Radiation from accelerated particles at strong coupling

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Outline

- **Introduction: Radiation from accelerated charges in classical electrodynamics and QED**
- **AdS/CFT correspondence:** D3-branes and D7-branes setups
- **Power radiated by an accelerated quark in AdS/CFT:** the trailing string
- **Motivation for further analysis:** synchrotron radiation in QED, strong magnetic fiels, and dimensional reduction
- **Magnetic solutions in AdS:** radiation from gauge-invariant states in a magnetic field and dimensional reduction.
- **Conclusions**

Maxwell equations with a moving charge as the source:

$$\partial_\mu F^{\mu\nu} = J^\nu \ , \qquad J^\nu(x) = q \int \mathrm{d}\tau \, U^\nu(\tau)\delta^{(4)}[x - r(\tau)] \ .$$

$$\Rightarrow \qquad A^\mu_{\mathrm{rad}}(x) = \int \mathrm{d}^4 y \, G_{\mathrm{R}}(x-y) J^\mu(y) \quad , \quad G_{\mathrm{R}}(x-y) = \theta(x^0 - y^0)\delta[(x-y)^2]$$

$$\boldsymbol{E} = -\nabla A^0 - \frac{\partial \boldsymbol{A}}{\partial t} \ , \quad \boldsymbol{B} = \nabla \times \boldsymbol{A}$$

$$\boldsymbol{E}(t, \boldsymbol{x}) = q \left[\frac{\boldsymbol{n} - \boldsymbol{\beta}}{\gamma^2 (1 - \boldsymbol{\beta}\cdot\boldsymbol{n})^3 R^2} \right]_{\mathrm{ret}} + q \left[\frac{\boldsymbol{n} \times \{(\boldsymbol{n} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}}\}}{(1 - \boldsymbol{\beta}\cdot\boldsymbol{n})^3 R} \right]_{\mathrm{ret}}$$

$$\boldsymbol{B} = \boldsymbol{n} \times \boldsymbol{E}\big|_{\mathrm{ret}}$$

Radiation from accelerated charges in classical electrodynamics

Flux of energy (Poynting vector):

$$\boldsymbol{S} = \frac{c}{4\pi}\,\boldsymbol{E} \times \boldsymbol{B}$$

Total power radiated by an accelerated charge in the non-relativistic case (*Larmor's formula*):

$$P = \frac{2}{3}\,q^2\,\dot{\beta}^2$$

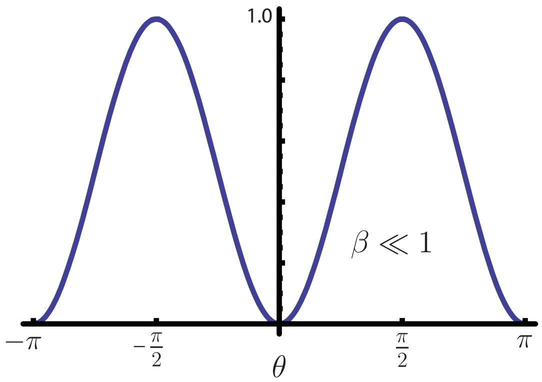
Relativistic generalization (*Liénard's formula*, *1898*):

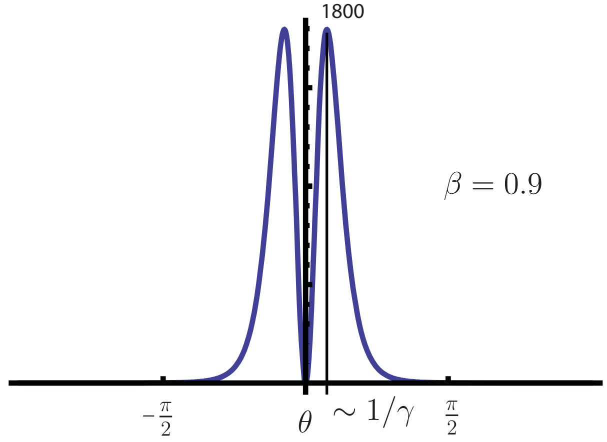
$$P = \frac{2}{3}\,q^2\,\gamma^6[\dot{\beta}^2 - (\beta \times \dot{\beta})^2]$$

Angular distribution of radiation in classical electrodynamics

Rectilinear motion with parallel velocity and acceleration:

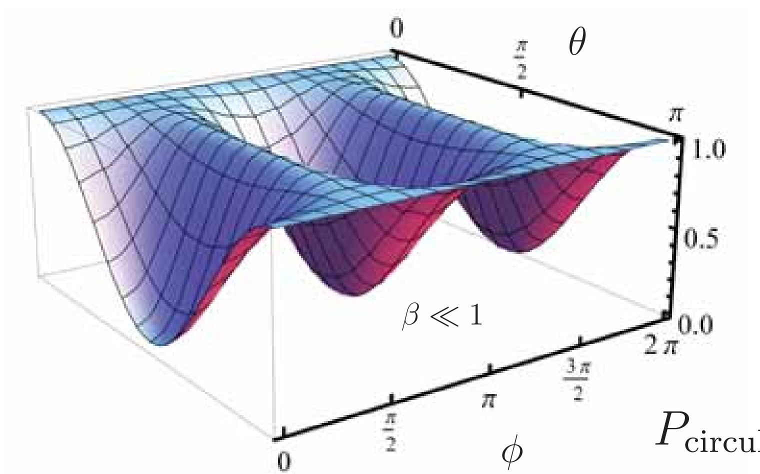
$$\frac{\mathrm{d}P}{\mathrm{d}\Omega} = \frac{q^2\dot{\beta}^2}{4\pi}\frac{\sin^2\theta}{(1-\beta\cos\theta)^5}$$



