Multi-Jet Matching @NLO

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Introduction

- Parton Showers
- Tree-level multi-jet matching
- NLO multi-jet matching

Disclaimer: ME+PS matching technically complicated.
In 12 minutes I can only give a flavour of what is involved.

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Parton Showers

- Exclusive final states
- (N)LL Resummation to all orders
- Soft and collinear approximation
- Crappy hard wide-angle emissions
- Unitary procedure
- Crappy total cross section

Exclusive n-jet cross section, Parton Shower style

$$d\sigma_n^{ex} = F_0 |\mathcal{M}_0|^2 \, d\phi_0 \times \left[\prod_{i=1}^{n} \alpha_S \frac{F_i}{F_{i-1}} P_i d\rho_i dz_i \Pi_{i-1} \right] \Pi_n(\rho_n, \rho_{\mathrm{MS}})$$

- $|\mathcal{M}_0|^2 \, d\phi_0$: Born-level ME and phase space.
- F_i: PDF's from both sides for the i-parton state.
- $P_i(\rho, z) d\rho dz \approx \frac{|\mathcal{M}_i|^2 d\phi_i}{|\mathcal{M}_{i-1}|^2 d\phi_{i-1}} \equiv P_i^{ME}(z) d\rho dz$
- ρ, z: Splitting variables. Assume ρ is a suitable jet scale.
- ρ_{MS}: jet resolution scale.
- $\Pi_i(\rho_{i-1}, \rho_i)$: No-emission probabilities.
- Ignore running of α_S and PDF's for now.

No-emission probabilities

$$\Pi_i(\rho_i, \rho_{i+1}) = \exp\left(-\int_{\rho_{i+1}}^{\rho_i} d\rho dz\, \alpha_S \frac{F_{i+1}}{F_i} P_{i+1}\right)$$

The probability of not having any splittings above the scale ρ_{i+1} starting the shower from the state i at scale ρ_i.

With the veto-algorithm it is easy to generate a hardest emission using the no-emission probabilities, even if the integrand is complicated and we have several possible processes.

Also, if we generate one emission (ρ, z) from a given state, i, starting from a maximum scale ρ_i

$P(\rho < \rho_{i+1}) = \Pi_i(\rho_i, \rho_{i+1})$

In addition, if we get a $\rho > \rho_{i+1}$, we can continue generating another emission from state i below ρ (discarding the first one), and then another one, etc. The average number of emissions above ρ_{i+1} is then given by the exponent

$$\langle n \rangle = \int_{\rho_{i+1}}^{\rho_i} d\rho dz\, \alpha_S \frac{F_{i+1}}{F_i} P_{i+1} = -\log \Pi_i(\rho_i, \rho_{i+1})$$

and

$$\langle n(n-1) \rangle = \left(\int_{\rho_{i+1}}^{\rho_i} d\rho dz\, \alpha_S \frac{F_{i+1}}{F_i} P_{i+1} \right)^2$$

Fixed-order expansion of a parton shower

(using $\mathcal{P}_i = \frac{F_i}{F_{i-1}} P_i$)

$$\frac{d\sigma_0^{ex}}{d\phi_0} = F_0 \left|\mathcal{M}_0\right|^2 \left[1 - \alpha_S \int_{\rho_{MS}}^{\rho_0} d\rho dz\, \mathcal{P}_1 + \frac{\alpha_S^2}{2} \left(\int_{\rho_{MS}}^{\rho_0} d\rho dz\, \mathcal{P}_1 \right)^2 \right]$$

$$\frac{d\sigma_1^{ex}}{d\phi_0} = F_0 \left|\mathcal{M}_0\right|^2 \alpha_S \mathcal{P}_1 d\rho_1 dz_1 \times \left[1 - \alpha_S \int_{\rho_1}^{\rho_0} d\rho dz\, \mathcal{P}_1 - \alpha_S \int_{\rho_{MS}}^{\rho_1} d\rho dz\, \mathcal{P}_2 \right]$$

$$\frac{d\sigma_2}{d\phi_0} = F_0 \left|\mathcal{M}_0\right|^2 \alpha_S^2 \mathcal{P}_1 d\rho_1 dz_1 \mathcal{P}_2 d\rho_2 dz_2 \Theta(\rho_1 - \rho_2)$$

Unitary to all orders in α_S.

