

MBR (Minimum-Bias Rockefeller) simulation in Pythia8



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Introduction



- About 30% of the total pp cross section at LHC is attributed to diffractive events (rapidity gaps).
- Large uncertainties in the modelling of diffractive component.
- MBR (Minimum-Bias Rockefeller) Monte Carlo simulation - an event generator addressing the contribution from diffractive processes:
 - Predicts energy dependence of the total, elastic and total-inelastic cross sections.
 - Fully simulates diffractive components (SD, DD, CD → next slide) of total-inel. cross section.
 - Originally written for, and tested at CDF (Tevatron).
 - Recently implemented in Pythia8.165.

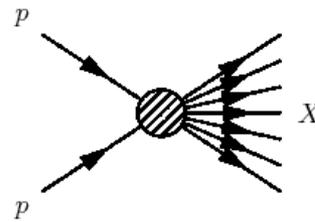
Main processes contributing to the total pp cross section



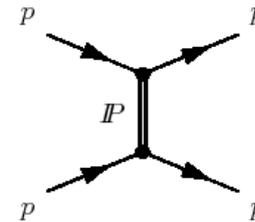
Non-diffractive:

$$pp \rightarrow X$$

(exponentially-suppressed rapidity gap)



(a)

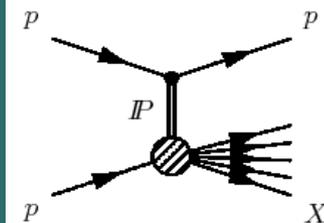


(b)

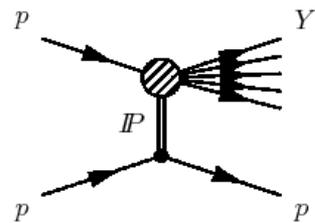
:Elastic

$$pp \rightarrow pp$$

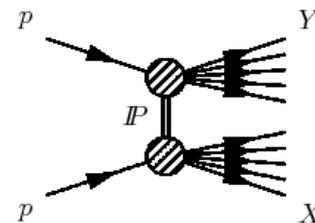
Diffractive:



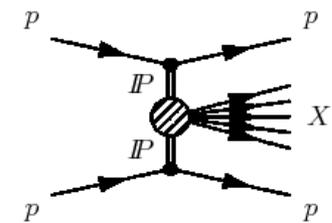
(c)



(d)



(e)



(f)

Single dissociation (SD),

$$pp \rightarrow Xp, \quad pp \rightarrow pY$$

Double dissociation (DD),

$$pp \rightarrow XY$$

Central dissociation (CD)

$$pp \rightarrow pXp$$

or double-Pomeron exchange (DPE)

(a) and (c)-(f) contribute to the total-inelastic cross section

Total, elastic and total-inelastic cross sections

- Total pp cross section

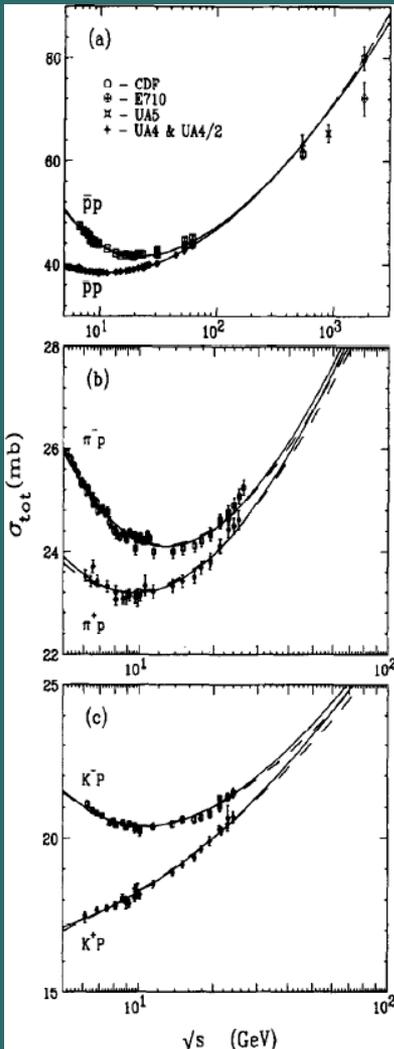
Energy dependence:

$$\sigma_{\text{tot}}^{p^\pm p} = \begin{cases} 16.79s^{0.104} + 60.81s^{-0.32} \mp 31.68s^{-0.54} & \text{for } \sqrt{s} < 1.8 \text{ TeV,} \\ \sigma_{\text{tot}}^{\text{CDF}} + \frac{\pi}{s_0} \left[\left(\ln \frac{s}{s_F} \right)^2 - \left(\ln \frac{s}{s_F} \right)^{\text{CDF}} \right]^2 & \text{for } \sqrt{s} \geq 1.8 \text{ TeV,} \end{cases}$$

- For $\sqrt{s} < 1.8$ TeV – global fit to pre-LHC data on $p^\pm p$, $K^\pm p$, $\pi^\pm p$ cross-sections. [Phys.Lett B389, 176 \(1996\)](#)

- For $\sqrt{s} > 1.8$ TeV (LHC and beyond) – model based on a saturated Froissart bound. [arXiv:1105.4916](#)

Froissart bound with two parameters: $s_F = 22$ GeV, $s_0 = 3.7 \pm 1.5$ GeV⁻², normalized to the CDF measurement @1.8 TeV: 80.03 ± 2.24 mb.



