Bootstrap Dynamical Symmetry Breaking
with New Heavy Chiral Quarks

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I. The (non)Motivational

abstract $\Rightarrow$ 125 GeV @ end ...
scattering / self-energy / Gap Eq.

II. Dynamical Symmetry Breaking: Two Examples

* NJL * Strong Scale-inv. QED — Setup

III. Yukawa “Bootstrap” Gap Eq.: Dyn. EWSB

$\lambda_Q$ Bootstrap DSB

IV. Discussion and Conclusion
I. The (non)Motivational

Title: Bootstrap Dynamical Symmetry Breaking with New Heavy Chiral Quarks

Abstract: Despite the hint for a 125 GeV Higgs boson, we consider the other option of MH > 600 GeV, noting that the existence of the Higgs boson itself is not yet an established fact. What we do know is that the Goldstone bosons of electroweak symmetry breaking exist as longitudinal components of the weak bosons. The Goldstone boson coupling to a new heavy chiral quark doublet Q (assuming it exist), the G-Q-Q(bar) Yukawa coupling, would now be in the strong coupling regime, given the LHC limit of MQ > 600 GeV is already beyond the perturbative partial-wave unitarity bound. Such strong Yukawa couplings could induce Q-Q(bar) condensation, which might take the role of the Higgs condensate. Guided by a Bethe-Salpeter ...

Should I stop here and now?

Return to 125 GeV towards end ...
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Abstract:
Despite the hint for a 125 GeV Higgs boson, we consider the other option of $M_H > 600$ GeV, noting that the existence of the Higgs boson itself is not yet an established fact. What we do know is that the Goldstone bosons of electroweak symmetry breaking exist as longitudinal components of the weak bosons. The Goldstone boson coupling to a new heavy chiral quark doublet $Q$ (assuming it exist), the $G$-$Q$-$Q$($\bar{Q}$) Yukawa coupling, would now be in the strong coupling regime, given the LHC limit of $M_Q > 600$ GeV is already beyond the perturbative partial-wave unitarity bound. Such strong Yukawa couplings could induce $Q$-$Q$($\bar{Q}$) condensation, which might take the role of the Higgs condensate. Guided by a Bethe-Salpeter ...
II. Dynamical Symmetry Breaking: Two Examples

* NJL
* Strong Scale-inv. QED — Setup
**Gap Equation: Nambu–Jona-Lasinio Model**

Self-energy Does Not depend on momentum

\[ m_N = \frac{N_C}{8\pi^2} G \int_0^\Lambda^2 dq^2 \frac{q^2}{q^2 + m_N^2} m_N \]

\[ = \frac{N_C}{8\pi^2} G \Lambda^2 \left( 1 - \frac{m_N^2}{\Lambda^2} \log \left( 1 + \frac{\Lambda^2}{m_N^2} \right) \right) m_N \]

Eventually trade \( G \) and \( \Lambda \) for \( f_\pi \) and \( m_N \)

\[ 1 - \frac{G_{\text{crit}}}{G} = \frac{m_N^2}{\Lambda^2} \log \left( 1 + \frac{\Lambda^2}{m_N^2} \right) \]

\[ G_{\text{crit}} = \frac{8\pi^2}{N_C\Lambda^2} \]
Gap equation for QED in the ladder approx. w/ $m_0 = 0$

\[ S(p)^{-1} = A(p^2) \not{p} - B(p^2) \]

\[ S(p)^{-1} = \not{p} - ie^2 \int \frac{d^4q}{(2\pi)^4} \gamma^\mu D_{\mu\nu}(p-q)S(q)\gamma^\nu \]

Self-energy momentum-dep. w. fn. renorm
Gap Equation: Strong Scale-invariant QED

Set up Notation

Gap equation for QED in the **ladder approx.**

with $m_0 = 0$

\[
S(p)^{-1} = A(p^2) p - B(p^2)
\]