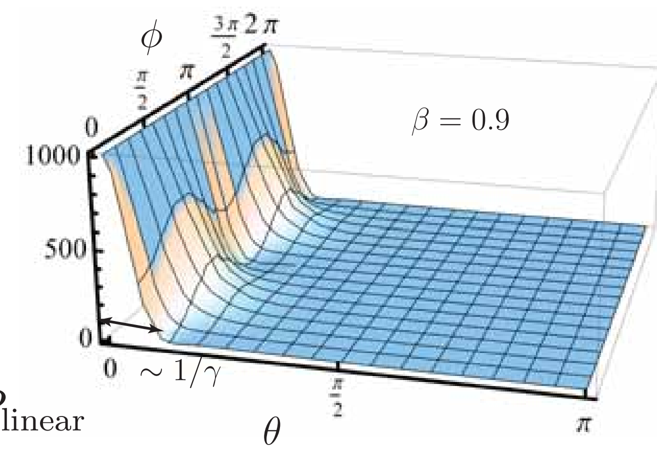


Particle in circular motion:

$$\frac{\mathrm{d}P}{\mathrm{d}\Omega} = \frac{q^2}{4\pi}\frac{\dot{\beta}^2}{(1-\beta\cos\theta)^3}\left[1-\frac{\sin^2\theta\cos^2\phi}{\gamma^2(1-\beta\cos\theta)^2}\right]$$



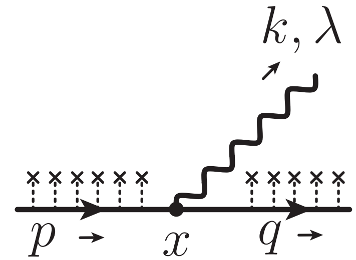
$$P_{\mathrm{circular}} = \gamma^2 P_{\mathrm{linear}}$$



Synchrotron radiation by an external field in QED

Three ways of calculating it perturbatively:

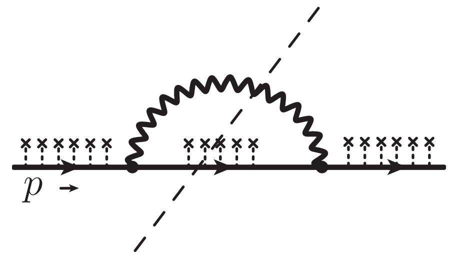
- Landau-Lifshitz/Sokolov-Ternov approach (easiest):



$$P = \frac{1}{\Delta t}\sum_f |\boldsymbol{k}|\,|\langle \mathrm{out}|\mathrm{in}\rangle|^2$$

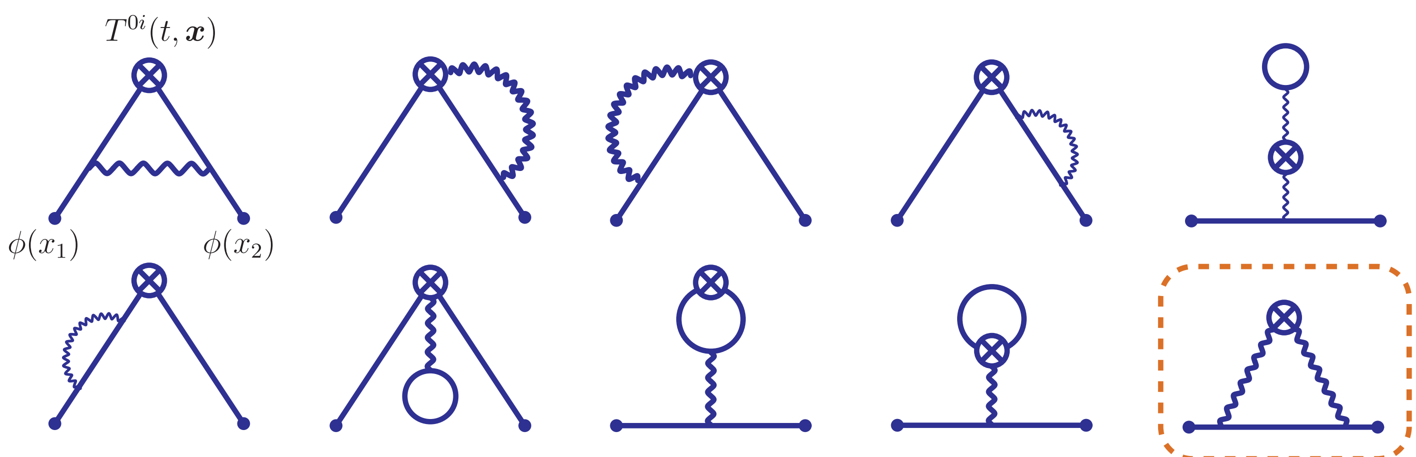
- Schwinger's approach (more involved):

Schwinger, PR (1951), PRD (1973).