Total, elastic and total-inelastic cross sections



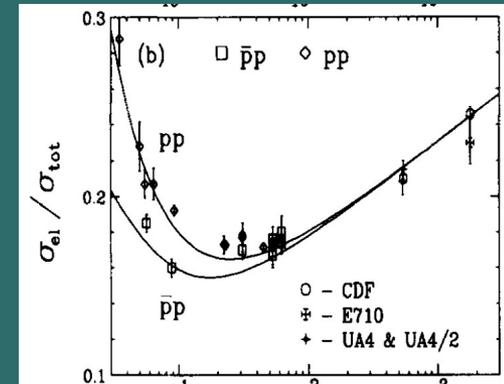
- Elastic cross section

$$\sigma_{\text{el}} = r \cdot \sigma_{\text{tot}}$$

with $r = \sigma_{\text{el}} / \sigma_{\text{tot}}$ from the global fit. \longrightarrow

Linear $\log(s)$ dependence at higher energies not expected to obey black-disk limit \rightarrow model valid up to energies of O(50 TeV).

Phys.Lett B389, 176 (1996)



- Total-inelastic cross section

$$\sigma_{\text{inel}} = \sigma_{\text{tot}} - \sigma_{\text{el}}$$

Diffractive cross sections



hep-ph/0407035
arXiv:hep-ph/020314

- Calculated based on renormalized-Regge theory.
- Differential cross sections vs. rapidity gap width, Δy , and 4-momentum transfer squared, t :

$$\frac{d^2\sigma_{SD}}{dt d\Delta y} = \frac{1}{N_{\text{gap}}(s)} \left[\frac{\beta^2(t)}{16\pi} e^{2[\alpha(t)-1]\Delta y} \right] \cdot \left\{ \kappa \beta^2(0) \left(\frac{s'}{s_0} \right)^\epsilon \right\},$$

$$\frac{d^3\sigma_{DD}}{dt d\Delta y dy_0} = \frac{1}{N_{\text{gap}}(s)} \left[\frac{\kappa \beta^2(0)}{16\pi} e^{2[\alpha(t)-1]\Delta y} \right] \cdot \left\{ \kappa \beta^2(0) \left(\frac{s'}{s_0} \right)^\epsilon \right\},$$

$$\frac{d^4\sigma_{DPE}}{dt_1 dt_2 d\Delta y dy_c} = \frac{1}{N_{\text{gap}}(s)} \left[\Pi_i \left[\frac{\beta^2(t_i)}{16\pi} e^{2[\alpha(t_i)-1]\Delta y_i} \right] \right] \cdot \kappa \left\{ \kappa \beta^2(0) \left(\frac{s'}{s_0} \right)^\epsilon \right\},$$

$\Delta y = \Delta y_1 + \Delta y_2$

DD: y_0 – center of rapidity gap, DPE: y_c – rapidity of dissociated system

$$\alpha(t) = 1 + \epsilon + \alpha' t = 1.104 + 0.25 (\text{GeV}^{-2}) \cdot t$$

$$\beta^2(t) = \beta^2(0) F^2(t)$$

$$\kappa \equiv g(t) / \beta(0)$$

$$\xi = e^{-\Delta y} \quad \xi_{SD} = M^2/s$$

$$\xi_{DD} = M_1^2 M_2^2 / (s \cdot s_0)$$

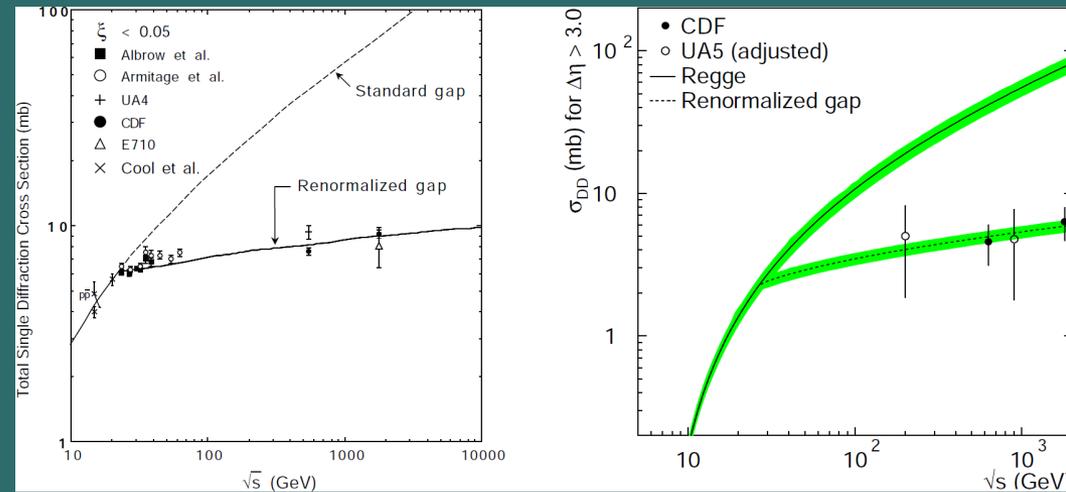
$$\text{DPE } \xi = \xi_1 \xi_2 = M^2/s$$

- Term in { } brackets: total Pomeron-p cross section at a reduced energy $s' = s \cdot e^{-dy}$.
- Term in [] brackets: Pomeron flux.
- $N_{\text{gap}}(s)$: renormalization factor: $\min(1, f)$, with $f :=$ integral of Pomeron flux
→ allows to interpret the flux as (diffractive) gap-formation probability.

