Quantum corrections to broken N=8 supergravity

Based on arXiv:1205.4711, with Gianguido Dall'Agata

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Motivations

Higgs observation as conference opening! Triumph of the SM as renormalizable gauge quantum field theory

Higgs boson expected with other new physics at the TeV scale, but so far data have not been encouraging

Next step may be connected with quantum gravity, whose scale is the nearest one we can be sure of

Naturalness different in the presence of gravity: vacuum energy density much more unnatural than weak scale

Try to study theories: (i) highly constrained; (ii) where well-defined calculations can be carried out

String theory? Still many assumptions made in string phenomenology, see e.g. de Sitter vacua

A constrained and calculable toy theory, even if non-realistic, might teach us some useful lessons

Among four-dimensional quantum field theories, the most constrained one is N=8 supergravity

N=8 supergravity: field content

unique multiplet: $2^8 = 256 = 128_B + 128_F$ d.o.f.

$\vert +2\rangle$:	1 graviton
$\vert +3/2, i\rangle = Q_i \vert +2\rangle$:	8 gravitini
$\vert +1, [ij]\rangle = Q_i Q_j \vert +2\rangle$:	28 vectors
$\vert +1/2, [ijk]\rangle = Q_i Q_j Q_k \vert +2\rangle$:	56 fermions
$\vert 0, [ijkl]\rangle = Q_i Q_j Q_k Q_l \vert +2\rangle$:	70 scalars

and similarly for the CPT conjugates of negative helicity, ending with $\vert -2\rangle$

The ungauged theory

[De Wit-Freedman 1977; Cremmer-Julia 1978+1979]

$E_{7(7)}$ **duality group**, invariance of B.I. & E.O.M.

Gauge group $U(1)^{28}$, no charged fields

Minkowski background with exact N=8
is solution of the classical E.O.M.
Vanishing potential, all fields massless

Remarkable UV properties: on-shell finiteness up to 4 loops, perhaps to all perturbative orders?

The gauged theories

A subgroup G of $E_{7(7)}$ is made local (dim ≤ 28)

Theory fully determined by embedding tensor

[DeWit-Samtleben-Trigiante 2003+2007]

gauge generators

$$X_M = \Theta_M{}^\alpha t_\alpha$$

E_7 generators

M=1,…,56 counts electric and magnetic vectors

$$[X_M, X_N] = -X_{MN}{}^P X_P \qquad X_{MN}{}^P = \Theta_M{}^\alpha (t_\alpha)_N{}^P$$

Supersymmetry ➔ linear constraints on Θ

Gauge invariance ➔ quadratic constraints on Θ

Effects of gauging

Gauge coupling constant g deformation parameter:

$$\partial_\mu \quad \longrightarrow \quad D_\mu \equiv \partial_\mu - g\,A_\mu^M\,X_M$$

Scalar potential and mass terms are generated

Possibility of partial or total SUSY breaking

Critical points with positive, zero or negative energy

Famous examples:

SO(8) gauging with stable N=8 AdS vacuum

CSS gauging with N=0 Minkowski vacuum

(classically stable, positive semi-definite V_0)

No locally stable dS vacuum found so far

The focus of our study

Consider the gaugings leading to **classical Minkowski vacua with fully broken (N=0) supersymmetry**

Study the **1-loop corrections** to the theory around these vacua (no arguments from ungauged theory apply):

1-loop finiteness? 1-loop stable Mink or dS vacua?

One-loop effective potential V_1 controlled by **supertraces**

$$Str\, M^{2k} \equiv \sum_a (2J_a+1)\,(-1)^{2J_a}\,(M_a^2)^k$$

k=0 quartic divergence

k=1 quadratic divergence

k=2 logarithmic divergence

Our general result

At any classical Minkowski vacuum of gauged N=8

$$Str\,M^0 = Str\,M^2 = Str\,M^4 = 0$$

Skip here all technicalities. Our proof makes use of:

1. Critical point condition
2. Vanishing vacuum energy
3. Quadratic constraints

Implication: one-loop effective potential V_1 is finite

$$V_1 = \frac{1}{64\pi^2}\, Str\left(M^4 \log M^2\right)$$

(provided that no tachyons in classical spectrum)

Old and new gaugings

Until 2011, the only known gauging leading to classically stable N=0 Minkowski vacua was the one by Cremmer-Scherk-Schwarz (1979)

