J/ψ production in NRQCD: A global analysis of yield and polarization

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J/ψ Production with NRQCD

Factorization theorem: $$\sigma_{J/\psi} = \sum_n \sigma_{c\bar{c}[n]} \cdot \langle O^{J/\psi}[n] \rangle$$

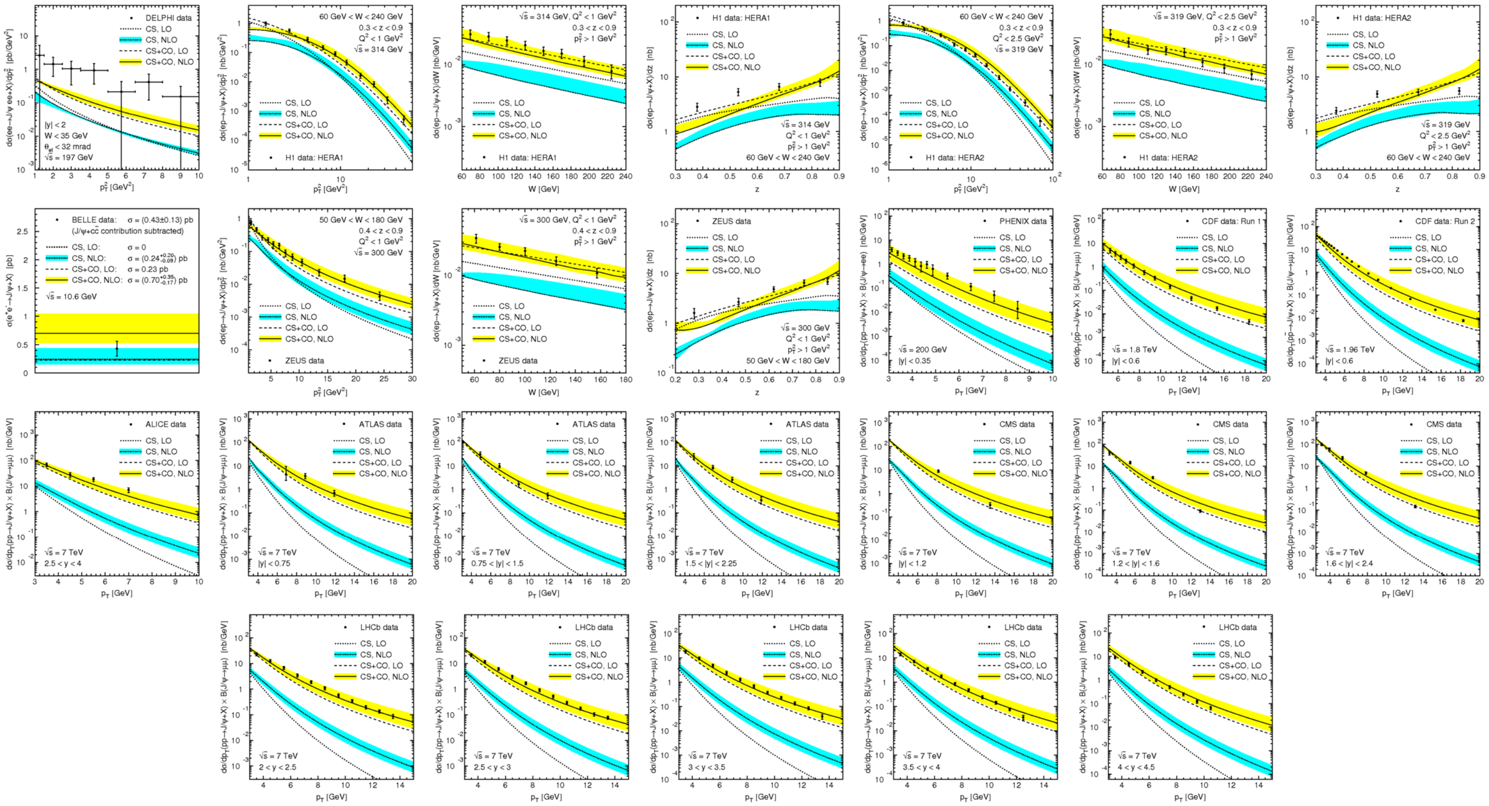
- ***n***: Every possible Fock state, including **color-octet** (CO) states.
- $\boldsymbol{\sigma_{c\bar{c}[n]}}$: Production rate of $c\bar{c}[n]$, calculated in perturbative QCD
- $\boldsymbol{\langle O^{J/\psi}[n] \rangle}$: Long distance matrix elements (LDMEs): describe $c\bar{c}[n] \rightarrow J/\psi$, universal, extracted from experiment.

Scaling rules: LDMEs scale with definite power of v ($v^2 \approx 0.2$):

scaling	v^3	v^7 ("CO states")	v^{11}
n	$^3S_1^{[1]}$	$^1S_0^{[8]}$, $^3S_1^{[8]}$, $^3P_J^{[8]}$	...

- **Double expansion** in v and α_s
- Leading term in v (n = $^3S_1^{[1]}$) equals **color-singlet model**.

