Dynamical Dark Matter

Collider Signatures and Direct Detection

Brooks Thomas
(University of Hawaii)

Work done in collaboration with Keith Dienes:

[arXiv:1106.4546]
[arXiv:1107.0721]
[arXiv:1203.1923]
[arXiv:1204.4183] also with Shufang Su
[arXiv:1207.xxxx] also with Jason Kumar

Dynamical Dark Matter (DDM)

- The dark-matter candidate is an **ensemble** consisting of a vast number of constituent particle species whose collective behavior transcends that of traditional dark-matter candidates.
- Dark-matter stability is not a requirement; rather, the individual abundances of the constituents are **balanced against decay rates** across the ensemble in manner consistent with observational limits.
- Cosmological quantities like the total dark-matter relic abundance, the composition of the dark-matter ensemble, and even the dark-matter equation of state exhibit a **non-trivial time-dependence** beyond that associated with the expansion of the universe.

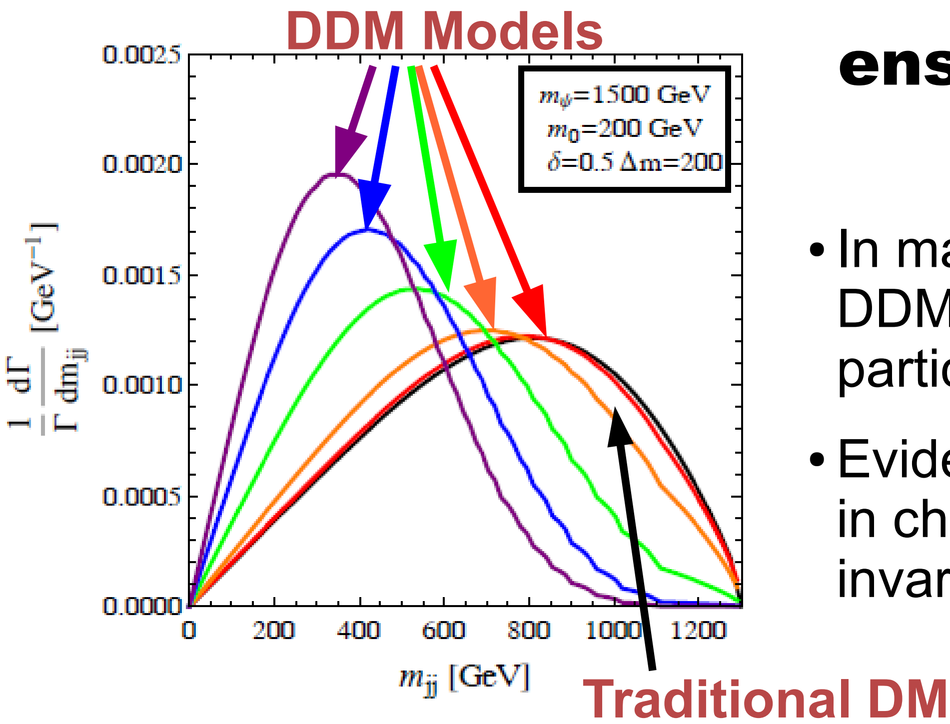
Keith's talk:

- General features of the DDM framework
- Characterizing the cosmology of DDM model
- An explicit realization of the DDM framework which satisfies all applicable constraints

This talk:

- Phenomenological consequences of DDM ensembles and methods of distinguishing them from traditional DM candidates experimentally.

Overview:



In this talk, I'll discuss methods for distinguishing DDM ensembles at the LHC...

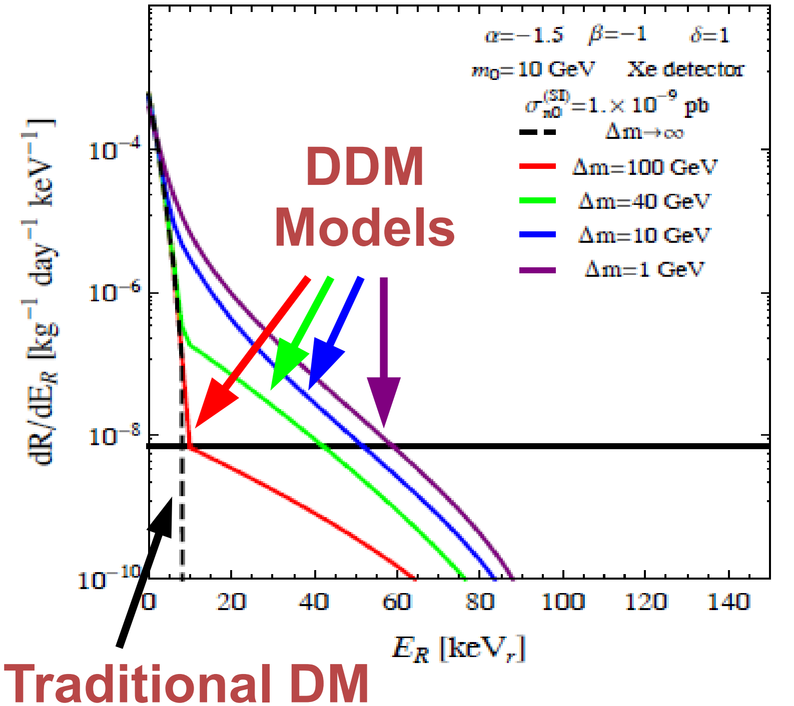
K. Dienes, S. Su, BT [arXiv:1204.4183]

- In many DDM models, constituent fields in the DDM ensemble can be produced alongside SM particles by the decays of additional heavy fields.
- Evidence of a DDM ensemble can be ascertained in characteristic features imprinted on the invariant-mass distributions of these SM particles.

and direct-detection experiments.

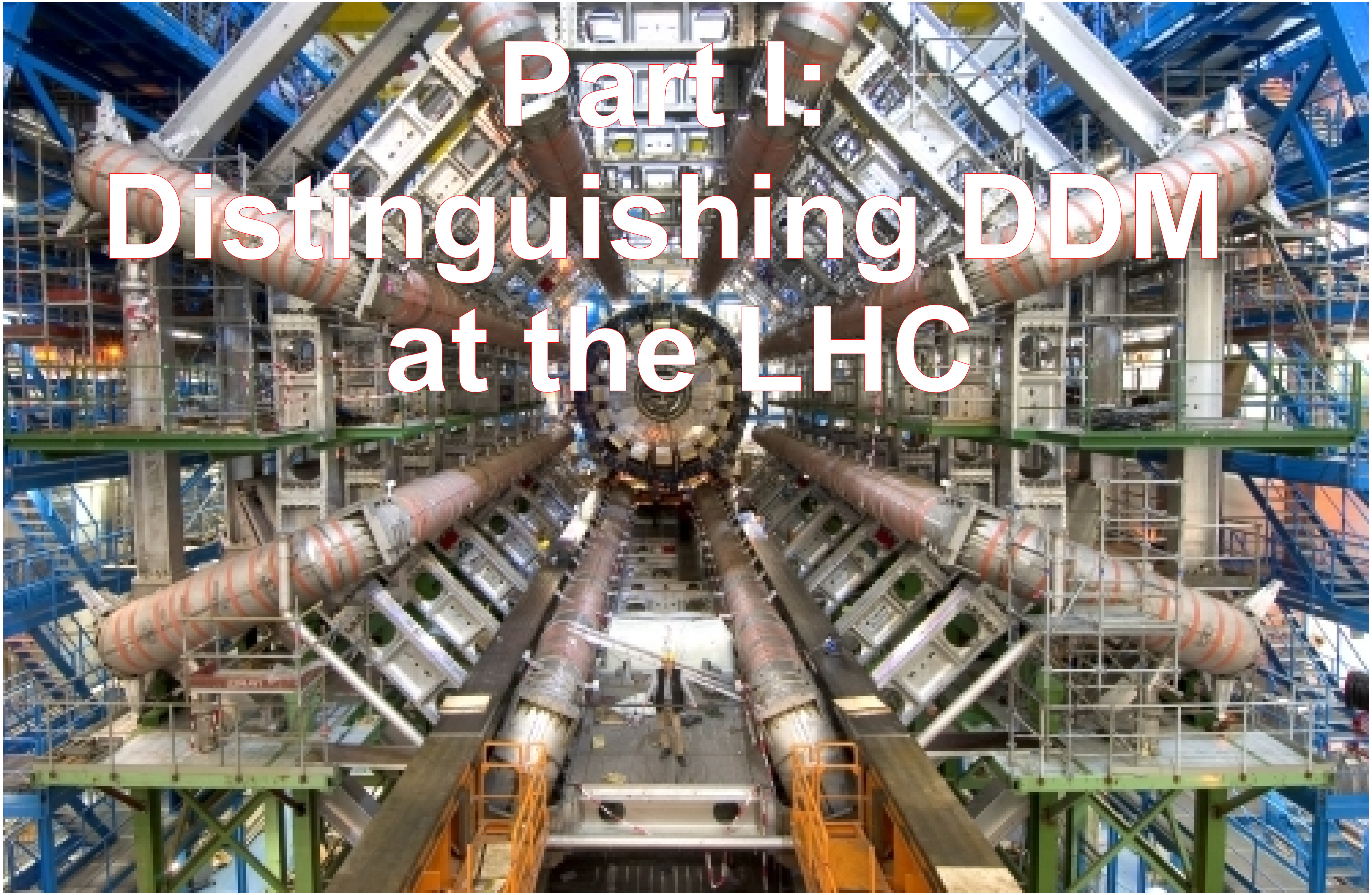
K. Dienes, J. Kumar, BT [arXiv:1205.xxxx]

- DDM ensembles can also give rise to distinctive features in recoil-energy spectra.



