# Near BPS Skyrmions: Non-shell configurations and Coulomb effects

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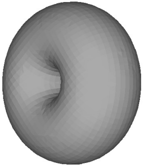
## The Skyrme Model : Motivations

- Skyrme Model = low-energy effective mesonic field theory of QCD
- Baryons and nuclei= topological solitons: conserved topological charge = baryon number
- Success: Hadron properties predicted within 30% (some quantities within a few %)
- Difficulties: Numerically challenging, Link to QCD,
  ...AND...
  Multibaryon physics (nuclear binding energy, configurations),[1]

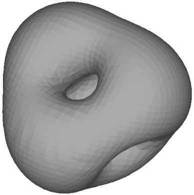
[1] [based on works in : Phys.Rev. D82 (2010) 054023, arXiv:1205.1414 (2012)]

# The Problem : Multibaryons

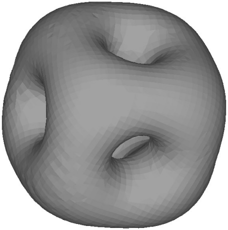
- Binding energies too large especially for small nuclei ( e.g. deuteron $\simeq 80 \times$ observed value)
- Shell-like baryon and mass density configurations



$A = 2$



$A = 3$



$A = 4$

## Multibaryons : Alternatives:

- **Different potential (mass) terms**
- Rotational deformations
- More higher order terms in derivatives
- More mesons (e.g. $\omega$, $\rho$, ...)

$\Longrightarrow$ Previous results: Similar configurations and binding energies

# New Model : Near BPS Skyrme Model

Basic idea:

Nuclei: $M \approx A \cdot M_{nucleon}$ and almost constant density

$$\Downarrow$$

Construct model where **Skyrmions** $\approx$ **BPS-solitons with NON-shell density**

## New Model : Near BPS Skyrme Model

- The pion fields described by matrix $U \in SU(2)$

$$U = exp(-\frac{2i}{F_\pi}\vec{\tau}\cdot\vec{\pi}) = \phi_0 + i\vec{\tau}\cdot\vec{\phi}$$

  such that $\phi_0^2 + \vec{\phi}^2 = 1$

- Our model: Lagrangian

$$\mathcal{L}_{\text{NBPS}} = \underbrace{\mathcal{L}_2 + \mathcal{L}_4}_{\text{Skyrme}} + \underbrace{\mathcal{L}_0 + \mathcal{L}_6}_{\text{BPS-solitons}}$$

## New Model : Near BPS Skyrme Model

1. Quadratic NL$\sigma$ term: kinetic term (here $L_\mu = U^\dagger \partial_\mu U$)

$$\mathcal{L}_2 = -\alpha \mathrm{Tr}\left[L_\mu L^\mu\right] \qquad \left(\alpha = \frac{F_\pi^2}{16}\right)$$

2. Quartic Skyrme term (necessary to stabilize soliton: Derrick Theorem)

$$\mathcal{L}_4 = \beta \mathrm{Tr}\left(\left[L_\mu, L_\nu\right]^2\right) \qquad \left(\beta = \frac{1}{32e^2}\right)$$

3. Potential term ($\chi$SB term): responsible for pion mass

$$\mathcal{L}_0 = -\mu^2 V(U) \qquad \left(= \frac{m_\pi^2 F_\pi^2}{8} \mathrm{Tr}\left(1 - U\right)\right)$$

## New Model : Near BPS Skyrme Model ...

4. Sextic term: **quadratic** in time derivatives ⇒ standard hamiltonian formulation.

$$\mathcal{L}_6 = -\frac{3}{2}\frac{\lambda^2}{16^2}\mathrm{Tr}\left(\left[L_\mu, L_\nu\right]\left[L^\nu, L^\lambda\right]\left[L_\lambda, L_\mu\right]\right) = \lambda^2\pi^4\mathcal{B}^\mu\mathcal{B}_\mu$$

where $\mathcal{B}^\mu$ is the baryon current

$$\mathcal{B}^\mu = \frac{1}{24\pi^2}\epsilon^{\mu\nu\rho\sigma}\mathrm{Tr}\left(L_\nu L_\rho L_\sigma\right)$$

- Winding number = baryon number = atomic number = $A$

$$A = \int d^3r\mathcal{B}^0 = -\frac{1}{24\pi^2}\epsilon^{ijk}\int d^3r\mathrm{Tr}\left(L_i L_j L_k\right).$$

## Near BPS Skyrme Model : The Strategy

Get closer to saturation of Bogomol'nyi bound without losing link with pion physics:

$$\mathcal{L}_{\text{NBPS}} = \underbrace{\mathcal{L}_0 + \mathcal{L}_6}_{\text{BPS-solitons}} + \underbrace{\mathcal{L}_2 + \mathcal{L}_4}_{\text{Skyrme}}$$

- Assume $\mathcal{L}_0 + \mathcal{L}_6$ dominate ( $\Longrightarrow$ BPS-solitons) and
- Treat $\mathcal{L}_2$ and $\mathcal{L}_4$ are small as perturbations.