CKKW(-L)

We can generate inclusive tree-level states with eg. MADGRAPH for up to a hand-full of jets using a cutoff ρ_{MS}.

These are then made exclusive by reweighting with no-emission probabilities (in CKKW-L generated by the shower itself)

Also fix the running of α_S and PDF's.

Add normal shower emissions below ρ_{MS}.

Add all samples together.

- Dependence on the merging scale cancels to the precision of the shower.
- If the merging scale is not defined in terms of the shower ordering variable, we need vetoed and truncated showers.
- Breaks the unitarity of the shower.

Higher-order tree-level matching

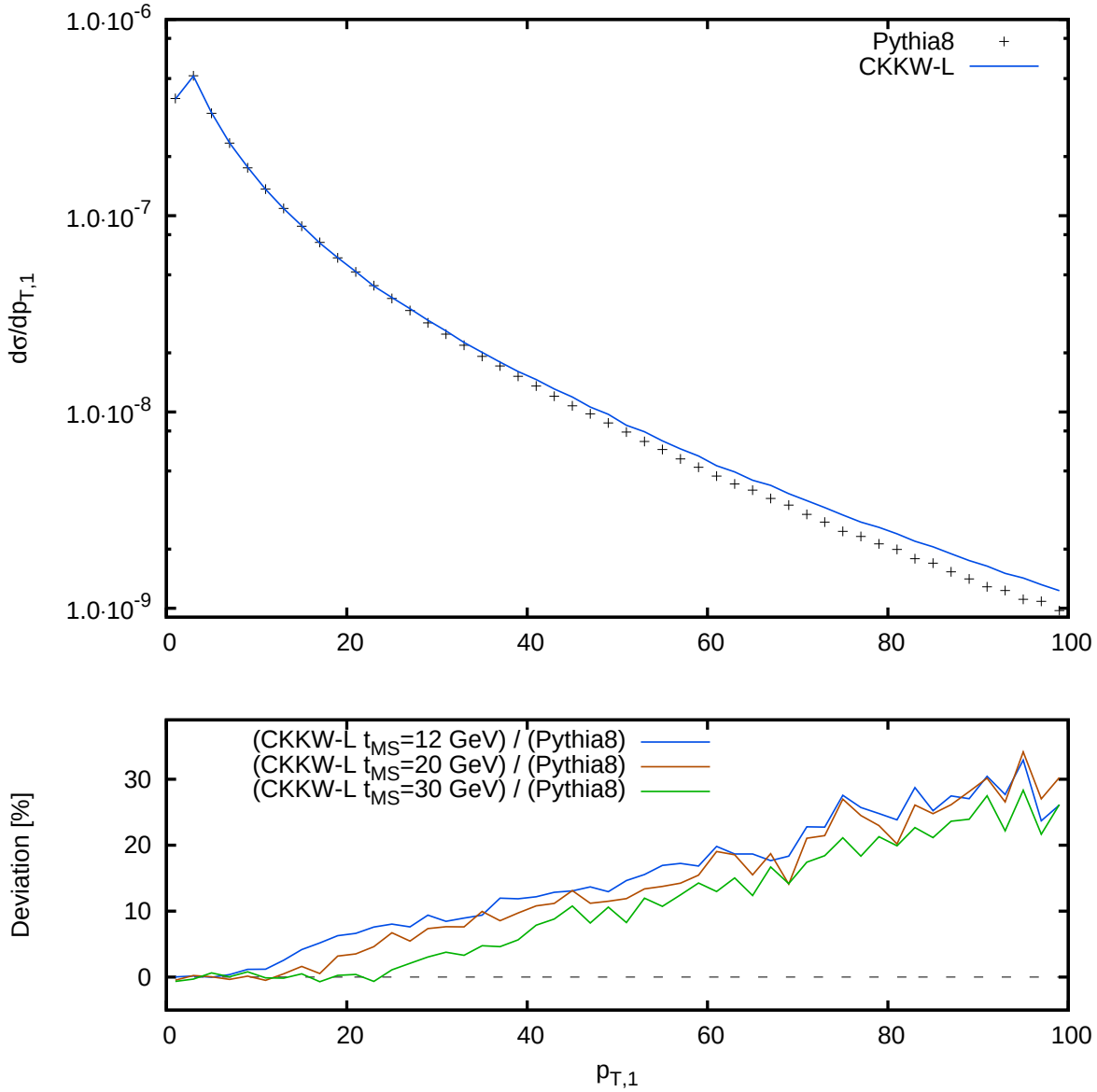
$$\frac{d\sigma_0^{ex}}{d\phi_0} = F_0 \left|\mathcal{M}_0\right|^2 \left[1 - \alpha_S \int_{\rho_{MS}}^{\rho_0} d\rho dz\, \mathcal{P}_1 + \frac{\alpha_S^2}{2}\left(\int_{\rho_{MS}}^{\rho_0} d\rho dz\, \mathcal{P}_1\right)^2\right]$$

