Bootstrap Dynamical Symmetry Breaking with New Heavy Chiral Quarks

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4G!?

I. The (non)Motivational

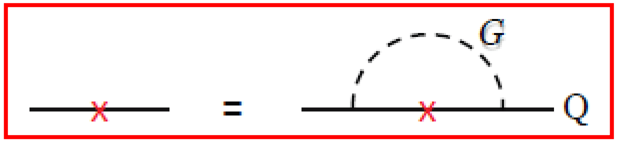
abstract $\Longrightarrow$ 125 GeV @ end ...

scattering / self-energy / Gap Eq.

II. Dynamical Symmetry Breaking: Two Examples

* NJL * Strong Scale-inv. QED — Setup

III. Yukawa "Bootstrap" Gap Eq.: Dyn. EWSB



IV. Discussion and Conclusion

Mimura, WSH, Kohyama, arXiv:1206.6063

I. The (non)Motivational

Title: Bootstrap Dynamical Symmetry Breaking with New Heavy Chiral Quarks

Abstract:

Should I stop here and now?

Despite the hint for a 125 GeV Higgs boson, we consider the other option of MH > 600 GeV, noting that the existence of the Higgs boson itself is not yet an established fact. What we do know is that the Goldstone bosons of electroweak symmetry breaking exist as longitudinal components of the weak bosons. The Goldstone boson coupling to a new heavy chiral quark doublet Q (assuming it exist), the G-Q-Q(bar) Yukawa coupling, would now be in the strong coupling regime, given the LHC limit of MQ > 600 GeV is already beyond the perturbative partial-wave unitarity bound. Such strong Yukawa couplings could induce Q-Q(bar) condensation, which might take the role of the Higgs condensate. Guided by a Bethe-Salpeter ...

Return to 125 GeV towards end ...

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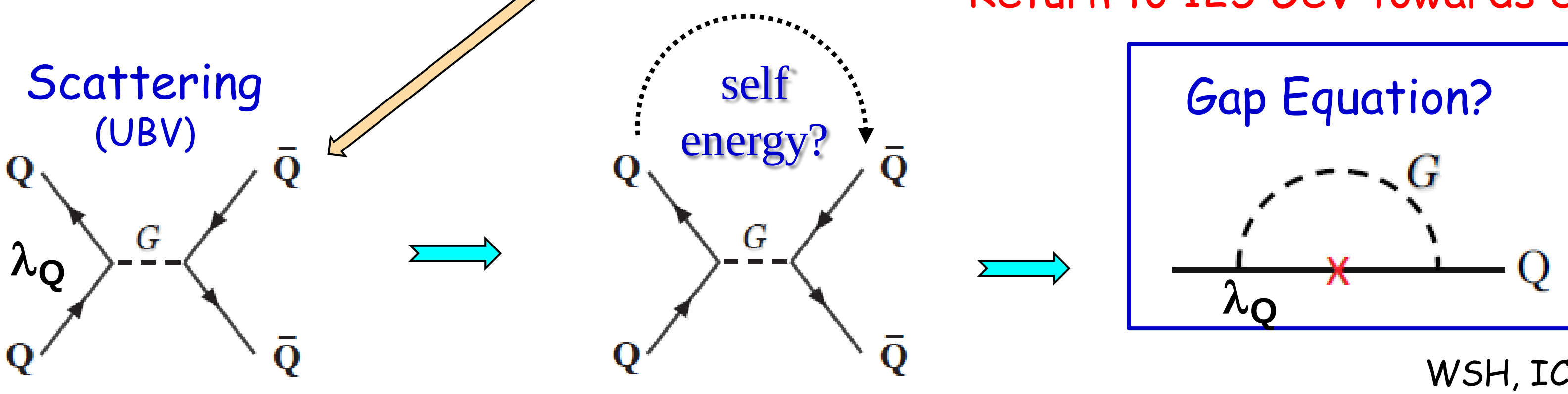
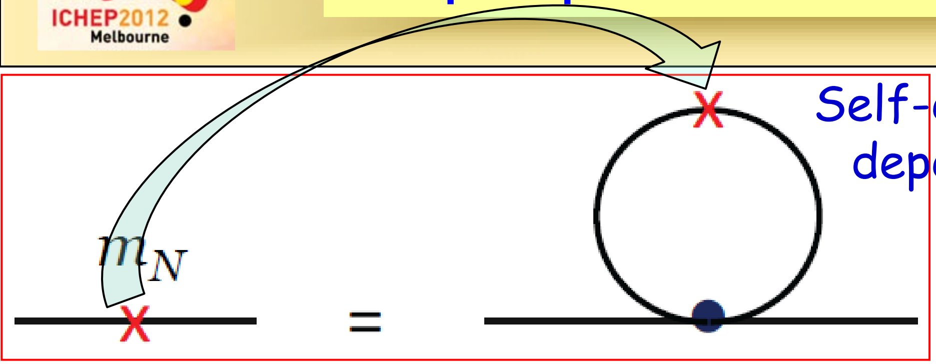