These are just two examples which illustrate that DDM ensembles give rise to **observable effects** which can serve to distinguish them from traditional DM candidates

Part I: Distinguishing DDM at the LHC



Searching for Signs of DDM at the LHC

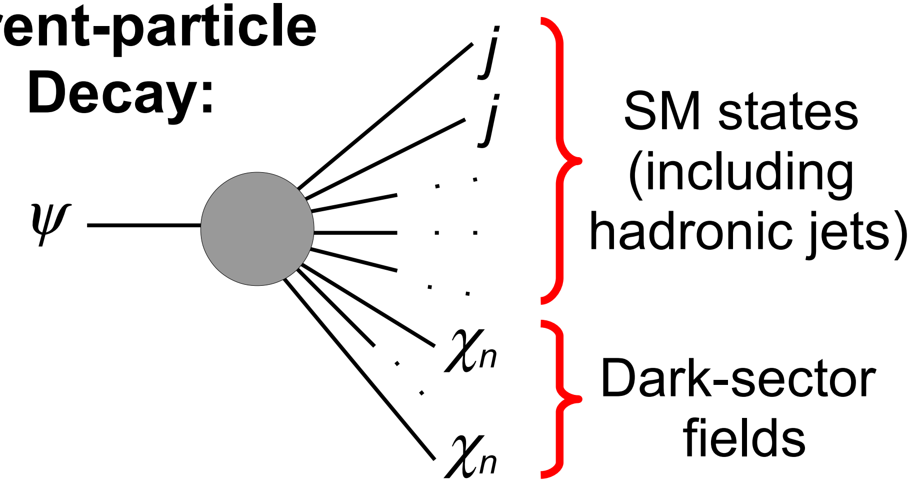
- In a wide variety of DM models, dark-sector fields can be produced via the decays of some heavy **"parent particle"** ψ.

- Strongly interacting ψ can be produced copiously at the LHC. $SU(3)_c$ invariance requires that such ψ decay to final states including not only dark-sector fields, but **SM quarks and gluons** as well.

- In such scenarios, the initial signals of dark matter will generically appear at the LHC in channels involving jets and $\not{E}_T$.

Further information about the dark sector or particles can <u>**also**</u> be gleaned from examining the <u>**kinematic distributions**</u> of visible particles produced alongside the DM particles.

Parent-particle Decay:

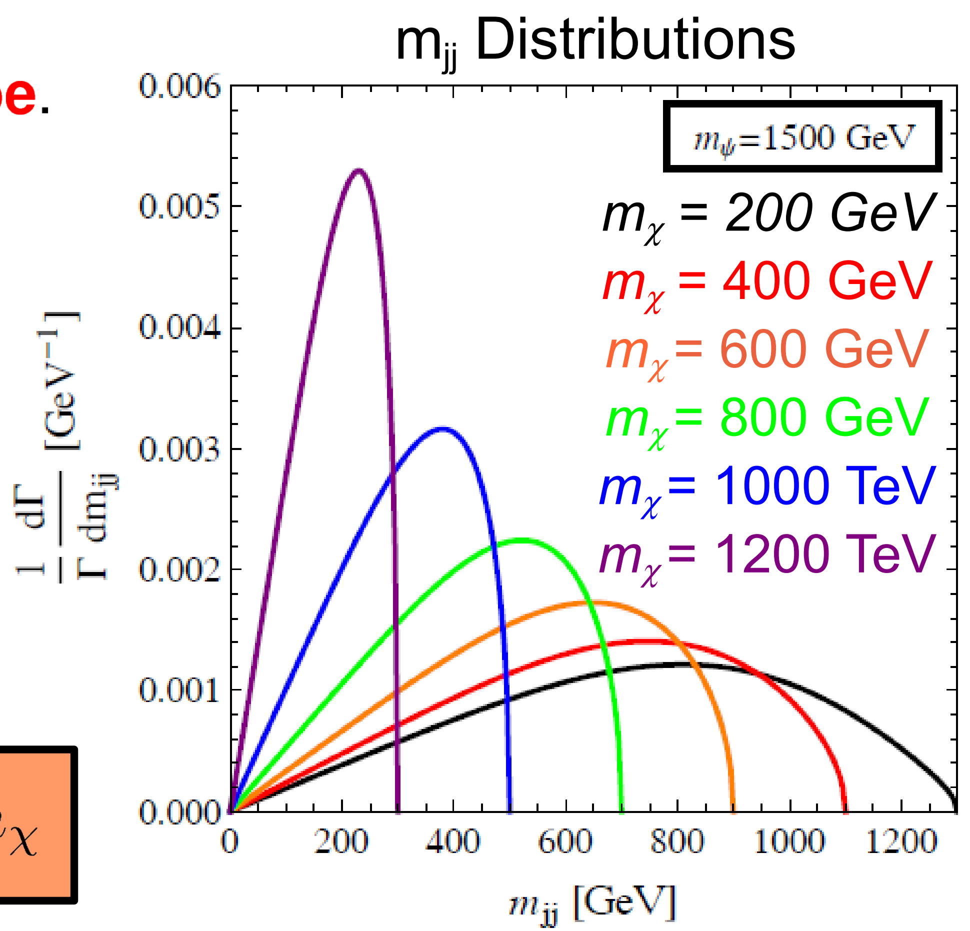


As we shall see, such information can be used to distinguish DDM ensembles from traditional DM candidates on the basis of LHC data.

Traditional DM Candidates

- Let's begin by considering a dark sector which consists of a traditional dark-matter candidate χ — a **stable** particle with a mass m_χ.
- For concreteness, consider the case in which ψ decays primarily via the **three-body** process $\psi \to jj\chi$ (no on-shell intermediary).
- Invariant-mass distributions for such decays manifest a **characteristic shape**.
- Different coupling structures between ψ, χ, and the SM quark and gluon fields, different representations for ψ, *etc.* have only a small effect on the distribution.
- m_{jj} distributions characterized by the presence of a **mass "edge"** at the kinematic endpoint:

$$\boxed{m_{jj} \leq m_\psi - m_\chi}$$



Parent Particles and DDM Daughters

In general, the constituent particles χ_n in a DDM ensemble and other fields in the theory through some set of effective operators $O_n^{(\alpha)}$:

$$\mathcal{L}_{\text{eff}} = \sum_{\alpha} \sum_{n=0}^{N} \frac{c_{n\alpha}}{\Lambda^{d_\alpha - 4}} \mathcal{O}_n^{(\alpha)} + \ldots$$

As an example, consider a theory in which the masses and coupling coefficients of the χ_n scale as follows:

γ_α : scaling indices for couplings

Including coupling between ψ and the dark-sector fields χ_n.

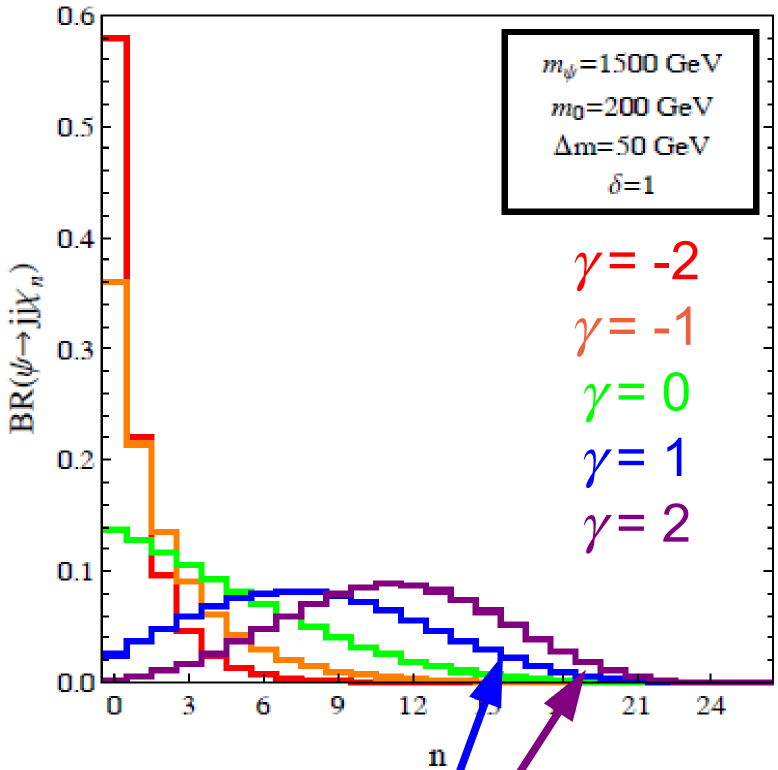
m_0 : mass of lightest constituent

$$\boxed{\begin{aligned} c_{n\alpha} &= c_{0\alpha}\left(\frac{m_n}{m_0}\right)^{\gamma_\alpha} \\ m_n &= m_0 + n^\delta \Delta m \end{aligned}}$$