Diffractive cross sections



- Flux renormalization procedure brings the standard Regge theory predictions in agreement with the CDF data.



from arXiv:hep-ph/020314

- Small gap widths, diffractive limit

Cross section formulae are used to generate events with large (diffractive) rapidity gaps. Small gaps are suppressed by convoluting the formulae with the error function (cumulative Gauss distribution) centered at $\Delta y_s = 2$ with a width of $\sigma_s = 0.5$:

$$S = \frac{1}{2} \left[1 + \operatorname{erf} \left(\frac{\Delta y - \Delta y_s}{\sigma_s} \right) \right]$$

SD events: coherence limit ($\xi \leq 0.135$)

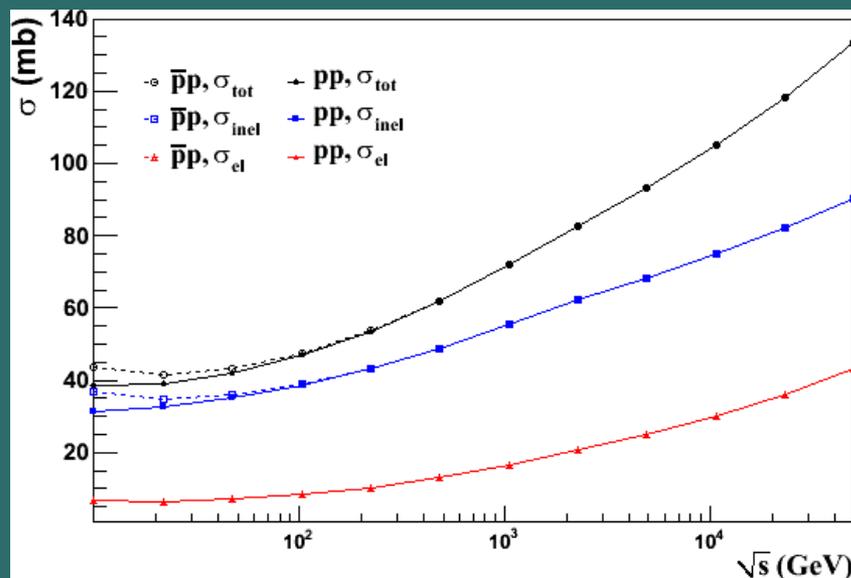
DD events: arbitrary choice, because small gaps in DD and ND events cannot be unambiguously distinguished.

CD events: suppression applied on total gap width, $\Delta y = \Delta y_1 + \Delta y_2$.

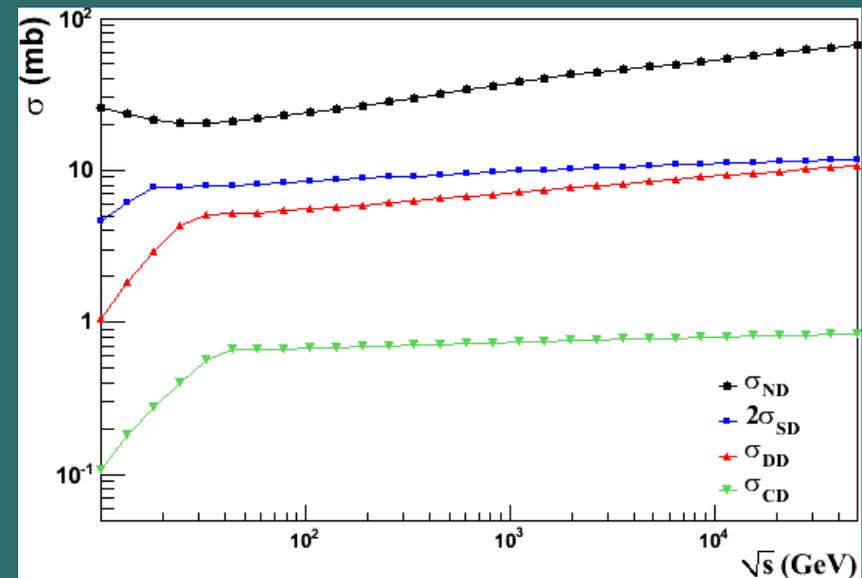
Cross section predictions



Total, elastic, total-inel. xsecs



Components of total-inel. xsec: diffractive (SD,DD,CD) and non-diffractive (ND) xsecs



Cross section values [mb] vs. energy

| \sqrt{s} (TeV) | 0.3 | 0.9 | 1.96 | 2.76 | 7 | 8 | 14 |
|------------------------|-------|-------|-------|-------|-------|--------|--------|
| σ_{tot} | 56.50 | 69.87 | 81.03 | 85.25 | 98.29 | 100.35 | 109.49 |
| σ_{el} | 11.28 | 15.83 | 19.97 | 21.70 | 27.20 | 28.09 | 32.10 |
| σ_{inel} | 45.23 | 54.04 | 61.06 | 63.55 | 71.10 | 72.26 | 77.39 |
| σ_{ND} | 29.19 | 36.50 | 42.41 | 44.39 | 50.57 | 51.54 | 55.84 |
| $\sigma_{2\text{SD}}$ | 9.10 | 9.76 | 10.22 | 10.41 | 10.91 | 10.98 | 11.26 |
| σ_{DD} | 6.21 | 7.03 | 7.67 | 7.97 | 8.82 | 8.94 | 9.47 |
| σ_{CD} | 0.718 | 0.746 | 0.766 | 0.776 | 0.800 | 0.804 | 0.818 |