~ "long-distance" (vs $1/m_Q$)

WSH, ICHEP2010;
arXiv:1201.6029
(Chin. J. Phys., 6/2012)

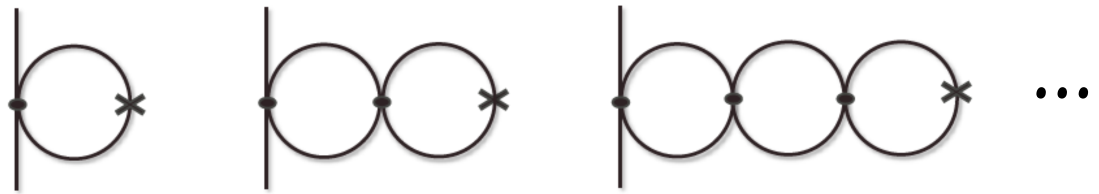
II. Dynamical Symmetry Breaking: Two Examples

$*$ NJL $*$ Strong Scale-inv. QED — Setup



Self-energy Does Not depend on momentum

→ infinite number of diagrams



$$m_N = \frac{N_C}{8\pi^2} G \int_0^{\Lambda^2} dq^2\, q^2 \frac{m_N}{q^2 + m_N^2}$$

$$= \frac{N_C}{8\pi^2} G\Lambda^2 \left(1 - \frac{m_N^2}{\Lambda^2} \log\left(1 + \frac{\Lambda^2}{m_N^2}\right)\right) m_N$$

factor out m_N

$$\boxed{1 - \frac{G_{\mathrm{crit}}}{G} = \frac{m_N^2}{\Lambda^2} \log\left(1 + \frac{\Lambda^2}{m_N^2}\right)}$$

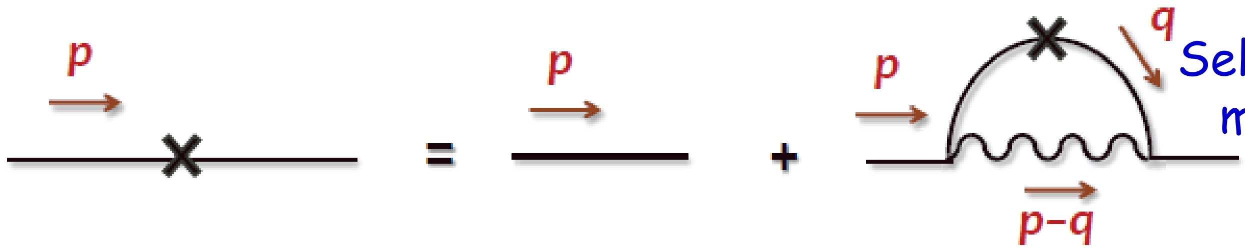
$$G_{\mathrm{crit}} = \frac{8\pi^2}{N_C \Lambda^2}$$

Eventually trade G and Λ for f_π and m_N

Gap Equation: Strong Scale-invariant QED

Gap equation for QED in the ladder approx.

w/ $m_0 = 0$



Self-energy
momentum-dep.

w. fn. renorm

$$\boxed{S(p)^{-1} = A(p^2)\,\not{p} - B(p^2)}$$

$$D_{\mu\nu}(q) = \frac{-g_{\mu\nu} + q^\mu q^\nu / q^2}{q^2} - \xi \frac{q_\mu q_\nu}{q^4}$$

$$\boxed{S(p)^{-1} = \not{p} - ie^2 \int \frac{d^4q}{(2\pi)^4} \gamma^\mu D_{\mu\nu}(p-q) S(q) \gamma^\nu}$$

Gap Equation: Strong Scale-invariant QED

Set up Notation

Gap equation for QED in the ladder approx.