δ : scaling index for the density of states

Δm : mass-splitting parameter

Parent-Particle Branching Fractions



- Once again, let's consider the simplest non-trivial case in which ψ couples to each of the χ_n via a four-body interaction, e.g.:

$$\mathcal{L}_{\rm eff} = \sum_n \left[\frac{c_n}{\Lambda^2} (\overline{q}_i t^a_{ij} \psi^a)(\overline{\chi}_n q_j) + \text{h.c.} \right]$$

- Assume partent's total width Γ_ψ dominated by decays of the form $\psi \to jj\chi_n$.

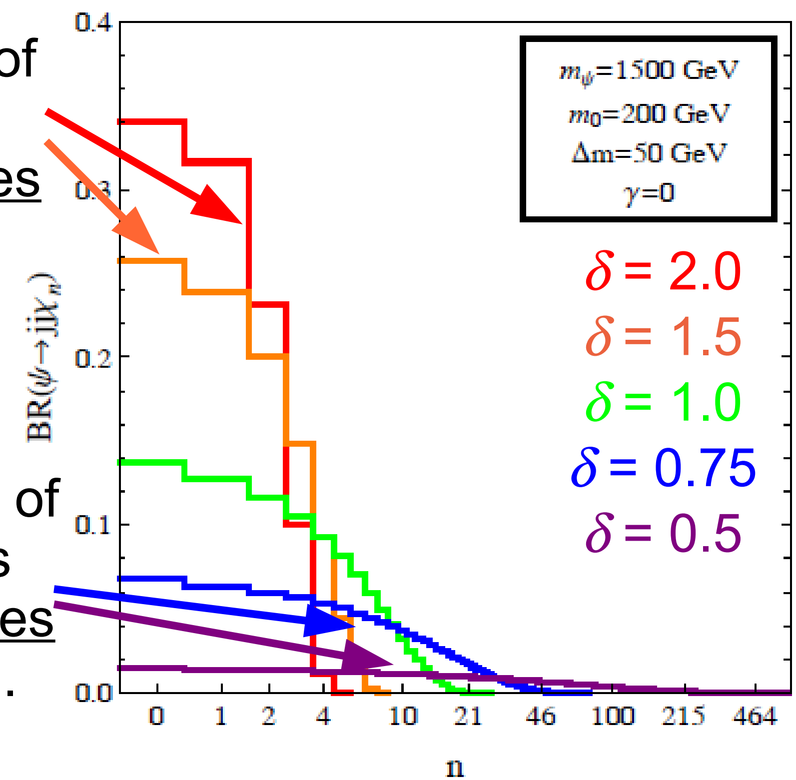
Coupling stength increases with n for γ>0...

…but phase space always decreases with n.

- **Branching fractions** of ψ to the different χ_n controlled by Δm, δ, and γ.

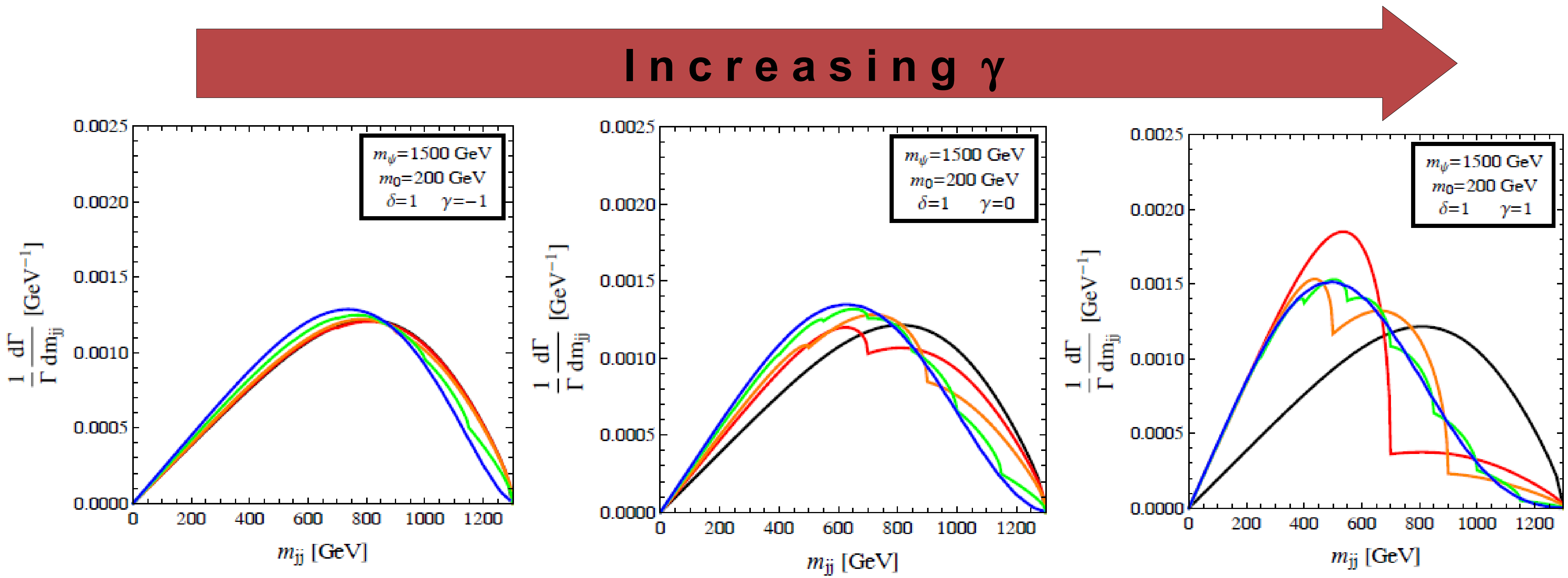
Density of states decreases with n.

Density of states increases with n.



DDM Ensembles & Kinematic Distributions

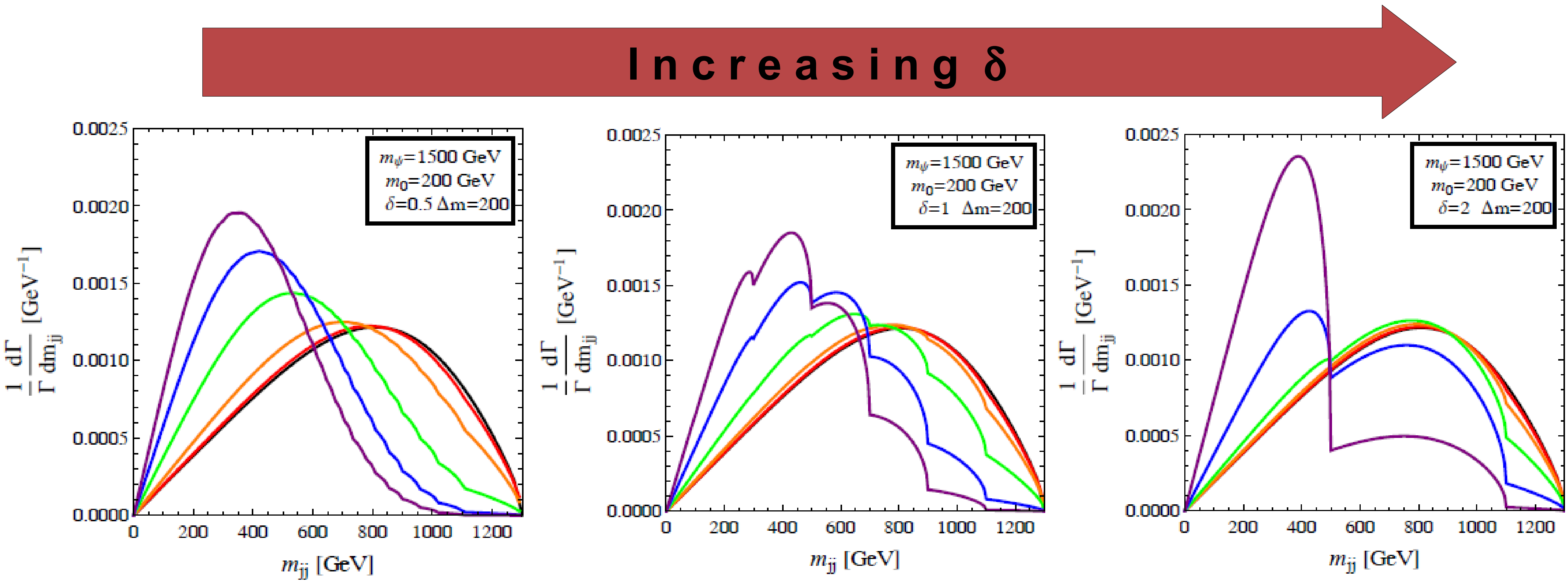
- Evidence of a DDM ensemble can be ascertained from characteristic features imprinted on the kinematic distributions of these SM particles.