**Self-energy momentum-dep.**

\[
D_{\mu \nu}(q) = -g_{\mu \nu} + q^\mu q^\nu / q^2 - \xi q^\mu q^\nu / q^4
\]

**“Mass” = $B/A$**

\[
S(p)^{-1} = p - ie^2 \int \frac{d^4q}{(2\pi)^4} \gamma^\mu D_{\mu \nu}(p - q) S(q) \gamma^\nu
\]

**Angular Wick rotation**

\[
x = p^2, \ y = q^2
\]

**Simplification**

\[
\xi = 0 \quad \text{(Landau gauge)} \quad \Rightarrow \quad A(p^2) = 1
\]
Integral equation

\[ B(x) = \frac{3\alpha}{4\pi} \left( \frac{1}{x} \int_{\Lambda_{IR}^2}^x dy \frac{yB(y)}{y + B(y)^2} + \int_{x}^{\Lambda_{UV}^2} dy \frac{B(y)}{y + B^2(y)} \right) \]

noting \[ B'(x) = -\frac{3\alpha}{4\pi} \frac{1}{x^2} \int_{\Lambda_{IR}^2}^x dy \frac{yB(y)}{y + B(y)^2} \]

\[ (xB(x))' = \frac{3\alpha}{4\pi} \int_{x}^{\Lambda_{UV}^2} dy \frac{B(y)}{y + B(y)^2} \]

Differential form

\[ x \frac{d^2B(x)}{dx^2} + 2 \frac{dB(x)}{dx} + \frac{3\alpha}{4\pi} \frac{B(x)}{x + B^2(x)} = 0 \]

plus B.C.:
\[ \left. \frac{dB(x)}{dx} \right|_{x=\Lambda_{IR}^2} = 0, \quad \left. \frac{d(xB(x))}{dx} \right|_{x=\Lambda^2} = 0 \]

Solution:
\[ B(x) \approx C_1 \sqrt{x^{-1 + \sqrt{1-3\alpha}/\pi}} + C_2 \sqrt{x^{-1 - \sqrt{1-3\alpha}/\pi}} \]

BCs \(\rightarrow\) oscillation solutions \(\rightarrow\) \(\alpha > \alpha_{crit} = \frac{\pi}{3}\)

Miransky scaling ... ?
III. Yukawa “Bootstrap” Gap Eq.: Dyn. EWSB

\[ \lambda_Q \]

\[ \text{Dyn. EWSB} \]
Gap equation for large Yukawa in the \textit{ladder approx.} (neglected gauge coupling) w/ $m_0 = 0$

\[ S(p)^{-1} = A(p^2) p - B(p^2) \]

Self-energy momentum-dep.

Goldstone propagator: $D(q) = 1/q^2$

\[ B(p^2) = \frac{3\lambda_Q^2}{2} \int \frac{d^4q}{i(2\pi)^4} \frac{1}{(p-q)^2} \frac{B(q^2)}{A^2(q^2)q^2 - B^2(q^2)} \]
\[ - \frac{\lambda_Q^2}{2} \int \frac{d^4q}{i(2\pi)^4} \frac{1}{(p-q)^2 - m_h^2} \frac{B(q^2)}{A^2(q^2)q^2 - B^2(q^2)} \]

\[ A(p^2)p^2 = p^2 + \frac{3\lambda_Q^2}{2} \int \frac{d^4q}{i(2\pi)^4} \frac{p \cdot q}{(p-q)^2} \frac{B(q^2)}{A^2(q^2)q^2 - B^2(q^2)} \]
\[ + \frac{\lambda_Q^2}{2} \int \frac{d^4q}{i(2\pi)^4} \frac{p \cdot q}{(p-q)^2 - m_h^2} \frac{B(q^2)}{A^2(q^2)q^2 - B^2(q^2)} \]