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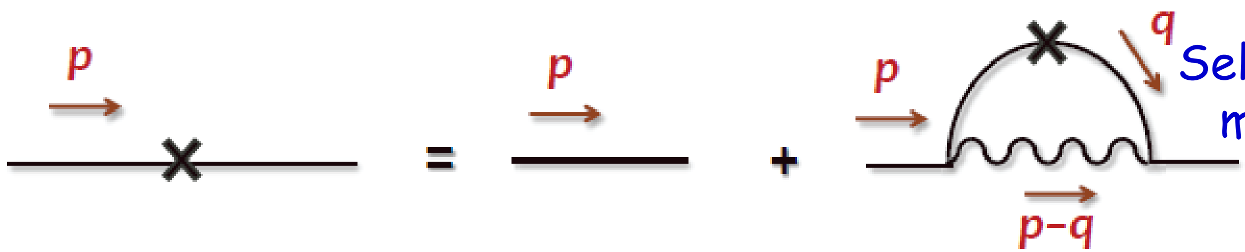

Self-energy momentum-dep.

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$$\boxed{S(p)^{-1} = A(p^2)\,\not p - B(p^2)}$$

"Mass" = B/A

$$D_{\mu\nu}(q) = \frac{-g_{\mu\nu} + q^\mu q^\nu/q^2}{q^2} - \xi\frac{q_\mu q_\nu}{q^4}$$

$$S(p)^{-1} = \not p - ie^2\int\frac{d^4q}{(2\pi)^4}\gamma^\mu D_{\mu\nu}(p-q)S(q)\gamma^\nu$$

angular integration

Wick rotation

$$B(x) = (3+\xi)\frac{\alpha}{4\pi}\int_{\Lambda_{IR}^2}^{\Lambda_{UV}^2} dy\frac{yB(y)}{yA(y)^2 + B(y)^2}\left(\frac{1}{x}\theta(x-y) + \frac{1}{y}\theta(y-x)\right)$$

$$A(x) = 1 + \xi\frac{\alpha}{4\pi}\int_{\Lambda_{IR}^2}^{\Lambda_{UV}^2} dy\frac{y^2B(y)}{y^2A(y)^2 + B(y)^2}\left(\frac{1}{x^2}\theta(x-y) + \frac{1}{y^2}\theta(y-x)\right)$$

$x = p^2,\ y = q^2$

Simplification

$\xi = 0$ (Landau gauge) ➡ $A(p^2) = 1$

Gap Equation: Strong Scale-invariant QED

Integral equation

$$B(x) = \frac{3\alpha}{4\pi}\left(\frac{1}{x}\int_{\Lambda_{IR}^2}^{x} dy \frac{yB(y)}{y+B(y)^2} + \int_x^{\Lambda_{UV}^2} dy \frac{B(y)}{y+B^2(y)}\right)$$

noting $B'(x) = -\frac{3\alpha}{4\pi}\frac{1}{x^2}\int_{\Lambda_{IR}^2}^{x} dy \frac{yB(y)}{y+B(y)^2}$ $\quad (xB(x))' = \frac{3\alpha}{4\pi}\int_x^{\Lambda_{UV}^2} dy \frac{B(y)}{y+B(y)^2}$

Differential form

$$\boxed{x\frac{d^2B(x)}{dx^2} + 2\frac{dB(x)}{dx} + \frac{3\alpha}{4\pi}\frac{B(x)}{x+B^2(x)} = 0}$$

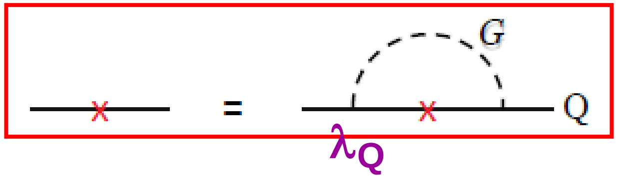
plus B.C.: $\left.\frac{dB(x)}{dx}\right|_{x=\Lambda_{\rm IR}^2} = 0, \quad \left.\frac{d(xB(x))}{dx}\right|_{x=\Lambda^2} = 0$

Solution: $B(x) \simeq C_1\sqrt{x}^{\,-1+\sqrt{1-3\alpha/\pi}} + C_2\sqrt{x}^{\,-1-\sqrt{1-3\alpha/\pi}}$

BCs → oscillation solutions → $\alpha > \alpha_{crit} = \frac{\pi}{3}$

Miransky scaling ... ?

