Scale invariance and the electroweak symmetry breaking

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with R. Foot, K.L. McDonald and R. R. Volkas:

Phys. Lett. B655 (2007)156-161
Phys. Rev. D76 (2007) 075014
Phys. Rev. D77 (2008) 035006
Phys. Rev. D82 (2010) 035005
Phys. Rev. D84 (2011) 075010
arXiv:1112.0607

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Motivation

- Scale invariance is a global symmetry under rescaling of coordinates and fields according their canonical dimensions

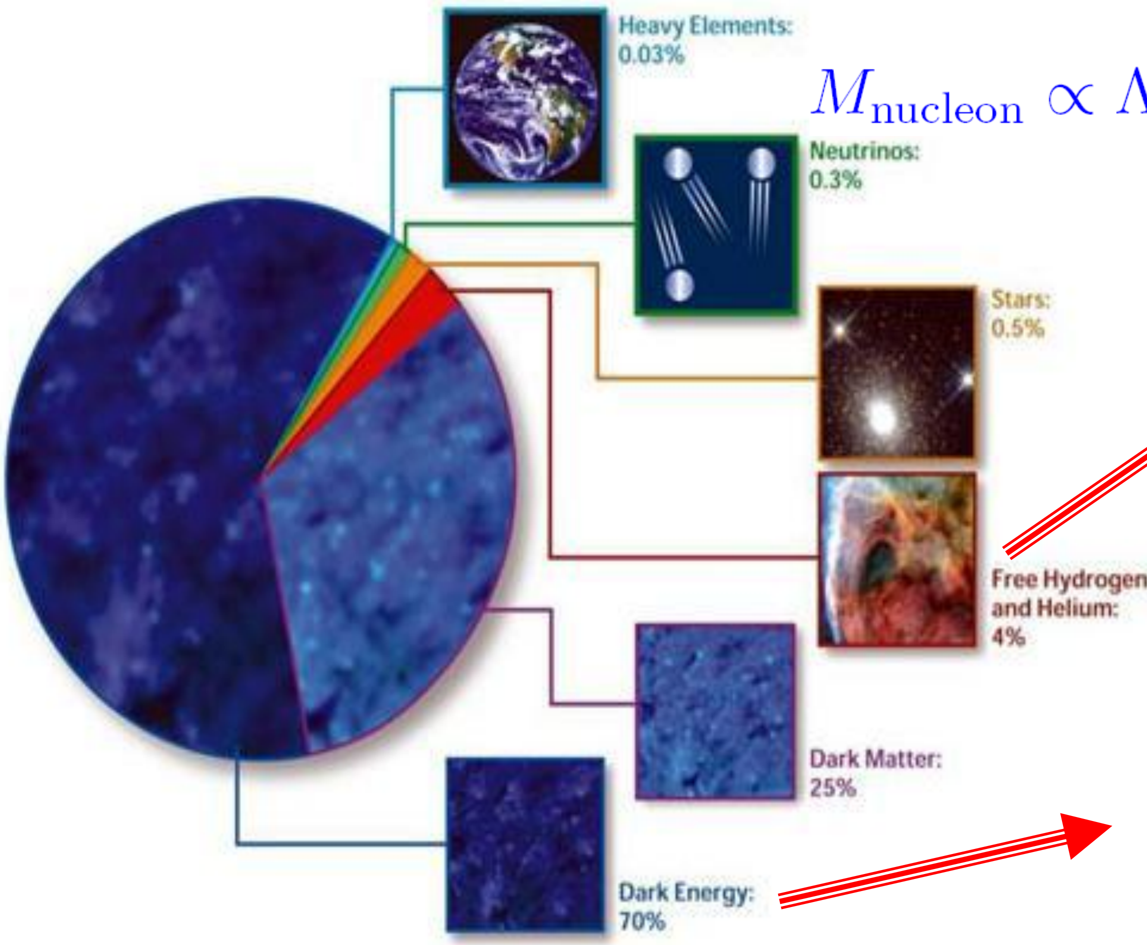
$$x_\mu \to t x_\mu, \;\; S(x) \to tS(tx), \;\; V_\mu(x) \to tV_\mu(tx), \;\; F(x) \to t^{3/2}F(tx)$$

- Scale invariance is typically an anomalous symmetry, that is, in most of the physically relevant theories it is an exact symmetry of the classical action [~$\mathcal{O}(\hbar^0)$] and is broken by quantum corrections [~$\mathcal{O}(\hbar)$].

- If the scale invariance is not an exact symmetry of the full theory why does it matter?