- Inspired by Hofman & M aldacena's calculation at strong coupling (most involved perturbatively): Hofman & Maldacena, JHEP (2008)

$\langle \hat{T}^{0i}(t,\boldsymbol{x})\rangle$ $\mathcal{O}(e^2)$



Synchrotron radiation in sQED: result with a magnetic field

In a magnetic field, the energy levels of the scalar particle are discrete (*Landau levels*):

$$E_p^2 = m^2 + (p^3)^2 + 2|e|B\left(n + \frac{1}{2}\right) , \quad n = 0, 1, 2, \ldots .$$

To leading order:

$$P = \sum_{\boldsymbol{k},n'} |\boldsymbol{k}| e^2 \frac{|e|B}{L^3} \delta(E_k + E_q - E_p) 2^{n-n'-3} \frac{1}{E_k E_p E_q} \frac{1}{n'!n!} \mathrm{e}^{-(k^2)^2/2|e|B} I_{\kappa,\gamma} \Big|_{q^3 = p^3 - k^3} ,$$

with

$$I_{\kappa,\gamma} \equiv \frac{1}{|\boldsymbol{k}|^2} \{[(k^2)^2 + (k^3)^2]\kappa^2 + [(k^1)^2((2p^2 - k^2)^2 + (2p^3 - k^3)^2) + 4(k^3p^2 - k^2p^3)^2]\gamma^2\} ,$$

$$\kappa \equiv -2(n'!) L_{n'}^{n+n'+1}((k^2)^2/2|e|B) + (n'+1) L_{n'+1}^{n}((k^2)^2/2|e|B) ,$$

$$\gamma \equiv \frac{1}{\sqrt{|e|B}} (n'!) L_{n'}^{n-n'}((k^2)^2/2|e|B) .$$

A great simplification happens in the *semi-classical limit*: $\xi \to 0^+ , \quad \xi \equiv \frac{3}{2}\frac{|e|BE}{m^3}$

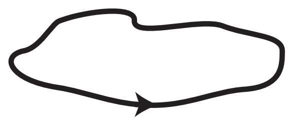
$$P = P_{\mathrm{cl}} \int_0^\infty \mathrm{d}y\, f(y) , \qquad P_{\mathrm{cl}} = \frac{2}{3}\frac{e^4B^2}{E^2} , \qquad f(y) \equiv \frac{9\sqrt{3}}{8\pi} \frac{y}{(1+\xi y)^3} \int_y^\infty \mathrm{d}x\, K_{5/3}(x) .$$

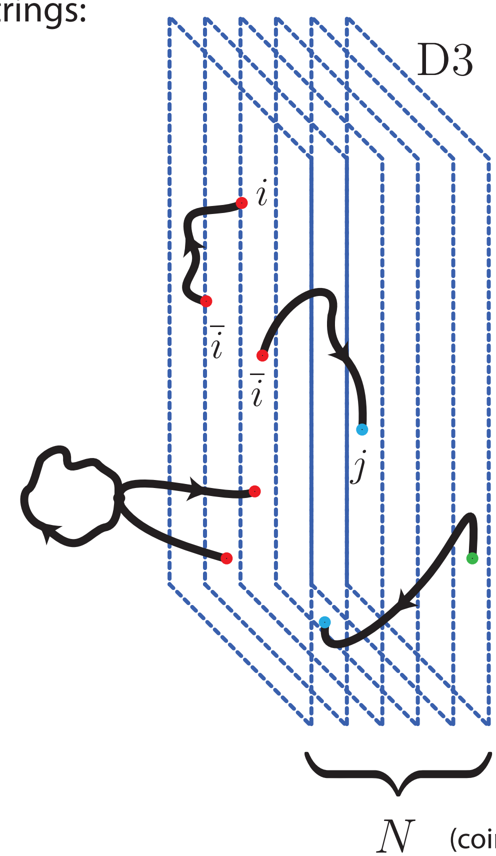
$$\Rightarrow \quad P = P_{\mathrm{cl}} \left[1 - 3.97\frac{|e|BE}{m^3} + O(\xi^2)\right] .$$

Sokolov & Ternov, Pergamon (1968)

Two ways of looking at D3-branes: perturbative point of view

Type IIB string theory in **10-dimensional** Minkowski space + open strings:





Massless spectrum of the **closed** string:

NS sector: G_2 (graviton), B_2 , Φ (dilaton),

RR sector: C_0 , C_2 , C_4

Massless spectrum of the **open** string: A_μ

Action of the system: $S = S_{\text{closed}} + S_{\text{open}} + S_{\text{int}}$

At low enegies $(E, p \ll 1/l_{\text{s}})$: $S_{\text{int}} \to 0$, $S_{\text{closed}} \to S_{\text{closed}}(\text{free})$ (***decoupling limit***)

In addition, we need: $g_{\text{s}} \to 0$, with $4\pi g_{\text{s}} N = g_{\text{YM}}^2 N \equiv \lambda$ constant (***'t Hooft limit***)

$$\Rightarrow \quad S \simeq S_{\mathcal{N}=4 \text{ SYM}} + S_{\text{closed}}(\text{free})$$

Two ways of looking at D3-branes: gravity point of view

C_4 in the massless spectrum of SUGRA must couple to (3+1)-dimensional **dynamical** objects:

Look for non-trivial **classical** solutions from the SUGRA action

$$S = \frac{1}{2\kappa^2}\int \mathrm{d}^{10}x\sqrt{-g}\left(R - \frac{1}{2}g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi - \frac{1}{4(5!)}F_5^2\right)$$

We want solutions which extend in 4 dims and are spherical in 6 dims. The result is:

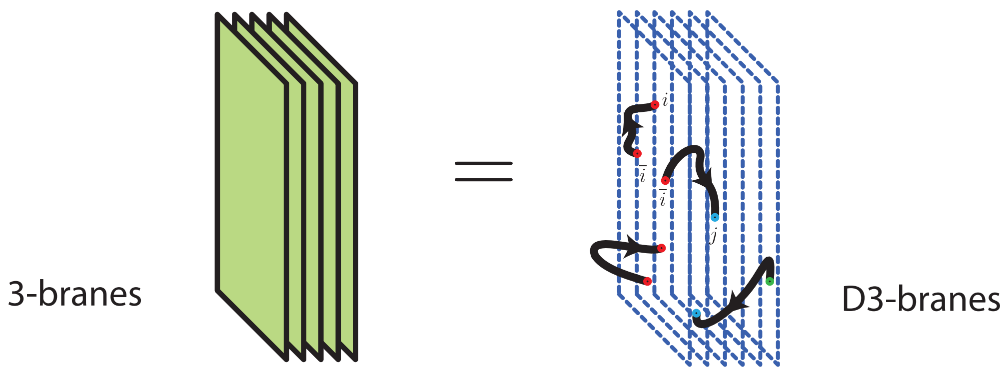
$$\mathrm{d}s^2 = f(r)^{-1/2}(-\mathrm{d}t^2 + \mathrm{d}\boldsymbol{x}^2) + f(r)^{1/2}(\mathrm{d}r^2 + r^2\mathrm{d}\Omega_5^2)$$

$$F_5 = (1+*)\mathrm{d}f^{-1}\wedge \mathrm{d}x^0\wedge \mathrm{d}x^1\wedge \mathrm{d}x^2\wedge \mathrm{d}x^3$$

$$\Phi = \mathrm{const.}$$

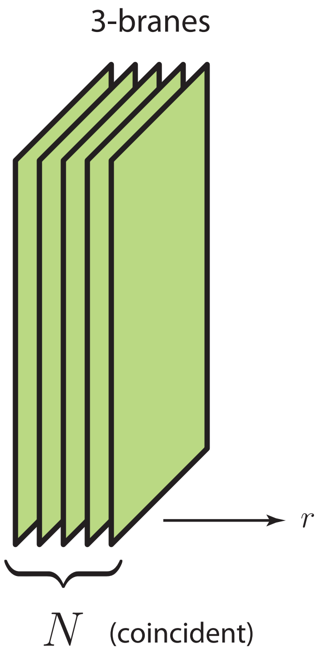
$$f(r) = 1 + \frac{L^4}{r^4}\ , \qquad L^4 = 4\pi\, g_s\, l_s^4\, N\ , \quad N\in\mathbb{Z}$$

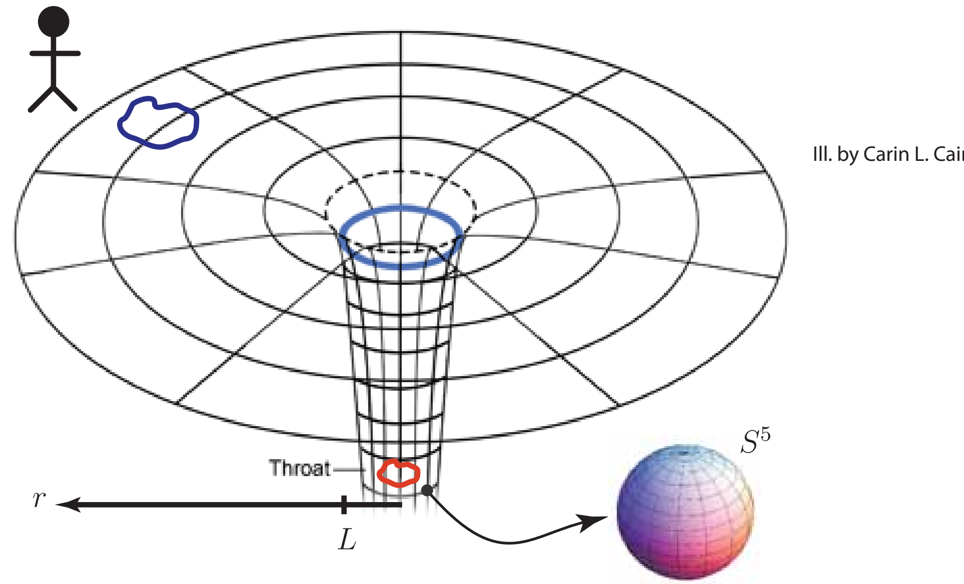
From calculations of the RR charge and dilaton absortion cross sections: Polchinski, PRL (1995) Klebanov, 97 Das, Gibbons, Mathur, 97



open-closed duality

Decoupling limit in SUGRA





$$\mathrm{d}s^2 = \left(1 + \frac{L^4}{r^4}\right)^{-1/2} (-\mathrm{d}t^2 + \mathrm{d}\boldsymbol{x}^2) + \left(1 + \frac{L^4}{r^4}\right)^{1/2} (\mathrm{d}r^2 + r^2 \mathrm{d}\Omega_5^2) \ , \qquad L^4 = 4\pi g_\mathrm{s} N \, l_\mathrm{s}^4$$

(radius of curvature)

Decoupling between modes at $r \gg L$ and $r \lesssim L$:

$$E_\infty = [g_{tt}(r)/g_{tt}(\infty)]^{1/2} E_r = \frac{r}{L} E_r \quad \Rightarrow \quad S \simeq S_\mathrm{closed}(\mathrm{Minkowski}) + S_\mathrm{closed}(AdS_5 \times S_5)$$

For $r \to 0^+$: $\boxed{\mathrm{d}s^2 \simeq \frac{L^2}{z^2} \left(-\mathrm{d}t^2 + \mathrm{d}\boldsymbol{x}^2 + \mathrm{d}z^2\right) + L^2 \mathrm{d}\Omega_5^2 \ ,}$ $\qquad z \equiv \frac{L^2}{r}$

In order for the **classical** SUGRA approximation to be valid, we need: $L \gg l_s \ \Rightarrow \ \lambda \gg 1$

Maldacena's conjecture

"Type IIB string theory on $(AdS_5 \times S^5)_N$ plus some appropriate boundary conditions (and possibly also some boundary degrees of freedom) is dual to $\mathcal{N} = 4$ $d = 3{+}1$ $U(N)$ super Yang-Mills theory."