Drop Higgs for now; return later.
Gap Equation: Yukawa “Bootstrap”

\[ B(p^2) = \frac{3\lambda_Q^2}{2} \int \frac{d^4q}{(2\pi)^4} \frac{1}{(p-q)^2} \frac{B(q^2)}{A^2(q^2)q^2 - B^2(q^2)} \]

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• If follow HX’11 and ignore \( A(p^2) \) equation, then \( \sim \) Scale-inv. QED, w/ critical \( \alpha_Q = \pi/2 \), or \( \lambda_Q \sim \sqrt{2\pi} \), and \( m_Q \sim 770 \) GeV

• If we use correct sign, but continue to use \( A(p^2) = 1 \) in \( B(p^2) \) equation, then \( \lambda_Q^c = 2\pi \), or \( m_Q^c \sim 1.1 \) TeV, i.e. \( \sqrt{2} \) higher

• But one should not ignore \( A(p^2) \) equation!
Gap Equation: Yukawa “Bootstrap”

So,

\[ B(x) = \kappa_b \left( \frac{1}{x} \int_0^x dy \frac{yB(y)}{yA^2(y) + B^2(y)} + \int_x^{\Lambda^2} dy \frac{B(y)}{yA^2(y) + B^2(y)} \right) \]

\[ A(x) = 1 + \kappa_a \left( \frac{1}{x^2} \int_0^x dy \frac{y^2 A(y)}{yA^2(y) + B^2(y)} + \int_x^{\Lambda^2} dy \frac{A(y)}{yA^2(y) + B^2(y)} \right) \]

\[ p^2 = x = e^{2t} \]

\[ xB'' + 2B' + \frac{\kappa_b B}{xA^2 + B^2} = 0, \]

\[ xA'' + 3A' + \frac{2\kappa_a A}{xA^2 + B^2} = 0, \]

the boundary conditions

\[ B'(x)|_{x=\Lambda^2_{IR}} = 0, \quad (xB'(x) + B(x))|_{x=\Lambda^2} = 0, \]

\[ A'(x)|_{x=\Lambda^2_{IR}} = 0, \quad \left( \frac{1}{2} xA'(x) + A(x) \right)|_{x=\Lambda^2} = 1 \]

We find, numerically, \( \kappa_b = 2\kappa_a = \frac{3\alpha_Q}{8\pi} \approx 1.4 \)

\[ \lambda_Q^c \approx 12 \]

\[ m_Q^c > 2.1 \text{ TeV}, \quad \text{(No Higgs)} \]
• Gap Eq.: Integrate over $q^2$, so $\Lambda < 2m_Q$ for $G$ to remain Goldstone

$\Rightarrow \Lambda$ cannot be taken arbitrarily large

This raises $\lambda_Q^c \approx 12$ to $\lambda_Q^c \approx 17.7$,

OR $m_Q \approx 3$ TeV, \hspace{1cm} (No Higgs; $\Lambda = 2m_Q$)

*depressingly large!*
• Gap Eq.: Integrate over $q^2$, so $\Lambda < 2m_Q$ for $G$ to remain Goldstone

$\rightarrow$ $\Lambda$ cannot be taken arbitrarily large

This raises $\lambda^c_Q \simeq 12$ to $\lambda^c_Q \sim 17.7$,

OR

$m_Q \sim 3 \text{ TeV}, \quad (\text{No Higgs}; \Lambda = 2m_Q)$

• Put back the light Higgs (125 GeV close to 80 GeV)?