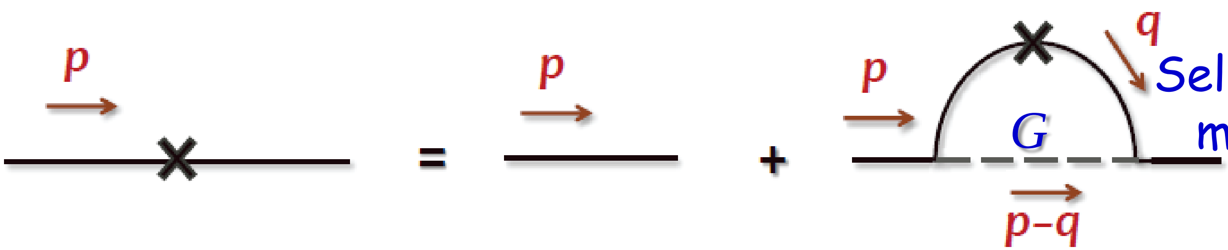
III. Yukawa "Bootstrap" Gap Eq.: Dyn. EWSB



Gap Equation: Yukawa "Bootstrap"

Gap equation for large Yukawa in the ladder approx. (neglected gauge coupling)

w/ $m_0 = 0$



Self-energy momentum-dep.

$$S(p)^{-1} = A(p^2)\,\not{p} - B(p^2)$$

$$\text{Goldstone propagator}: \; D(q) = 1/q^2$$

"Mass" = B/A

$$B(p^2) = +\frac{3\lambda_Q^2}{2}\int \frac{d^4q}{i(2\pi)^4}\frac{1}{(p-q)^2}\frac{B(q^2)}{A^2(q^2)q^2 - B^2(q^2)} \cancel{-\frac{\lambda_Q^2}{2}\int \frac{d^4q}{i(2\pi)^4}\frac{1}{(p-q)^2 - m_h^2}\frac{B(q^2)}{A^2(q^2)q^2 - B^2(q^2)}}$$

$$A(p^2)p^2 = p^2 + \frac{3\lambda_Q^2}{2}\int \frac{d^4q}{i(2\pi)^4}\frac{p\cdot q}{(p-q)^2}\frac{B(q^2)}{A^2(q^2)q^2 - B^2(q^2)} \cancel{+\frac{\lambda_Q^2}{2}\int \frac{d^4q}{i(2\pi)^4}\frac{p\cdot q}{(p-q)^2 - m_h^2}\frac{B(q^2)}{A^2(q^2)q^2 - B^2(q^2)}}$$

Drop Higgs for now; return later.

Gap Equation: Yukawa "Bootstrap"

$$B(p^2) = +\frac{3\lambda_Q^2}{2}\int\frac{d^4q}{i(2\pi)^4}\frac{1}{(p-q)^2}\frac{B(q^2)}{A^2(q^2)q^2-B^2(q^2)}$$
$$-\frac{\lambda_Q^2}{2}\int\frac{d^4q}{i(2\pi)^4}\frac{1}{(p-q)^2-m_h^2}\frac{B(q^2)}{A^2(q^2)q^2-B^2(q^2)}$$

but, vs. Hung-Xiong '11

massless ϕ doublet

$+$

"Mass" = B/A

$$A(p^2)p^2 = p^2+\frac{3\lambda_Q^2}{2}\int\frac{d^4q}{i(2\pi)^4}\frac{p\cdot q}{(p-q)^2}\frac{B(q^2)}{A^2(q^2)q^2-B^2(q^2)}$$
$$+\frac{\lambda_Q^2}{2}\int\frac{d^4q}{i(2\pi)^4}\frac{p\cdot q}{(p-q)^2-m_h^2}\frac{B(q^2)}{A^2(q^2)q^2-B^2(q^2)}$$

$$\boxed{A(p^2) = 1}$$

- If follow HX'11 and ignore $A(p^2)$ equation, then ~ Scale-inv. QED, w/ critical $\alpha_Q = \pi/2$, or $\lambda_Q \sim \sqrt{2}\pi$, and $m_Q \sim 770\ \mathrm{GeV}$
- If we use correct sign, but continue to use $A(p^2) = 1$ in $B(p^2)$ equation, then $\lambda_Q^c = 2\pi$, or $m_Q^c \sim 1.1\ \mathrm{TeV}$, i.e. $\sqrt{2}$ higher
- But one should not ignore $A(p^2)$ equation!