Maldacena, Adv. Theor. Math. Phys., 1998

Calculation of correlation functions

$$\left\langle \exp \int \mathrm{d}^4x \, \phi_0(x) \mathcal{O}(x) \right\rangle_{\mathrm{CFT}} = \mathrm{e}^{-I_{\mathrm{SUGRA}}[\phi]} \Big|_{\phi=\phi[\phi_0]} \; .$$

Gubser, Klebanov, Polyakov (1998)

Witten (1998)

Scalar operators of scale dimension Δ will couple to massive scalar fields:

Scale transformation: $x \mapsto \lambda x$

$\mathcal{O}(x) \mapsto \lambda^{\Delta} \mathcal{O}(\lambda x)$

AdS/CFT correspondence $\Rightarrow$ $\Delta = \frac{1}{2}(d + \sqrt{d^2 + 4m^2}) \; .$

Correlation functions of the energy-momentum tensor can be obtained by coupling it to the graviton:

$$\left\langle \exp \int \mathrm{d}^4x \, \tilde{h}_{\mu\nu}(x) T^{\mu\nu}(x) \right\rangle_{\mathrm{CFT}} = \mathrm{e}^{-I_{\mathrm{SUGRA}}[h]} \Big|_{h=h[\tilde{h}]} \; .$$

$\mathcal{N}=4$ super Yang-Mills

Supersymmetry algebra: $Q^A|\mathrm{boson}\rangle = |\mathrm{fermion}\rangle$

$$\{Q^A_\alpha, \bar{Q}_{\dot\beta B}\} = 2\sigma^\mu_{\alpha\dot\beta}P_\mu\delta^A_B\ , \quad \{Q^A_\alpha, Q^B_\beta\} = 0\ .$$

Field content:

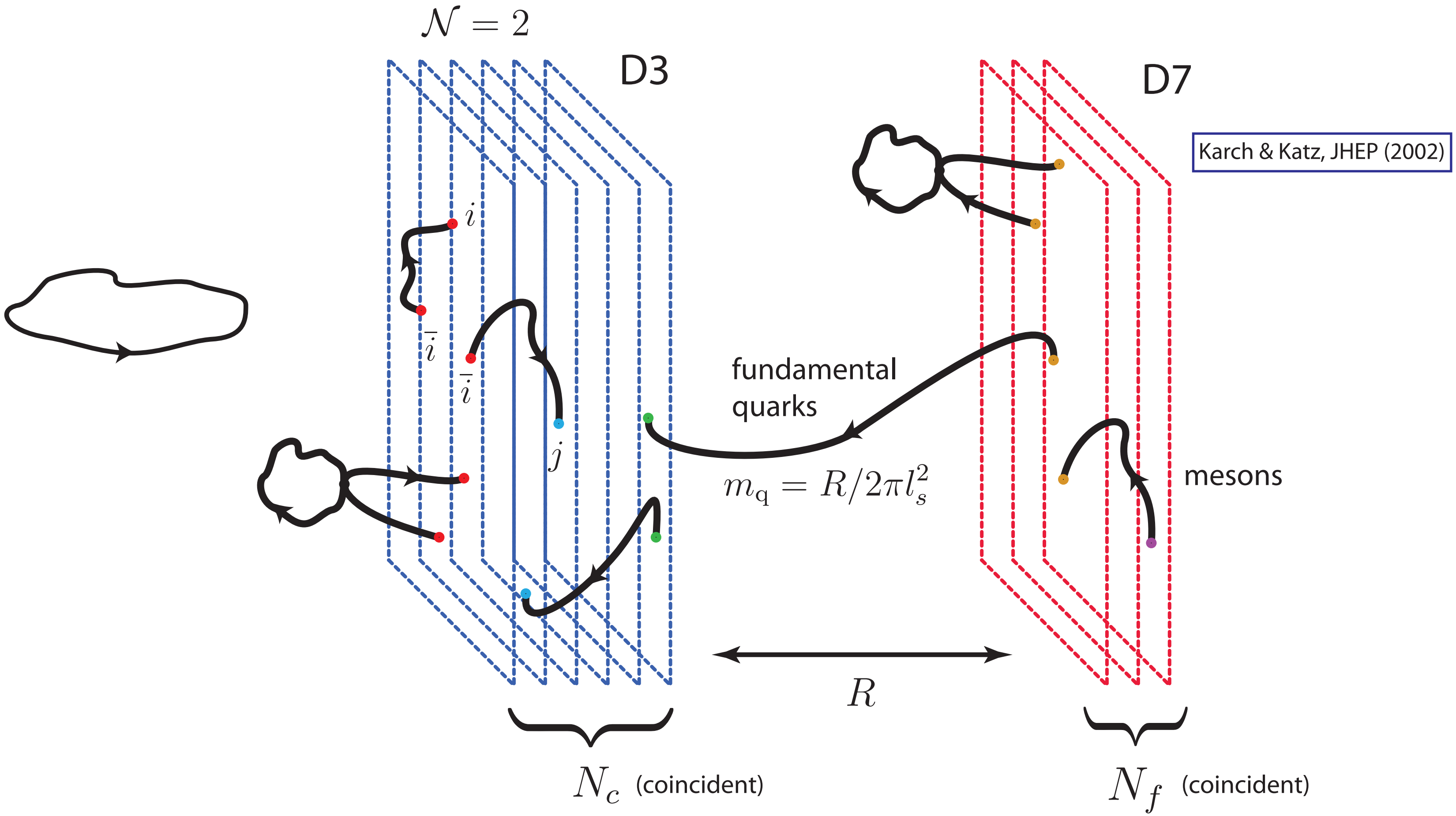
h	$\mathcal{N}=1$ vector	$\mathcal{N}=1$ chiral	$\mathcal{N}=2$ vector	$\mathcal{N}=2$ hyper	$\mathcal{N}=3$ vector	$\mathcal{N}=4$ vector
1	1	0	1	0	1	1
1/2	1	1	2	2	3+1	4
0	0	1+1	1+1	4	3+3	6
-1/2	1	1	2	2	1+3	4
-1	1	0	1	0	1	1