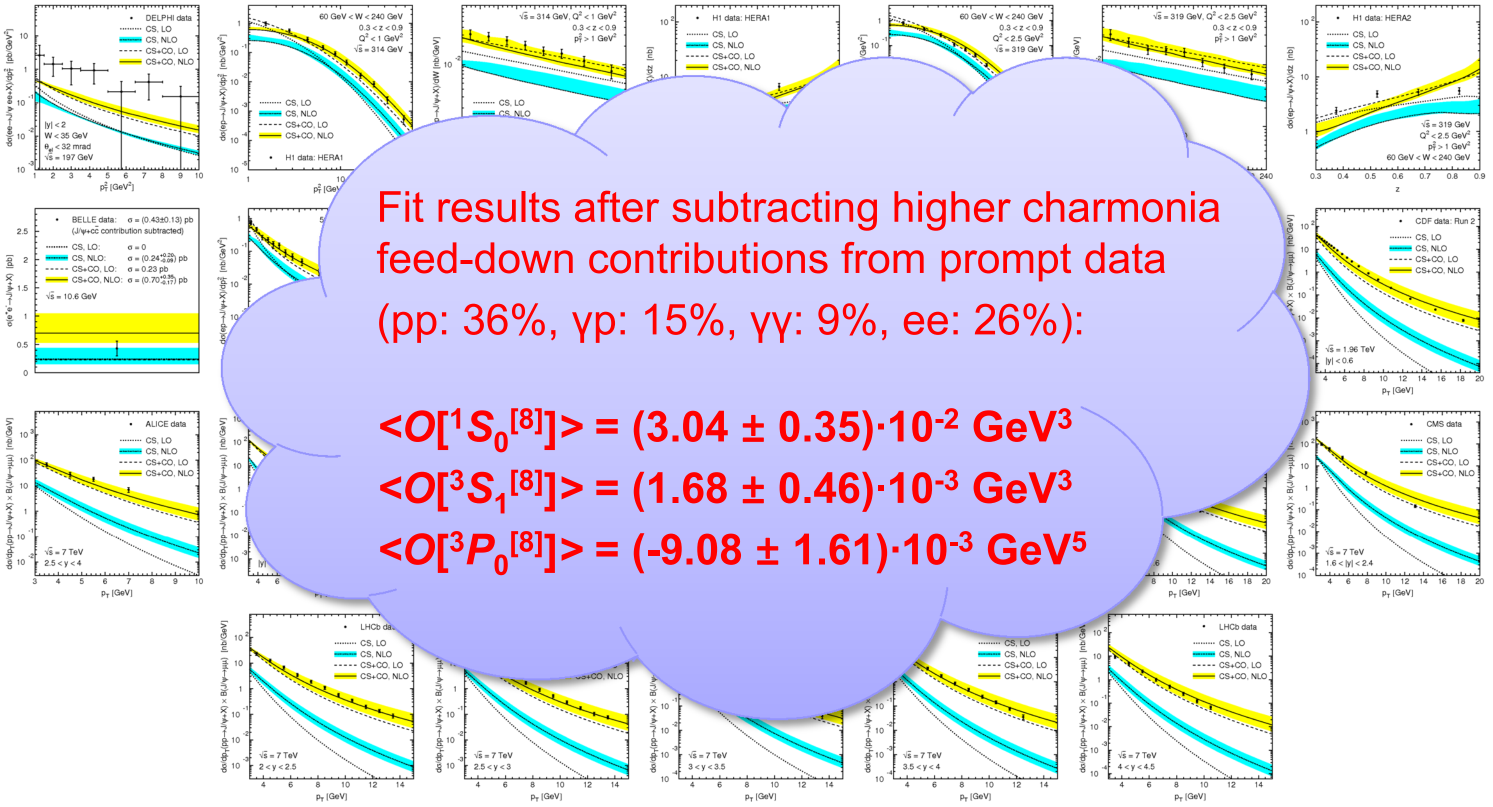
Global Fit to Unpolarized Data



$\langle O[^1S_0^{[8]}]\rangle$ = (4.97 ± 0.44)·10^{-2} GeV^3 **$\langle O[^3S_1^{[8]}]\rangle$** = (2.24 ± 0.59)·10^{-3} GeV^3

$\langle O[^3P_0^{[8]}]\rangle$ = (-1.61 ± 0.20)·10^{-2} GeV^5

Global Fit to Unpolarized Data



Fit results after subtracting higher charmonia feed-down contributions from prompt data (pp: 36%, γp: 15%, γγ: 9%, ee: 26%):

$\langle O[{}^1S_0^{[8]}]\rangle = (3.04 \pm 0.35)\cdot 10^{-2}$ GeV^3

$\langle O[{}^3S_1^{[8]}]\rangle = (1.68 \pm 0.46)\cdot 10^{-3}$ GeV^3

$\langle O[{}^3P_0^{[8]}]\rangle = (-9.08 \pm 1.61)\cdot 10^{-3}$ GeV^5

$\langle O[{}^1S_0^{[8]}]\rangle$ $= (4.97 \pm 0.44)\cdot 10^{-2}$ GeV^3 **$\langle O[{}^3S_1^{[8]}]\rangle$** $= (2.24 \pm 0.59)\cdot 10^{-3}$ GeV^3

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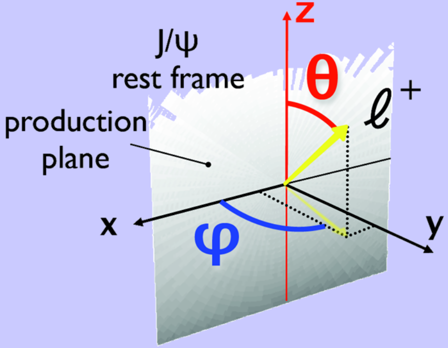
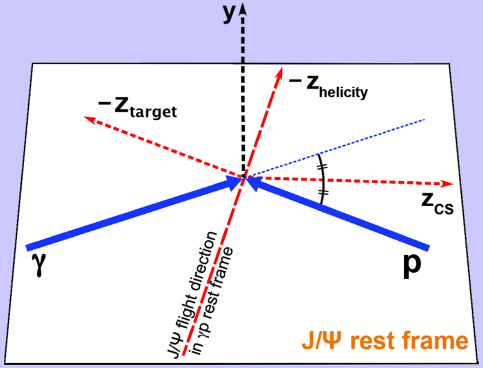
J/ψ Polarization

- **Angular distribution** of decay lepton l^+ in J/ψ rest frame
  ➡ Polarization observables ***λ*, *μ*, *ν*:**

$$\frac{d\Gamma(J/\psi \to l^+l^-)}{d\cos\theta\, d\phi} \propto 1 + \lambda\cos^2\theta + \mu\sin(2\theta)\cos\phi + \frac{\nu}{2}\sin^2\theta\cos(2\phi)$$



- Depends on choice of **coordinate system:**
  - ☐ Helicity frame: z axis $\parallel -(\vec{p}_\gamma + \vec{p}_p)$
  - ☐ Collins-Soper frame: z axis $\parallel \vec{p}_\gamma/|\vec{p}_\gamma| - \vec{p}_p/|\vec{p}_p|$
  - ☐ Target frame: z axis $\parallel -\vec{p}_p$

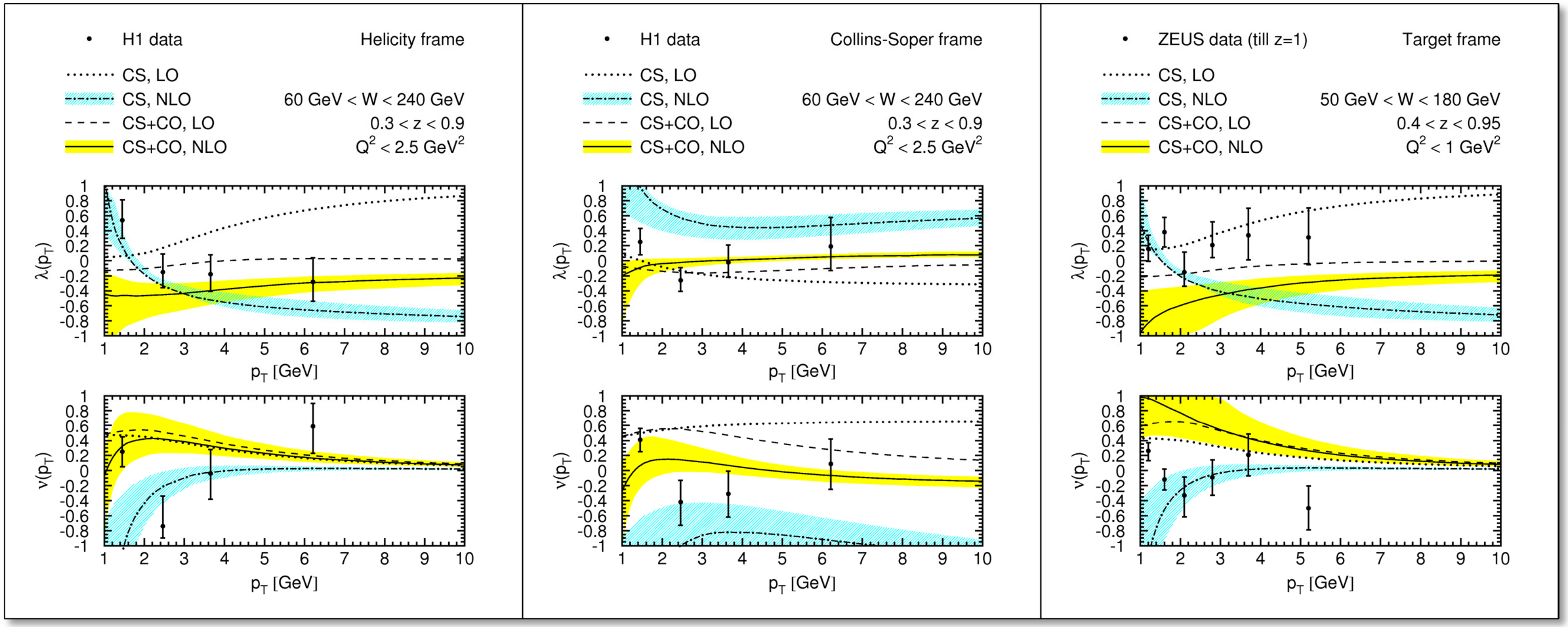


- **In Calculation:** Plug in explicit expressions for $c\bar{c}[n]$ spin polarization vectors according to

$$\lambda = \frac{d\sigma_{11} - d\sigma_{00}}{d\sigma_{11} + d\sigma_{00}}, \quad \mu = \frac{\sqrt{2}\,\mathrm{Re}\, d\sigma_{10}}{d\sigma_{11} + d\sigma_{00}}, \quad \nu = \frac{2 d\sigma_{1,-1}}{d\sigma_{11} + d\sigma_{00}}$$