Gap Equation: Yukawa "Bootstrap"

So, $$B(x)=\kappa_b\left(\frac{1}{x}\int_0^x dy\frac{yB(y)}{yA^2(y)+B^2(y)}+\int_x^{\Lambda^2}dy\frac{B(y)}{yA^2(y)+B^2(y)}\right)$$

$$A(x)=1+\kappa_a\left(\frac{1}{x^2}\int_0^x dy\frac{y^2A(y)}{yA^2(y)+B^2(y)}+\int_x^{\Lambda^2}dy\frac{A(y)}{yA^2(y)+B^2(y)}\right)$$

$$\boxed{p^2=x=e^{2t}}$$

$$xB''+2B'+\frac{\kappa_b B}{xA^2+B^2}=0,$$

$$xA''+3A'+\frac{2\kappa_a A}{xA^2+B^2}=0,$$

the boundary conditions

$$B'(x)|_{x=\Lambda_{\rm IR}^2}=0,\ \ (xB'(x)+B(x))\,|_{x=\Lambda^2}=0,$$

$$A'(x)|_{x=\Lambda_{\rm IR}^2}=0,\ \ \left(\frac{1}{2}xA'(x)+A(x)\right)|_{x=\Lambda^2}=1$$

$$\ddot{B}+2\dot{B}+\frac{4\kappa_b B}{A^2+B^2e^{-2t}}=0,$$

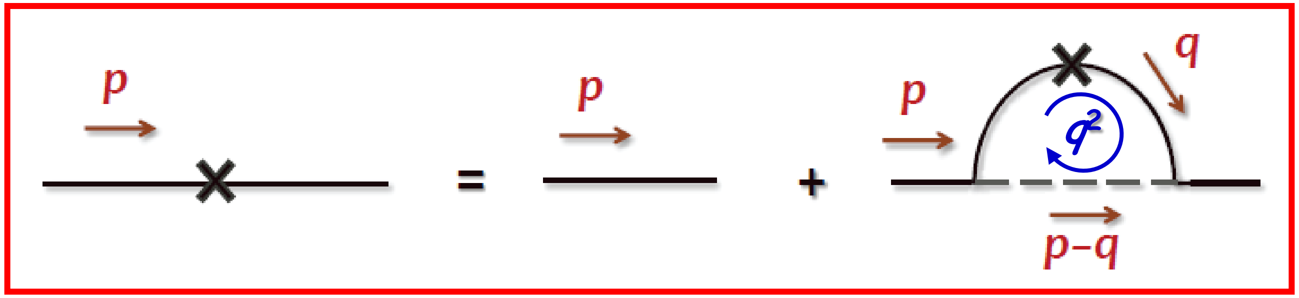
$$\ddot{A}+4\dot{A}+\frac{8\kappa_a A}{A^2+B^2e^{-2t}}=0,$$

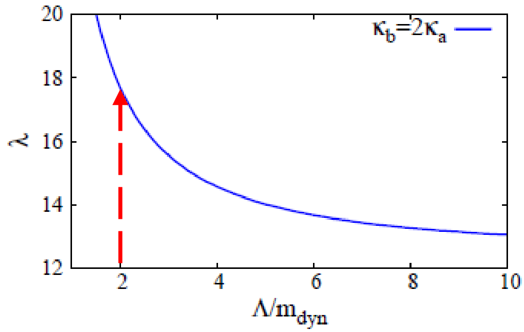
$$\dot{B}(t_{\rm IR})=0,\qquad \dot{B}(t_{\rm UV})+B(t_{\rm UV})=0$$

$$\dot{A}(t_{\rm IR})=0,\quad \frac{1}{4}\dot{A}(t_{\rm UV})+A(t_{\rm UV})=1$$

We find, numerically, $\kappa_b=2\kappa_a=3\alpha_Q/8\pi$ ~ 1.4

→ $\boxed{\lambda_Q^c\simeq 12}$ → $\boxed{m_Q^c>2.1\ \text{TeV},\qquad \text{(No Higgs)}}$



