Higher Spins & Strings

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Plan

§ I. Motivations & context

§ II. Basics of higher-spin interactions

§ III. Maxwell-like Lagrangians for higher spins

§ I

Motivations & context

Higher spins & Relativistic QM

Symmetry group of space-time

$\Rightarrow$

fundamental particles (fields) labeled by two quantum numbers

- *mass:* $m \geq 0$
- *spin:* $s = 0,\ 1/2,\ 1,\ 3/2,\ 2,\ 5/2,\ 3,\ \ldots$

(more labels in $D \geq 5$)

why there should be ``preferred'' subset of values?

[no selection rules from first principles!]

Majorana '32, Dirac '36, Fierz-Pauli '39, Wigner '39, ...

Higher spins & Field Theory

Known interactions fill the first levels of Wigner's scheme:

spin 0:	Higgs boson
spin 1:	electroweak & strong interactions
spin 2:	gravitational force

looks like the beginning of a sequence. . .

locality vs geometry

at the free level, all gauge symmetries look alike

spin 1: $$\delta A_\mu = \partial_\mu \Lambda$$

spin 2: $$\delta h_{\mu_1 \mu_2} = \partial_\mu \Lambda_\nu + \partial_\nu \Lambda_\mu$$

spin 3: $$\delta \varphi_{\mu_1 \mu_2 \mu_3} = \partial_{\mu_1} \Lambda_{\mu_2 \mu_3} + \cdots$$

pictorially (at least):

hints of a unifying geometrical framework?

Difficulties

However:

- No phenomenological inputs for (elementary) higher-spins

(high-spin ``particles'' do exist as hadronic resonances)

- No-go arguments against their interactions

Weinberg '64, Coleman-Mandula '67, Velo-Zwanziger '69, Aragone-Deser '79, . . .

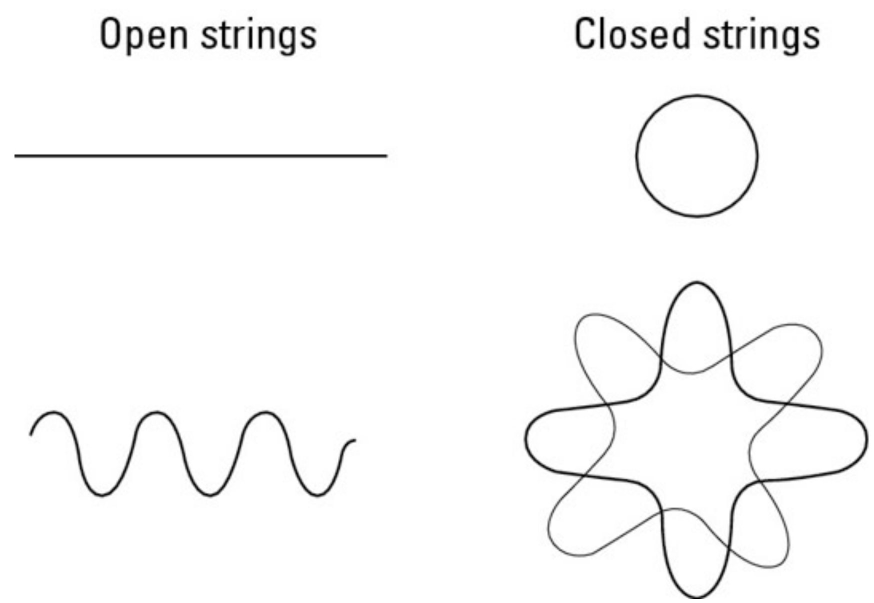
Higher spins & String Theory

String Theory ``predicts'' higher spins

1st quantisation of its vibrating modes $\longrightarrow$ relativistic particles

massless
vectors
(gauge bosons)

massive spin 2, 3, ...