Lagrangian:
$$\mathcal{L} = \mathrm{tr}\left\{ -\frac{1}{2g^2}F_{\mu\nu}F^{\mu\nu} + \frac{\theta_I}{8\pi^2}F_{\mu\nu}\tilde{F}^{\mu\nu} - \mathrm{i}\bar\lambda^a\bar\sigma^\mu D_\mu\lambda_a - D_\mu X^i D^\mu X^i + gC^{ab}_i\lambda_a[X^i,\lambda_b] \right.$$
$$\left. + g\bar{C}_{iab}\bar\lambda^a[X^i,\bar\lambda^b] + \frac{g^2}{2}[X^i,X^j]^2 \right\}\ ,$$

$$F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu + \mathrm{i}[A_\mu, A_\nu]\ , \quad \tilde{F}_{\mu\nu} \equiv \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}\ , \quad D_\mu\lambda \equiv \partial_\mu\lambda + \mathrm{i}[A_\mu,\lambda]\ .$$

- R-symmetry: rotations of supercharges, $U(1)_R$ and $SU(4)_R \simeq SO(6)_R$.

D7-branes and fundamental quarks



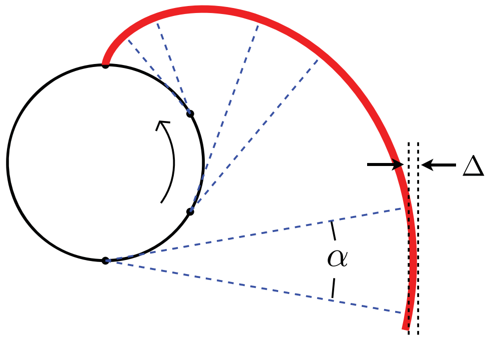
Field content: $\mathcal{N}=4$ SYM $+$ N_f hypermultiplets in the f. rep.

Probe limit: $N_c \gg N_f$ $\Rightarrow$

- 7-7 modes decouple at low energies, $\lambda_7 = \lambda_3 (2\pi l_s)^4 N_f/N_c$
- The theory remains conformal, $\beta \propto \lambda_3^2 N_f/N_c$
- Quark loops are suppressed (no influence on the geometry)

Radiation from a heavy quark in circular motion at strong coupling

Consider a charged particle forced to go in circular motion in classical electrodynamics:

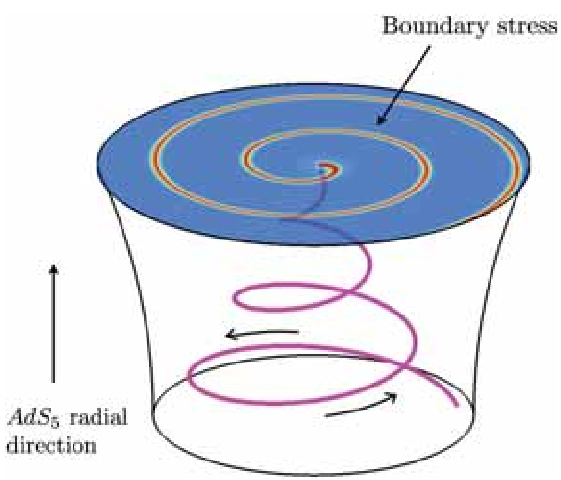


$$\alpha \sim \frac{1}{\gamma}$$

$$\Delta \sim \frac{1}{\gamma^3}$$

Trailing string in circular motion with a D3-D7 setup:

Athanasiou, Chesler, Liu, Nickel, & Rajagopal, PRD (2010)



$$P = \frac{\sqrt{\lambda}}{2\pi}\, a^2 \,, \qquad a = v\gamma^2\omega_0$$

Other works with similar results:

Mikhailov (2003)

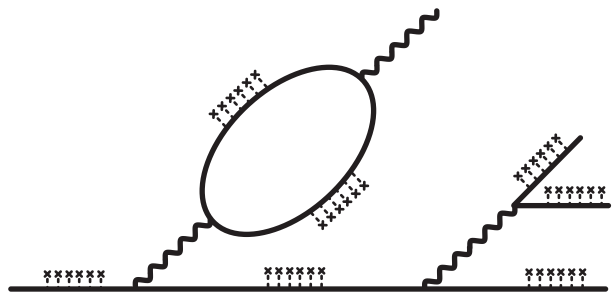
Chernicoff, Antonio Garcia, Guijosa, & Pedraza (2011)

Hatta, Iancu, Mueller, & Triantafyllopoulos (2011)

Baier (2011)

Motivation for further study

- Quantum effects present in the standard leading-order perturbative evaluation of synchrotron radiation in QED:
  - Quantization of the trajectory for strong magnetic fields: $E, E' \cancel{\gg} \sqrt{|e|B}$
  - Recoil of the electron for emission of high-energy photons: $E' \cancel{\gg} |\boldsymbol{k}|$
- Loops and dimensional reduction:

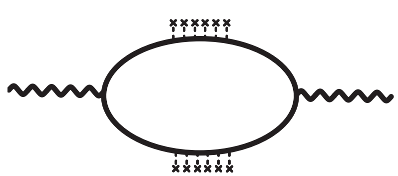


Fermion propagator in a magnetic field: Schwinger (1951) Gusynin, Miranski, & Shovkovy (1996)

$$S(x,y) = \exp\left[\frac{\mathrm{i}e}{2}(x-y)^\mu A_\mu^{\mathrm{ext}}(x+y)\right]\tilde{S}(x-y)\ , \qquad \tilde{S}(k) = \mathrm{i}\exp\left(-\frac{\boldsymbol{k}_\perp^2}{|eB|}\right)\sum_{n=0}^{\infty}(-1)^n\frac{D_n(eB,k)}{k_0^2-k_3^2-m^2-2|e|Bn}\ ,$$

For strong magnetic field, the lowest Landau-level dominates:

$$\tilde{S}^{(0)}(k) = \mathrm{i}\exp\left(-\frac{\boldsymbol{k}_\perp^2}{|eB|}\right)\frac{k^0\gamma^0-k^3\gamma^3+m}{k_0^2-k_3^2-m^2}(1-\mathrm{i}\gamma^1\gamma^2\,\mathrm{sgn}(eB))\ .$$

$\Rightarrow$  $\sim \int \mathrm{d}^2k_\perp\,\mathrm{e}^{-2k_\perp^2/|e|B} = \frac{|e|B}{4}\ .$ $\Rightarrow$ 1+1 dimensional dynamics

Magnetic brane solutions

- Non-trivial solutions of the Einstein-Maxwell action: D'Hoker & Kraus, JHEP (2009)

$$S = -\frac{1}{16\pi G_5}\int \mathrm{d}^5x\,\sqrt{-g}\left(R + F^{MN}F_{MN} - \frac{12}{L^2}\right) + S_{\mathrm{bndy}}\ .$$

- Equivalent to introduce a magnetic field in $\mathcal{N}=4$ SYM theory associated to the $U(1)_R$ R-charge

- They interpolate between AdS_5 at high energies and a 1+1 dimensional conformal field theory at low energies:

$$\mathrm{d}s^2 = -\mathrm{e}^{2W(r)}\mathrm{d}t^2 + \mathrm{e}^{-2W(r)}\mathrm{d}r^2 + \mathrm{e}^{2V(r)}[(\mathrm{d}x^1)^2 + (\mathrm{d}x^2)^2] + \mathrm{e}^{2W(r)}(\mathrm{d}x^3)^2\ ,$$

For $r \to \infty$: AdS_5

For $r \to 0^+$: $\mathrm{AdS}_3 \times T^2$

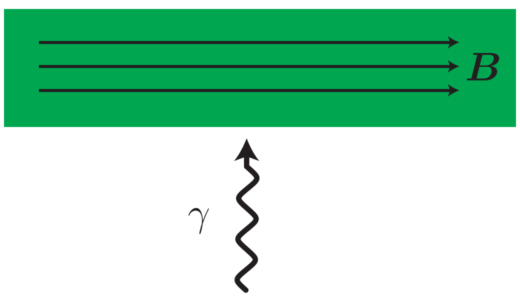
$$\mathrm{d}s^2 = -3r^2\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{3r^2} + \frac{B}{\sqrt{3}}[(\mathrm{d}x^1)^2 + (\mathrm{d}x^2)^2] + 3r^2(\mathrm{d}x^3)^2\ .$$

The magnetic field as an effective medium

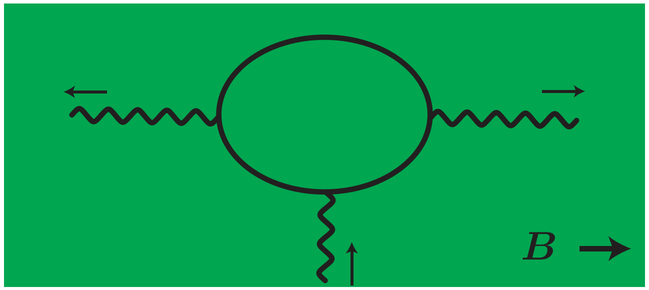
- Modifiying the background geometry is equivalent to considering the effect of the magnetic field to arbitrary loop orders.

- This implies that even neutral particles will be affected by the magnetic field.

QED analogy: photon splitting



$\Rightarrow$



Probability: $$P \sim \alpha^3 \left(\frac{\omega}{m}\right)^5 \left(\frac{B\sin\theta}{B_e}\right)^6 m$$

Adler *et al.*, PRL 25, 1061 (1970)

Bialynicka-Birula *et al.*, PRD 2, 2341 (1970)

for $(B/B_e)(\omega/m)\sin\theta \ll 1$

- The dispersion relation of the massless excitations will be modified by the magnetic field:

  ➡ The vacuum behaves effectively as a birefringent medium

Dittrich & Gies, hep-ph/9806417

Energy-flux à la Hofman-Maldacena

- We want to calculate the expectation value of the energy-momentum tensor in a well localizes state:

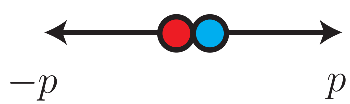
$$\langle 0|\hat{\mathcal{O}}_q^\dagger \hat{T}^{0i}(x)\hat{\mathcal{O}}_q|0\rangle \sim \text{ flux of energy}$$

$$\hat{\mathcal{O}}_q = \int \mathrm{d}^4x\, \mathrm{e}^{\mathrm{i}q\cdot x} \exp\left(-\frac{x_0^2+\boldsymbol{x}^2}{\sigma^2}\right)\hat{\mathcal{O}}(x)\ , \qquad q\sigma \gg 1$$

