Chris White, University of Glasgow

New insights into soft gluons and gravitons

ICHEP 2012
Overview

- Brief introduction to IR singularities.
- QED, QCD and gravity.
- AdS space techniques: QED $\leftrightarrow$ gravity.
- Outlook.
Infrared divergences

- In scattering amplitudes, get singularities due to gluon emission at large distances.

- Due to integrals over gluon positions:

\[ \int d^n x \]

- Uncertainty principle \( \Rightarrow \) equivalent to emission of zero energy gluons.

- Common to abelian / non-abelian gauge theories, including gravity.
IR singularities

- Related to large logarithms in perturbation theory ("resummation").
- Various unsolved conjectures regarding IR singularity structures.
- Scattering amplitudes factorise into a hard and soft part (Mueller, Collins, Sen, Korchemsky, Magnea, Sterman), where the latter has an exponential form. Schematically:

  \[ S \sim \exp \left( \sum_{W} W \right). \]

- Here \( W \) are certain special diagrams called webs, and differ between theories.
Webs in QED

- In QED, one may show that the exponent of the soft function contains only connected subdiagrams ("QED webs"), for any number of lines:

- Originally derived using combinatoric methods (Yennie, Frautschi, Suura), and recently rederived using path integral methods (Laenen, Stavenga, White).

- Gives IR singularities to all orders in perturbation theory!
Webs in QCD

- Things are more complicated in QCD, due to non-commuting colour matrices (Gatheral; Frenkel, Taylor; Gardi, Laenen, Stavenga, White; Mitov, Sterman; Sung).

- Each of these has a kinematic factor $\mathcal{F}(D)$ ($D = a, b$) and a colour factor $\tilde{C}(D)$.

- The contribution to the exponent of the soft function turns out to be

$$\left( \begin{array}{c} \mathcal{F}(a) \\ \mathcal{F}(b) \end{array} \right)^T \left( \begin{array}{c} \tilde{C}(a) \\ \tilde{C}(b) \end{array} \right) = \left( \begin{array}{c} \mathcal{F}(a) \\ \mathcal{F}(b) \end{array} \right)^T \frac{1}{2} \left( \begin{array}{cc} 1 & -1 \\ -1 & 1 \end{array} \right) \left( \begin{array}{c} C(a) \\ C(b) \end{array} \right).$$
Multiparton webs in QCD

- The set of diagrams mixes in the exponent.
- Colour and kinematic information is entangled in a non-trivial way.
- Higher order diagrams also form closed sets, which contribute to the exponent of the soft amplitude according to

\[
\sum_{D,D'} \mathcal{F}_D R_{DD'} C_{D'},
\]

where \( R_{DD'} \) is a web-mixing matrix.
- These encode a huge amount of physics!
A four loop example
### Four loop mixing matrix ($\times 24$)