Motivation

Precedent in Nature: The mass of the visible matter is essentially emerges from the quantum dynamics of (nearly) scale-invariant strong interactions



$$M_{\mathrm{nucleon}} \propto \Lambda_{\mathrm{QCD}} \Longrightarrow \Lambda_{\mathrm{QCD}} \approx \mu e^{\frac{2\pi}{b\alpha_s(\mu)}}$$

(dimensional transmutation)

$$\Lambda_{\mathrm{exp}} = 10^{-120}\Lambda_{\mathrm{theor}}$$

(vac. flucts. of quantum fields)

Motivation

- It is conceivable to think that all the masses are generated quantum mechanically through the mechanism of dimensional transmutation in classically scale invariant theories.

- Non-perturbative scenarios – technicolour models and alike. However, typically predict (super)heavy Higgs boson, incompatible with the LHC discovery of a Higgs-like particle of mass $m_h \approx 125.5$ GeV announced at this conference.

- Dimensional transmutation is a generic phenomenon which exist in perturbative theories as well [S.R. Coleman and E.J. Weinberg, 1973].

- Classical scale invariance maybe a low-energy remnant of a fundamental theory. E.g., in string theory:

~~conformal invariance~~ + ~~supersymmetry~~

- Classical scale invariance resolves the problem of quadratic “divergences”

Scale invariance and quadratic “divergences”

- Wilsonian effective theory with cut-off scale Λ:

$$Z_\Lambda[J_S] = \int DS \; \exp\left(i\int d^4x \; [\mathcal{L}_\Lambda + J_S S]\right)$$

$$\mathcal{L}_\Lambda = \frac{1}{2}(\partial_\mu S)^2 - \frac{1}{2}m^2(\Lambda)S^2 - \frac{\lambda(\Lambda)}{4}S^4 + ...$$

- Compute quantum corrections:

$$m_R^2(\mu) = m^2(\Lambda) + \frac{3\lambda}{16\pi^2}\left[\Lambda^2 - m^2(\Lambda)\log\left(\Lambda^2/\mu^2\right)\right]$$

- Thus, a light scalar,

$$m_R^2 << \Lambda^2$$

is “unnatural” (hierarchy problem)

Scale invariance and quadratic "divergences"

- Suppose the underline theory is scale-invariant:

$$Z[J_S, J_H] = \int [DS\ DH] \exp\left(i \int d^4x\ [\mathcal{L} + J_S S + J_H H]\right)$$

$$\mathcal{L}[tS, tH] = t^4 \mathcal{L}[S, H]$$

- Then

$$m_R^2(\Lambda) = 0 = m^2(\Lambda) + \frac{3\lambda}{16\pi^2}\Lambda^2$$

is a natural renormalization condition which is imposed due to the absence of a mass parameter in the scale-invariant bare Lagrangian [Foot, A.K., Volkas; Meissner, Nicolai, 2007].

- (Anomalous) Ward-Takahashi identity [W.A. Bardeen, 1995]:

$$T_\mu^\mu = \sum_i \beta_i(\lambda) \mathcal{O}_i$$

- Masses are generated from spontaneous breaking of scale invariance through the mechanism of dimensional transmutation.

Scale-invariance: how does it work?

A. Classical (tree-level) approximation

- Scale invariance is an exact global symmetry.

$$x_\mu \to t x_\mu, \quad S(x) \to tS(tx), \quad V_\mu(x) \to tV_\mu(tx), \quad F(x) \to t^{3/2}F(tx)$$

- Scale-invariant classical potential $V_0(S_i) = \sum_{i,j,k,l} \lambda_{ijkl}\, S_i S_j S_k S_l$

- Hyper-spherical parameterization:

$$S_i(x) = r(x)\cos\theta_i(x)\prod_{k=1}^{i-1}\sin\theta_k(x) \qquad V_0(r,\theta_i) = r^4 f(\lambda_{ijkl},\theta_i)$$

- VEV equation:

$$\left.\frac{\partial V_0}{\partial r}\right|_{r=\langle r\rangle,\ \theta_i=\langle\theta_i\rangle} = 0 \Longrightarrow f(\lambda_{ijkl},\langle\theta_i\rangle) = 0, \langle r\rangle \neq 0$$

a. *r(x)* is massless (Goldstone theor.): $m_r^2 \equiv \left.\frac{d^2V_0}{dr^2}\right|_{r=\langle r\rangle,\theta_i=\langle\theta_i\rangle} = 12\langle r\rangle^2 f(\lambda_{ijkl},\langle\theta_i\rangle) = 0$

b. Cosmological const. is zero: $\Lambda_0 \equiv V_0(\langle r\rangle,\langle\theta_i\rangle) = \langle r\rangle^4 f(\lambda_{ijkl},\langle\theta_i\rangle) = 0$

Scale-invariance: how does it work?