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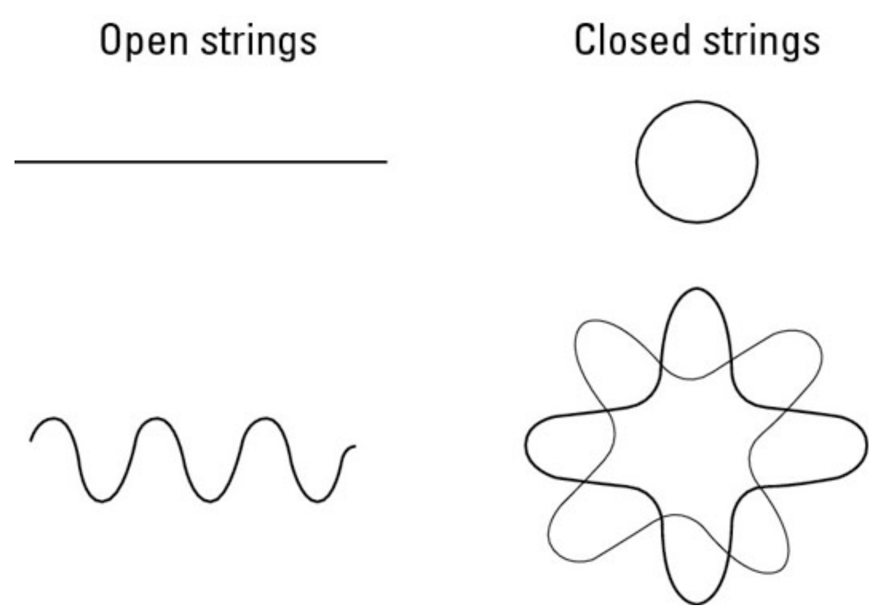

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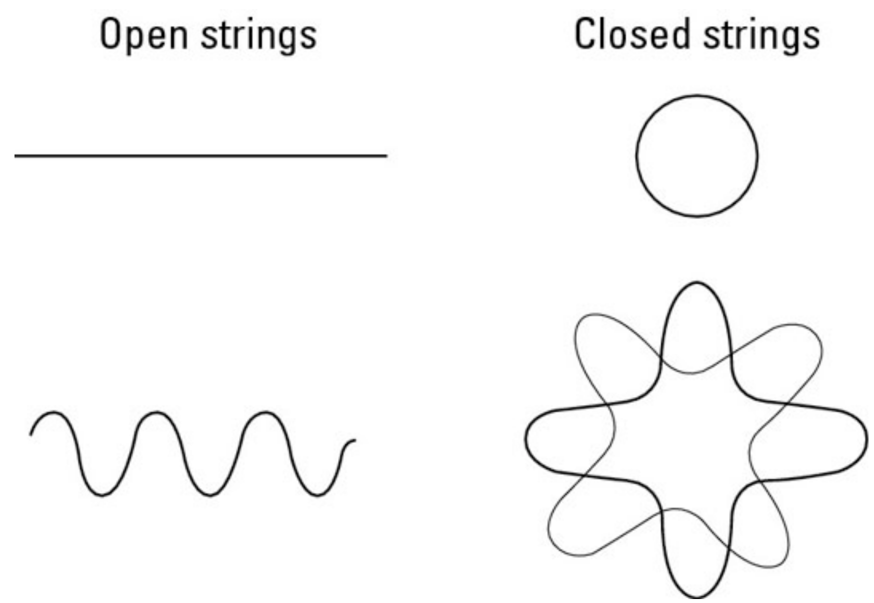

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- string solution to unification of interactions and UV finiteness involves particles with all possible spins

- *what is the dynamical origin of particle mass/string tension?*
  is there an underlying (huge) broken gauge symmetry ?

Higher spins & holography

In Vasiliev's formulation of hsp theory:
- massless spin 2 particle
- bkg space-time (A)dS; *(not flat!)*

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➳ AdS_4/CFT_3 Conformal, 3D theories of N scalars in the fundamental of O(N) conjectured to be dual, for large N, to massless hsp in AdS_4

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- AdS_4/CFT_3 — Conformal, 3D theories of N scalars in the fundamental of O(N) conjectured to be dual, for large N, to massless hsp in AdS_4

- AdS_3/CFT_2 — 3D higher-spin theories are very special:

  they admit an action principle, which is Chern-Simons-like

  they admit consistent truncations to finite number of spins!

interesting laboratory to test a number of issues including, in particular the study of asymptotic symmetries

§ II

Basics of higher-spin interactions

Aragone-Deser: hsp & minimal couplings

spin-s massless fields $\leftrightarrow$ *interaction with gravity*

minimal coupling inconsistent as soon as $s > 2$

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- *spin 3/2* in gravitational backgrounds - *(toy model for supergravity)*

$$\delta E_{\psi_\mu} = \frac{1}{2} \gamma^{\mu\nu\rho} [D_\nu, D_\rho] \varepsilon \ \sim R^\mu{}_\nu \gamma^\nu \varepsilon \longrightarrow \delta E_{\psi_\mu} = 0 \qquad \textit{if Ricci} = 0$$

$$(but\ Riemann\ can\ be \neq 0)$$

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- *spin 5/2* in gravitational backgrounds: Aragone - Deser '79

$$\delta E_{\psi_{\mu\nu}} \sim \text{``Ricci terms''} \cdot \varepsilon + \text{``Riemann terms''} \cdot \varepsilon \quad \Rightarrow$$ **no Hypergravity**