$$
\begin{pmatrix}
6 & -6 & 2 & 2 & -2 & 4 & -4 & 2 & -2 & -2 & -4 & 4 & -4 & 4 & 0 & 0 \\
-6 & 6 & -2 & -2 & 2 & -4 & 4 & -2 & 2 & 2 & 4 & -4 & 4 & -4 & 0 & 0 \\
2 & -2 & 6 & -2 & 2 & 4 & -4 & -2 & 2 & -6 & 4 & 4 & -4 & -4 & 0 & 0 \\
2 & -2 & -2 & 6 & 2 & 4 & -4 & -2 & -6 & 2 & -4 & -4 & 4 & 4 & 0 & 0 \\
-2 & 2 & 2 & 2 & 6 & 4 & -4 & -6 & -2 & -2 & 4 & -4 & 4 & -4 & 0 & 0 \\
2 & -2 & 2 & 2 & 2 & 4 & -4 & -2 & -2 & -2 & 0 & 0 & 0 & 0 & 0 & 0 \\
-2 & 2 & -2 & -2 & -2 & -2 & -4 & 4 & 2 & 2 & 2 & 2 & 0 & 0 & 0 & 0 \\
2 & -2 & -2 & -2 & -6 & -4 & 4 & 6 & 2 & 2 & 4 & -4 & 4 & -4 & 4 & 0 & 0 \\
-2 & 2 & 2 & -6 & -2 & -4 & 4 & 2 & 6 & -2 & 4 & 4 & -4 & -4 & 0 & 0 \\
-2 & 2 & -6 & 2 & -2 & -4 & 4 & 2 & -2 & 6 & -4 & -4 & 4 & 4 & 0 & 0 \\
-2 & 2 & 2 & -2 & 2 & 0 & 0 & -2 & 2 & -2 & 4 & 0 & 0 & -4 & 0 & 0 \\
2 & -2 & 2 & -2 & -2 & 0 & 0 & 2 & 2 & -2 & 0 & 4 & -4 & 0 & 0 & 0 \\
-2 & 2 & -2 & 2 & 2 & 0 & 0 & -2 & -2 & 2 & 0 & -4 & 4 & 0 & 0 & 0 \\
2 & -2 & -2 & 2 & -2 & 0 & 0 & 2 & -2 & 2 & -4 & 0 & 0 & 4 & 0 & 0 \\
-18 & -6 & -6 & -6 & -18 & 12 & 12 & -6 & -18 & -18 & 12 & 12 & 12 & 12 & 24 & 0 \\
-6 & -18 & -18 & -18 & -6 & 12 & 12 & -18 & -6 & -6 & 12 & 12 & 12 & 12 & 0 & 24 \\
\end{pmatrix}
$$
Multiparton webs

- Web mixing matrices have been shown to have interesting properties (e.g. zero sum rows, idempotence).
- We are beginning to translate these properties into physics (Gardi, White, Smillie).
- Also, the mathematics of web mixing matrices can be mapped to interesting combinatoric problems in computer science (Dukes, Gardi, McAslan, Scott, Steingrimsson, White), to do with posets and order-preserving maps.
- Progress can be made with or without any physics knowledge!
The dipole formula

- For massless particles, it is conjectured that correlations between three or more particles vanish in the soft limit.
- This is known as the *dipole formula* (Becher, Neubert; Gardi, Magnea).
- Known corrections may exist at three loops and beyond (Dixon, Gardi, Magnea, Becher, Neubert, Vernazza).
- Recent additional constraints were derived from the high energy ("Regge") limit (Del Duca, Duhr, Gardi, Magnea, White).
- Web-mixing matrices may also have something to say about this!
IR singularities in gravity were studied by Weinberg (1965), and more recently by Naculich, Schnitzer; White; Akhoury, Saotome, Sterman.

The exponent of the soft amplitude contains only one-loop graphs!

One can also continuously relate QED and gravity in the soft limit, using AdS methods.
AdS methods

- The emission of soft gauge bosons is described by Wilson line operators:

\[
\exp \left[ ig_s \int_0^\infty dx_\mu A_\mu(x) \right] \leftrightarrow \exp \left[ i \frac{\kappa}{2} p^\mu \int_0^\infty dx_\nu h^{\mu\nu}(x) \right]
\]

- A new way to think about Wilson lines is to look at them in a Euclidean AdS space (Chien, Schwartz, Simmons-Duffin, Stewart).

- There they become point charges, whose potential energy corresponds to the structure of IR singularities (cusp anomalous dimension) in Minkowski space!
General formulation

- The potential energy of a pair of spin $n$ Wilson lines in AdS space is (White, Miller)

$$\tilde{H}(\beta) = A_1 \left( \frac{\sinh(n\beta)}{\sinh \beta} \right) + A_2 \left( \frac{\cosh(n\beta)}{\sinh \beta} \right).$$

- QED and gravity are $n = 1$ and $n = 2$.
- Can relate this solution to the known cusp anomalous dimensions.
- However, $n$ could be continuous: QED and gravity are continuously related in the soft limit!
Infrared singularities remain a fertile subject of research in a variety of field theories.

QCD has a rich structure of singularities, underpinned by web mixing matrices.

Gravitational singularities, on the other hand, are extremely simple!

Intriguing relations exist between QED/ QCD and gravity.