Implementation in Pythia8.165

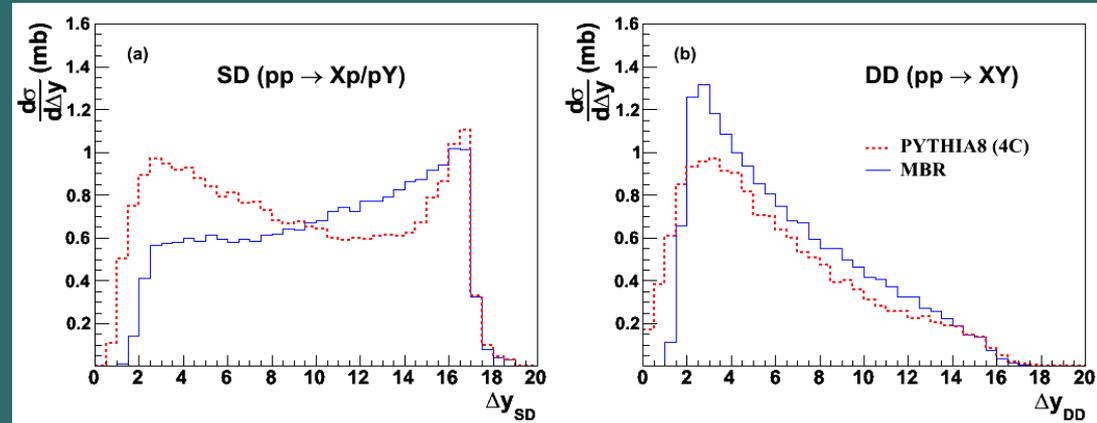


- MBR simulation activated with `Diffraction:PomFlux = 5`

- SD, DD processes

MBR code added to the already existing simulation of processID =103,104 and 105.

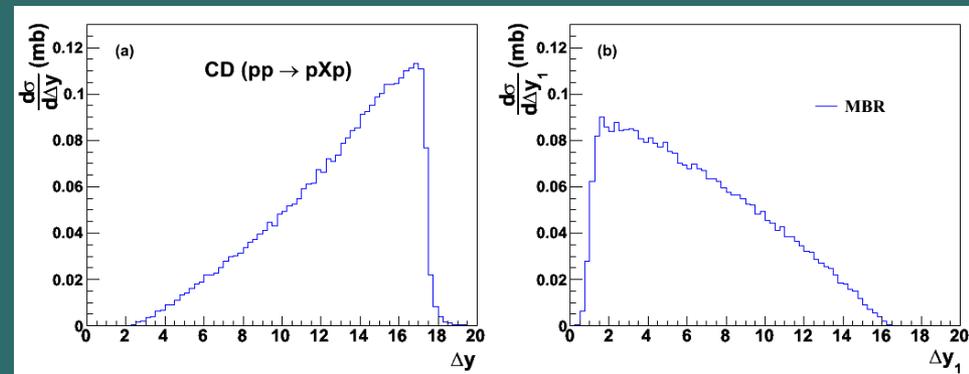
Comparison with Pythia8-4C (rescaled Schuler&Sjostrand model, `Diffraction:PomFlux=1`) →



- CD (DPE) process

Implemented in PYTHIA for the first time. ProcessID=106, `SoftQCD:centralDiffractive = on`.

4-vector of centrally-produced inclusive hadronic system propagated to PYTHIA for hadronization



→ possible to extend the simulation to CD processes such as exclusive di-hadron or exclusive di-jet production (hadronization step).

Implementation in Pythia8.165



When option 5 is selected, the following parameters of the MBR model [Cie12] are used:

parm **Diffraction:MBRepsilon** (default = **0.104**; minimum = 0.02; maximum = 0.15)

parm **Diffraction:MBRalpha** (default = **0.25**; minimum = 0.1; maximum = 0.4)
the parameters of the Pomeron trajectory.

parm **Diffraction:MBRbeta0** (default = **6.566**; minimum = 0.0; maximum = 10.0)

parm **Diffraction:MBRsigma0** (default = **2.82**; minimum = 0.0; maximum = 5.0)
the Pomeron-proton coupling, and the total Pomeron-proton cross section.

parm **Diffraction:MBRm2Min** (default = **1.5**; minimum = 0.0; maximum = 3.0)
the lowest value of the mass squared of the dissociated system.

parm **Diffraction:MBRdyminSDflux** (default = **2.3**; minimum = 0.0; maximum = 5.0)

parm **Diffraction:MBRdyminDDflux** (default = **2.3**; minimum = 0.0; maximum = 5.0)

parm **Diffraction:MBRdyminCDflux** (default = **2.3**; minimum = 0.0; maximum = 5.0)
the minimum width of the rapidity gap used in the calculation of $N_{gap}(s)$ (flux renormalization).