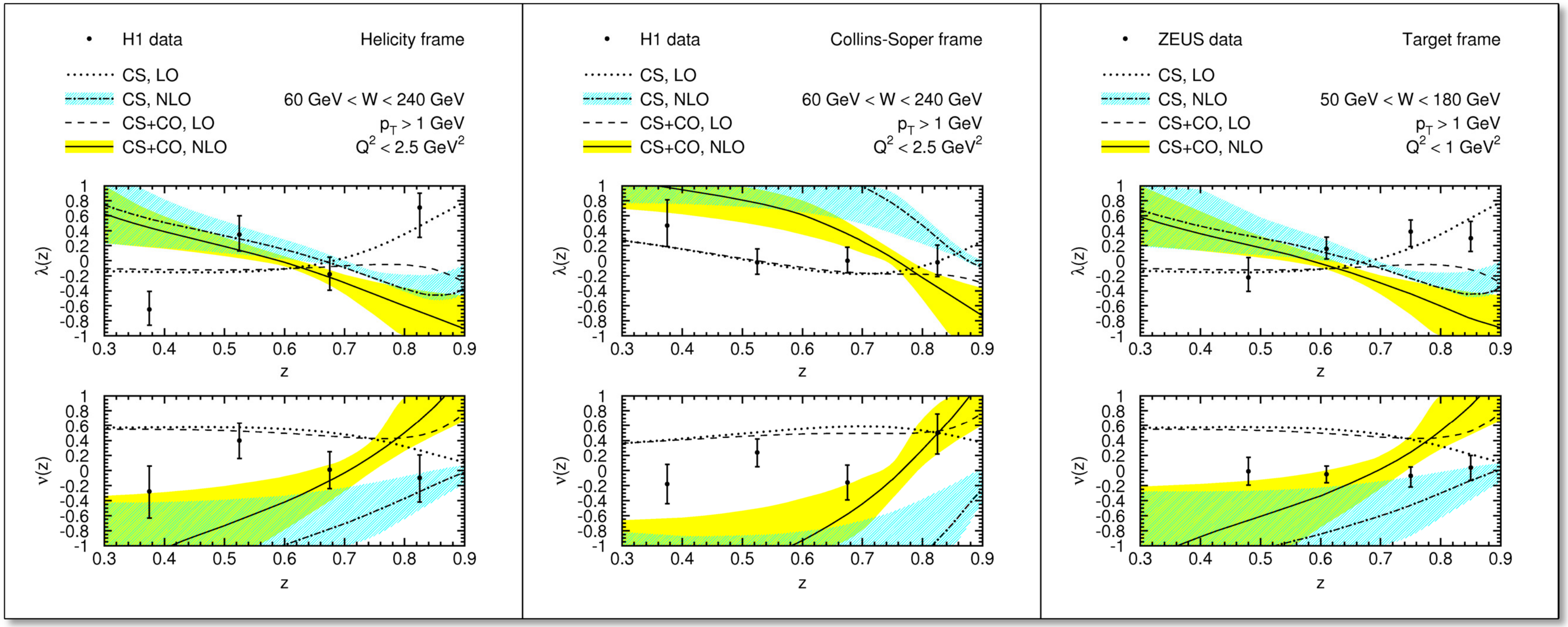
- We use the CO LDME set with feed-down contributions subtracted.

J/ψ Polarization in Photoproduction: p_T Distribution



- Bands: Uncertainties due to scale variation and CO LDMEs.
- **CSM** predicts **longitudinal** *J/ψ* at high p_T.
- **CS+CO:** largely **unpolarized** *J/ψ* at high p_T. α_s expansion converges better.
- H1 and ZEUS **data not precise** enough to discriminate CSM / NRQCD.

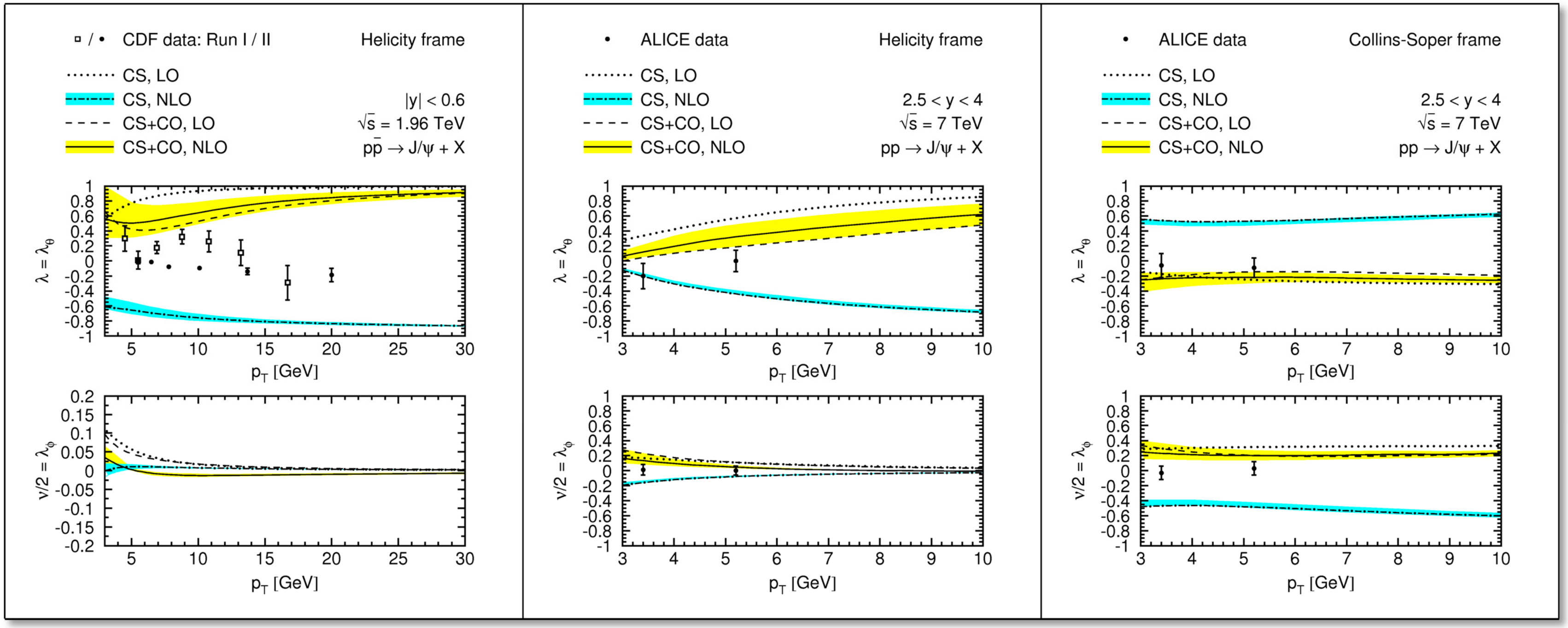
J/ψ Polarization in Photoproduction: z Distribution



- Bands: Uncertainties due to scale variation and CO LDMEs.
- **Scale** uncertainties very large.
- **Error bands** of CSM and NRQCD largely **overlap**.