Then:

## Near BPS Skyrme Model : The Strategy ...

1. Setting $\alpha = \beta = 0$, choose an appropriate potential $V(U)$ for **NON-shell configuration**

$$-\mu^2 V(U) = \frac{\mu^2}{576}\mathrm{Tr}\left[\frac{\left(2I - U - U^\dagger\right)\left(2I + U + U^\dagger\right)^3}{\ln\left(\left(2I + U + U^\dagger\right)/4\right)}\right]$$

2. Solution = BPS-soliton with axial symmetry

$$U = \cos F(r) + i\hat{\mathbf{n}} \cdot \tau \sin F(r)$$

$$\hat{\mathbf{n}} = (\sin\theta\cos n\varphi, \sin\theta\sin n\varphi, \cos\theta)$$

with $n = A =$ integer and **analytical** form

$$F(r) = \mp 2\left|\arccos\left(\exp[-ar^2]\right)\right| \qquad \text{with } a = \left(\frac{\mu}{18n\lambda}\right)^{2/3}$$

# Near BPS Skyrme Model : The Strategy ...

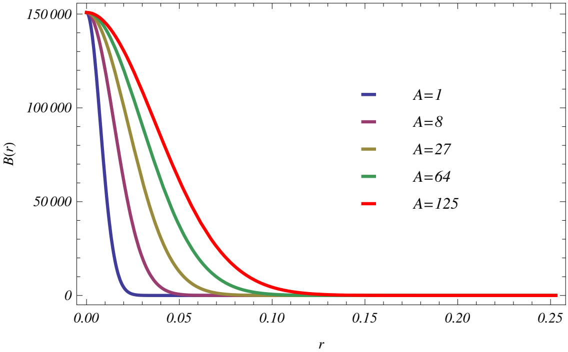



Figure 1: Baryon density $B(r)$ ($r$ in MeV$^{-1}$)

## Near BPS Skyrme Model : The Strategy ...

3. Switch on the NL$\sigma$ and Skyrme terms as small perturbations
4. Add rotational and iso-rotational energies $E_{\text{rot}}$: well-known procedure

$$E_{\text{rot}} = \frac{1}{2}\left[\frac{j(j+1)}{V_{11}} + \frac{i(i+1)}{U_{11}} + \left(\frac{1}{U_{33}} - \frac{1}{U_{11}} - \frac{n^2}{V_{11}}\right)\kappa^2\right]$$

$i, j, \kappa =$ lab. isospin, spin, max. of B.F. $3^{rd}$ comp. of isospin.

$U_{ij}, W_{ij}, V_{ij} =$ moments of inertia.

## Near BPS Skyrme Model : The Strategy ...

5. Add Coulomb energy $E_C$ using charge density

$$\rho(\mathbf{r}) = J^0_{EM} \equiv \frac{1}{2}\mathcal{B}^0(\mathbf{r}) + i_3 \frac{\mathcal{U}_{33}(\mathbf{r})}{U_{33}}$$

   where $\mathcal{B}^0(r)$ and $\mathcal{U}_{33}(\mathbf{r})$ have an analytical form

6. Add isospin breaking term $E_I$ (p-n mass difference):

$$E_I = a_I i_3$$

7. Fit **4 parameters** of the model $\mu, \alpha, \beta, \lambda$ w.r.t. nuclear data and mass of the multiskyrmions:

$$M = E_{\text{stat}} + E_{\text{rot}} + E_C + E_I$$

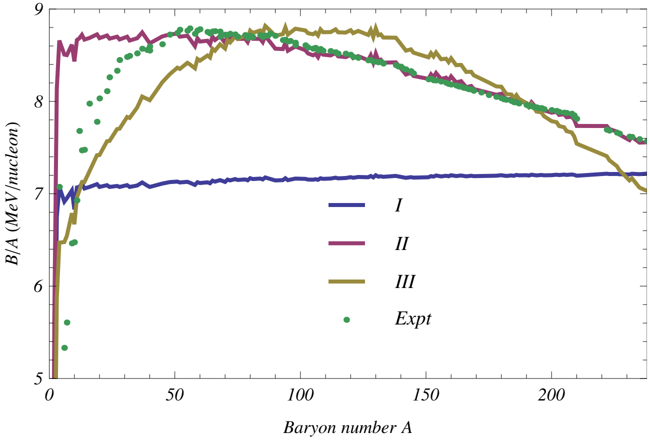
   and compute other properties of the nuclei.

# Near BPS Skyrme Model : The Strategy

Set I: $\alpha = \beta = 0$, input = H and He masses
Set II: input = masses of 144 most stable isotopes
Set III: input = $B/A$ of 144 most stable isotopes

| | Set I | Set II | Set III |
|---|---|---|---|
| $\mu$ ($10^4$ MeV$^2$) | 1.49 | 1.51 | 1.73 |
| $\alpha$ ($10^{-3}$ MeV$^2$) | 0 | 5.88 | 22.1 |
| $\beta$ ($10^{-6}$ MeV$^0$) | 0 | $-1.85$ | $-5.81$ |
| $\lambda$ ($10^{-3}$ MeV$^{-1}$) | 6.41 | 6.34 | 5.54 |

# Binding Energies per Nucleon ($B/A$) : Set I, II and III



## Conclusions : Summary and Outlook

Near-BPS Skyrme Model

- Non shell-like configurations for baryon and charge densities for all $A$ possible
- Surprisingly good fit for $B/A$
- Other important results: e.g. size of nuclei $\propto A^{1/3}$

Next:

- Improve choice of potential $\Longrightarrow$ Constant charge density with constant skin thickness for nuclei
- Study more properties: magnetic moments, form factors, vibrational modes,...
- etc...

# Conclusions : Summary and Outlook

Questions