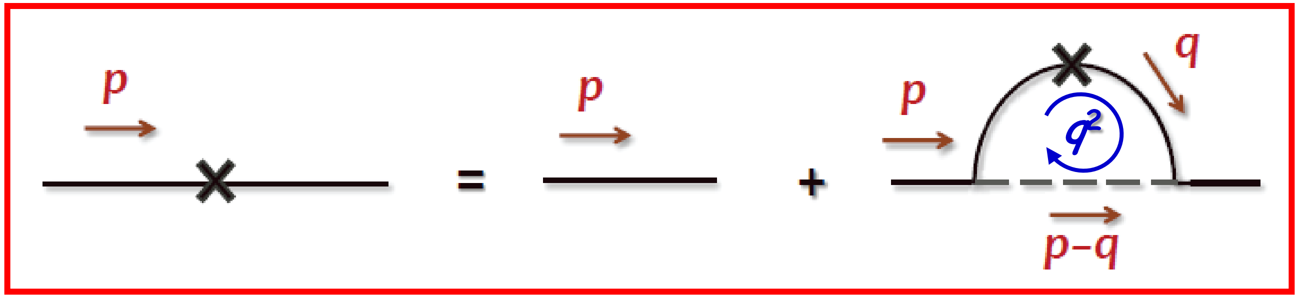


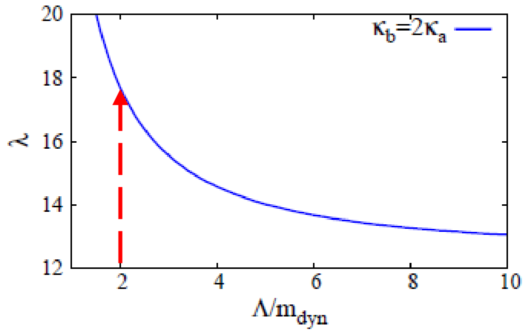
- Gap Eq.: Integrate over q^2, so $\Lambda < 2m_Q$ for G to remain Goldstone

→ Λ cannot be taken arbitrarily large

This raises $\lambda_Q^c \simeq 12$ to $\lambda_Q^c \sim 17.7$,

OR $\boxed{m_Q \sim 3 \text{ TeV}, \qquad (\text{No Higgs; } \Lambda = 2m_Q)}$ depressingly large !





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- Put back the light Higgs (125 GeV close to 80 GeV) ?

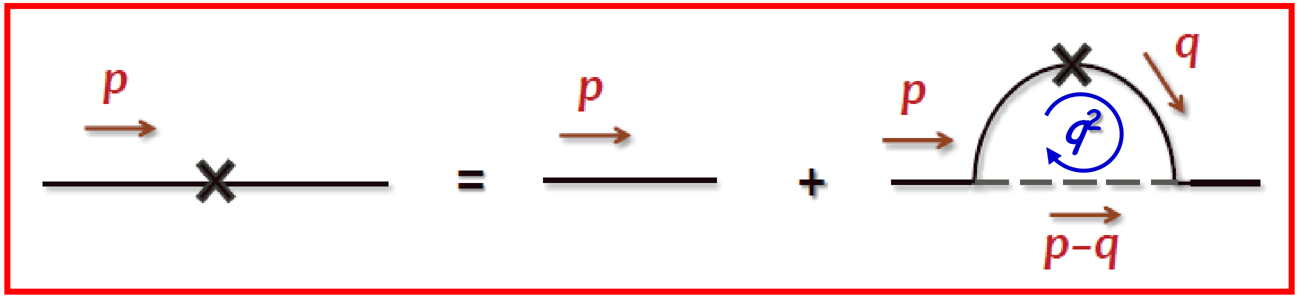
$\kappa_b = 2\kappa_a = 3\alpha_Q/8\pi$ ~ 1.4

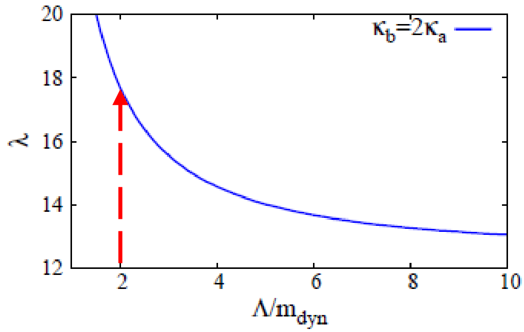
→ $\boxed{\lambda_Q^c \simeq 12}$ → $\boxed{m_Q^c > 2.1\ \text{TeV}, \qquad (\text{No Higgs})}$

$\kappa_b = \kappa_a = \alpha_Q/4\pi$ ~ 13.7 !

No Good !