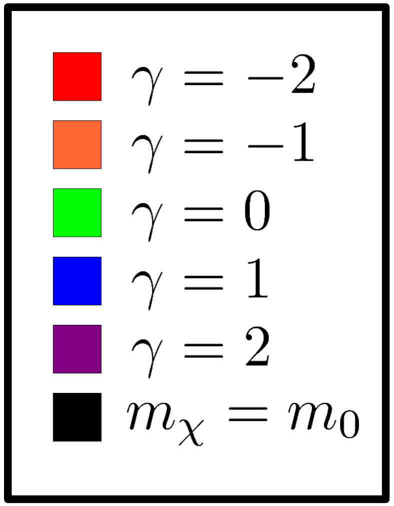


- $\Delta m = 600$ GeV
- $\Delta m = 400$ GeV
- $\Delta m = 150$ GeV
- $\Delta m = 20$ GeV
- $m_\chi = m_0$

- For example, in the scenarios we're considering here, the (normalized) dijet invariant-mass distribution is given by

$$\frac{1}{\Gamma_\psi}\frac{d\Gamma_\psi}{dm_{jj}} = \sum_{n=0}^{n_{\max}} \left(\frac{1}{\Gamma_{\psi n}}\frac{d\Gamma_{\psi n}}{dm_{jj}} \times \mathrm{BR}_{\psi n} \right)$$





Two Characteristic Signatures:

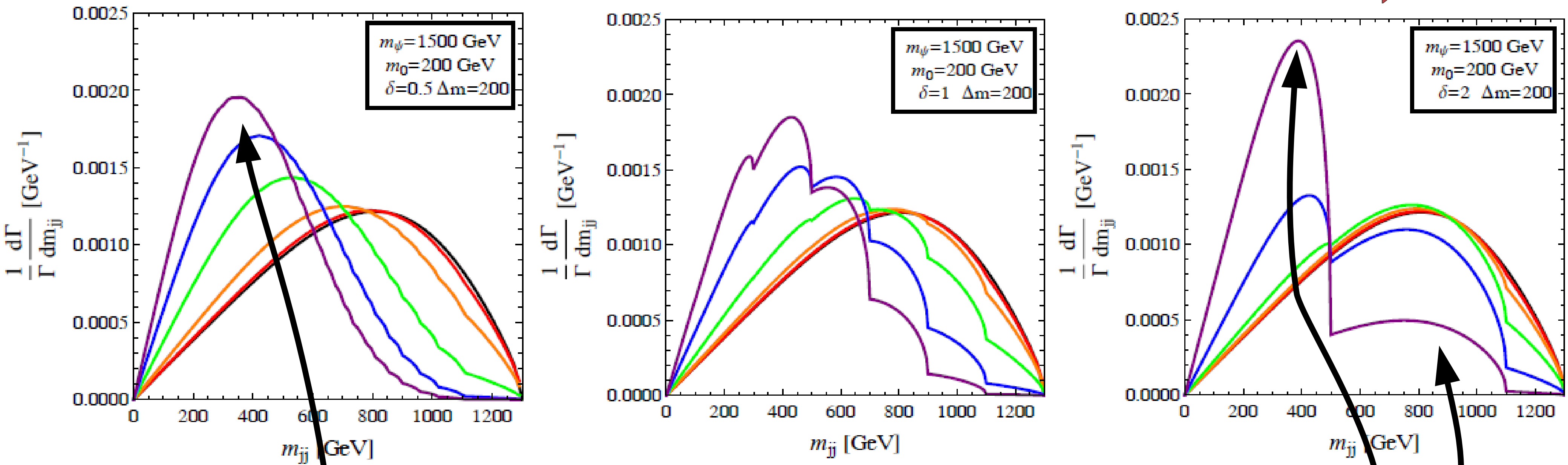
1. Multiple distinguishable peaks

Large δ, Δm: individual contributions from two or more of the χ_n can be resolved.

2. The Collective Bell

Small δ, Δm: Individual peaks cannot be distinguished, mass edge "lost," m_{jj} distribution assumes a characteristic shape.

Increasing δ



- $\gamma = -2$ (red)
- $\gamma = -1$ (orange)
- $\gamma = 0$ (green)
- $\gamma = 1$ (blue)
- $\gamma = 2$ (purple)
- $m_\chi = m_0$ (black)

Two Characteristic Signatures:

1. Multiple distinguishable peaks

Large δ, Δm: individual contributions from two or more of the χ_n can be resolved.

2. The Collective Bell

Small δ, Δm: Individual peaks cannot be distinguished, mass edge "lost," m_{jj} distribution assumes a characteristic shape.

But the REAL question is...

How well can we distinguish these features in practice?

In other words: to what degree are the characteristic kinematic distributions to which DDM ensembles give rise truly **distinctive**, in the sense that they cannot be reproduced by **any** traditional DM model?

The Procedure:

- Survey over traditional DM models with different DM-candidate masses m_χ and coupling structures.
- Divide the into bins with width determined by the invariant-mass resolution Δm_{jj} of the detector (dominated by jet-energy resolution ΔE_j).
- For each value of m_χ in the survey, define a χ^2 statistic $\chi^2(m_\chi)$ to quantify the degree to which the two resulting m_{jj} distributions differ.

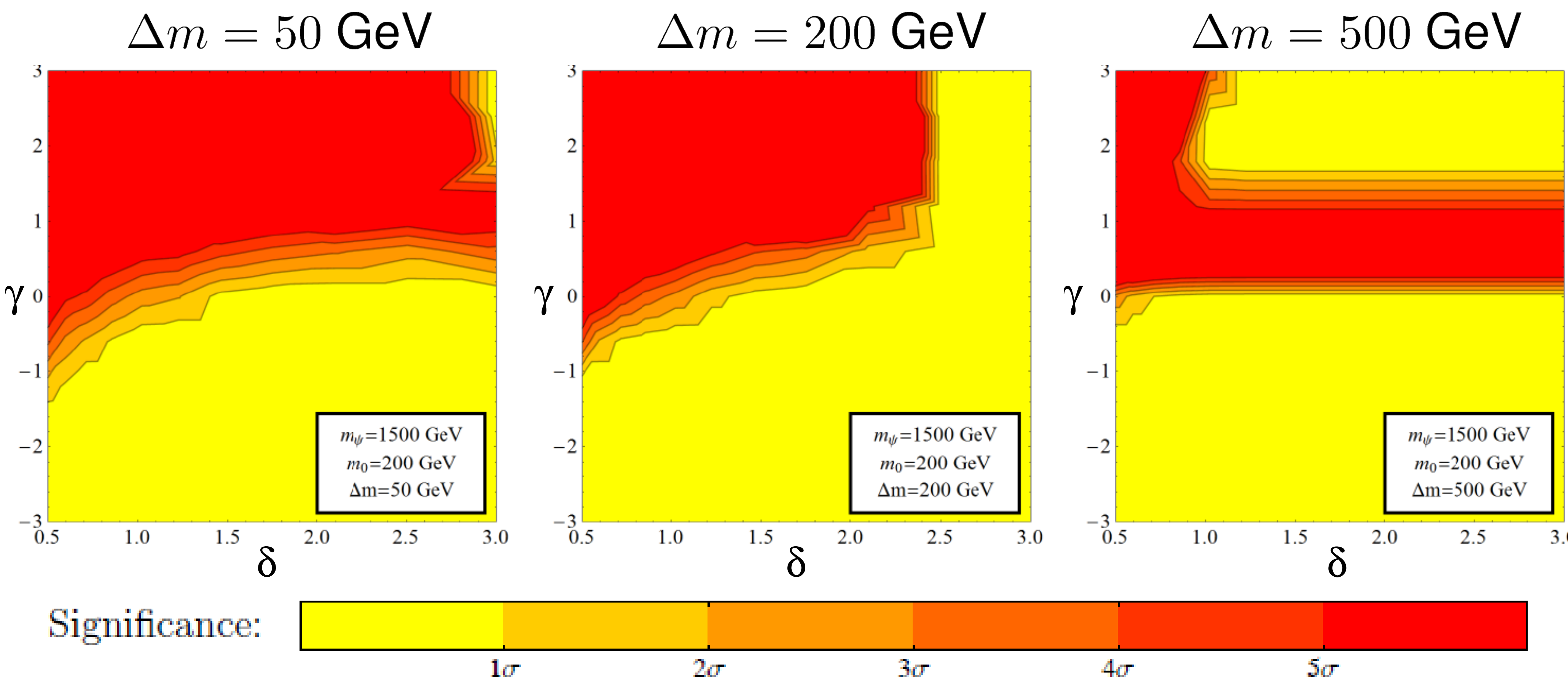
$$\chi^2(m_\chi) = \sum_k \frac{[X_k - \mathcal{E}_k(m_\chi)]^2}{\sigma_k^2}$$

$$\chi^2_{\rm min} = \min_{m_\chi} \{\chi^2(m_\chi)\}$$

- The **minimum** χ^2 value from among these represents the degree to which a DDM ensemble can be distinguished from **any** traditional DM candidate.

