Radiation from accelerated particles
at strong coupling

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Introduction: Radiation from accelerated charges in classical electrodynamics and QED

AdS/CFT correspondence: D3-branes and D7-branes setups

Power radiated by an accelerated quark in AdS/CFT: the trailing string

Motivation for further analysis: synchrotron radiation in QED, strong magnetic fields, and dimensional reduction

Magnetic solutions in AdS: radiation from gauge-invariant states in a magnetic field and dimensional reduction.

Conclusions
Radiation from accelerated charges in classical electrodynamics

Maxwell equations with a moving charge as the source:

\[ \partial_\mu F^{\mu\nu} = J^\nu, \quad J^\nu(x) = q \int d\tau U^\nu(\tau) \delta^{(4)}[x - r(\tau)]. \]

\[ \Rightarrow A_\text{rad}^\mu(x) = \int d^4 y G_R(x - y) J^\mu(y), \quad G_R(x - y) = \theta(x^0 - y^0) \delta[(x - y)^2] \]

\[ E = -\nabla A^0 - \frac{\partial A}{\partial t}, \quad B = \nabla \times A \]

\[ E(t, x) = q \left[ \frac{n - \beta}{\gamma^2 (1 - \beta \cdot n)^3 R^2} \right]_{\text{ret}} + q \left[ \frac{n \times \{(n - \beta) \times \dot{\beta}\}}{(1 - \beta \cdot n)^3 R} \right]_{\text{ret}} \]

\[ B = n \times E|_{\text{ret}} \]
Radiation from accelerated charges in classical electrodynamics

Flux of energy (Poynting vector):

\[ S = \frac{c}{4\pi} \mathbf{E} \times \mathbf{B} \]

Total power radiated by an accelerated charge in the non-relativistic case (Larmor’s formula):

\[ P = \frac{2}{3} q^2 \dot{\beta}^2 \]

Relativistic generalization (Liénard’s formula, 1898):

\[ P = \frac{2}{3} q^2 \gamma^6 [\dot{\beta}^2 - (\beta \times \dot{\beta})^2] \]
Angular distribution of radiation in classical electrodynamics

Rectilinear motion with parallel velocity and acceleration:

\[
\frac{dP}{d\Omega} = \frac{q^2 \beta^2}{4\pi} \frac{\sin^2 \theta}{(1 - \beta \cos \theta)^5}
\]

\[\beta = 0.9\]

Particle in circular motion:

\[
\frac{dP}{d\Omega} = \frac{q^2 \beta^2}{4\pi (1 - \beta \cos \theta)^3} \left[ 1 - \frac{\sin^2 \theta \cos^2 \phi}{\gamma^2 (1 - \beta \cos \theta)^2} \right]
\]

\[P_{\text{circular}} = \gamma^2 P_{\text{linear}}\]
Three ways of calculating it perturbatively:

- **Landau-Lifshitz/Sokolov-Ternov approach (easiest):**
  \[
  P = \frac{1}{\Delta t} \sum_f |k| |\langle \text{out} | \text{in} \rangle|^2
  \]

- **Schwinger’s approach (more involved):**

- **Inspired by Hofman & Maldacena’s calculation at strong coupling (most involved perturbatively):**
Synchrotron radiation in sQED: result with a magnetic field

In a magnetic field, the energy levels of the scalar particle are discrete (Landau levels):

\[ E_p^2 = m^2 + (p^3)^2 + 2|e|B \left(n + \frac{1}{2}\right), \quad n = 0, 1, 2, \ldots . \]

To leading order:

\[
P = \sum_{k,n'} |k| e^2 \frac{|e|B}{L^3} \frac{1}{E_k E_p E_q} n'^! n! \frac{1}{n'^! n!} e^{- (k^2)^2 / 2 |e|B} \left| I_{\kappa, \gamma} \right|_{q^3 = p^3 - k^3},
\]

with

\[
I_{\kappa, \gamma} = \frac{1}{|k|^2} \left\{ [(k^2)^2 + (k^3)^2] \kappa^2 + [(k^1)^2((2p^2 - k^2)^2 + (2p^3 - k^3)^2) + 4(k^3 p^2 - k^2 p^2)^2] \gamma^2 \right\},
\]

\[
\kappa = -2(n'^!) L_{n'^n'+1}((k^2)^2 / 2 |e|B) + (n' + 1) L_{n + 1}^n((k^2)^2 / 2 |e|B),
\]

\[
\gamma = \frac{1}{\sqrt{|e|B}}(n'^!) L_{n'^-n}^{n' - n'}((k^2)^2 / 2 |e|B).
\]