parm **Diffraction:MBRdyminSD** (default = **2.0**; minimum = 0.0; maximum = 5.0)

parm **Diffraction:MBRdyminDD** (default = **2.0**; minimum = 0.0; maximum = 5.0)

parm **Diffraction:MBRdyminCD** (default = **2.0**; minimum = 0.0; maximum = 5.0)
the minimum width of the rapidity gap used in the calculation of cross sections, i.e. the parameter dy_S , which suppresses the cross section at low dy (non-diffractive region).

parm **Diffraction:MBRdyminSigSD** (default = **0.5**; minimum = 0.001; maximum = 5.0)

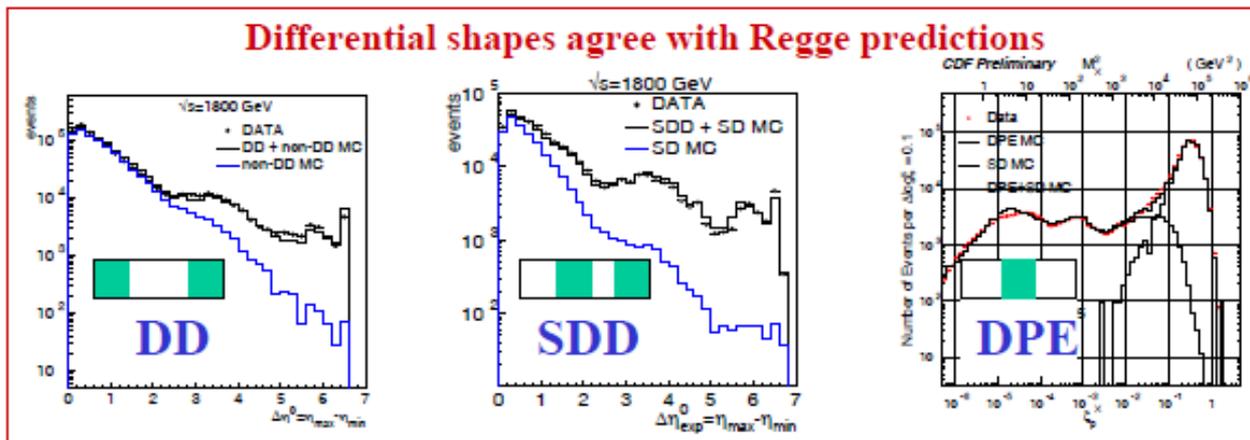
parm **Diffraction:MBRdyminSigDD** (default = **0.5**; minimum = 0.001; maximum = 5.0)

parm **Diffraction:MBRdyminSigCD** (default = **0.5**; minimum = 0.001; maximum = 5.0)
the parameter σ_{S} , used for the cross section suppression at low dy (non-diffractive region).