$$\frac{d\sigma_1^{ex}}{d\phi_0} = F_0 \left|\mathcal{M}_0\right|^2 \alpha_S \mathcal{P}_1^{ME} d\rho_1 dz_1$$

$$\times \left[1 - \alpha_S \int_{\rho_1}^{\rho_0} d\rho dz\, \mathcal{P}_1 - \alpha_S \int_{\rho_{MS}}^{\rho_1} d\rho dz\, \mathcal{P}_2\right]$$

$$\frac{d\sigma_2}{d\phi_0} = F_0 \left|\mathcal{M}_0\right|^2 \alpha_S^2 \mathcal{P}_1^{ME} d\rho_1 dz_1 \mathcal{P}_2^{ME} d\rho_2 dz_2 \Theta(\rho_1 - \rho_2)$$

NOT unitary. Gives aritficial dependence of ρ_{MS}.
(Beyond (N)LL but may still be large.)


Pythia8
CKKW-L
dσ/dp_T,1
1.0·10^-6
1.0·10^-7
1.0·10^-8
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Deviation [%]
30
20
10
0
p_T,1

Mature procedure. Available in

- (HERWIG++)
- SHERPA
- PYTHIA8
- Also MLM-procedure, ALPGEN + HERWIG/PYTHIA

Why worry about unitarity when cross section anyway is only leading order?

Parton Showers get many shapes of observables right. Maybe it is enough to just calculate a NLO K-factor

$$K_0 = \frac{\int d\sigma_0^{NLO}}{\int d\sigma_0^{LO}}$$

But we want to do better than that.

Cf. POWHEG

$$K(\phi_0) = \frac{d\sigma_0^{NLO}}{d\sigma_0^{LO}}$$

where σ_0^{NLO} is the inclusive NLO cross section projected down to the Born-level phase space.

Let's look at the 0-jet exclusive cross section in CKKW-L in more detail (adding a flat K-factor).

$$\frac{d\sigma_0^{ex}}{d\phi_0} = K_0 F_0 \left|\mathcal{M}_0\right|^2 \Pi_0(\rho_0, \rho_{\text{MS}})$$
$$= K_0 F_0 \left|\mathcal{M}_0\right|^2 \left[1 - \alpha_S(\rho) \int_{\rho_{\text{MS}}}^{\rho_0} d\rho dz \, \frac{F_1(\rho)}{F_0(\rho)} P_1 + \ldots\right]$$

We want to replace these terms by the exact *exclusive* cross section

Generate a Born-level sample point using POWHEG's $K(\phi_0)$.

$$\frac{d\sigma_0^{NLO}}{d\phi_0} = K(\phi_0) F_0(\mu_F) \left|\mathcal{M}_0\right|^2$$

Generate a one-jet sample using the tree-level ME

$$\frac{d\sigma_1}{d\phi_0} = F_0(\mu_F) \left|\mathcal{M}_0\right|^2 \alpha_S(\mu_R) \int_{\rho_{MS}}^{\rho_0} \frac{F_1(\mu_F)}{F_0(\mu_F)} P_1^{ME}$$

But recluster it to a 0-jet state.
This corresponds directly to the α_S term in the PS no-emission probability.

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$\mathbf{NL}^3_{SP}$

If we now take the CKKW-L sample but reweight it by subtracting

$$F_0 \left|\mathcal{M}_0\right|^2 \left[K_0 \Pi_0(\rho_0, \rho_{\text{MS}}) - K_0 + \alpha_S(\mu_R) \int_{\rho_{\text{MS}}}^{\rho_0} d\rho dz \, \frac{F_1(\mu_F)}{F_0(\mu_F)} P_1\right]$$

Then we add the POWHEG sample and the reclustered 1-jet sample Only adding a parton shower below ρ_{MS}.

