-violating asymmetries in ν-oscillations.