Getting the best out of T2K and NO\(\nu\)A


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$P_{\mu e}$: algebraic expression

$\nu_\mu \to \nu_e$ oscillation probability, expanded perturbatively in $\alpha = \Delta_{21}/\Delta_{31}$:

$$P_{\mu e} = \sin^2 2\theta_{13} \sin^2 \theta_{23} \frac{\sin^2 \Delta (1 - \hat{A})}{(1 - \hat{A})^2}$$

$$+ \alpha \cos \theta_{13} \sin 2\theta_{12} \sin 2\theta_{13} \sin 2\theta_{23} \cos(\hat{\Delta} + \delta_{CP}) \frac{\sin \hat{\Delta} \hat{A} \sin \hat{\Delta} (1 - \hat{A})}{\hat{A} (1 - \hat{A})}$$

$$+ \alpha^2 \sin^2 2\theta_{12} \cos^2 \theta_{13} \cos^2 \theta_{23} \frac{\sin^2 \hat{\Delta} \hat{A}}{\hat{A}^2}$$

where

- $\hat{\Delta} = \Delta_{31} L/4E$
- $\hat{A} = A/\Delta_{31}$

Wolfenstein matter term: $A(eV^2) = 0.76 \times 10^{-4} \rho(\text{gm/cc}) E(\text{GeV})$. 
Updated results from Reactor Neutrino Experiments (reported at Neutrino 2012):

- **Daya Bay**: $\sin^2 2\theta_{13} = 0.089 \pm 0.011^{\text{stat}} \pm 0.005^{\text{syst}}$, $7.7\sigma$ for non-zero $\theta_{13}$
- **RENO**: $\sin^2 2\theta_{13} = 0.113 \pm 0.013^{\text{stat}} \pm 0.019^{\text{syst}}$, $4.9\sigma$ for non-zero $\theta_{13}$
- **Double CHOOZ**: $\sin^2 2\theta_{13} = 0.109 \pm 0.030^{\text{stat}} \pm 0.025^{\text{syst}}$, $3.1\sigma$ for non-zero $\theta_{13}$

Global $\sin^2 2\theta_{13}$ best-fit: $0.095$ [Fogli et.al, arXiv:hep-ph/1205.5254v3]

Best final precision on $\sin^2 2\theta_{13}$: Daya Bay: $\sim 5\%$
$P_{\mu e}$: degeneracies

$P_{\mu e}$ depends on all oscillation parameters. A measurement of $P_{\mu e}$ therefore, has a number of degenerate solutions.

- We assume $\theta_{23} = 45^\circ$ so degeneracies involving $\theta_{23}$ are not relevant.
- Because of precise measurement of $\sin^2 2\theta_{13}$ by reactor neutrino experiments, degeneracies involving $\theta_{13}$ are also not relevant.
- We care about the hierarchy-$\delta_{CP}$ degeneracy only.
## Details of LBL Superbeam experiments considered

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<tr>
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$P_{\mu e}$: hierarchy and $\delta_{CP}$ effects

$P_{\mu e}$ (as a band in $\delta_{CP}$) vs. Energy.
For neutrinos (left panel) and anti-neutrinos (right panel).
Favourable and unfavourable $\delta_{CP}$ planes for NO$\nu$A

Notice:

- $(\text{NH,} \delta_{CP} = -90^\circ)$ is farthest from IH $\delta_{CP}$ band and $(\text{IH,} \delta_{CP} = 90^\circ)$ is farthest from NH $\delta_{CP}$ band.
- $(\text{NH,} \delta_{CP} \sim 90^\circ)$ is closest to IH $\delta_{CP}$ band and $(\text{IH,} \delta_{CP} \sim -90^\circ)$ is closest to NH $\delta_{CP}$ band.

Therefore,

- lower half plane (LHP) of $\delta_{CP}$ forms the favourable plane for NH
- upper half plane (UHP) of $\delta_{CP}$ forms the favourable plane for IH
Hierarchy exclusion ability of NO$\nu$A

NO$\nu$A with various design statistics

For $\sin^2 2\theta_{13} = 0.1$, 1.5*NO$\nu$A can determine the hierarchy for entire favourable half plane, for both NH and IH.
Breaking hierarchy-$\delta_{CP}$ degeneracy: Combining data from two LBL experiments

- Given a $P_{\mu e}$ measurement, two degenerate solutions: (correct hierarchy, correct $\delta_{CP}$) and (wrong hierarchy, wrong $\delta_{CP}$)

- For a given experiment, $[\sin(\text{correct } \delta_{CP}) - \sin(\text{wrong } \delta_{CP})]$ is proportional to the matter term $A$ for that experiment [O. Mena and S. Parke, arXiv:hep-ph/0408070]

- For T2K, this difference is small and is about 0.7 for $\sin^2 2\theta_{13} = 0.1$

- For NO$\nu$A it is three times larger.