MBR vs. CDF data

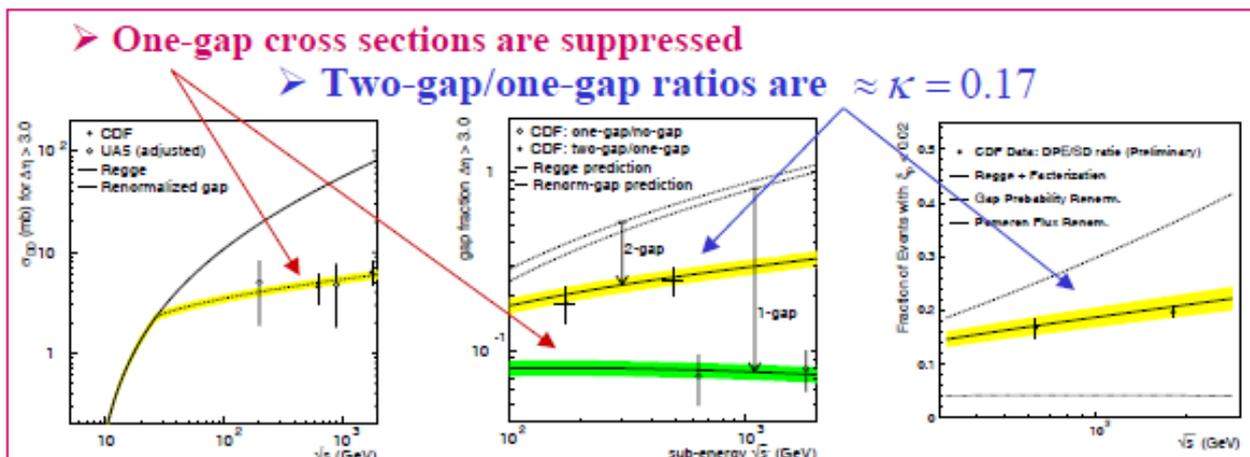
Central & Double-Gap CDF Results

Differential shapes agree with Regge predictions

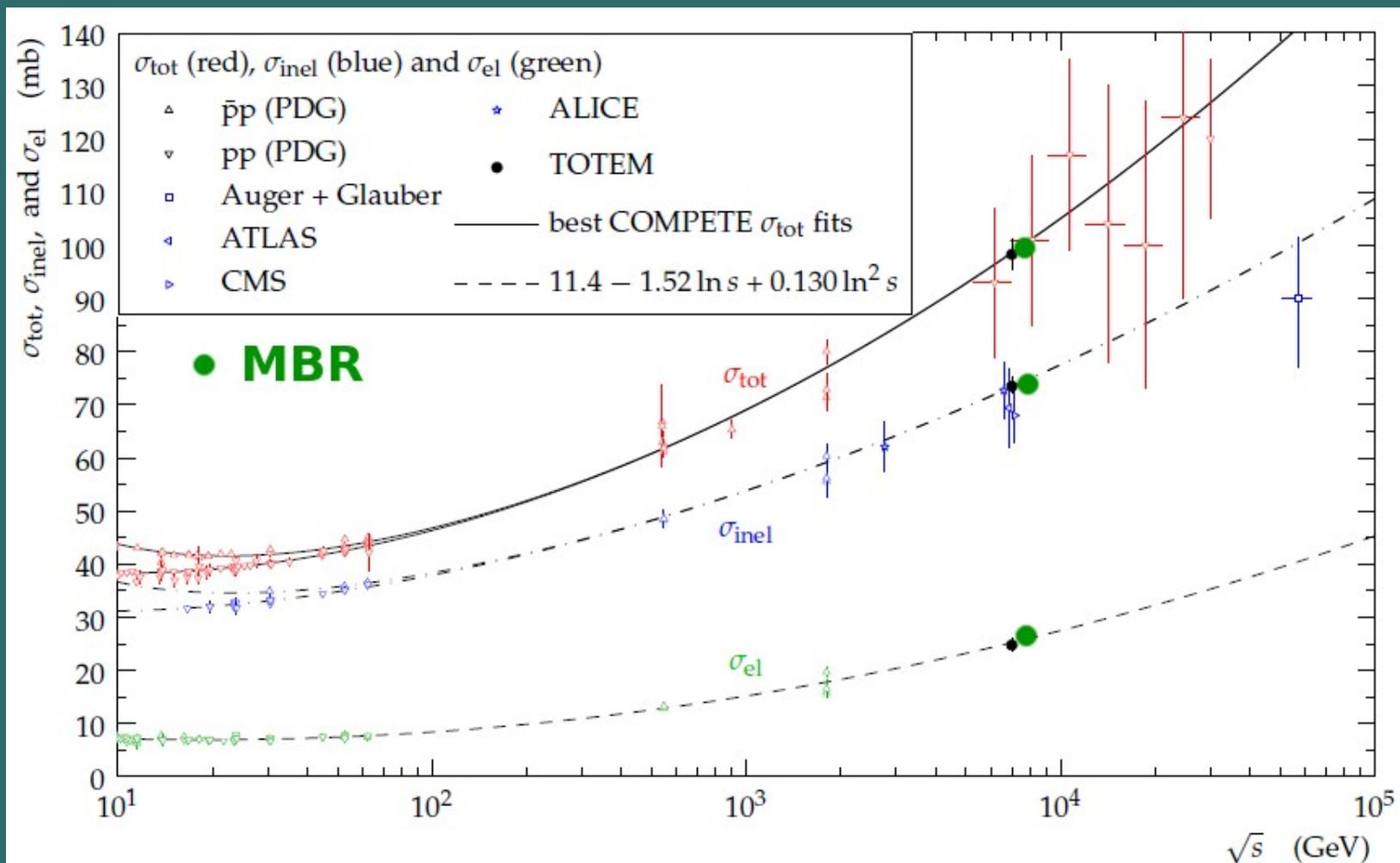


➤ One-gap cross sections are suppressed

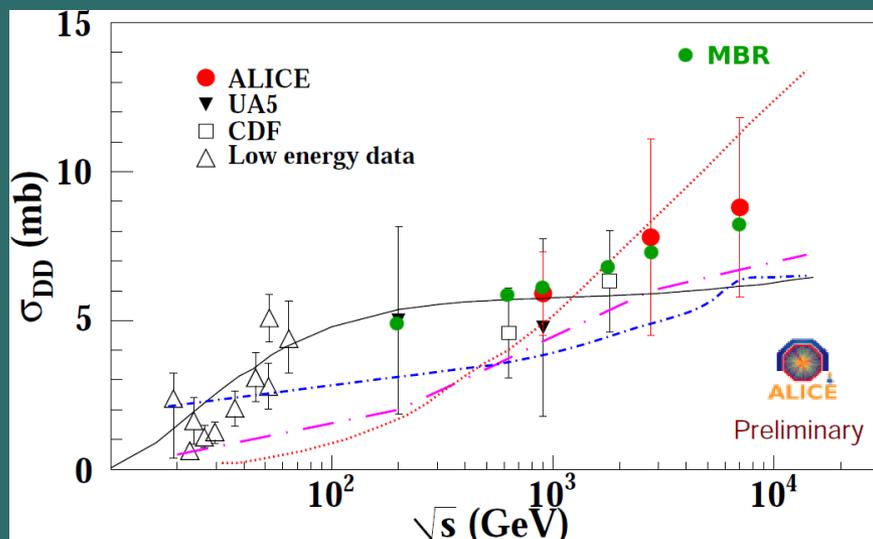
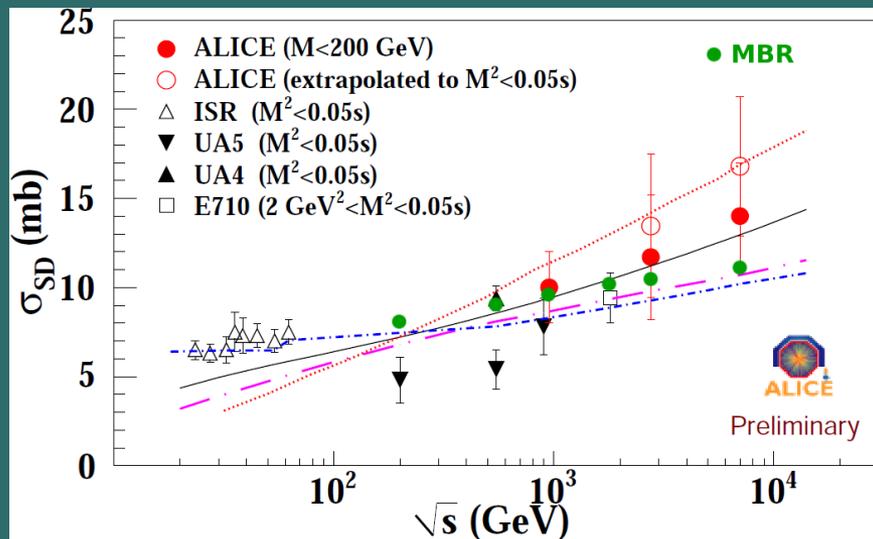
➤ Two-gap/one-gap ratios are $\approx \kappa = 0.17$