$\kappa_b = 2\kappa_a = \frac{3\alpha_Q}{8\pi} \sim 1.4$

$\rightarrow$ $\lambda^c_Q \simeq 12$ $\rightarrow$ $m^c_Q > 2.1 \text{ TeV}, \quad (\text{No Higgs})$

$\kappa_b = \kappa_a = \frac{\alpha_Q}{4\pi} \sim 13.7!$ $\rightarrow$ $\lambda^c_Q \simeq 46$ $\rightarrow$ $m^c_Q > 8.1 \text{ TeV}, \quad (\text{massless Higgs})$
• Gap Eq.: Integrate over $q^2$, so $\Lambda < 2m_Q$ for $G$ to remain Goldstone

$\rightarrow \Lambda$ cannot be taken arbitrarily large

This raises $\lambda_Q^c \approx 12$ to $\lambda_Q^c \approx 17.7$, OR $m_Q \approx 3$ TeV, (No Higgs; $\Lambda = 2m_Q$)

• Can this be made Lower?
Gap Eq.: Integrate over $q^2$, so $\Lambda < 2m_Q$ for $G$ to remain Goldstone

$\rightarrow \Lambda$ cannot be taken arbitrarily large

This raises $\lambda_Q^c \approx 12$ to $\lambda_Q^c \approx 17.7$.

OR

$m_Q \approx 3$ TeV, (No Higgs; $\Lambda = 2m_Q$)

Can this be made Lower? Perhaps:

Other Tight Bound States $\Rightarrow$ Heavy “meson”
IV. Discussion and Conclusion
So, what about 125 GeV object?

4G (chiral Q) bites the dust, again ... (not that it did not happen before)

But our Gap Eq. is nominally Scale-inv. (no scale):

\[ x \rightarrow a^2 x \quad (t \rightarrow t + \log a), \]
\[ \Lambda_{UV,IR} \rightarrow a\Lambda_{UV,IR}, \]
\[ B \rightarrow aB, \quad A \rightarrow A. \]

Used in aid of numerical solution.

\[ p^2 = x = e^{2t} \]

Dynamical Mass Generation also means breaking of scale invariance.

Could there be a Dilaton? (Radion via AdS/CFT or warped)

Goldberger, Grinstein, Skiba, '08
Barger, Ishida, Keung, '12
Campbell, Ellis, Olive, '12
Coleppa, Gregoire, Logan, '12
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etc.
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• Is 2-3 TeV Quark Mass troubling for sake of DSB?

Yes, but no. Pion-Nucleon very analogous!
Possible Phenomenology: $Q\bar{Q} \rightarrow nV_L$

$g_{\pi NN} \approx \lambda_{\pi NN} \equiv \sqrt{2m_N/f_\pi} \approx 14 \quad \Rightarrow \quad m_Q \gtrsim 2 \text{ TeV}$

An Intriguing Analogy

annihilation “fireball”

- Size of order $1/m_\pi$;
- Temperature $T \approx 120 \text{ MeV}$;
- Average number of emitted pions $\langle n_\pi \rangle \approx 5$;
- A soft-pion $p_\pi^2/E_\pi^2$ factor modulates the Maxwell–Boltzmann distribution for the pions.

Sample

$T \sim \frac{2}{3} v \sim 160 \text{ GeV}$

$\langle |p_G| \rangle \sim 310 \text{ GeV}$,

$\langle n_G \rangle \sim 6.25 \pm 12.5$,

$P(n_G) \approx 0.319 e^{-\frac{(n_G-6.25)^2}{3.13}} \left(0.226 e^{-\frac{(n_G-12.5)^2}{6.25}}\right)$

wsh, arXiv:1206.1453 [poster]
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A New Fermi-Yang Model of $G$ as $Q\bar{Q}$ boundstate?
Conclusions

- A Dynamical Gap Eq., by Goldstone, or Longitudinal V exch. with Strong Yukawa Coupling is constructed, and solved. This can, in principle, Replace Scalar Condensation mech.

- The Needed Yukawa Coupling is above $10^{10}$!! This implies 4G masses in 2-3 TeV Range.

  Boundstate Resonance Production (Yokoya talk), with Decay into Multi-$V_L$, should be considered.

- The new 125 GeV Boson poses difficulties for 4G. Could Dilaton save the day? Our Gap Eq. is nominally Scale-invariant.