B. Quantum corrections

- Scale invariance is not an exact symmetry in quantum world. E.g., quantum corrections modify the scalar potential [S.R. Coleman and E.J. Weinberg, 1973]:

$$V = A(g_a(\mu), m_x(\mu), \theta_i(\mu), \mu) r^4(\mu) + B(g_a(\mu), m_x(\mu), \theta_i(\mu), \mu) r^4(\mu) \log\left(\frac{r^2(\mu)}{\mu^2}\right)$$
$$+ C(g_a(\mu), m_x(\mu), \theta_i(\mu), \mu) r^4(\mu) \left[\log\left(\frac{r^2(\mu)}{\mu^2}\right)\right]^2 + \ldots$$

- μ is an arbitrary renormalization scale; physics does not depend on μ, that is,

$$\mu\frac{dV}{d\mu} \equiv \left(\mu\frac{\partial}{\partial\mu} + \sum_a \beta_a \frac{\partial}{\partial g_a} + \sum_x \gamma_x \frac{\partial}{\partial m_x} - \gamma_r \frac{\partial}{\partial r} - \sum_i \gamma_i \theta_i \frac{\partial}{\partial \theta_i}\right) V = 0$$

- Take $\mu = \langle r \rangle$, then RG equations are:

$$B(\mu = \langle r \rangle) = \frac{1}{2}\mu\frac{\partial A}{\partial \mu}\bigg|_{\mu=\langle r \rangle},$$
$$C(\mu = \langle r \rangle) = \frac{1}{4}\mu\frac{\partial B}{\partial \mu}\bigg|_{\mu=\langle r \rangle}$$

Scale-invariance: how does it work?

B. Quantum corrections

- The VEV equation:

$$\frac{dV}{dr} = 0 \quad \Rightarrow \quad A(\mu = \langle r \rangle) + 2B(\mu = \langle r \rangle) = 0,$$

determines $\langle r \rangle$ in terms of running dimensional coupling(s) (dimensional transmutation)

- PGB dilaton mass:

$$m_r^2 = (2B + 8C)\langle r \rangle^2$$

Evidently, the vacuum stability requires: $B + 4C \geqslant 0$

- Cosmological constant: $\Lambda = -2B\langle r \rangle^4$
- Perturbative series:

$$A \approx A^{(0)} = f(\lambda_{ijkl}, \theta_i), \quad B \approx B^{(1)} = \frac{1}{64\pi^2\langle r \rangle^4}[3\mathrm{Tr}m_V^4 + \mathrm{Tr}m_S^4 - 4\mathrm{Tr}m_F^4],$$

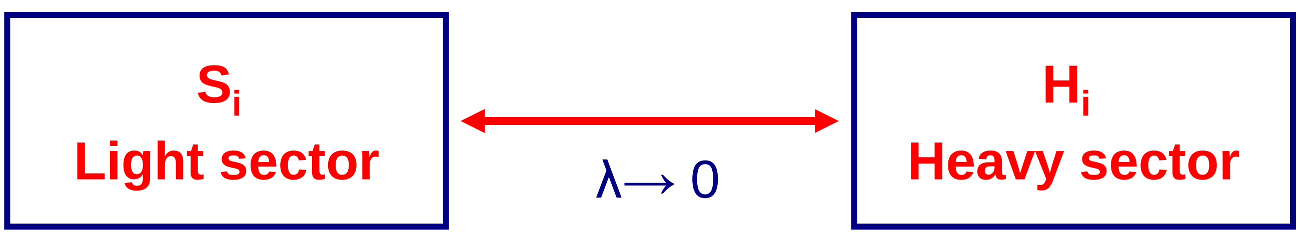
$$C \approx C^{(2)} = \frac{1}{64\pi^2\langle r \rangle^4}\left[3\mathrm{Tr}m_V^4\gamma_V + \mathrm{Tr}m_S^4\gamma_S - 4\mathrm{Tr}m_F^4\gamma_F\right]$$

Scale-invariance: how does it work?

- $\langle r \rangle$ -- defines the overall scale in the theory

- Hierarchies of scales are defined by $\langle \theta_i \rangle = \phi_i(\lambda)$

- Mass hierarchies in the scale-invariant theories can be maintained only through the hierarchies in dimensionless couplings. These hierarchies are technically natural if decoupling limit can be achieved. This is a limit, when heavy sector fields completely decouple from the light sector fields when certain couplings are taken to zero [R. Foot, A. K., K.L. McDonald, R.R. Volkas, Phys. Rev. D77 (2008) 035006; R. Foot, A. K., R.R. Volkas, Phys. Rev. D82 (2010) 035005].