➡ p_T distribution better suited to discriminate production mechanisms than z.

J/ψ Polarization in Hadroproduction



- **Helicity frame**: NRQCD predicts strong **transverse** polarization at high p_T.
- **Collins-Soper frame**: NRQCD predicts slightly longitudinal *J/ψ*.
- **Disagreement** with CDF Run II data, rough agreement with early ALICE data.

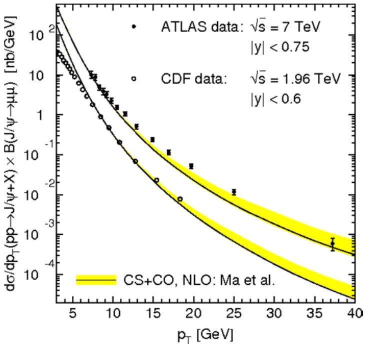
  ➡ Following high precision LHC data: **Confirm/rule out** LDME universality!

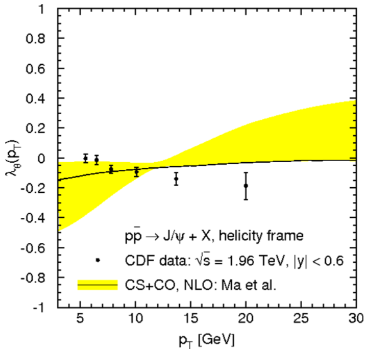
Polarization in Hadroproduction: Ma et al.

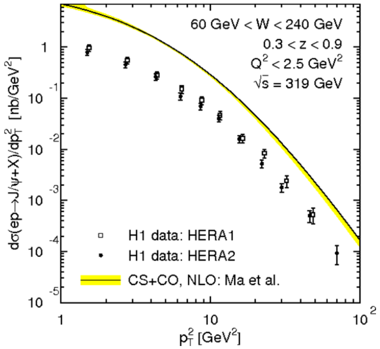
- Chao, Ma, Shao, Wang, Zhang (2012)
- **Fit** to CDF Tevatron *J/ψ* **yield and polarization** data with p_T>7 GeV:

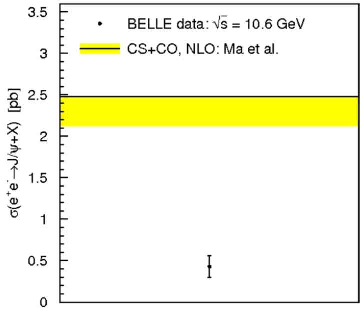
$$\langle O_8^{J/\psi}({}^1S_0)\rangle = 0.089\ \mathrm{GeV}^3 \qquad \langle O_8^{J/\psi}({}^3S_1)\rangle = 0.003\ \mathrm{GeV}^3 \qquad \langle O_8^{J/\psi}({}^3P_0)\rangle = 0.0126\ \mathrm{GeV}^5$$

- **Describes** CDF Run II polarization data **and** *J/ψ* hadroproduction yield up to **highest measured p_T** values, not below 7 GeV.
- But: **Disagreement** with photoproduction at **HERA** and e^+e^- at **BELLE**:









- **Bands**: Two alternative LDME sets specified in Ma *et al.*:

$$\langle O_8^{J/\psi}({}^1S_0)\rangle = 0 \qquad \langle O_8^{J/\psi}({}^3S_1)\rangle = 0.014\ \mathrm{GeV}^3 \qquad \langle O_8^{J/\psi}({}^3P_0)\rangle = 0.054\ \mathrm{GeV}^5$$

$$\langle O_8^{J/\psi}({}^1S_0)\rangle = 0.11\ \mathrm{GeV}^3 \qquad \langle O_8^{J/\psi}({}^3S_1)\rangle = 0 \qquad \langle O_8^{J/\psi}({}^3P_0)\rangle = 0$$

Polarization in Hadroproduction: Gong et al.

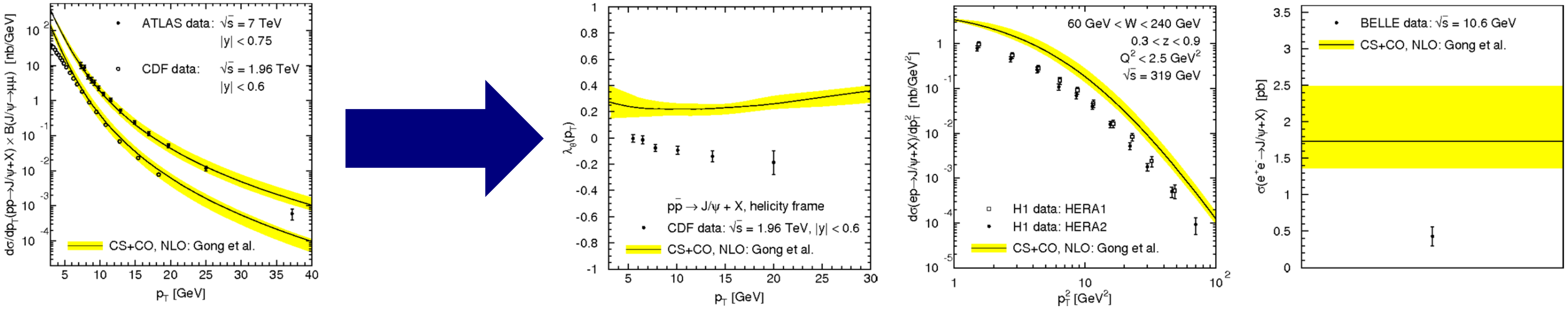
- Gong, Wan, Wang, Zhang (2012)
- **Fit only hadroproduction yield**, but consider **also *ψ′* and *χ$_{cj}$*** contributions:
  - ☐ Fit χ_{c0} CO LDME to LHCb data
  - ☐ Fit ψ' CO LDMEs to CDF and LHCb data (p_T>7 GeV)
  - ☐ Subtract ψ' and χ_{cj} feddowns, fit J/ψ LDMEs to CDF and LHCb data (p_T>7 GeV):

$$\langle O_8^{J/\psi}(^1S_0)\rangle = 0.097\ \mathrm{GeV}^3 \qquad \langle O_8^{J/\psi}(^3S_1)\rangle = -0.0046\ \mathrm{GeV}^3 \qquad \langle O_8^{J/\psi}(^3P_0)\rangle = -0.0214\ \mathrm{GeV}^5$$

$$\langle O_8^{\psi'}(^1S_0)\rangle = -0.0001\ \mathrm{GeV}^3 \qquad \langle O_8^{\psi'}(^3S_1)\rangle = 0.0034\ \mathrm{GeV}^3 \qquad \langle O_8^{\psi'}(^3P_0)\rangle = 0.0095\ \mathrm{GeV}^5$$

$$\langle O_8^{\chi_0}(^3S_1)\rangle = 0.0022\ \mathrm{GeV}^3$$

- **Predict** *J/ψ*, *ψ′* and *χ$_{cj}$* **polarization** in prompt hadroproduction (first time!)
- Predicts **moderate** transverse *J/ψ* polarization, **contrary to** CDF Run II data
- Also: In **disagreement** with photoproduction at **HERA** and e^+e^- at **BELLE**:

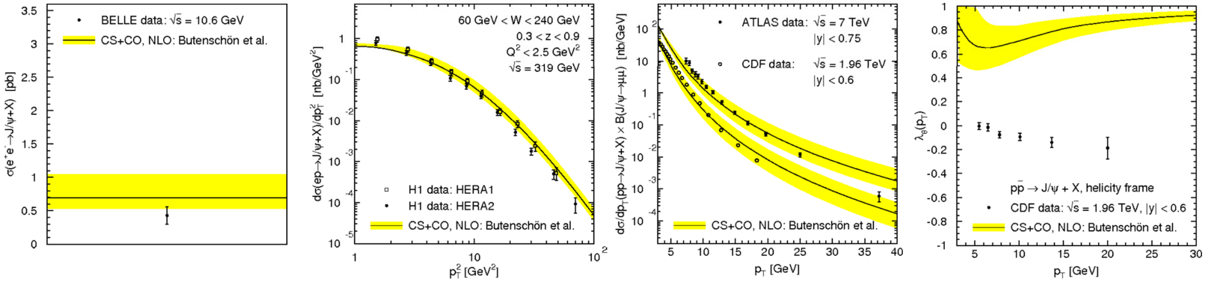


Overview: Three J/ψ Production Works

e+e- yield: **γp yield:** **pp yield:** **CDF polariz.:**

Butenschön, Kniehl:

$\langle O_8^{J/\psi}({}^1S_0)\rangle = 0.0497\ \mathrm{GeV}^3$
$\langle O_8^{J/\psi}({}^3S_1)\rangle = 0.0022\ \mathrm{GeV}^3$
$\langle O_8^{J/\psi}({}^3P_0)\rangle = -0.0161\ \mathrm{GeV}^5$



Gong, Wan, J.-X. Wang, H.-F. Zhang:

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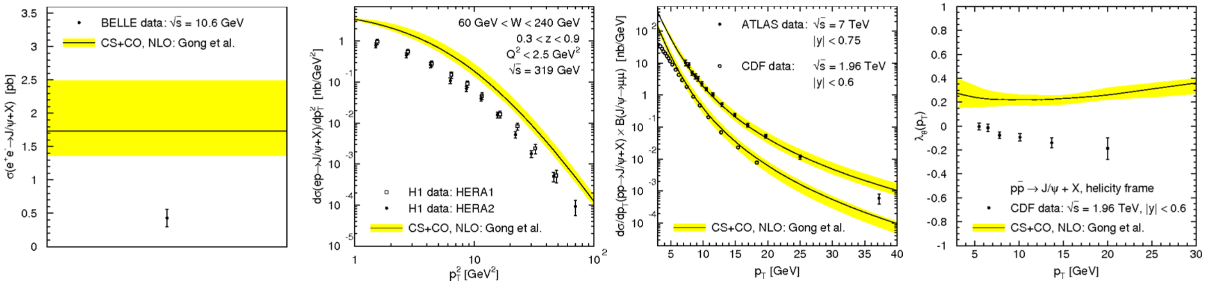

Chao, Ma, Shao, K. Wang, Y.-J. Zhang:

$\langle O_8^{J/\psi}({}^1S_0)\rangle = 0.089\ \mathrm{GeV}^3$
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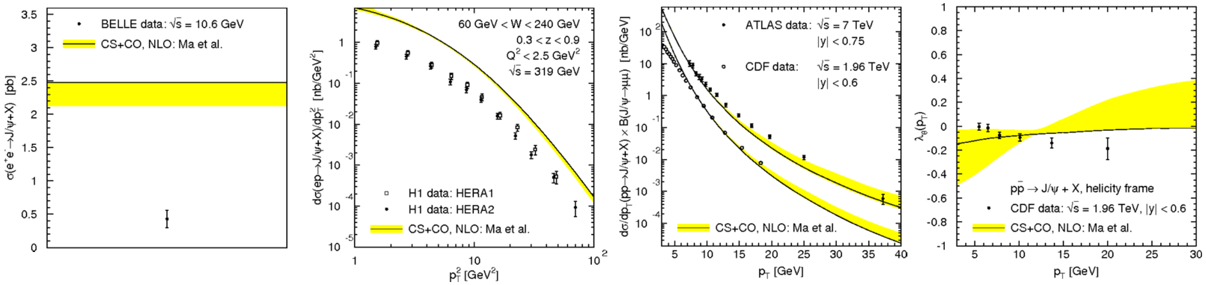


Overview: Three J/ψ Production Works

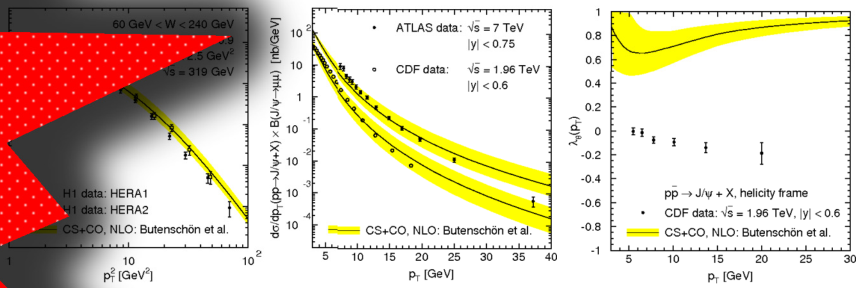
AGREEMENT:

Can NOT describe e^+e^-, γp, pp yield and CDF polarization with same LDMEs.

e... d: | ***γp* yield:** | ***pp* yield:** | **CDF polariz.:**

Butenschön, K

$\langle O_8^{J/\psi}(^1S$
$\langle O_8^{J/\psi}(^3S_1)\rangle$
$\langle O_8^{J/\psi}(^3P_0)\rangle = -$



Gong, W
H.-F. Zh

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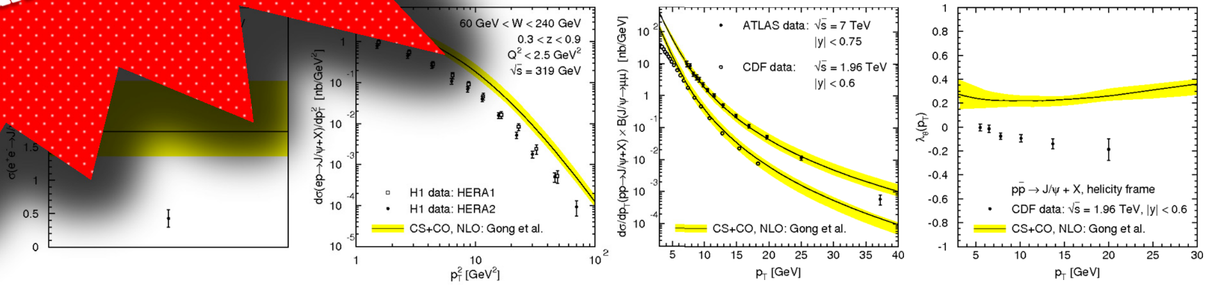
.0095
$\langle O_8^{\chi_0}(^3S_1)\rangle = 0.$ V^3