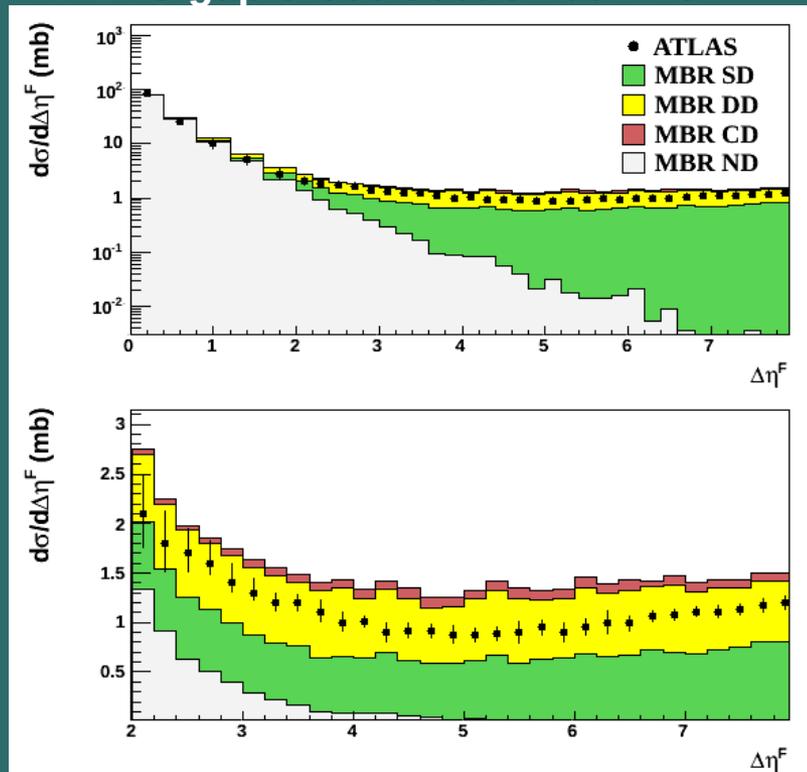
MBR vs. LHC data



MBR vs. LHC data



ATLAS gap cross section at 7 TeV



Default Pythia8-4C hadronization

Summary



- MBR (Minimum-Bias Rockefeller) simulation, developed and successfully tested at CDF, has been implemented in Pythia8.165.
- MBR predicts the total, elastic and total-inelastic cross sections and fully simulates diffractive processes with single-, double- and central-dissociation.
- Central-dissociation (double-Pomeron exchange) process included in PYTHIA for the first time.
- Good agreement with CDF and LHC data.
- Future development: tune of hadronization step using original MBR hadronization model, based on pre-LHC and pre-Tevatron low-energy data.

Thank you for your attention!

Backup slides



3.1 Single-diffractive events

Events are generated by first choosing the rapidity-gap width, Δy , according to Eq. (3) integrated over t :

$$\frac{d\sigma_{SD}}{d\Delta y} \sim e^{\epsilon\Delta y} \cdot \left(\frac{a_1}{b_1 + 2\alpha'\Delta y} + \frac{a_2}{b_2 + 2\alpha'\Delta y} \right) \cdot S. \quad (9)$$

The range of the generation is defined by $\Delta y_{min} = 0$ and $\Delta y_{max} = -\ln M_0^2/s$, where $M_0^2 = \text{MBRm2Min}$. The term:

$$S = \frac{1}{2} \left[1 + \text{erf} \left(\frac{\Delta y - \text{MBRdyminSD}}{\text{MBRdyminSigSD}} \right) \right], \quad (10)$$

is added to suppress events at low values of Δy , as explained in Sec. 2.2

A value of t is then chosen according to:

$$\frac{d\sigma_{SD}}{dt} \sim F^2(t) \cdot e^{2\alpha'\Delta yt}, \quad (11)$$

where $F^2(t)$ is given by Eq. (7) and the integration is performed up to $t_{max} = -m_p^2 \cdot \frac{\xi^2}{1-\xi}$, with $\xi = e^{-\Delta y}$. The diffractive mass is calculated as $M = \sqrt{s\xi}$. The four-momenta of the outgoing proton and the dissociated mass system are calculated using Mandelstam variables for a two-body scattering process, as implemented in PYTHIA8 for other `Diffraction:PomFlux` options.