Distinguishing DDM Ensembles: Results

Results for N_e = 1000 signal events (e.g., $pp \to \psi\psi$ for TeV-scale parent, $L_{int} < 30\ fb^{-1}$)



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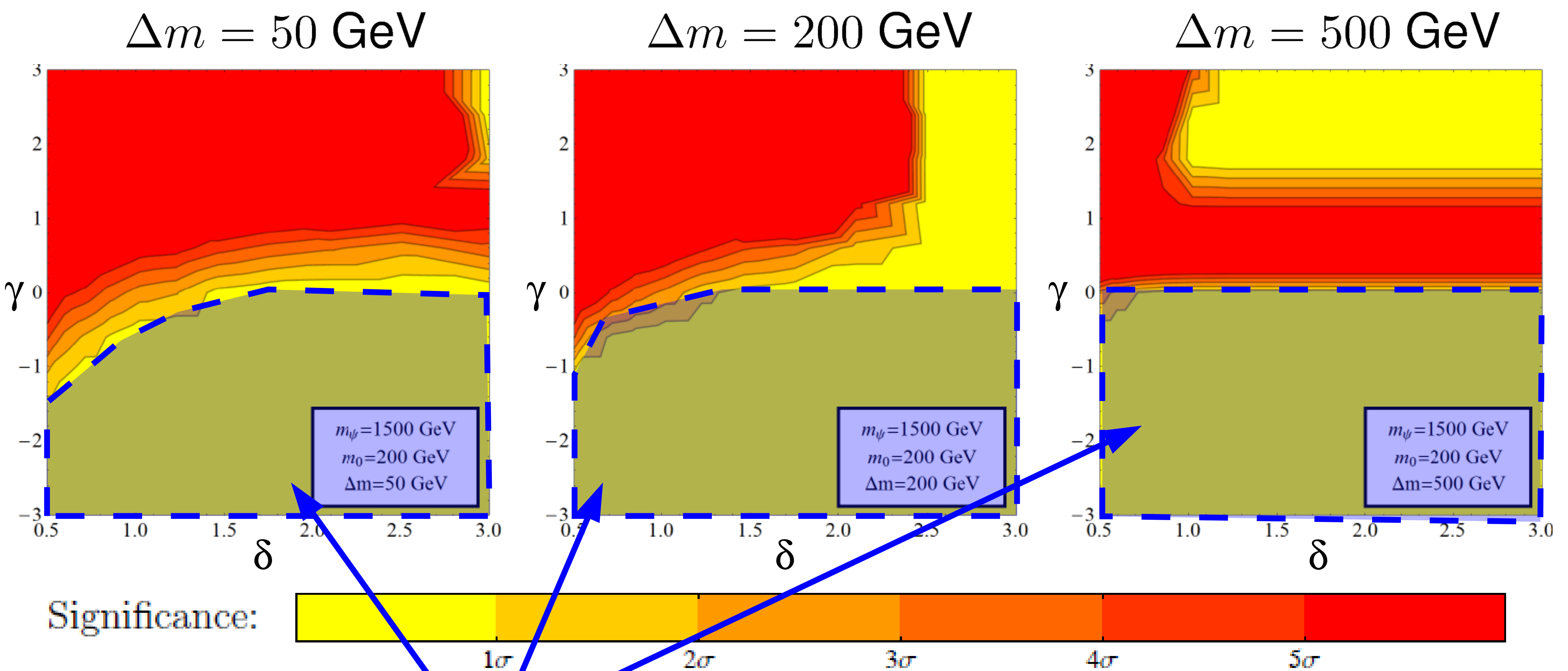

BRs to all χ_n with n > 1 suppressed: lightest constituent dominates the width of ψ.

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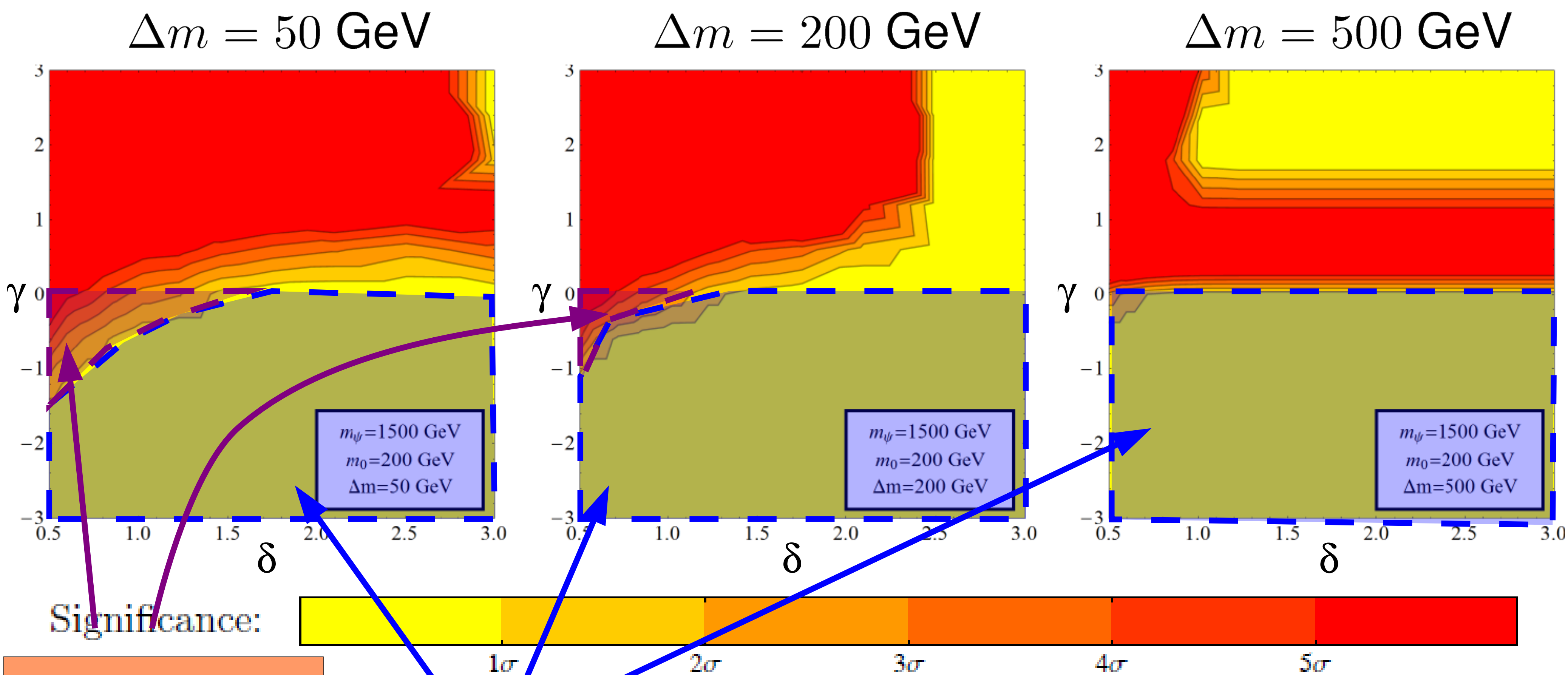

Density of states large enough to overcome γ suppression for small δ.