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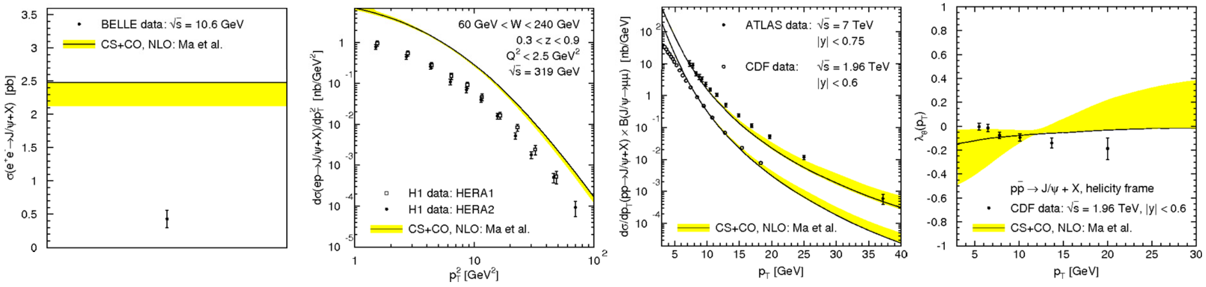


Summary

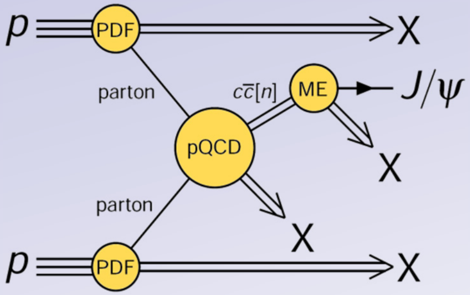
- NRQCD provides rigorous **factorization theorem** for heavy quarkonium production. But: Need to proof **LDME universality**.
- **Combined NLO fit** of NRQCD LDMEs to inclusive J/ψ production data from ALICE, ATLAS, BELLE, CDF, CMS, DELPHI, H1, LHCb, PHENIX, ZEUS.
- Good agreement for **CS+CO** with data except perhaps for $\gamma\gamma \rightarrow J/\psi+X$.
- **CSM** predictions fall **short of data** everywhere except for $e^+e^- \rightarrow J/\psi+X$.
- Fit constrained. CO LDMEs in accordance with **velocity scaling rules**.
- NLO calculations of **polarized** J/ψ cross section including CO states: Direct photoproduction at HERA and hadroproduction at Tevatron and LHC.
- **CDF Tevatron** Run II data in disagreement with our NRQCD prediction, early low-p_T **ALICE** data however still in agreement.

- **Two later** analyses **also show** that e^+e^-, γp, pp **yield** and **CDF Run II** polarization data can **not** be described with same LDME set.
  ➡ Following LHC measurements: Hopefully **clarify** LDME universality!

BACKUP SLIDES

Calculate Inclusive J/ψ Production within NRQCD

Factorization formulas (here hadroproduction):



- Convolute partonic cross section with **proton PDFs:** $\sigma_{\text{hadr}} = \sum_{i,j} \int dx\, dy\, f_{i/p}(x)\, f_{j/p}(y) \cdot \sigma_{\text{part,i,j}}$
- **NRQCD factorization:** $\sigma_{\text{part,i,j}} = \sum_{n} \sigma(ij \to c\bar{c}[n] + X) \cdot \langle O^{J/\psi}[n] \rangle$

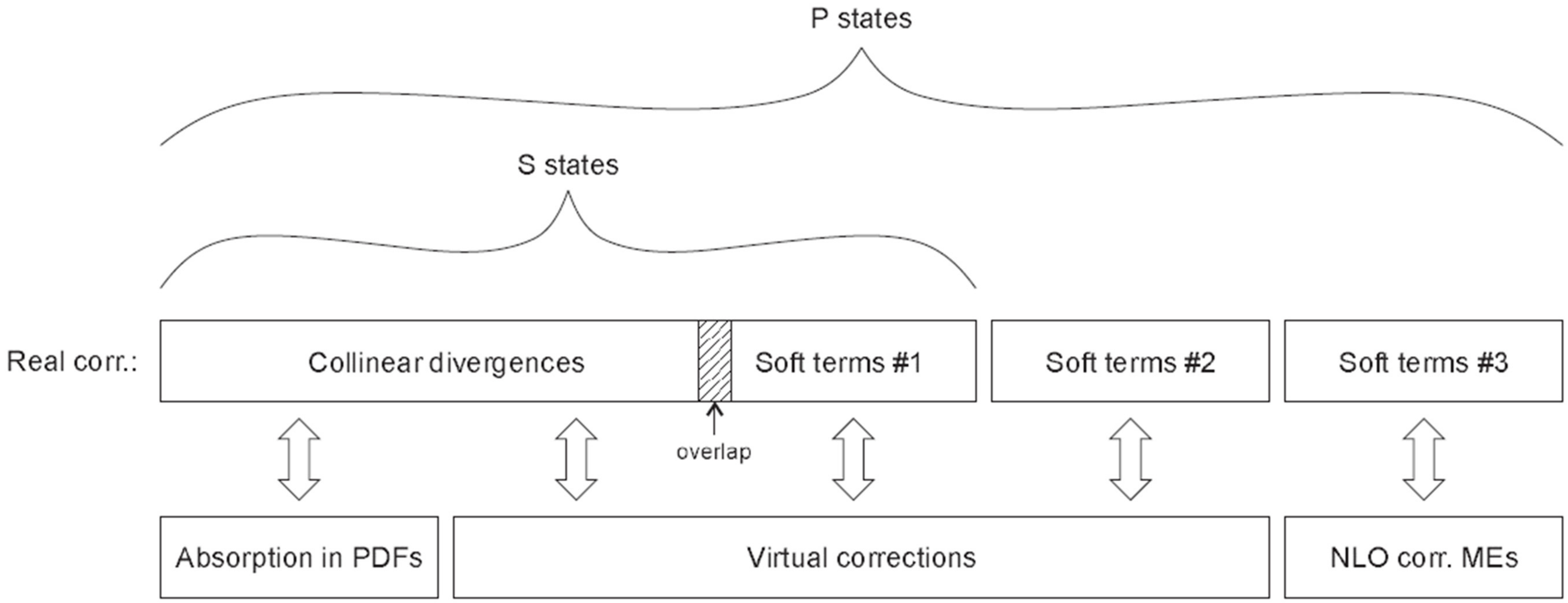
Amplitudes for $c\bar{c}[n]$ production by projector application, e.g.:

$$A_{c\bar{c}[^3S_1^{[1/8]}]} = \varepsilon_\alpha(m_S)\, \mathrm{Tr}\,[C\, \Pi^\alpha A_{c\bar{c}}]\,|_{q=0}$$