A combined analysis of data from T2K and NO$\nu$A will pick out the correct hierarchy and a range of $\delta_{CP}$ around the correct value, provided the statistics from each experiment are large enough. We find that $1.5*\text{NO$\nu$A} + 2*\text{T2K}$ is required for the present best-fit of $\sin^2 2\theta_{13}$
How does combining data help?

$\chi^2$ vs. $\delta_{CP}(\text{test})$ plots for NO$\nu$A

Irrespective of true $\delta_{CP}$, $\chi^2_{min}$ always occurs around $-90^\circ$. $\chi^2_{min}$ large for LHP, very small for UHP.
How does combining data help?

$\chi^2$ vs. $\delta_{CP}(test)$ plots for T2K

Left panel: $\chi^2_{min}$ occurs at $-90^\circ$ and is very small.

Right panel: $\chi^2$ values are relatively large around $-90^\circ$. $\chi^2_{min}$ occurs near CP conserving test $\delta_{CP}$ in the right half plane.
How does combining data help?

\( \chi^2 \) vs. \( \delta_{CP}(\text{test}) \) plots for 1.5*NO\( \nu \)A +2*T2K

From the right panel, we see that \( \chi^2 \) dips just below 2.71 when IH is the test hierarchy. Similar features are observed for other unfavourable \( \delta_{CP} \) values and also when IH is the true hierarchy.
Hierarchy exclusion with NO$\nu$A and T2K combined

Hierarchy exclusion plots for NO$\nu$A and T2K with various design statistics

True: NH / Test: IH, 90% C.L.

True: IH / Test: NH, 90% C.L.

From these plots, we see that for $\sin^2 2\theta_{13} = 0.1$, 1.5*NO$\nu$A + 2*T2K is able to exclude wrong hierarchy for all $\delta_{CP}$ except some small regions in the unfavourable plane.
Hierarchy exclusion with NO\(\nu\)A and T2K combined

- It seems as if \((1.5 \times \text{NO\(\nu\)A} + 2 \times \text{T2K})\) is insufficient to exclude wrong hierarchy for some \(\delta_{CP}\) values in the unfavourable plane.
- These are those \(\delta_{CP}\) values for which \(\chi^2\) dips just below the cut-off for 90\% C.L.
- With a small increase in statistics, hierarchy can be excluded at 90\% C.L. for these troublesome regions also.
- Thus, \((1.5 \times \text{NO\(\nu\)A} + 2 \times \text{T2K})\) can essentially determine hierarchy for the entire \(\delta_{CP}\) plane for both NH and IH for \(\sin^2 2\theta_{13} \geq 0.1\).
Can $\delta_{CP}$ be measured before hierarchy?

Allowed $\delta_{CP}$ plots for 1*T2K[3+3]

Left panel: allowed range of test $\delta_{CP}$ is around true $\delta_{CP}$.
Right panel: allowed range of test $\delta_{CP}$ includes a large region of the wrong half-plane also. Best-fits, in general, are far away from true point.
Can $\delta_{CP}$ be measured before hierarchy?

Allowed $\delta_{CP}$ plots for 10*T2K[3+3]

Not knowing hierarchy gives incorrect $\delta_{CP}$ solutions.

Right panel: For true $\delta_{CP}$ around CP conserving values, test $\delta_{CP}$ range around maximum CP violating values and vice versa.
$\delta_{CP}$ measurement with T2K and NO$\nu$A

Allowed $\delta_{CP}$ plots for 1.5*NO$\nu$A + 2*T2K[5+0]

True:NH/Test:NH, 90% C.L.

True:NH/Test:IH, 90% C.L.

1.5*NO$\nu$A + 2*T2K can measure $\delta_{CP}$ with an accuracy of $\pm 40^\circ$ for true $\delta_{CP} = 0$ and $\pm 60^\circ$ for true $\delta_{CP} = \pm 90^\circ$. 
Hierarchy - $\delta_{CP}$ degeneracy severely limits the ability of any single experiment to determine hierarchy as well as $\delta_{CP}$.

The observed moderately large value of $\theta_{13}$ is certainly a very good news for the upcoming NO$\nu$A.

1.5*NO$\nu$A by itself can resolve the hierarchy at 90% C.L. if $\delta_{CP}$ is in the favourable half-plane and $\sin^2 2\theta_{13} \geq 0.1$.

When $\delta_{CP}$ is in the unfavourable half-plane, the data from NO$\nu$A and T2K beautifully complement each other to rule out the wrong hierarchy.

1.5*NO$\nu$A + 2*T2K can *essentially* resolve mass hierarchy at 90% C.L. for the entire $\delta_{CP}$ range.