The 1-jet and higher order exclusive CKKW-samples stays the same.

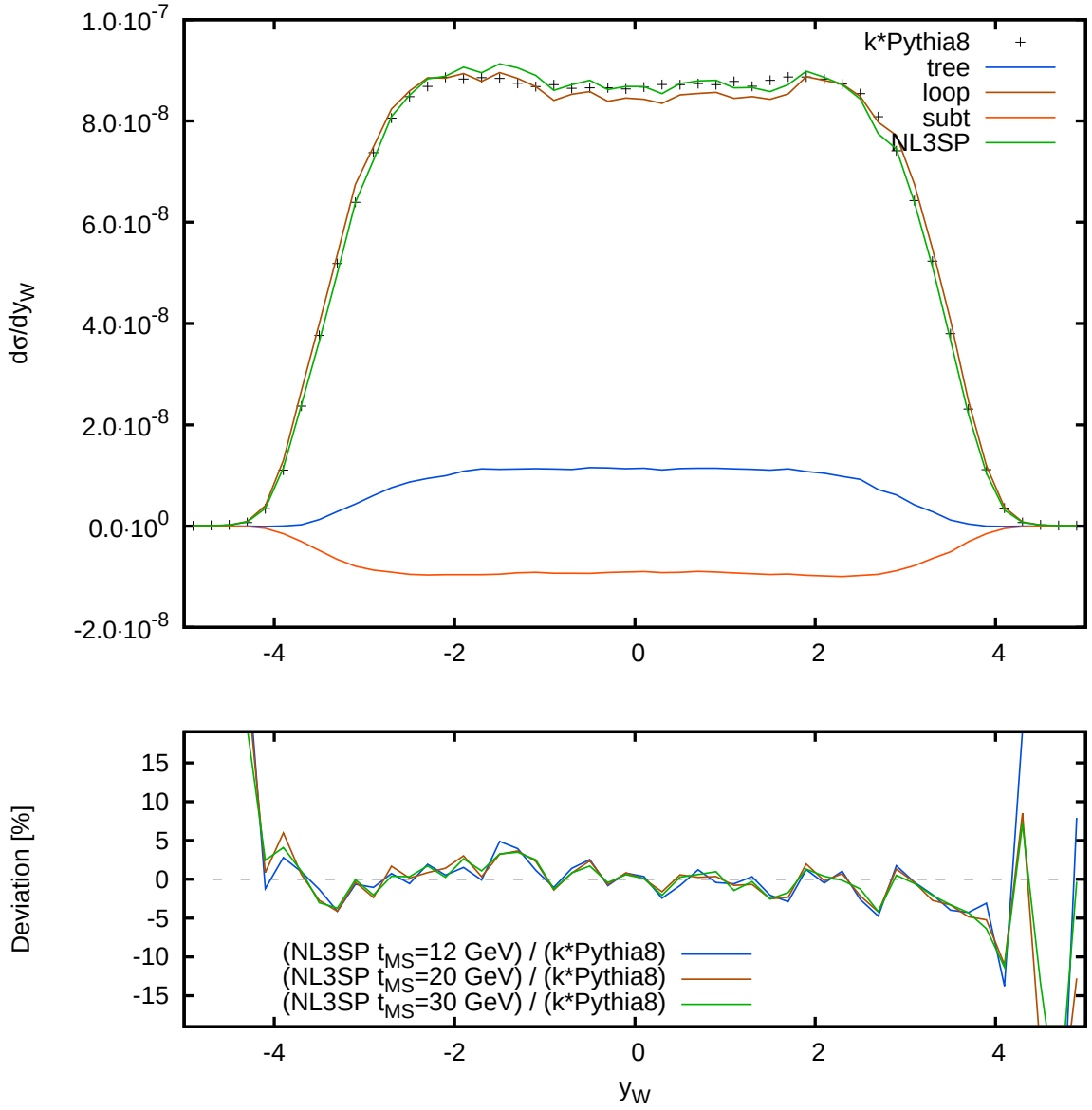
We then get something which is correct to NLO but with all higher order terms taken from the Parton Shower.