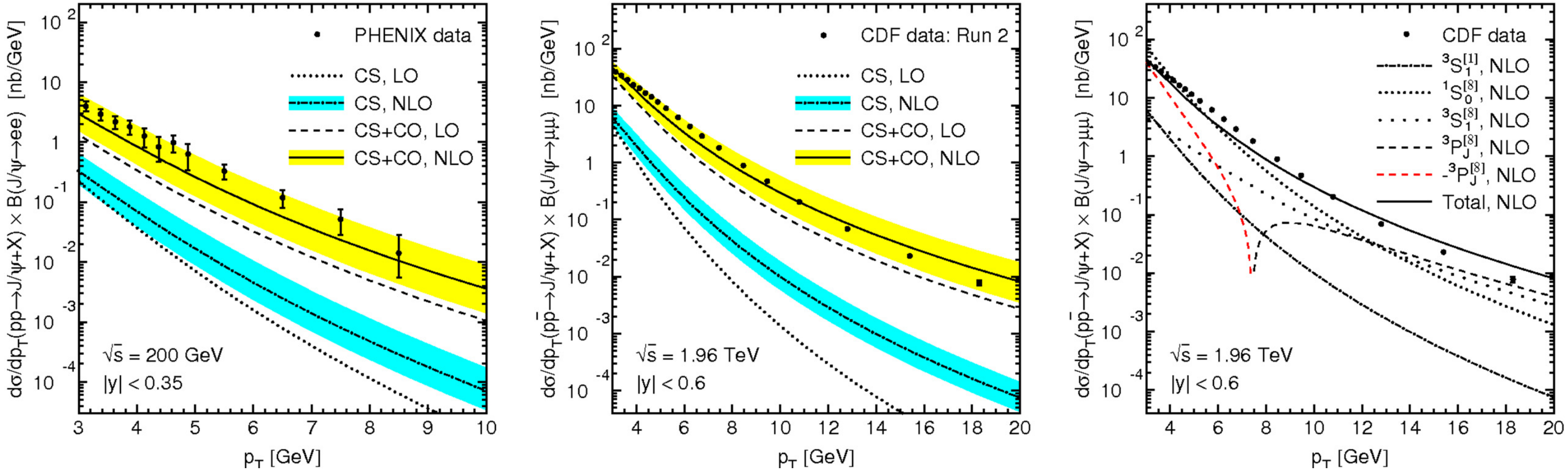
$$A_{c\bar{c}[^3P_J^{[8]}]} = \varepsilon_\alpha(m_S)\, \varepsilon_\beta(m_l)\, \frac{d}{dq_\beta}\, \mathrm{Tr}\,[C\, \Pi^\alpha A_{c\bar{c}}]\,|_{q=0}$$

- **$A_{c\bar{c}}$:** Amputated pQCD amplitude for open $c\bar{c}$ production.
- **q:** Relative momentum between c and $\bar{c}$. **ε:** Polarization vectors.

Overview of IR Singularity Structure

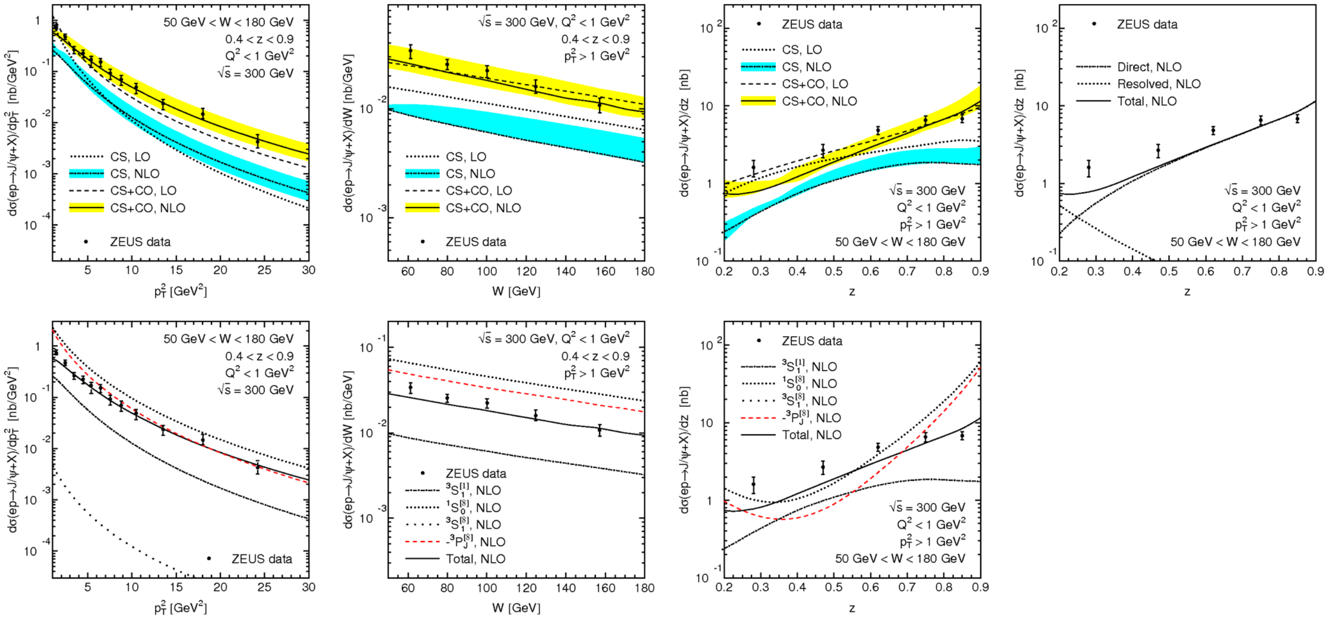


In Detail: Hadroproduction (RHIC, Tevatron)



- Color singlet model **not enough** to describe data (although increase from Born to NLO)
- **CS+CO** can describe data.
- ${}^3P_J^{[8]}$ short distance cross section **negative** at $p_T > 7$ GeV.
- But: Short distance cross sections and LDMEs **unphysical**
  ➡ No problem!

In Detail: Photoproduction (ZEUS HERA1)



- **Distributions:** Transverse momentum (p_T), photon-proton c.m. energy (W), and z = Fraction of photon energy going to J/ψ.
- Again: Color singlet alone **below** the data, **CS+CO** describes data well.
- Calculation includes **resolved** photon contributions: Important at low z.
- **Good description at high z:** No increase like in older Born analyses!

Hadroproduction-only Fit

Global fit to hadroproduction data alone, vary low-p_T cut:

	$p_T > 1$ GeV	$p_T > 2$ GeV	$p_T > 3$ GeV	$p_T > 5$ GeV	$p_T > 7$ GeV
$\langle O[^1S_0^{[8]}]\rangle$ $[10^{-2}\ \mathrm{GeV}^3]$	8.54 ± 0.52	16.85 ± 1.23	11.02 ± 1.67	1.68 ± 2.20	2.18 ± 2.56
$\langle O[^3S_1^{[8]}]\rangle$ $[10^{-3}\ \mathrm{GeV}^3]$	-2.66 ± 0.69	-13.36 ± 1.60	-5.56 ± 2.19	8.75 ± 2.98	10.34 ± 3.55
$\langle O[^3P_0^{[8]}]\rangle$ $[10^{-2}\ \mathrm{GeV}^5]$	-3.63 ± 0.23	-7.70 ± 0.61	-4.46 ± 0.87	2.20 ± 1.23	3.50 ± 1.50
M_0 $[10^{-2}\ \mathrm{GeV}^3]$	**2.25 ± 0.12**	**3.51 ± 0.19**	**3.29 ± 0.20**	**5.50 ± 0.29**	**8.24 ± 0.58**
M_1 $[10^{-3}\ \mathrm{GeV}^3]$	**6.37 ± 0.19**	**5.80 ± 0.19**	**5.54 ± 0.20**	**3.27 ± 0.29**	**1.63 ± 0.43**

- Fit **underconstrained.** Therefore give two linear combinations of Ma *et al.*:

$$M_0 = \langle O(^1S_0^{[8]})\rangle + 3.9\,\langle O(^3P_0^{[8]})\rangle/m_c^2 \qquad M_1 = \langle O(^3S_1^{[8]})\rangle - 0.56\,\langle O(^3P_0^{[8]})\rangle/m_c^2$$

- Fit results **depend strongly** on low-p_T cut.