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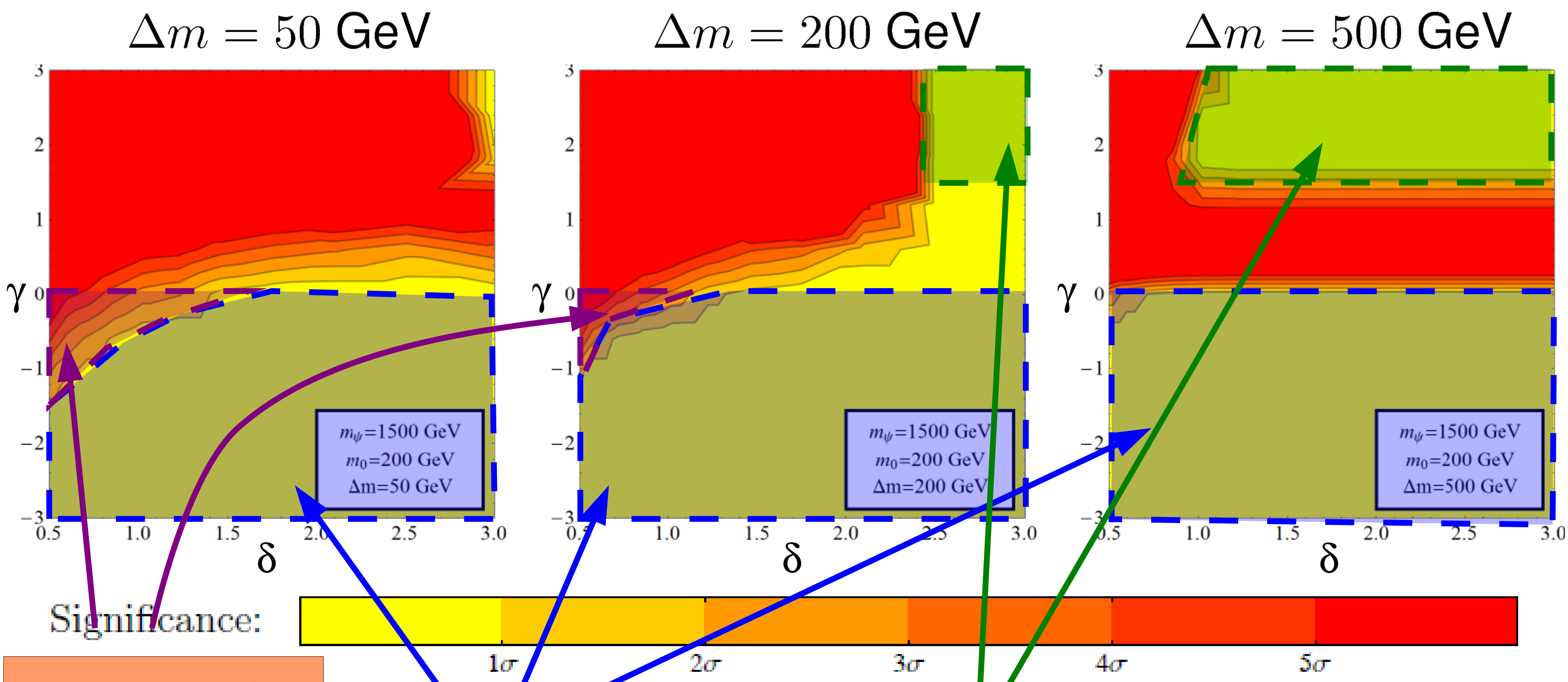

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Next-to-lightest constituent χ_1 dominates the width of ψ.

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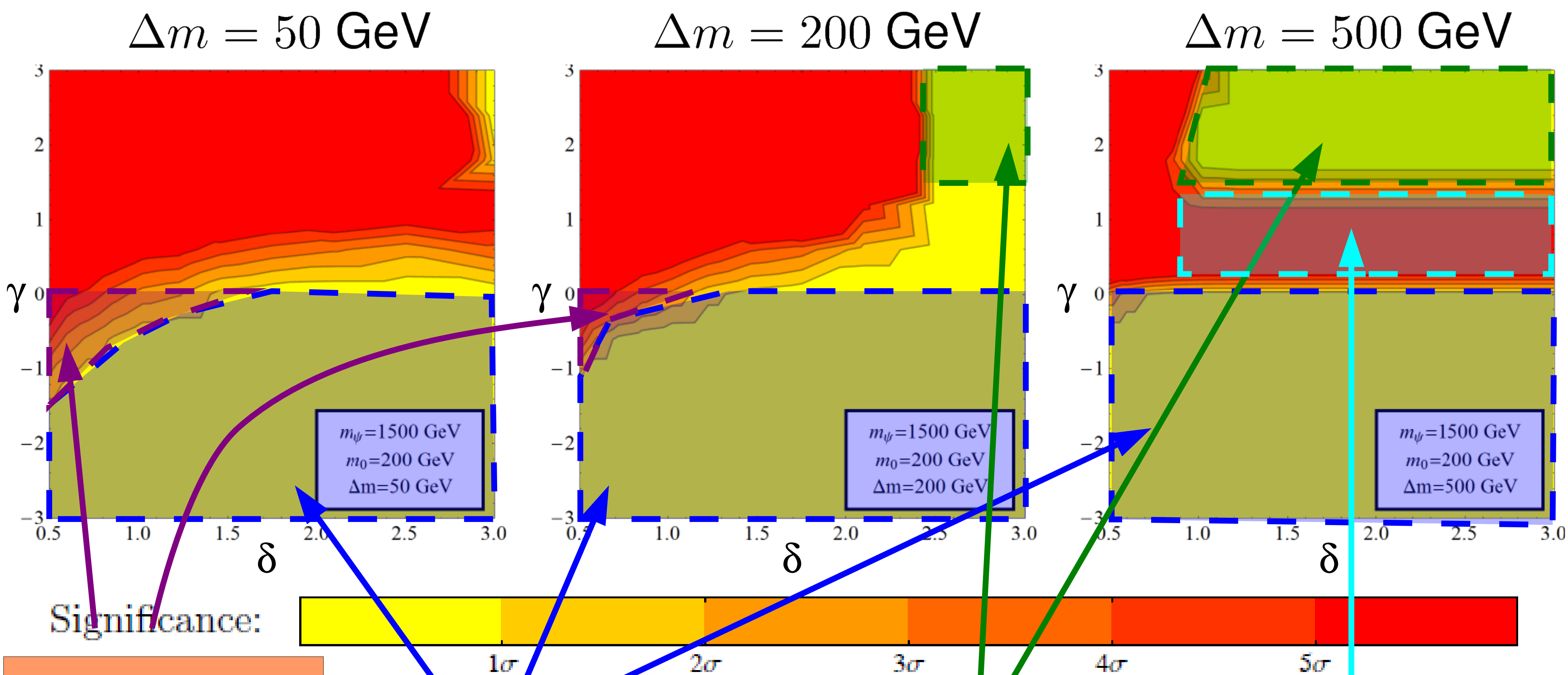

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Next-to-lightest constituent χ_1 dominates the width of ψ.

BR($\psi \rightarrow jj\chi_0$) ≈ BR($\psi \rightarrow jj\chi_1$): two distinct m_{jj} peaks.

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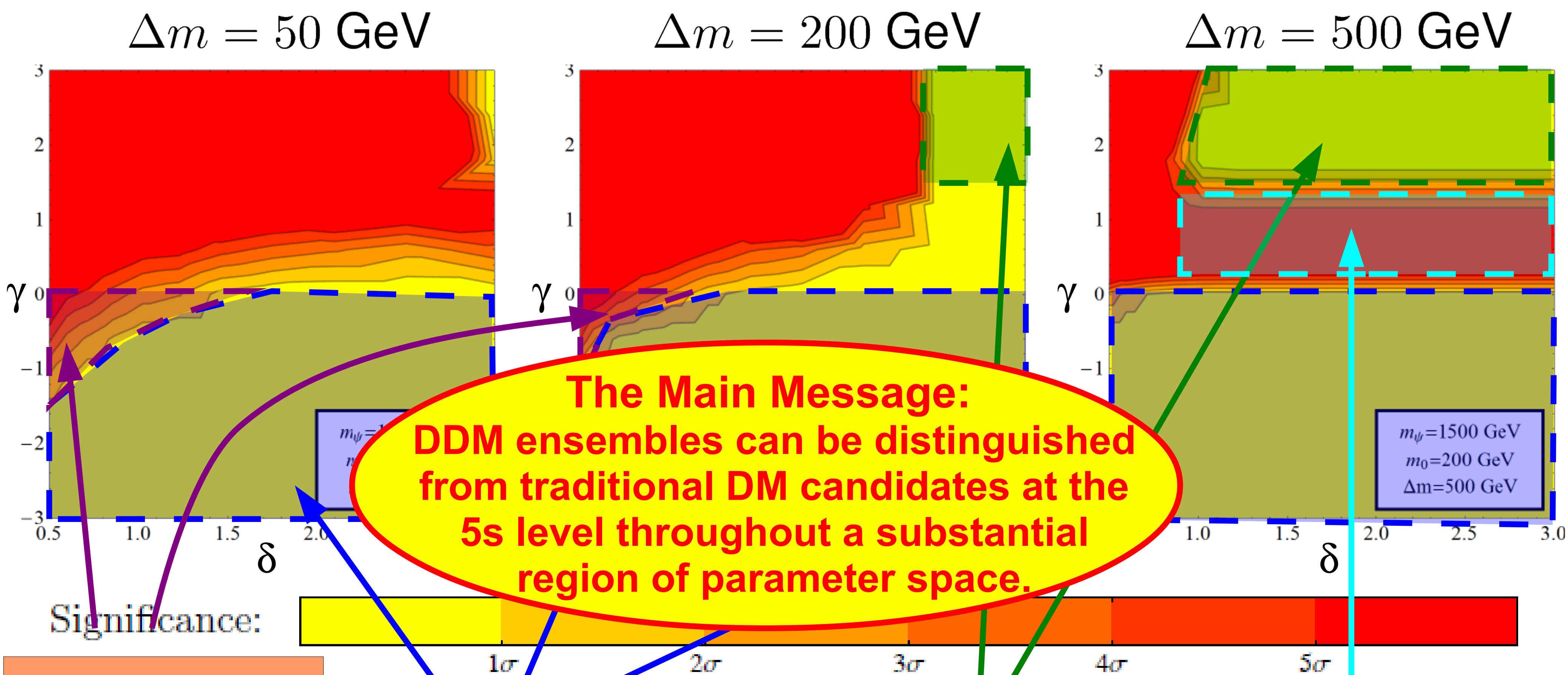

The Main Message:
DDM ensembles can be distinguished from traditional DM candidates at the 5s level throughout a substantial region of parameter space.