A recent development [Dall'Agata-Inverso (2011)]

New way of generating gaugings and vacua, including N=0 Minkowski, but also others: stay at the origin of Φ field space, solve quadratic constraints on Θ to identify consistency, gauge group, vacuum energy, masses

Old and new models

- $U(1) \ltimes T^{24}$ ➔ U(1) [x U(1)^3] (CSS)
- SO(6,2) ➔ SO(6) x SO(2) ➔ … ➔ U(1)^4
- SO(2,2) x SO(4) $\ltimes T^{16}$ ➔ …
- U(1)^2 $\ltimes T^{20}$ ➔ …

Some new features:

- Possible **tachyonic instabilities** along flat directions
- Always at least one **unbroken U(1)** factor [in SU(8)]
- Always at least 4 vectors and 6 scalars massless

An intriguing pattern emerging…

Supercharges Q_i (i=1,…,8) transform non-trivially under at most 4 unbroken U(1) factors in SU(8)

Charge vectors: $\vec{q}_i \equiv (q_i^1, \ldots, q_i^n)$

8 supercharges Q_i always come in pairs:

$$\vec{q}_1 = -\vec{q}_2 \quad \vec{q}_3 = -\vec{q}_4 \quad \vec{q}_5 = -\vec{q}_6 \quad \vec{q}_7 = -\vec{q}_8$$

Neutral graviton |±2>: $\sum_{i=1}^{8} \vec{q}_i = \vec{0}$

Charges of all other states fully determined:

|±3/2,i>: $\pm q_i$

|±1,[ij]>: $\pm(q_i+q_j)$

|±1/2,[ijk]>: $\pm(q_i+q_i+q_k)$

|0,[ijkl]>: $q_i+q_j+q_k+q_l$

Spectrum controlled by U(1) charges

$$\begin{aligned}
|2\rangle : &\quad M^2 = 0\,, \\
|3/2\,,\, i\rangle : &\quad M_i^2 = (\vec{q}_i)^2\ , \\
|1\,,\,[ij]\rangle : &\quad M_{ij}^2 = (\vec{q}_i + \vec{q}_j)^2\ , \\
|1/2\,,\,[ijk]\rangle : &\quad M_{ijk}^2 = (\vec{q}_i + \vec{q}_j + \vec{q}_k)^2\ , \\
|0\,,\,[ijkl]\rangle : &\quad M_{ijkl}^2 = (\vec{q}_i + \vec{q}_j + \vec{q}_k + \vec{q}_l)^2
\end{aligned}$$

Scalar products of charge vectors taken with suitable **field-dependent real diagonal metric**:

$$\vec{q}_i \cdot \vec{q}_j = \sum_{A=1}^{n} q_i^A\, q_j^A\ \mu_A^2$$

Not necessarily positive definite (when so, absence of tachyons in the classical spectrum guaranteed)

Some important consequences

(not proven in general, but valid for all classical N=0 Minkowski vacua identified so far)

$$Str\, M^6 = 0$$

$$Str\, M^8 = 40320 \sum_{A=1}^{n} \left(\prod_{i=1}^{8} q_i^A \right) \mu_A^8 > 0$$

Empirically, we find it implies that the one-loop effective potential V_1 is negative definite:

$$V_1 < 0$$

no stable N=0 Minkowski or dS vacua at 1 loop

Summary of conclusions

- Quadratic and quartic supertraces vanish at any classical Minkowski vacuum → **finite one-loop effective potential**

- All known gaugings leading to N=0 Minkowski vacua have at least one **unbroken U(1) factor** in the gauge group, with the **spectrum determined by the corresponding charges**

- As a consequence, **$\mathrm{Str}\, M^6 = 0$** and **$\mathrm{Str}\, M^8 > 0$**

- In turn, this implies a **negative definite 1-loop potential**,

$$V_1 < 0$$

no locally stable 1-loop Mink or dS vacuum found

Open questions and outlook

- **More N=0 Minkowski or any dS classical vacua?** More examples could be within reach with the new method…
- General proof of at least an **unbroken U(1)** factor in the gauge group, and of the **relation between classical spectrum and U(1) charges** (or counterexamples)?
- General proof of the model-dependent results on **Str M^6 and Str M^8** (or counterexamples)?
- General proof of **relation between Str $M^8 > 0$ and $V_1 < 0$**?
- **Extensions to generalized flux compactifications?**
- **Nature of the obstructions to locally stable dS vacua?**