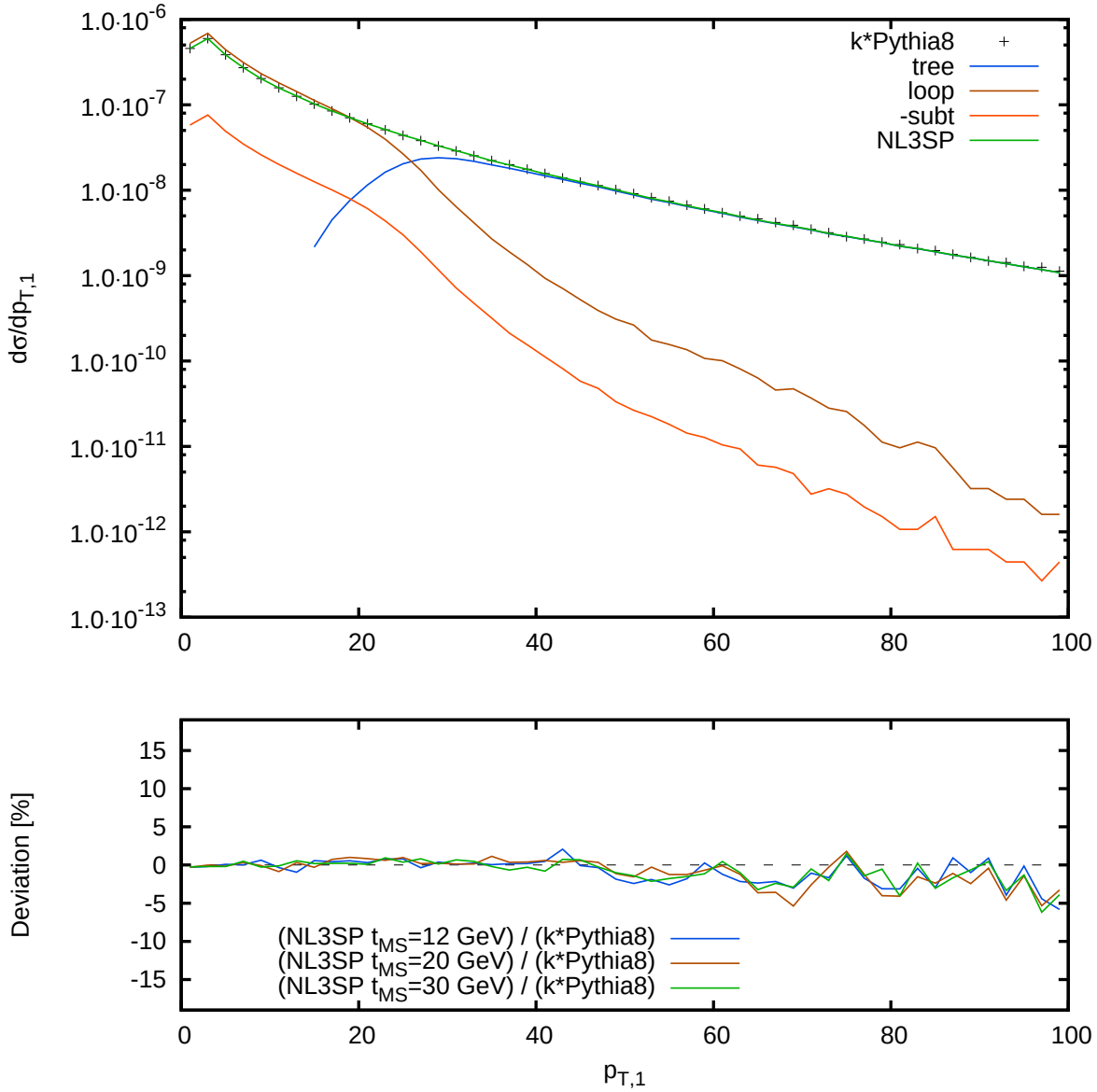
$\mathbf{NL}^3_{SP}$

$$\frac{d\sigma_0^{ex}}{d\phi_0} = F_0\,|\mathcal{M}_0|^2\left[K(\phi_0) - \alpha_S\int_{\rho_{\rm MS}}^{\rho_0} d\rho dz\,\mathcal{P}_1^{ME} + K_0\frac{\alpha_S^2}{2}\left(\int_{\rho_{\rm MS}}^{\rho_0} d\rho dz\,\mathcal{P}_1\right)^2\right]$$