Density of states large enough to overcome γ suppression for small δ.

BRs to all χ_n with $n > 1$ suppressed: lightest constituent dominates the width of ψ.

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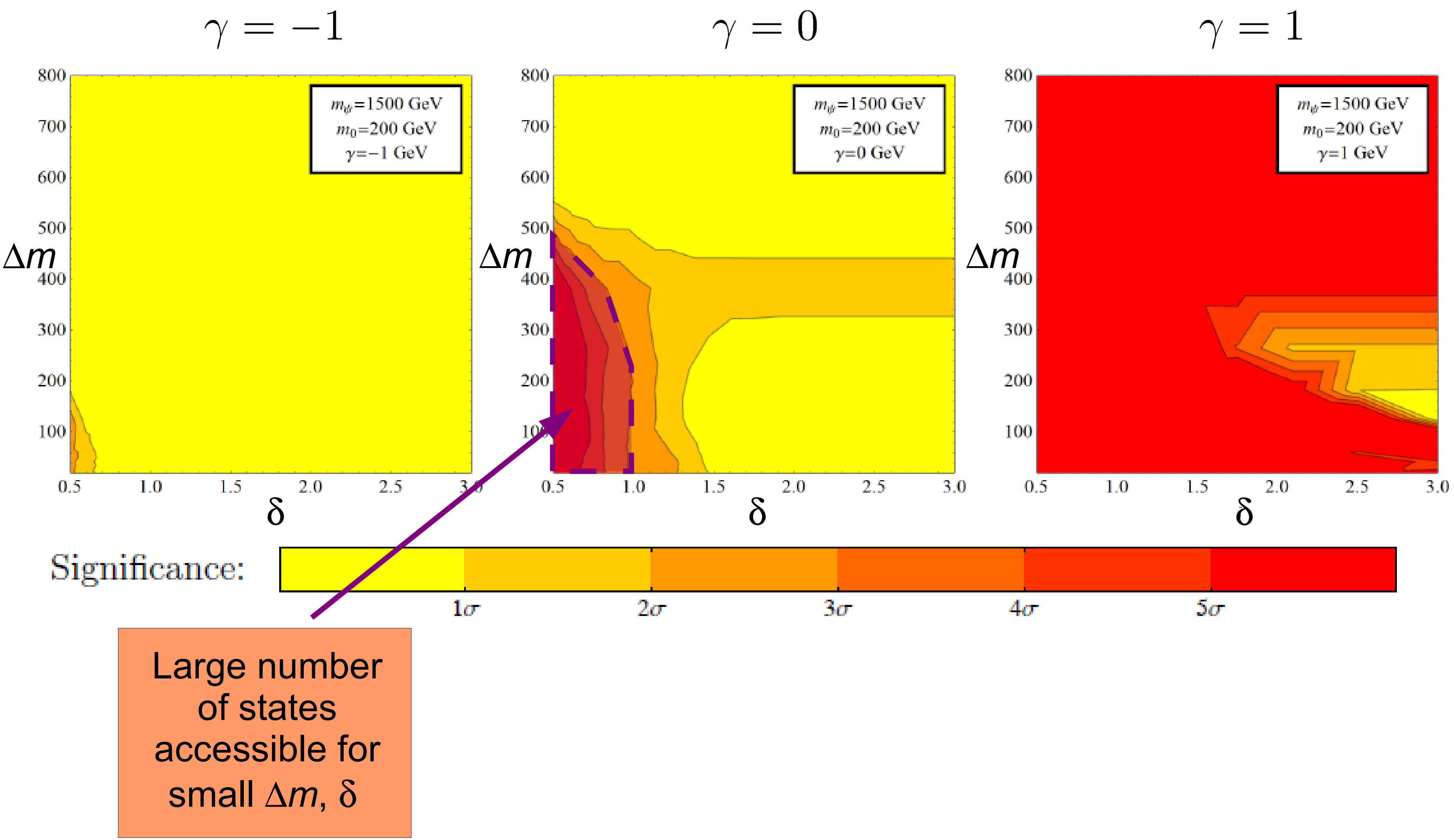

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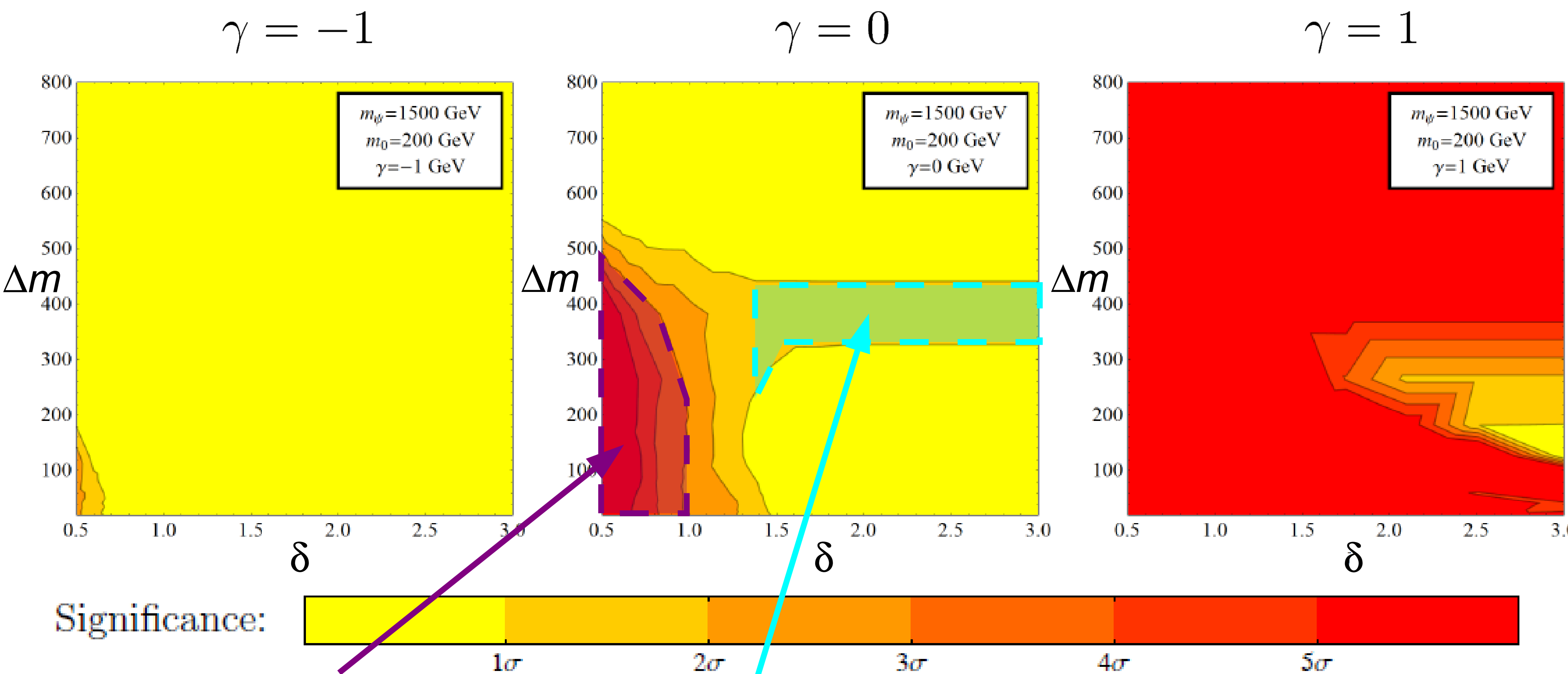