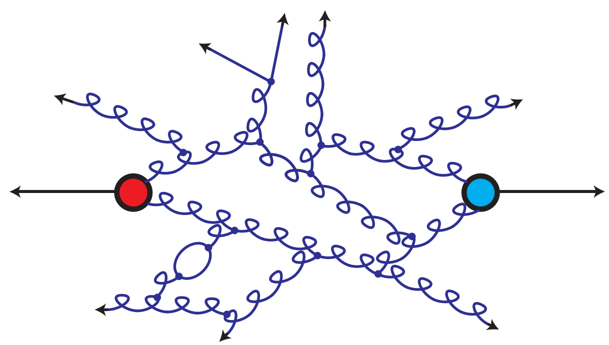
(well defined momentum and position)

- In a conformal field theory, isotropic energy distribution for scalar states in the COM frame

$$\mathcal{O} \sim \bar{q}q$$



$$\Longrightarrow$$



$\Longrightarrow$ no jets from scalar states in the COM for a conformal field theory

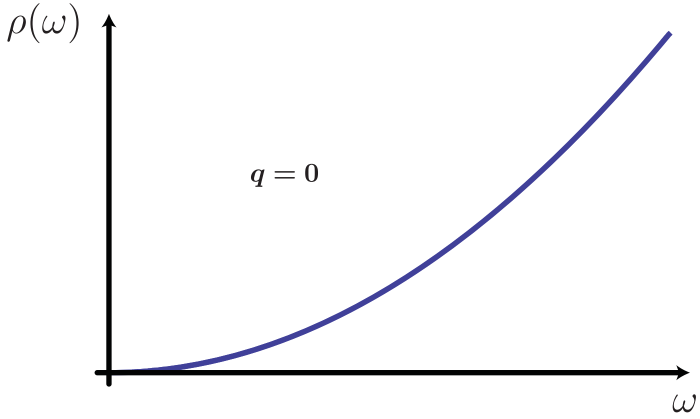
Hofman & Maldacena, JHEP (2008)

- This should be different in the presence of a magnetic field

Accelerated mesons

- Fundamental fields are not gauge-invariant quantities. According to the AdS/CFT correspondence, we have to rely on (composite) chiral fields.

- In $\mathcal{N}=4$ SYM theory, the chiral fields have no quasi-particle structure:



$$\rho(\omega,\boldsymbol{q}) = -2\,\mathrm{Im}\,G_{\mathrm{R}}(\omega,\boldsymbol{q})$$

$$G_{\mathrm{R}}(\omega,\boldsymbol{q}) = -\mathrm{i}\int \mathrm{d}^4x\,\mathrm{e}^{\mathrm{i}(-\omega t+\boldsymbol{q}\cdot\boldsymbol{x})}\,\theta(t)\langle[\hat{\mathcal{O}}(x),\hat{\mathcal{O}}(0)]\rangle$$

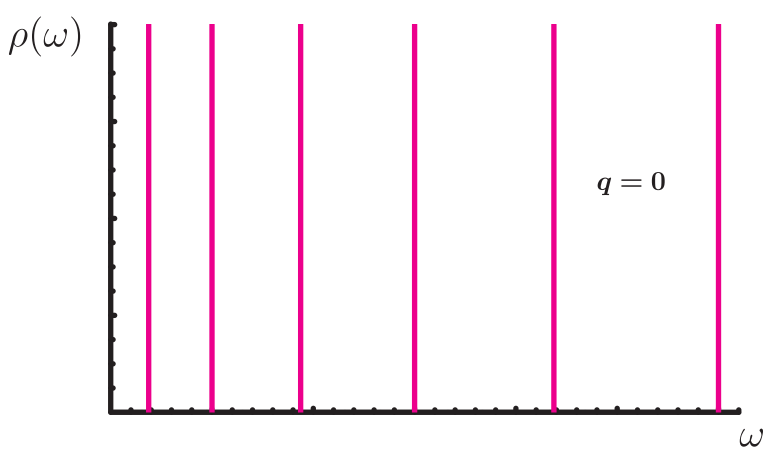
$$\rho(\omega,\mathbf{0}) \sim \omega^{2\Delta-4}$$

Son & Starinets, JHEP 09, 042 (2002)

- Using the D3-D7 set-up we can construct (stable) meson states:

Kruczenski *et al.*, JHEP 07, 049 (2003)

Myers *et al.*, JHEP 11, 091 (2007)



$$\rho(\omega) \sim \sum_n \frac{\Gamma}{(\omega-\omega_n)^2+\Gamma^2}$$

$$\omega_n^2 = 16\pi^2 n(n+1)\frac{M_{\mathrm{q}}^2}{\lambda}\ , \quad n=1,2,\ldots$$

$$\Gamma \sim \frac{1}{N_{\mathrm{c}}}$$

Conclusions

- The distribution of radiation from an accelerated charge in a strongly-coupled field theory should be qualitatively different from the classical case once all the quantum effects are considered.
- We suspect that the classical distribution of radiation is obtained in a strongly-coupled conformal field theory once we force the test particle to follow a classical trajectory.
- Dimensional reduction in a magnetic field is a non-perturbative effect due to the contributions from loops involving particles which interact with the external field ($\Longrightarrow$ neutral particles also suffer dimensional reduction)
- Applications: strongly-coupled systems or systems with extremely large magnetic fields: heavy-ion collisions ($B \sim 10^{19}$ G), neutron stars ($B \sim 10^{15}$ G), ...