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New insights into soft gluons and gravitons

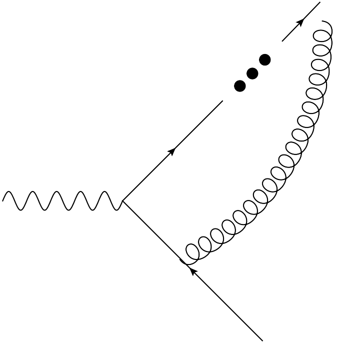
ICHEP 2012

Overview

- Brief introduction to IR singularities.
- QED, QCD and gravity.
- AdS space techniques: QED $\leftrightarrow$ gravity.
- Outlook.

Infrared divergences

- In scattering amplitudes, get singularities due to gluon emission at large distances.



- Due to integrals over gluon positions:

$$\int d^n x$$

- Uncertainty principle $\Rightarrow$ equivalent to emission of zero energy gluons.

- Common to abelian / non-abelian gauge theories, including gravity.

IR singularities

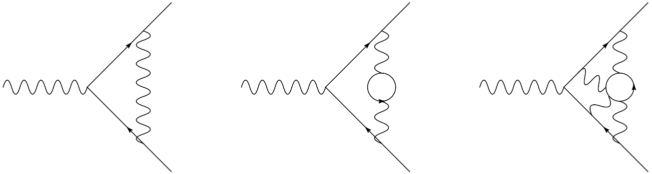
- Related to large logarithms in perturbation theory ("resummation").
- Various unsolved conjectures regarding IR singularity structures.
- Scattering amplitudes factorise into a hard and soft part (Mueller, Collins, Sen, Korchemsky, Magnea, Sterman), where the latter has an exponential form. Schematically:

$$\mathcal{S} \sim \exp\left[\sum_{W} W\right].$$

- Here W are certain special diagrams called *webs*, and differ between theories.

Webs in QED

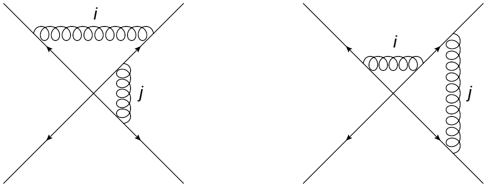
- In QED, one may show that the exponent of the soft function contains only connected subdiagrams ("QED webs"), for any number of lines:



- Originally derived using combinatoric methods (Yennie, Frautschi, Suura), and recently rederived using path integral methods (Laenen, Stavenga, White).
- Gives IR singularities to all orders in perturbation theory!

Webs in QCD

- Things are more complicated in QCD, due to non-commuting colour matrices (Gatheral; Frenkel, Taylor; Gardi, Laenen, Stavenga, White; Mitov, Sterman; Sung).



(a) (b)

- Each of these has a kinematic factor $\mathcal{F}(D)$ $(D = a,\, b)$ and a colour factor $C(D)$.
- The contribution to the exponent of the soft function turns out to be

$$\left(\begin{array}{c}\mathcal{F}(a)\\ \mathcal{F}(b)\end{array}\right)^T\left(\begin{array}{c}\tilde{C}(a)\\ \tilde{C}(b)\end{array}\right)=\left(\begin{array}{c}\mathcal{F}(a)\\ \mathcal{F}(b)\end{array}\right)^T\frac{1}{2}\left(\begin{array}{cc}1 & -1\\ -1 & 1\end{array}\right)\left(\begin{array}{c}C(a)\\ C(b)\end{array}\right).$$

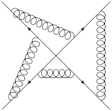
Multiparton webs in QCD

- The set of diagrams mixes in the exponent.
- Colour and kinematic information is entangled in a non-trivial way.
- Higher order diagrams also form closed sets, which contribute to the exponent of the soft amplitude according to

$$\sum_{D,D'} \mathcal{F}_D R_{DD'} C_{D'},$$

  where $R_{DD'}$ is a web-mixing matrix.
- These encode a huge amount of physics!

A four loop example



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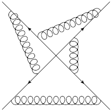

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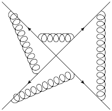

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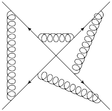

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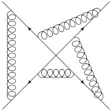

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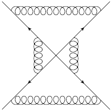

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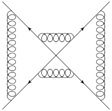

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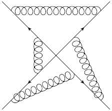

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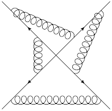

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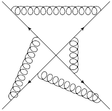

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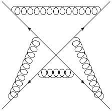

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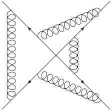

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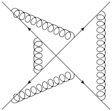

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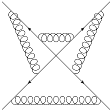

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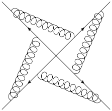

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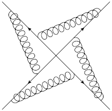

[[1,2],[2,3],[3,4],[4,1]]