3.2 Double-diffractive events

Events are generated by first choosing the rapidity-gap width according to Eq. (4) integrated over t . Eq. (4) is divergent as $\Delta y \rightarrow 0$. In order to remove the divergence, the integration over t is performed within the limits from $t_{min} = -e^{\Delta y}$ to $t_{max} = -e^{-\Delta y}$. Then, Δy is chosen from the distribution:

$$\frac{d\sigma_{DD}}{d\Delta y} \sim e^{\epsilon\Delta y} \cdot \frac{\ln \frac{ss_0}{M_0^4} - \Delta y}{2\alpha'\Delta y} \left(e^{-2\alpha'\Delta ye^{-\Delta y}} - e^{-2\alpha'\Delta ye^{\Delta y}} \right) \cdot S, \quad (12)$$

and the range of the generation is defined by $\Delta y_{min} = 0$ and $\Delta y_{max} = -\ln M_0^4/(ss_0)$, where $M_0^2 = \text{MBRm2Min}$ and $s_0 = 1 \text{ GeV}^2$. To further suppress events at low values of Δy the term:

$$S = \frac{1}{2} \left[1 + \text{erf} \left(\frac{\Delta y - \text{MBRdyminDD}}{\text{MBRdyminSigDD}} \right) \right], \quad (13)$$

is used as explained in Sec. 2.2.

The variable t is chosen according to:

$$\frac{d\sigma_{DD}}{dt} \sim e^{2\alpha'\Delta yt}, \quad (14)$$

in the range from $t_{min} = -e^{\Delta y}$ to $t_{max} = -e^{-\Delta y}$.

Then, the center of the rapidity gap, y_0 , is selected uniformly within the limits:

$$-\frac{1}{2} \left(\ln \frac{ss_0}{M_0^4} - \Delta y \right) < y_0 < \frac{1}{2} \left(\ln \frac{ss_0}{M_0^4} - \Delta y \right), \quad (15)$$

and the diffractive masses are calculated as:

$$M_1^2 = \sqrt{s \cdot e^{-\Delta y - y_0}}, \quad (16)$$

$$M_2^2 = \sqrt{s \cdot e^{-\Delta y + y_0}}. \quad (17)$$

The four-momenta of the outgoing dissociated mass systems are calculated using Mandelstam variables for a two-body scattering process, as implemented in PYTHIA8 for other options of Diffraction:PomFlux .

Event generation, CD (DPE)



3.3 Central-diffractive (DPE) events

Events are generated by first choosing the total rapidity gap width, Δy , according to Eq. (5), integrated over t_1 and t_2 :

$$\frac{d\sigma_{CD}}{d\Delta y} \sim e^{\epsilon\Delta y} \int_{-\Delta y/2+y_0}^{\Delta y/2-y_0} dy_0 f_- \cdot f_+ \cdot S_1 S_2, \quad (18)$$

where:

$$f_{\pm} = \left(\frac{a_1}{b_1 + \alpha' \Delta y \pm 2\alpha' y_0} + \frac{a_2}{b_2 + \alpha' \Delta y \pm 2\alpha' y_0} \right), \quad (19)$$

and the integration is performed from $\Delta y_{min} = 0$ to $\Delta y_{max} = -\ln M_0^2/s$, where $M_0^2 = \text{MBRm2Min}$. For events at low values of Δy we suppress individual gaps with the factor:

$$S = \frac{1}{2} \left[1 + \text{erf} \left(\frac{\Delta y - \text{MBRdyminCD}/2}{\text{MBRdyminSigCD}/\sqrt{2}} \right) \right]. \quad (20)$$

Then, the direction of the centrally-produced hadronic system, y_c , is selected uniformly within the region:

$$-\frac{1}{2}(\Delta y - \Delta y_{min}) < y_c < \frac{1}{2}(\Delta y - \Delta y_{min}), \quad (21)$$

and rapidity gaps corresponding to each of the two Pomerons are calculated as:

$$\Delta y_1 = \Delta y/2 + y_0, \quad (22)$$

$$\Delta y_2 = \Delta y/2 - y_0. \quad (23)$$

The four-momentum transfers squared at each proton vertex, t_1 and t_2 , are generated according to:

$$\frac{d\sigma_{CD,i}}{dt} \sim F^2(t_i) \cdot e^{2\alpha' \Delta y_i t_i}, \quad (24)$$

up to $t_{max,i} = -m_p^2 \cdot \frac{\xi_i^2}{1-\xi_i}$, where $\xi_i = e^{-\Delta y_i}$ and $i = 1, 2$. Then, the p_T and p_z of outgoing protons are calculated as $p_{T,i}^2 = (1-\xi_i)|t_i| - m_p^2 \xi_i^2$ and $|p_{z,i}| = p(1-\xi_i)$, where $p = \sqrt{s/4 - m_p^2}$ is the incoming proton momentum.

Finally, the four-momentum of the hadronic system is calculated from the sum of the four-momenta of the Pomerons, each calculated as a difference between the incoming and outgoing proton four-vectors.