Back-up slides

The CSS gauging

Until 2011, the only known explicit gauging leading to classically stable N=0 Minkowski vacua was the one found by Cremmer-Scherk-Schwarz (1979):

- Positive semidefinite potential (no-scale model)
- Gauge group $U(1) \ltimes T^{24}$ broken to U(1) [xU(1)^3]
- Four independent mass parameters:
  U(1) charge matrices in Sp(8,R) [Ferrara-Zumino 1979]
- Str M^2 = Str M^4 = Str M^6 = 0 , Str $M^8 \neq 0$
- One-loop finite, $V_1 < 0$ [Sezgin-Van Nieuwenhuizen 1982]
- Flux compactification of D=11 sugra with geometrical and non-geometrical fluxes
  [Scherk-Schwarz 1979; Catino-Dall'Agata-Inverso-FZ 2012, to appear]

A recent development [Dall'Agata-Inverso 2011]

New way of generating gaugings and vacua, including N=0 Minkowski, but also others

- Embedding tensor Θ and coset representatives L transform linearly under duality, classical potential $V_0(\Phi) = V_0(L^{-1}\Theta)$ depends on combination: $V_0[L(\Phi), \Theta'] = V_0[L(\Phi'), \Theta]$

➔ stay at the origin of Φ field space, solve simple quadratic constraints on Θ to identify consistent gaugings, gauge group, vacuum energy, masses

- Scalar manifold is a coset space

Example 1: CSS gauging [single unbroken U(1)]

$$q_1 = -q_2 = e_1\,, \quad q_3 = -q_4 = e_2\,, \quad q_5 = -q_6 = e_3\,, \quad q_7 = -q_8 = e_4$$

$\mu^2 = \phi^2$ (universal modulus giving scale of all masses)

Example 2: SO(6,2) gauging [unbroken $U(1)^4$]

$$\vec{q}_1 = -\vec{q}_2 = (+1, +1, +1, +1)\,, \qquad \vec{q}_3 = -\vec{q}_4 = (+1, +1, -1, -1)\,,$$
$$\vec{q}_5 = -\vec{q}_6 = (+1, -1, +1, -1)\,, \qquad \vec{q}_7 = -\vec{q}_8 = (+1, -1, -1, +1)\,,$$

Choosing some of the parameters to avoid tachyons:

$$\mu_1^2 = \frac{(x-y)^2(1+x^2y^2)}{8x^2y^2}\,, \quad \mu_2^2 = \mu_3^2 = 0\,, \quad \mu_4^2 = \frac{(1+x^2y^2)(1+xy^3)^2}{8x^2y^4}$$

[now unbroken $U(1)^2$ x SO(4) and dilaton-like modulus]

Evidence for Str $M^6=0$, Str $M^8>0$ ➔ $V_1 < 0$

$$F(x^2) = \frac{1}{64\,\pi^2}\,\mathrm{Str}\left[\mathcal{M}^4\ \log(\mathcal{M}^2 + x^2)\right]$$

$F(0)=V_1$; Str M^2=Str M^4=0 imply, for $x^2 >> M_a^2$:

$$F(x^2) \to \frac{1}{64\,\pi^2}\,\mathrm{Str}\left(\frac{\mathcal{M}^6}{x^2} - \frac{\mathcal{M}^8}{2\,x^4} + \ldots\right)$$

Moreover:

$$\frac{dF}{dx^2} = \frac{1}{64\,\pi^2}\,\mathrm{Str}\left(\mathcal{M}^4\,\frac{1}{\mathcal{M}^2 + x^2}\right) = \frac{1}{64\,\pi^2}\,\mathrm{Str}\left(\frac{x^4}{\mathcal{M}^2 + x^2}\right)$$

Goes to zero for x^2 ➔ 0, x^2 ➔ ∞, and for large x^2:

$$\frac{dF}{dx^2} \to \frac{1}{64\,\pi^2}\,\mathrm{Str}\left(-\frac{\mathcal{M}^6}{x^2} + \frac{\mathcal{M}^8}{x^4} + \ldots\right)$$

with $F<0$ and $dF/dx^2 > 0$ for large x^2. Other zeros?