→ $\boxed{\lambda_Q^c \simeq 46}$ → $\boxed{m_Q^c > 8.1\ \text{TeV}, (\text{massless Higgs})}$





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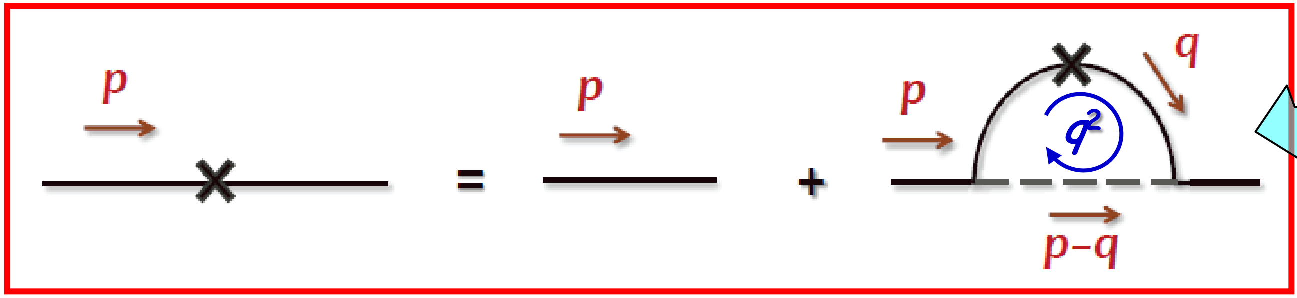
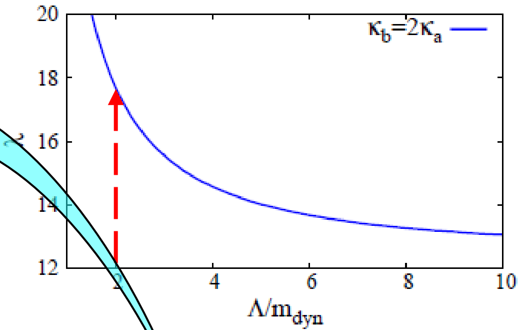
- Can this be made Lower?





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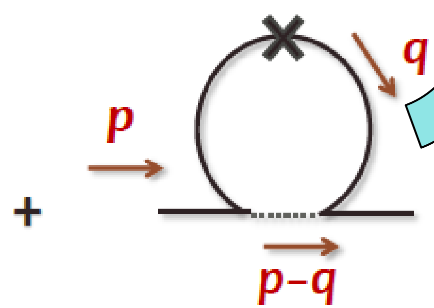
- Can this be made Lower? Perhaps:



Other Tight Bound States → Heavy “meson”

IV. Discussion and Conclusion

- So, what about 125 GeV object ?

4G (chiral Q) bites the dust, again ... (not that it did not happen before)

But our Gap Eq. is nominally Scale-inv. (no scale):

$$\boxed{\begin{aligned} &x \to a^2 x \qquad (t \to t + \log a), \\ &\Lambda_{\rm UV,IR} \to a\Lambda_{\rm UV,IR}, \\ &B \to aB, \quad A \to A. \end{aligned}}$$

Used in aid of numerical solution.

$$p^2 = x = e^{2t}$$

Dynamical Mass Generation also means breaking of scale invariance.

Could there be a Dilaton ?

Goldberger, Grinstein, Skiba, '08
Barger, Ishida, Keung, '12
Campbell, Ellis, Olive, '12
Coleppa, Gregroire, Logan, '12
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(Radion via AdS/CFT or warped)

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etc.

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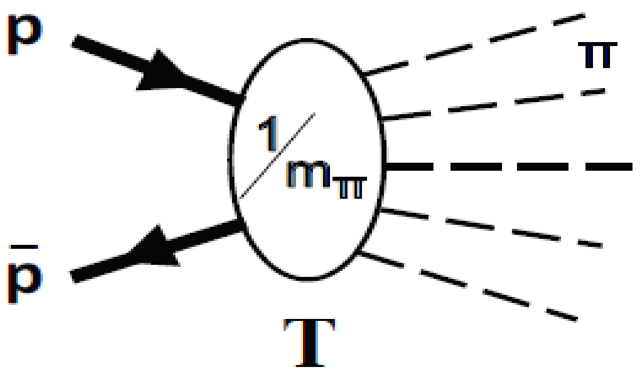
- Is 2-3 TeV Quark Mass troubling for sake of DSB ?