$$\frac{d\sigma_1^{ex}}{d\phi_0} = F_0\,|\mathcal{M}_0|^2\,K_0\alpha_S\mathcal{P}_1^{ME}d\rho_1 dz_1 \times\left[1 - \alpha_S\int_{\rho_1}^{\rho_0} d\rho dz\,\mathcal{P}_1 - \alpha_S\int_{\rho_{\rm MS}}^{\rho_1} d\rho dz\,\mathcal{P}_2\right]$$

$$\frac{d\sigma_2}{d\phi_0} = F_0\,|\mathcal{M}_0|^2\,K_0\alpha_S^2\mathcal{P}_1^{ME}d\rho_1 dz_1\mathcal{P}_2^{ME}d\rho_2 dz_2\Theta(\rho_1-\rho_2)$$


k*Pythia8
tree
loop
subt
NL3SP
dσ/dy_W
1.0·10^-7
8.0·10^-8
6.0·10^-8
4.0·10^-8
2.0·10^-8
0.0·10^0
-2.0·10^-8
-4
-2
0
2
4
Deviation [%]
15
10
5
0
-5
-10
-15
(NL3SP t_MS=12 GeV) / (k*Pythia8)
(NL3SP t_MS=20 GeV) / (k*Pythia8)
(NL3SP t_MS=30 GeV) / (k*Pythia8)
y_W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p_T,1

Now let's go to the 1-jet exclusive cross section in CKKW-L

$$\frac{d\sigma_1^{ex}}{d\phi_0} = F_1(\mu_F)\,|\mathcal{M}_0|^2\,P_1^{ME}\,\alpha_S(\mu_R)d\rho_1 dz_1$$
$$\times K\,\frac{F_0(\mu_F)}{F_1(\mu_F)}\frac{F_1(\rho_1)}{F_0(\rho_1)}\frac{\alpha_S(\rho_1)}{\alpha_S(\mu_R)}\Pi_0(\rho_0,\rho_1)\Pi_1(\rho_1,\rho_{\rm MS})$$
$$= F_1(\mu_F)\,|\mathcal{M}_0|^2\,P_1^{ME}\,\alpha_S(\mu_R)d\rho_1 dz_1$$
$$\times\left[K + A\alpha_S(\mu_R) + B\alpha_S(\mu_R) - \alpha_S(\mu_R)\int_{\rho_1}^{\rho_0} d\rho dz\,\frac{F_1(\mu_F)}{F_0(\mu_F)}P_1\right.$$
$$\left.-\alpha_S(\mu_R)\int_{\rho_{\rm MS}}^{\rho_1} d\rho dz\,\frac{F_2(\mu_F)}{F_1(\mu_F)}P_2\right] + \mathcal{O}\left(\alpha_S^2(\mu_R)\right)$$

A and B can be calculated from the leading order running of α_s and the PDF's.