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Let us reconsider the Aragone-Deser problem on AdS bkg

- Consider *fluctuations* of metric *over (A)dS bkg*

$$R_{\mu\nu,\,\rho\sigma} = R^{(AdS)}_{\mu\nu,\,\rho\sigma} + \hat{R}_{\mu\nu,\,\rho\sigma}\,, \qquad \textit{s.t.} \qquad (\hat{R}\,\mu\nu,\ \rho\sigma)^{\,2} \sim 0$$

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- exploiting the (A)dS radius L we can add a coupling:

$$E^{(cov)}_{\psi\,\mu\nu} \longrightarrow E^{(cov)}_{\psi\,\mu\nu} + k\,L^2\hat{\mathcal{R}}\not{\Delta}\,\psi + \cdots$$

s.t.
- $\delta\left(E^{(cov)}_{\psi\,\mu\nu} + kL^2\hat{\mathcal{R}}\not{\Delta}\,\psi + \cdots\right) = 0$
- at order $(\hat{R}\,\mu\nu,\ \rho\sigma)^2 \sim 0$

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- the corresponding cubic vertex contains

Fradkin - Vasiliev '87

$$V = \frac{i}{\Lambda}\int d^D x \sqrt{g}\{\bar{\psi}\hat{R}\not{\nabla}\,\psi + \bar{\psi}\,(\not{\nabla}\,\hat{R})\,\psi\}$$

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Flat limit not defined

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General lesson:

see also: Coleman-Mandula '67, Velo-Zwanziger '69, Benincasa-Cachazo '07, Porrati-Rahman-Sagnotti '08-'10 . . .

more generally, no room for higher-spin interactions if

- *couplings are minimal (or ``too simple")*
- *finitely many fields are involved*
- *space-time background is flat*

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what makes (A)dS and Minkowski bkgs essentially different?

Interactions: cubic vertices

general structure: $s_1 - s_2 - s_3$

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Crucial difference: bkg space-time s.t.

- cosmological constant $\Lambda = 0$
- cosmological constant $\Lambda \neq 0$

$\Lambda = 0$: *Metsaev bound & minimal couplings*

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under the assumptions of

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(A)dS bkg vs Minkowski bkg

see: Boulanger-Leclerq-Sundell ’08, Sagnotti-Taronna ’10, Joung-Lopez-Taronna ’12, Vasiliev ’11, ...

- *cubic couplings are consistent in both settings;*

- *however, in (A)dS, Metsaev’s lower bound does not apply (e.g. minimal coupling to Gravity exists);*

- *at present, difficult to make sense of Minkowski couplings beyond cubic level (perturbative non-localities);*

in (A)dS spaces there exist non-linear equations of motion (not yet a standard action) consistent to all orders: Vasiliev’s equations

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- spin 2 has non-trivial hsp gauge transformation: space-time distance has no covariant meaning in hsp theory in unbroken phase.

Comparing with ST

Interacting higher-spins in String Theory

Interacting higher-spins in Vasiliev's theory

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In (Anti)-de Sitter space-time even general massless Lagrangians proposed only very recently

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massive	massless
flat space-time	Anti-de Sitter space-time (cosmological constant $\neq 0$)
all possible species of allowed particles in D = 10, 26 (tensors with mixed symmetry)	only ``simplest'' particles (symmetric tensors)

In (Anti)-de Sitter space-time even general massless Lagrangians proposed only very recently

§ III

Maxwell-like Lagrangians for higher spins

A. Campoleoni, D.F.

arXiv: 1206.5877 [hep-th]

Object

We propose a class of Lagrangians for bosons of arbitrary spin and symmetry, both on Minkowski and (A)dS backgrounds, based on

Maxwell's operator

$$(M\varphi)_{\mu_1\cdots\mu_s} = \Box\,\varphi_{\mu_1\cdots\mu_s} - \partial_{(\mu_1}\partial^{\alpha}\varphi_{\alpha\,\mu_2\cdots\mu_s)}$$

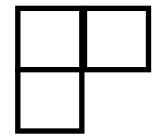
Their distinctive feature is *transverse gauge invariance:*

$$\delta\,\varphi_{\mu_1\cdots\mu_s} = \partial_{(\mu_1}\Lambda_{\mu_2\cdots\mu_s)}$$

s.t.

$$\partial^{\alpha}\Lambda_{\alpha\,\mu_2\cdots\mu_{s-1}} = 0$$

Mixed-symmetry tensors: AdS bkg

For simplicity, consider a {2, 1} field  :

$$\varphi = \varphi_{\mu\mu,\nu}$$

s.t.