A great simplification happens in the semi-classical limit: \( \xi \to 0^+ \), \( \xi \equiv \frac{3 |e|B E}{2 m^3} \)

\[
P = P_{cl} \int_0^\infty dy f(y), \quad P_{cl} = \frac{2 e^4 B^2}{3 E^2}, \quad f(y) = \frac{9 \sqrt{3}}{8 \pi} \frac{y}{(1 + \xi y)^3} \int_y^\infty dx K_{5/3}(x).
\]

\[
\Rightarrow P = P_{cl} \left[ 1 - 3.97 \frac{|e|B E}{m^3} + O(\xi^2) \right].
\]

Sokolov & Ternov, Pergamon (1968)
Two ways of looking at D3-branes: perturbative point of view

Type IIB string theory in **10-dimensional** Minkowski space + open strings:

Massless spectrum of the **closed** string:
- NS sector: $G_2$ (graviton), $B_2$, $\phi$ (dilaton),
- RR sector: $C_0$, $C_2$, $C_4$

Massless spectrum of the **open** string: $A_\mu$

Action of the system: $S = S_{\text{closed}} + S_{\text{open}} + S_{\text{int}}$

At low energies ($E, p \ll 1/l_s$): $S_{\text{int}} \to 0$, $S_{\text{closed}} \to S_{\text{closed}}(\text{free})$ (**decoupling limit**)

In addition, we need: $g_s \to 0$, with $4\pi g_s N = g_{\text{YM}}^2 N \equiv \lambda$ constant (**t Hooft limit**)

$\Rightarrow S \simeq S_{N=4 \text{ SYM}} + S_{\text{closed}}(\text{free})$
in the massless spectrum of SUGRA must couple to (3+1)-dimensional **dynamical** objects:

Look for non-trivial **classical** solutions from the SUGRA action

\[
S = \frac{1}{2\kappa^2} \int d^{10}x \sqrt{-g} \left( R - \frac{1}{2} g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi - \frac{1}{4(5!)} F_5^2 \right)
\]

We want solutions which extend in 4 dims and are spherical in 6 dims. The result is:

\[
ds^2 = f(r)^{-1/2}(-dt^2 + dx^2) + f(r)^{1/2}(dr^2 + r^2 d\Omega_5^2)
\]

\[
F_5 = (1 + *)df^{-1} \wedge dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3
\]

\[
\Phi = \text{const.}
\]

\[
f(r) = 1 + \frac{L^4}{r^4}, \quad L^4 = 4\pi g_s l_s^4 N, \quad N \in \mathbb{Z}
\]

From calculations of the RR charge and dilaton absorption cross sections:

- Polchinski, PRL (1995)
- Klebanov, 97
- Das, Gibbons, Mathur, 97

open-closed duality
Decoupling limit in SUGRA

In order for the classical SUGRA approximation to be valid, we need: \( L \gg l_s \Rightarrow \lambda \gg 1 \)
“Type IIB string theory on $(AdS_5 \times S^5)_N$ plus some appropriate boundary conditions (and possibly also some boundary degrees of freedom) is dual to $\mathcal{N} = 4 \ d = 3+1 \ U(N)$ super Yang-Mills theory.”

Calculation of correlation functions

\[
\left\langle \exp \int d^4x \, \phi_0(x) \mathcal{O}(x) \right\rangle_{\text{CFT}} = e^{-I_{\text{SUGRA}}[\phi]} \bigg|_{\phi = \phi[\phi_0]} .
\]

Gubser, Klebanov, Polyakov (1998)

Witten (1998)

Scalar operators of scale dimension $\Delta$ will couple to massive scalar fields:

\[
\text{Scale transformation:} \quad x \mapsto \lambda x \quad \Rightarrow \quad \Delta = \frac{1}{2} \left( d + \sqrt{d^2 + 4m^2} \right).
\]

AdS/CFT correspondence

Correlation functions of the energy-momentum tensor can be obtained by coupling it to the graviton:

\[
\left\langle \exp \int d^4x \, \tilde{h}_{\mu\nu}(x) T^{\mu\nu}(x) \right\rangle_{\text{CFT}} = e^{-I_{\text{SUGRA}}[h]} \bigg|_{h = h[\tilde{h}]} .
\]
Supersymmetry algebra:

\[ Q^A |\text{boson}\rangle = |\text{fermion}\rangle \]

\[ \{ Q^A_{\alpha}, \bar{Q}^B_{\beta} \} = 2\sigma^\mu_{\alpha\beta} P_\mu \delta^A_B, \quad \{ Q^A_{\alpha}, Q^B_{\beta} \} = 0. \]