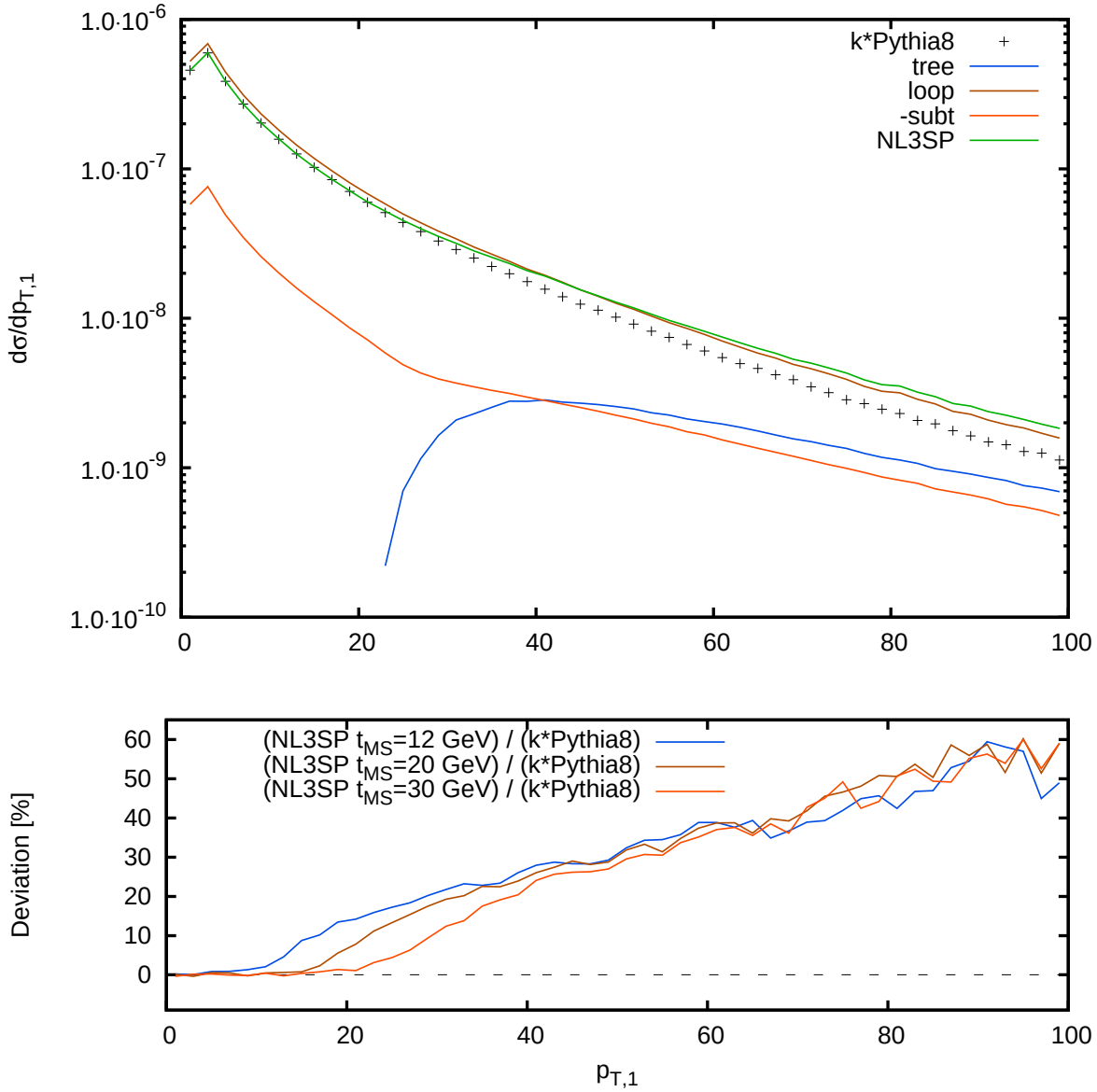
Again we get the 1-jet NLO inclusive sample from POWHEG's $K_1(\phi_1)$ (using $\rho_{\rm MS}$ as cutoff) and make it exclusive by subtracting the reclustered 2-jet tree-level ME

$$\frac{d\sigma_2^{rec}}{d\phi_0} = F_0(\mu_F)\,|\mathcal{M}_0|^2\,\alpha_S^2(\mu_R)\frac{F_1(\mu_F)}{F_0(\mu_F)}P_1^{ME}\,d\rho_1 dz_1\int_{\rho_{\rm MS}}^{\rho_0}\frac{F_2(\mu_F)}{F_1(\mu_F)}P_2^{ME}$$

(Note that this gives both ordered and un-ordered emissions)

- ▸ 0-jet LO sample, reweighted with subtracted CKKW
- \+ 0-jet NLO sample.
- \- 1-jet LO sample, reclustered to 0-jet
- ▸ 1-jet LO sample, reweighted with subtracted CKKW
- \+ 1-jet NLO sample.
- \- 2-jet LO sample, reclustered to 1-jet
- ▸ 2-jet LO sample, reweighted with CKKW
- ▸ ...

In principle we could go higher if we have an generator for n jets to NLO and tree-level $n+1$ jets .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The bottom line

If we have a generator for up to N-partona at NLO:

Any $n \leq N$ jet observable above ρ_{MS} will be correct to NLO resummed to the precision of the Parton Shower used.

NL^3_{SP} summary

- Has been implemented in PYTHIA8, but a lot more testing to be done.
- A bit complicated with many samples to be combined.
- Can be generalized to arbitrary jet multiplicities.
- Lots of technical details omitted here.
- There are some non-trivial ρ_{MS} dependencies left which may be large (α_S^2 term in 0-jet contribution).
- In principle we can go to NNLO 0-jet (need to recluster 2-jet ME twice).