Scale-invariant models

- Scale-invariant Standard Model [S.R. Coleman and E.J. Weinberg, 1973]:

$$\phi(x) = \frac{r(x)}{\sqrt{2}} \mathrm{e}^{i\pi^a(x)\tau^a}, \quad V_0 = \frac{\lambda}{2}\left(\phi^\dagger\phi\right)^2 = \frac{\lambda}{8} r^4$$

- Higgs is the PGB dilaton:

$$m_h^2 \approx \frac{1}{8\pi^2 v_{\mathrm{EW}}^2}\left[6m_W^4 + 3m_Z^4 - 12m_t^4\right] < 0$$

No electroweak symmetry breaking in the scale-invariant SM

- Successful EWSB in scale-invariant theories requires extension of the bosonic sector of the Standard Model.

R. Hempfling Phys. Lett. B379 (1996) 153;

W.-F. Chang *et al.* Phys.Rev. D75 (2007) 115016;

K.A. Meissner *et al.* Phys.Lett. B648 (2007) 312

T. Hambye *et al.* Phys.Lett. B659 (2008) 651;

S. Iso *et al.* Phys.Lett. B676 (2009) 81;

M. Holthausen *et al.* Phys.Rev. D82 (2010) 055002; L.Alexander-Nunneley *et al.* JHEP 1009 (2010) 021; …

Scale-invariant models

- Minimal scale-invariant Standard Model [R. Foot, A. K., R.R. Volkas, Phys. Lett. B655 (2007)156-161]:

$$V_0 = \frac{\lambda_1}{2}\left(\phi^\dagger\phi\right)^2 + \frac{\lambda_2}{4}S^4 + \frac{\lambda_3}{2}\left(\phi^\dagger\phi\right)S^2 = \frac{r^4}{8}\left(\lambda_1\cos^4\theta + 2\lambda_2\sin^4\theta + 2\lambda_3\sin\theta\cos\theta\right)$$

- Demanding cancellation of the cosmological constant the dilaton mass is generated at 2-loop. A light dilaton is a generic prediction of such class of models [R. Foot, A. K., arXiv:1112.0607].

$$V_{\text{min}} = -2B = 0 \Longrightarrow \quad m_r^2 = 8C\langle r\rangle^2$$

$$m_r \approx 7 - 10 \text{ GeV}$$

- Higgs mass prediction: $m_h \approx 12^{1/4} m_t \approx 300 \text{ GeV}$

Excluded by LHC

Dark matter: Scale-invariant mirror world

- Scale-invariant scalar potentials are automatically invariant under discrete Z_2 symmetry. If Z_2 is unbroken some heavy scalar states are stable and can play the role of dark matte.

- However, some recent experiments (DAMA/LIBRA, CoGeNT) provide evidence for light (7-15 GeV) dark matter particles. The best explanation of these experiments is provided by the mirror dark matter models [R. Foot, H. Lew and R. R. Volkas, Phys. Lett. B272, (191) 67; R. Foot, Phys. Rev. D 78 (2008) 043529; Phys. Lett. B692 (2010)].

- Scale-invariant mirror world models are discussed in: R. Foot, A.K. and R. R. Volkas Phys. Lett. B655 (2007)156-161; Phys. Rev. D82 (2010) 035005 and R. Foot, A. K., arXiv:1112.0607. Extended scalar sector has further motivation due to the mirror symmetry doubling.
- Higgs sector, besides the light dilaton, contains two neutral Higgs scalars:

$$m_H = (24m_t^4 - m_h^4)^{1/4} \approx 355 \text{ GeV}$$

Neutrino masses in scale-invariant models

- Different possibilities of neutrino mass generation is discussed in R. Foot, A. K., K.L. McDonald, R.R. Volkas, Phys. Rev. D76 (2007) 075014.

- One particular model contains extra electroweak triplet scalar particle Δ(type II see-saw) [R. Foot, A. K., arXiv:1112.0607]:

$$m_\Delta = (2m_t^4 - m_h^4/6)^{1/4} \approx 190 \text{ GeV}$$

Conclusions

- Scale invariance: all mass scales have purely quantum mechanical origin.
- Very simple models with classical scale invariance are capable to resolve the hierarchy problem without introducing supersymmetry and/or other exotics. Some of the predictions are testable at LHC and/or future linear collider
- Electroweak scale invariant models with vanishing CC generically predict a light dilaton.
- Realistic scale-invariant models require extended bosonic sector with masses correlated with the masses of the Higgs boson and the top quark.
- Hopefully, some of the features of scale-invariant theories described in this talk will be observed at LHC.