Four loop mixing matrix ($\times 24$)

$$
\begin{pmatrix}
6 & -6 & 2 & 2 & -2 & 4 & -4 & 2 & -2 & -2 & -4 & 4 & -4 & 4 & 0 & 0 \\
-6 & 6 & -2 & -2 & 2 & -4 & 4 & -2 & 2 & 2 & 4 & -4 & 4 & -4 & 0 & 0 \\
2 & -2 & 6 & -2 & 2 & 4 & -4 & -2 & 2 & -6 & 4 & 4 & -4 & -4 & 0 & 0 \\
2 & -2 & -2 & 6 & 2 & 4 & -4 & -2 & -6 & 2 & -4 & -4 & 4 & 4 & 0 & 0 \\
-2 & 2 & 2 & 2 & 6 & 4 & -4 & -6 & -2 & -2 & 4 & -4 & 4 & -4 & 0 & 0 \\
2 & -2 & 2 & 2 & 2 & 4 & -4 & -2 & -2 & -2 & 0 & 0 & 0 & 0 & 0 & 0 \\
-2 & 2 & -2 & -2 & -2 & -4 & 4 & 2 & 2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & -2 & -2 & -2 & -6 & -4 & 4 & 6 & 2 & 2 & -4 & 4 & -4 & 4 & 0 & 0 \\
-2 & 2 & 2 & -6 & -2 & -4 & 4 & 2 & 6 & -2 & 4 & 4 & -4 & -4 & 0 & 0 \\
-2 & 2 & -6 & 2 & -2 & -4 & 4 & 2 & -2 & 6 & -4 & -4 & 4 & 4 & 0 & 0 \\
-2 & 2 & 2 & -2 & 2 & 0 & 0 & -2 & 2 & -2 & 4 & 0 & 0 & -4 & 0 & 0 \\
2 & -2 & 2 & -2 & -2 & 0 & 0 & 2 & 2 & -2 & 0 & 4 & -4 & 0 & 0 & 0 \\
-2 & 2 & -2 & 2 & 2 & 0 & 0 & -2 & -2 & 2 & 0 & -4 & 4 & 0 & 0 & 0 \\
2 & -2 & -2 & 2 & -2 & 0 & 0 & 2 & -2 & 2 & -4 & 0 & 0 & 4 & 0 & 0 \\
-18 & -6 & -6 & -6 & -18 & 12 & 12 & -6 & -18 & -18 & 12 & 12 & 12 & 12 & 24 & 0 \\
-6 & -18 & -18 & -18 & -6 & 12 & 12 & -18 & -6 & -6 & 12 & 12 & 12 & 12 & 0 & 24
\end{pmatrix}
$$

Multiparton webs

- Web mixing matrices have been shown to have interesting properties (e.g. zero sum rows, idempotence).
- We are beginning to translate these properties into physics (Gardi, White, Smillie).
- Also, the mathematics of web mixing matrices can be mapped to interesting combinatoric problems in computer science (Dukes, Gardi, McAslan, Scott, Steingrimmsson, White), to do with posets and order-preserving maps.
- Progress can be made with or without any physics knowledge!

The dipole formula

- For massless particles, it is conjectured that correlations between three or more particles vanish in the soft limit.
- This is known as the *dipole formula* (Becher, Neubert; Gardi, Magnea).
- Known corrections may exist at three loops and beyond (Dixon, Gardi, Magnea, Becher, Neubert, Vernazza).
- Recent additional constraints were derived from the high energy ("Regge") limit (Del Duca, Duhr, Gardi, Magnea, White).
- Web-mixing matrices may also have something to say about this!

IR singularities in GR

- IR singularities in gravity were studied by Weinberg (1965), and more recently by Naculich, Schnitzer; White; Akhoury, Saotome, Sterman.
- The exponent of the soft amplitude contains only one-loop graphs!
- One can also continuously relate QED and gravity in the soft limit, using AdS methods.

AdS methods

- The emission of soft gauge bosons is described by *Wilson line* operators:

$$\exp\left[ig_s \int_0^\infty dx_\mu A^\mu(x)\right] \leftrightarrow \exp\left[i\frac{\kappa}{2}p^\mu \int_0^\infty dx_\nu h^{\mu\nu}(x)\right]$$

- A new way to think about Wilson lines is to look at them in a Euclidean AdS space (Chien, Schwartz, Simmons-Duffin, Stewart).
- There they become point charges, whose potential energy corresponds to the structure of IR singularities (cusp anomalous dimension) in Minkowski space!

General formulation

- The potential energy of a pair of spin n Wilson lines in AdS space is (White, Miller)

$$\tilde{H}(\beta) = A_1 \left(\frac{\sinh(n\beta)}{\sinh \beta} \right) + A_2 \left(\frac{\cosh(n\beta)}{\sinh \beta} \right).$$

- QED and gravity are $n = 1$ and $n = 2$.
- Can relate this solution to the known cusp anomalous dimensions.
- However, n could be continuous: QED and gravity are continuously related in the soft limit!

Summary

- Infrared singularities remain a fertile subject of research in a variety of field theories.
- QCD has a rich structure of singularities, underpinned by web mixing matrices.
- Gravitational singularities, on the other hand, are extremely simple!
- Intriguing relations exist between QED/ QCD and gravity.