Field content:

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<th>( h )</th>
<th>( \mathcal{N} = 1 ) vector</th>
<th>( \mathcal{N} = 1 ) chiral</th>
<th>( \mathcal{N} = 2 ) vector</th>
<th>( \mathcal{N} = 2 ) hyper</th>
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Lagrangian:

\[ \mathcal{L} = \text{tr} \left\{ -\frac{1}{2g^2} F_{\mu\nu} F^{\mu\nu} + \frac{\theta_I}{8\pi^2} F_{\mu\nu} \tilde{F}^{\mu\nu} - i\bar{\lambda}^a \sigma^\mu D_\mu \lambda_a - D_\mu X^i D^\mu X^i + gC^{ab}_i \lambda_a [X^i, \lambda_b] \\
+ g\tilde{C}_{iab} \bar{\lambda}^a [X^i, \bar{\lambda}^b] + \frac{g^2}{2} [X^i, X^j]^2 \right\}, \]

\[ F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu + i[A_\mu, A_\nu], \quad \tilde{F}_{\mu\nu} \equiv \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} F^{\rho\sigma}, \quad D_\mu \lambda \equiv \partial_\mu \lambda + i[A_\mu, \lambda]. \]

- R-symmetry: rotations of supercharges, \( U(1)_R \) and \( SU(4)_R \approx SO(6)_R \).
D7-branes and fundamental quarks

Probe limit: $N_c \gg N_f$  
The theory remains conformal, $\beta \propto \frac{\lambda_3^2 N_f}{N_c}$
Quark loops are suppressed (no influence on the geometry)

Field content: $\mathcal{N} = 4$ SYM $+ N_f$ hypermultiplets in the fundamental rep.

7-7 modes decouple at low energies, $\lambda_7 = \lambda_3 (2\pi l_s)^4 \frac{N_f}{N_c}$

$m_q = R/2\pi l_s^2$
Radiation from a heavy quark in circular motion at strong coupling

Consider a charged particle forced to go in circular motion in classical electrodynamics:

\[ \alpha \sim \frac{1}{\gamma} \]

\[ \Delta \sim \frac{1}{\gamma^3} \]

Trailing string in circular motion with a D3-D7 setup:

Other works with similar results:

- Athanasiou, Chesler, Liu, Nickel, & Rajagopal, PRD (2010)
- Chernicoff, Antonio Garcia, Guijosa, & Pedraza (2011)
- Hatta, Iancu, Mueller, & Triantafyllopoulos (2011)
- Baier (2011)
Motivation for further study

- Quantum effects present in the standard leading-order perturbative evaluation of synchrotron radiation in QED:
  - Quantization of the trajectory for strong magnetic fields: \( E, E' \gg \sqrt{|e|B} \)
  - Recoil of the electron for emission of high-energy photons: \( E' \gg |k| \)

- Loops and dimensional reduction:

Fermion propagator in a magnetic field:  

\[
S(x, y) = \exp \left[ \frac{ie}{2} (x - y) \mu \, A_\mu^{\text{ext}} (x + y) \right] \tilde{S}(x - y), \quad \tilde{S}(k) = i \exp \left( -\frac{k_\perp^2}{|eB|} \right) \sum_{n=0}^{\infty} (-1)^n \frac{D_n(eB, k)}{k_0^2 - k_3^2 - m^2 - 2|e|Bn},
\]

For strong magnetic field, the lowest Landau-level dominates:

\[
\tilde{S}^{(0)}(k) = i \exp \left( -\frac{k_\perp^2}{|eB|} \right) \frac{k^0 \gamma^0 - k^3 \gamma^3 + m}{k_0^2 - k_3^2 - m^2} (1 - i \gamma^1 \gamma^2 \text{sgn}(eB)).
\]

\[
\sim \int d^2 k_\perp e^{-2k_\perp^2/|e|B} = \frac{|e|B}{4}. \quad \Rightarrow \quad 1+1 \text{ dimensional dynamics}
\]
Magnetic brane solutions


\[ S = -\frac{1}{16\pi G_5} \int d^5x \sqrt{-g} \left( R + F^{MN} F_{MN} - \frac{12}{L^2} \right) + S_{\text{bndy}}. \]