Without knowing the hierarchy, $\delta_{CP}$ cannot be measured.

With 1.5*NO$\nu$A + 2*T2K, the correct half-plane of $\delta_{CP}$ can be determined at 90% C.L. for most values of $\delta_{CP}$.
Thank you?
Neutrino Oscillation Parameters:

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<th>3σ range</th>
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<td>3.07</td>
<td>2.59-3.59</td>
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<td>$\sin^2 2\theta_{13}$(NH)</td>
<td>0.094</td>
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<td>2.19-2.62</td>
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<td>2.17-2.61</td>
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<td>$\delta/\pi$(NH)</td>
<td>1.08</td>
<td>...</td>
</tr>
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Experiments considered

In this analysis we simulate data from the following experiments.

- NO$\nu$A
- T2K
- C2F

The details of these experiments are in the following table.
## Experiment Details

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Calculating $\chi^2$

- We use the software GLoBES for our analysis. [Huber et. al. arXiv:hep-ph/0407333v1]
- We calculated statistical $\chi^2$ including the systematical errors and priors.
- Solar parameters, $\Delta_{21}$ and $\theta_{12}$, were kept fixed at their best fit values.
- $\chi^2$ marginalised over $\Delta_{31}$, $\sin^2 2\theta_{13}$, $\sin^2 \theta_{23}$ and $\delta_{CP}$
- Priors in $\sin^2 2\theta_{13}$, $\sin^2 \theta_{23}$ and $|\Delta_{31}|$ have been added.
  $\sigma (\sin^2 2\theta_{13}) = 0.005$, $\sigma (\sin^2 2\theta_{23}) = 0.02$ and $\sigma (|\Delta_{31}|) = 0.03 \times |\Delta_{31}|$

Including Reactor Neutrino Experiments

A prior of $\sigma (\sin^2 2\theta_{13}) = 0.005$ effectively takes into account the data due to Reactor Neutrinos Experiments
Calculating $\chi^2$

We have taken care in defining $\Delta_{31}^{NH}$ and $\Delta_{31}^{IH}$ in terms of the measured quantity $\Delta_{\mu\mu}$.

$$\Delta m_{\mu\mu}^2 = \Delta_{31} - (\cos^2 \theta_{12} - \cos \delta \sin \theta_{13} \sin 2\theta_{12} \tan \theta_{23}) \Delta_{21}$$

- $\Delta m_{\mu\mu}^2 = \pm 2.4 \times 10^{-3}\text{eV}^2$ ; +: NH, -: IH


Mainly, 3 kinds of plots have been shown.

- Hierarchy exclusion plots
- Allowed $\sin^2 2\theta_{13}$-$\delta_{CP}$ plots
- Allowed $\delta_{CP}(test)$-$\delta_{CP}(true)$ plots
Probability of neutrinos
\[
\sin^2 2\theta_{13} = 0.1
\]
812 km

Probability of anti-neutrinos
\[
\sin^2 2\theta_{13} = 0.1
\]
812 km

\(\delta_{CP} = +90^\circ\)
\(\delta_{CP} = -90^\circ\)
$P_{\mu e}$: $\theta_{13}$ effect

\[ P_{\mu e} \] vs. $E_{\text{GeV}}$ for $\sin^2 2\theta_{13} = 0.05$ and 0.15, with NH and IH cases, for $\delta_{\text{CP}} = +90^\circ$ and $-90^\circ$. The baseline is 812 km.
Oscillation Parameters: Known best fit values

Known Parameters

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<tr>
<td>$\sin^2 \theta_{12}$</td>
<td>0.312</td>
<td>$(0.265 - 0.364)$</td>
</tr>
<tr>
<td>$\sin^2 2\theta_{13}$</td>
<td>0.1</td>
<td>$(0.02 - 0.19)$</td>
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<tr>
<td>$\sin^2 \theta_{23}$</td>
<td>0.42</td>
<td>$(0.34 - 0.64)$</td>
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<td>$</td>
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- Recent T2K results: $0.03(0.04) < \sin^2 2\theta_{13} < 0.28(0.34)$ for normal (inverted) hierarchy at 90% C.L. for $\delta_{CP} = 0$
- Recent MINOS results: $0.0(0.0) < \sin^2 2\theta_{13} < 0.12(0.19)$ for normal (inverted) hierarchy at 90% C.L. for $\delta_{CP} = 0$
- Latest DChooz results: $\sin^2 2\theta_{13} = 0.085 \pm 0.051$ at 68% C.L.
Will adding a shorter baseline help?

Allowed $\sin^2 2\theta_{13}$-$\delta_{CP}$ plots

It can be seen that adding C2F provides only a marginal improvement, over and above T2K.