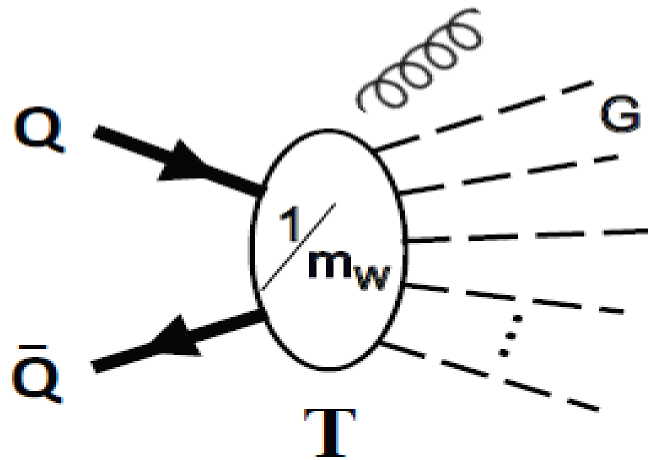
Yes, but no. Pion-Nucleon very analogous !

Possible Phenomenology: $Q\bar{Q} \to nV_L$

$$\boxed{g_{\pi NN} \simeq \lambda_{\pi NN} \equiv \sqrt{2} m_N / f_\pi \simeq 14} \quad \Rightarrow \quad \boxed{m_Q \gtrsim 2\ \text{TeV}}$$

An Intriguing Analogy





annihilation "fireball"

data

- Size of order $1/m_\pi$;
- Temperature $T \simeq 120$ MeV;
- Average number of emitted pions $\langle n_\pi \rangle \simeq 5$;
- A soft-pion p_π^2/E_π^2 factor modulates the Maxwell–Boltzman distribution for the pions.

Sample

$$T \sim \frac{2}{3} v \sim 160\ \text{GeV}$$

$$\langle |p_G| \rangle \sim 310\ \text{GeV},$$

$$\langle n_G \rangle \sim 6.25\ (12.5),$$

$$P(n_G) \simeq 0.319\, e^{-\frac{(n_G - 6.25)^2}{3.13}} \quad \left(0.226\, e^{-\frac{(n_G-12.5)^2}{6.25}}\right)$$

wsh, arXiv:1206.1453 [poster]

Possible Phenomenology: $Q\bar{Q} \to nV_L$

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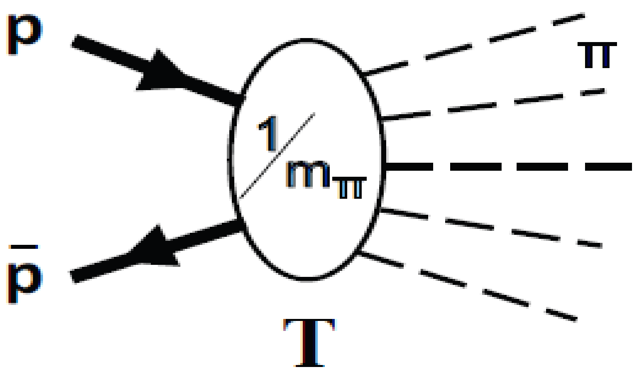
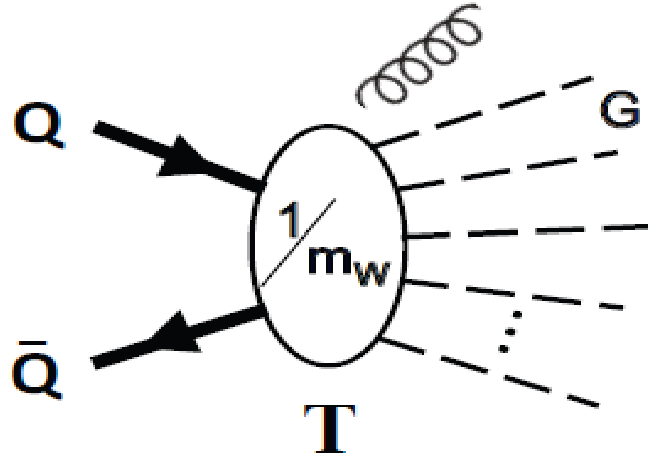
An Intriguing Analogy





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A New Fermi-Yang Model of G as $Q\bar{Q}$ boundstate?

wsh, arXiv:1206.1453 [poster]

Conculsion

- A Dynamical Gap Eq., by Goldstone, or Longitudinal V exch. with Strong Yukawa Coupling is constructed, and solved.

  This can, in principle, Replace Scalar Condensation mech.

- The Needed Yukawa Coupling is above 10!! [~ proton]
  This implies 4G masses in 2-3 TeV Range.

  Boundstate Resonance Production (Yokoya talk),
  with Decay into Multi-V_L, should be considered.

- The new 125 GeV Boson poses difficulties for 4G.

  Could Dilaton save the day?
  Our Gap Eq. is nominally Scale-invariant.