- Equivalent to introduce a magnetic field in $\mathcal{N} = 4$ SYM theory associated to the $U(1)_R$ R-charge

- They interpolate between $AdS_5$ at high energies and a 1+1 dimensional conformal field theory at low energies:

\[ ds^2 = -e^{2W(r)} dt^2 + e^{-2W(r)} dr^2 + e^{2V(r)} [(dx^1)^2 + (dx^2)^2] + e^{2W(r)} (dx^3)^2, \]

For $r \to \infty$: $AdS_5$

For $r \to 0^+$: $AdS_3 \times T^2$

\[ ds^2 = -3r^2 dt^2 + \frac{dr^2}{3r^2} + \frac{B}{\sqrt{3}} [(dx^1)^2 + (dx^2)^2] + 3r^2 (dx^3)^2. \]
The magnetic field as an effective medium

- Modifying the background geometry is equivalent to considering the effect of the magnetic field to arbitrary loop orders.

- This implies that even neutral particles will be affected by the magnetic field.

QED analogy: photon splitting

\[ P \sim \alpha^3 \left( \frac{\omega}{m} \right)^5 \left( \frac{B \sin \theta}{B_e} \right)^6 m \]

for \( (B/B_e)(\omega/m) \sin \theta \ll 1 \)

- The dispersion relation of the massless excitations will be modified by the magnetic field:

  The vacuum behaves effectively as a birefringent medium

\[ \text{Adler et al., PRL 25, 1061 (1970)} \]
\[ \text{Bialynicka-Birula et al., PRD 2, 2341 (1970)} \]
\[ \text{Dittrich & Gies, hep-ph/9806417} \]
We want to calculate the expectation value of the energy-momentum tensor in a well localized state:

\[ \langle 0 | \hat{\mathcal{O}}_q \hat{T}^{0i} (x) \hat{\mathcal{O}}_q | 0 \rangle \sim \text{flux of energy} \]

\[ \hat{\mathcal{O}}_q = \int d^4 x \, e^{i q \cdot x} \exp \left( -\frac{x_0^2 + x^2}{\sigma^2} \right) \hat{\mathcal{O}}(x), \quad q \sigma \gg 1 \] (well defined momentum and position)

In a conformal field theory, **isotropic** energy distribution for scalar states in the COM frame

\[ \mathcal{O} \sim \bar{q}q \]

\[ \implies \]

no jets from scalar states in the COM for a conformal field theory

Hofman & Maldacena, JHEP (2008)

This should be different in the presence of a magnetic field
Accelerated mesons

- Fundamental fields are not gauge-invariant quantities. According to the AdS/CFT correspondence, we have to rely on (composite) chiral fields.

- In $\mathcal{N} = 4$ SYM theory, the chiral fields have no quasi-particle structure:

$$\rho(\omega, q) = -2 \text{Im} G_R(\omega, q)$$

$$G_R(\omega, q) = -i \int d^4x \, e^{i(-\omega t + q \cdot x)} \theta(t) \langle [\hat{O}(x), \hat{O}(0)] \rangle$$

$$\rho(\omega, 0) \sim \omega^{2\Delta - 4}$$  Son & Starinets, JHEP 09, 042 (2002)

- Using the D3-D7 set-up we can construct (stable) meson states:

$$\rho(\omega) \sim \sum_n \frac{\Gamma}{(\omega - \omega_n)^2 + \Gamma^2}$$

$$\omega_n^2 = 16\pi^2n(n+1)\frac{M_4^2}{\lambda}, \quad n = 1, 2, \ldots$$

$$\Gamma \sim \frac{1}{N_c}$$  Kruczenski et al., JHEP 07, 049 (2003)  Myers et al., JHEP 11, 091 (2007)
Conclusions

- The distribution of radiation from an accelerated charge in a strongly-coupled field theory should be qualitatively different from the classical case once all the quantum effects are considered.

- We suspect that the classical distribution of radiation is obtained in a strongly-coupled conformal field theory once we force the test particle to follow a classical trajectory.

- Dimensional reduction in a magnetic field is a non-perturbative effect due to the contributions from loops involving particles which interact with the external field (neutral particles also suffer dimensional reduction).

- Applications: strongly-coupled systems or systems with extremely large magnetic fields: heavy-ion collisions \( (B \sim 10^{19} \text{ G}) \), neutron stars \( (B \sim 10^{15} \text{ G}) \), ...