Agreement with Ma *et al.*'s fit to Tevatron run II data with $p_T > 7$ GeV:

Default: Include feed-downs, directly fit M_0 and M_1:	**$M_0 = (7.4 \pm 1.9)\ 10^{-2}\ \mathrm{GeV}^3$**	**$M_1 = (0.5 \pm 0.2)\ 10^{-3}\ \mathrm{GeV}^3$**
Ignore feed-downs, directly fit M_0 and M_1:	**$M_0 = (8.92 \pm 0.39)\ 10^{-2}\ \mathrm{GeV}^3$**	**$M_1 = (1.26 \pm 0.23)\ 10^{-3}\ \mathrm{GeV}^3$**
Ignore feed-downs, M_0 and M_1 from 3-parameter fit:	**$M_0 = (8.54 \pm 1.02)\ 10^{-2}\ \mathrm{GeV}^3$**	**$M_1 = (1.67 \pm 1.05)\ 10^{-3}\ \mathrm{GeV}^3$**
[Ma, Wang, Chao: Table 1 of PRL 106, 042002 and Equation (18) of PRD 84, 114001]		

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[Ma, Wang, Chao: Table 1 of PRL 106, 042002 and Equation (18) of PRD 84, 114001

Global Fit: Dependence on Low-p_T Cuts (1)

Global fit: Vary low-p_T cut on hadroproduction data:

hadroproduction data left	$p_T > 1$ GeV 148 points	$p_T > 2$ GeV 134 points	$p_T > 3$ GeV 119 points	$p_T > 5$ GeV 86 points	$p_T > 7$ GeV 60 points
$\langle O[^1S_0^{[8]}]\rangle$ [10^{-2} GeV^3]	5.68 ± 0.37	4.25 ± 0.43	4.97 ± 0.44	4.92 ± 0.49	3.91 ± 0.51
$\langle O[^3S_1^{[8]}]\rangle$ [10^{-3} GeV^3]	0.90 ± 0.50	2.94 ± 0.58	2.24 ± 0.59	2.23 ± 0.62	2.96 ± 0.64
$\langle O[^3P_0^{[8]}]\rangle$ [10^{-2} GeV^5]	-2.23 ± 0.17	-1.38 ± 0.20	-1.61 ± 0.20	-1.59 ± 0.22	-1.16 ± 0.23
M_0 [10^{-2} GeV^3]	**1.81 ± 0.09**	**1.85 ± 0.09**	**2.18 ± 0.10**	**2.17 ± 0.12**	**1.89 ± 0.12**
M_1 [10^{-3} GeV^3]	**6.46 ± 0.17**	**6.37 ± 0.17**	**6.25 ± 0.17**	**6.18 ± 0.17**	**5.86 ± 0.18**

↑ **Our default fit**

- **Stabilizing** influence of **photoproduction** data.
- Fit **constrained** enough: Can now extract 3 CO LDMEs.
- Fit results now **almost independent** of low-p_T cut.
- Fit less stable with low-p_T cut below 2 GeV (nonperturbative effects).

Global Fit: Dependence on Low-p_T Cuts (2)

Global fit: Vary low-p_T cut on photoproduction (including γγ-scattering):

photoproduction data left	$p_T > 1$ GeV 74 points	$p_T > 2$ GeV 30 points	$p_T > 3$ GeV 15 points	$p_T > 5$ GeV 5 points	$p_T > 7$ GeV 1 point
$\langle O[{}^1S_0^{[8]}]\rangle$ [10^{-2} GeV^3]	4.97 ± 0.44	5.10 ± 0.92	4.05 ± 1.17	5.44 ± 1.27	9.56 ± 1.59
$\langle O[{}^3S_1^{[8]}]\rangle$ [10^{-3} GeV^3]	2.24 ± 0.59	2.11 ± 1.22	3.52 ± 1.56	1.73 ± 1.68	-3.66 ± 2.09
$\langle O[{}^3P_0^{[8]}]\rangle$ [10^{-2} GeV^5]	-1.61 ± 0.20	-1.58 ± 0.48	-0.97 ± 0.63	-1.63 ± 0.68	-3.73 ± 0.83
M_0 [10^{-2} GeV^3]	**2.18 ± 0.10**	**2.36 ± 0.12**	**2.37 ± 0.13**	**2.62 ± 0.15**	**3.10 ± 0.19**
M_1 [10^{-3} GeV^3]	**6.25 ± 0.17**	**6.05 ± 0.18**	**5.94 ± 0.19**	**5.78 ± 0.20**	**5.62 ± 0.20**

↑

Our default fit

- **Fit stable** against varying low-p_T cut in region 1 GeV ~ 3 GeV.
- Just 5 or 1 photoproduction against 119 hadroproduction points not enough to stabilize the fit. ➡ **Not stable** with low-p_T cut much larger than 3 GeV. (Would need more high-p_T photoproduction data.)

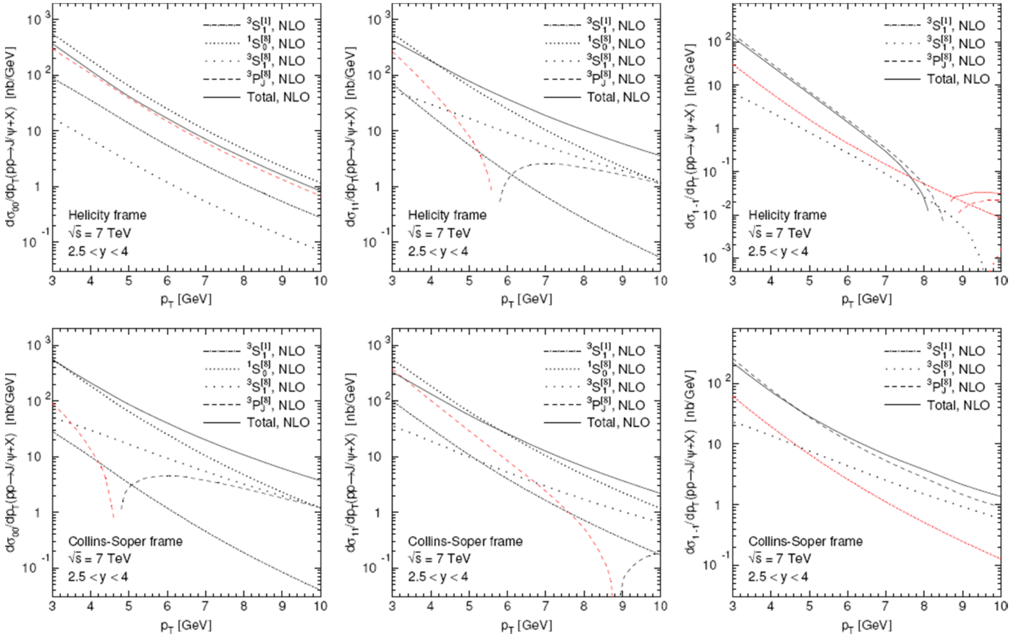
Polarization in Hadroproduction: Contributions

- **First:** Sum up contributions of intermediate states:

$d\sigma_{00}$ **$d\sigma_{11}$** **$d\sigma_{1\,-1}$**

Helicity frame:

Collins-Soper frame:



- **Then:** $\lambda_\theta = \dfrac{d\sigma_{11} - d\sigma_{00}}{d\sigma_{11} + d\sigma_{00}}$, $\lambda_{\theta\phi} = \dfrac{\sqrt{2}\,\mathrm{Re}\, d\sigma_{10}}{d\sigma_{11} + d\sigma_{00}}$, $\lambda_\phi = \dfrac{d\sigma_{1,-1}}{d\sigma_{11} + d\sigma_{00}}$