$$\delta_0\,\varphi_{\mu\mu,\nu} = \nabla_\mu \Lambda_{\mu,\nu} + \nabla_\nu \lambda_{\mu\mu} - \frac{1}{2}\nabla_\mu \lambda_{\mu\nu}$$

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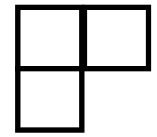
For simplicity, consider a {2, 1} field  :

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peculiarity of (A)dS massless particles:

gauge-per-gauge invariance-breaking

$$\begin{cases} \delta \Lambda_{\mu,\nu} = \nabla_\nu \theta_\mu - \nabla_\mu \theta_\nu \\ \delta \lambda_{\mu\mu} = -2 \nabla_\mu \theta_\mu \end{cases} \quad \rightarrow \quad \delta \varphi_{\mu\mu,\nu} \sim [\nabla_\mu, \nabla_\nu]\, \theta_\mu$$

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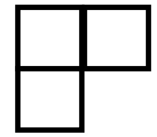
For simplicity, consider a {2, 1} field  :

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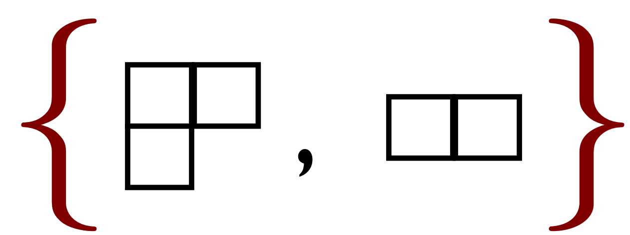
→ In order to ``neutralize'' the effect of θ_μ on the initial $\{2,1\}$ field one can promote it to a gauge parameter for a ***new field***: an additional graviton

massless particles in (A)dS possess more d.o.f. than flat-space particles with the same symmetries



``BMV multiplets''

Metsaev '95, '98; Brink, Metsaev, Vasiliev '00

Mixed-symmetry tensors: AdS bkg

➙ In the Stueckelberg Lagrangian, all flat-space fields and gauge parameters explicitly present

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Metsaev '95, '98

- In this setting, asking for the single parameter Λ to be divergence-free and traceless we can propose a full *Lagrangian for N-family mixed-symmetry fields on AdS*:

$$\mathcal{L} = \frac{1}{2}\,\varphi\left\{M - \frac{1}{L^2}\left[(s_1 - t_1 - 1)(D + s_1 - t_1 - 2) - \sum_{k=1}^{p} s_k\, t_k\right]\right\}\varphi$$

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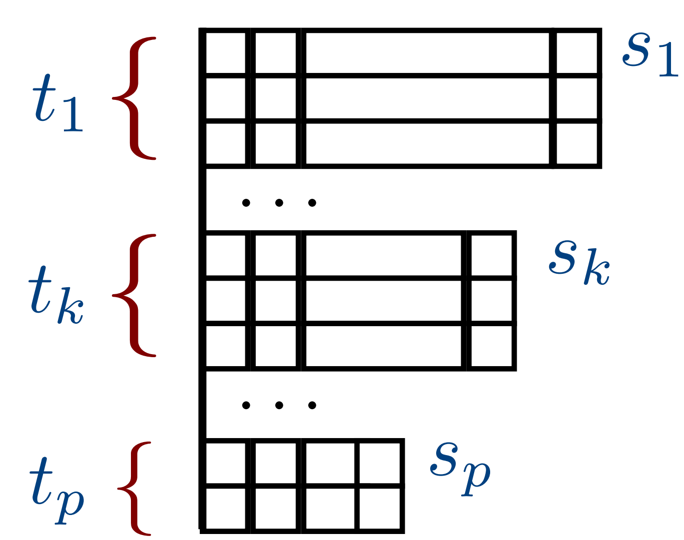
where

$$M \sim \Box - \nabla\,\nabla\cdot$$

Maxwell-like operator

$$\varphi \sim$$



Outlook

Higher-spin field theory:

* *bypass long-standing limitations to low spins, RQFT in its full generality;*

* *generalise notion of local invariance and geometry;*
  possible unifying framework for gravity and spin-1 gauge interactions.

Role of higher spins in String Theory:

* *Provide novel instances of holographic correspondences, in a setting which is still rich enough but also rather simpler than that of standard AdS/CFT;*

* *Key motivation from higher-spin side: possible symmetries underlying its high-energy regime, stressing space-time features of String Theory more than its world-sheet origin.*

(quantum) gravity and higher spins:
is String Theory the unique solution?