Large number of states accessible for small Δm, δ

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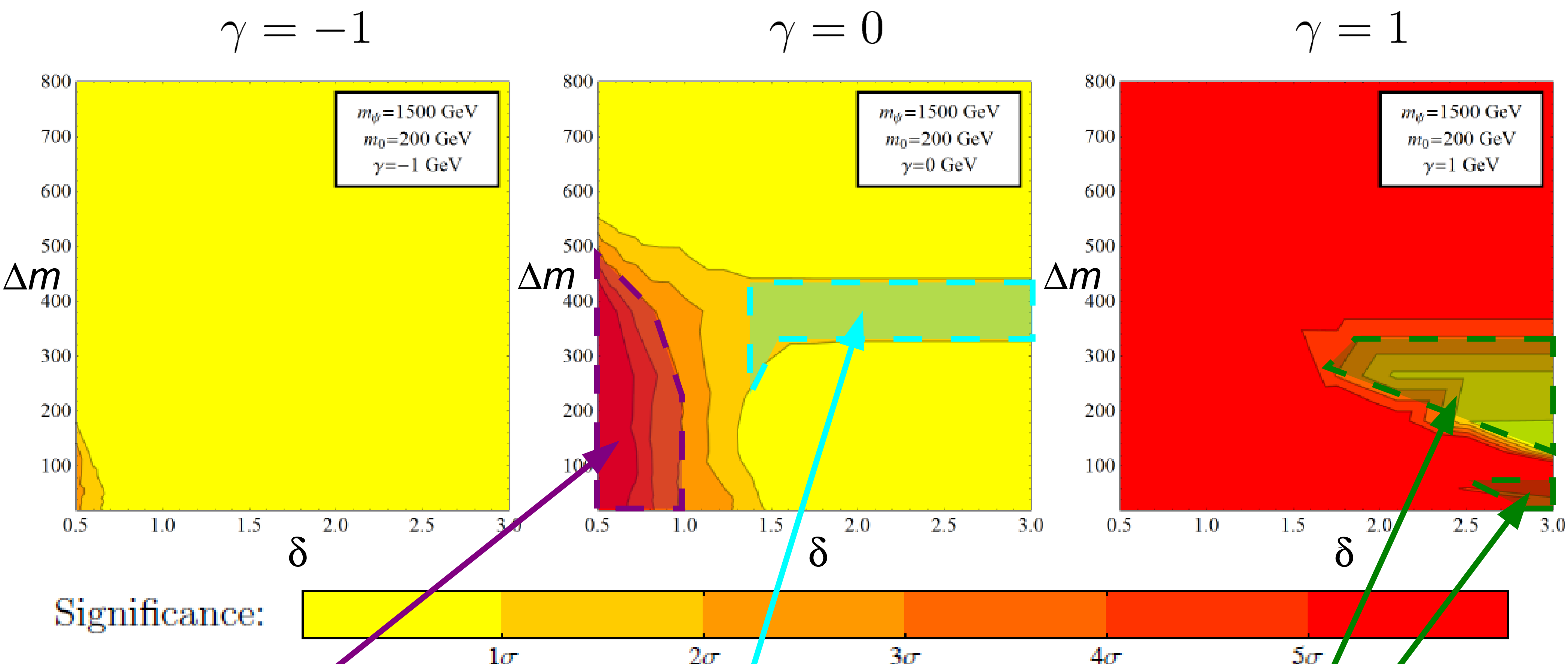
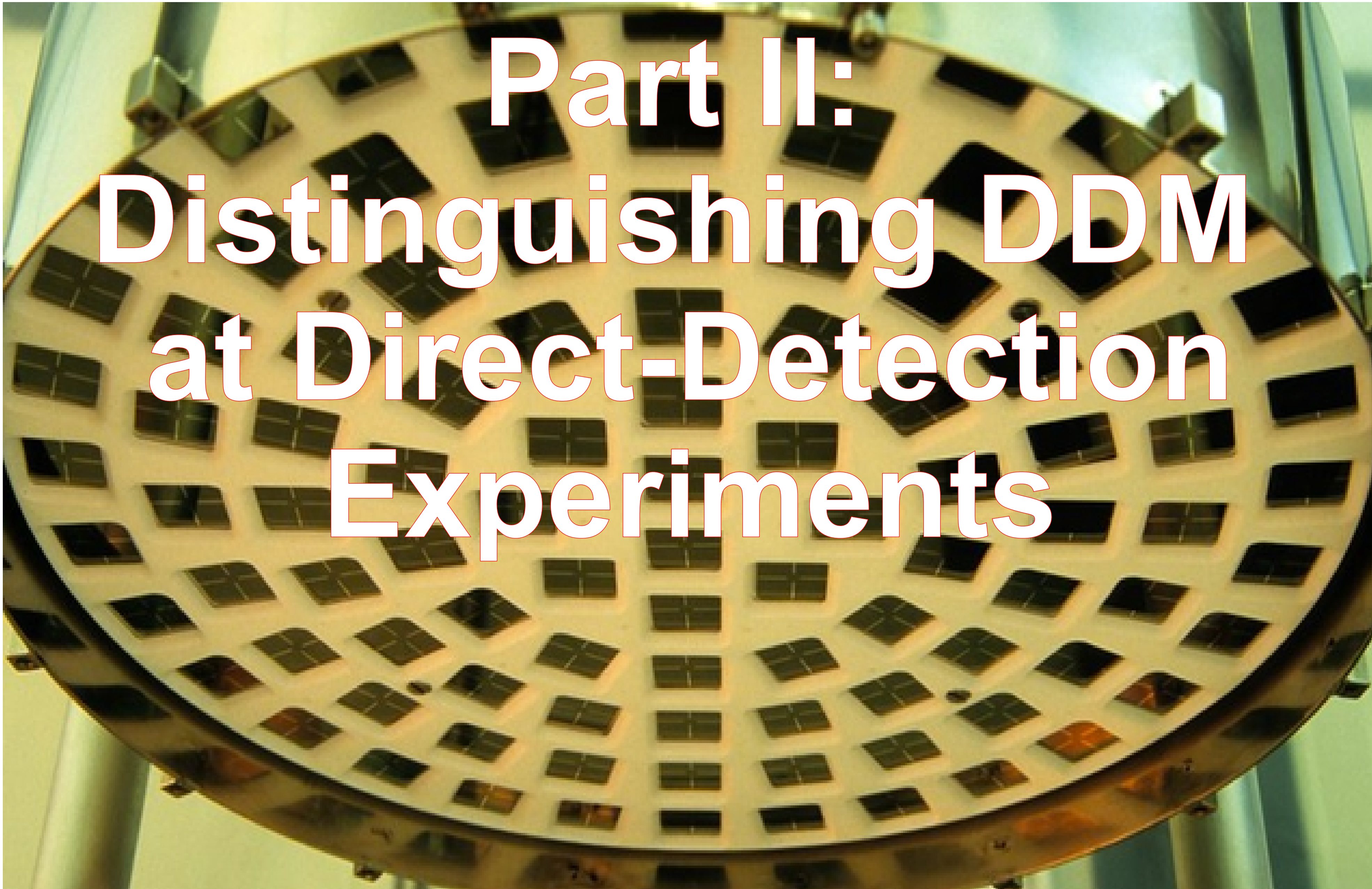


Large number of states accessible for small Δm, δ

BR($\psi \rightarrow jj\chi_0$) ≈ BR($\psi \rightarrow jj\chi_1$): two distinct m_{jj} peaks.

Only χ_0 and χ_1 kinematically accessible. One or the other dominates the width of ψ.



Part II: Distinguishing DDM at Direct-Detection Experiments

DDM Direct-Detection Experiments

- Direct-detection experiments offer another possible method for distinguishing DDM ensembles from traditional DM candidates.
- After the initial observation an excess of signal events at such an experiment, the shape of the **recoil-energy spectrum** associated with those events can provide additional information about the properties of the DM candidate.
- A number of factors impact the shape of the recoil-energy spectrum in a generic dark-matter scenario. **Particle physics**, **astrophysics**, *and* **cosmology** all play an important role.

χ_j-nucleus scattering cross-section

Form factor

Local energy density of χ_j

Halo-velocity distribution for χ_j

$$\boxed{\frac{dR}{dE_R} = \sum_j \underbrace{\frac{\sigma_{Nj}^{(0)}}{2m_j\mu_{Nj}^2}}_{\textbf{Particle physics}} \underbrace{F^2(E_R)}_{\textbf{Nuclear physics}} \underbrace{\rho_j^{\mathrm{loc}} \int_{v_{\mathrm{min}}^{(j)}}^{v_{\mathrm{esc}}} \frac{f_j(v)}{v}\,dv}_{\textbf{Astrophysics and cosmology}}}$$

Mass of χ_j

Reduced mass of χ_j-nucleon system

Direct Detection of DDM

In this talk, I'll adopt the following standard assumptions about the particles in the DM halo as a definition of the "**standard picture**" of DM:

- Total local DM energy density: $\rho_{\rm tot}^{\rm loc} \approx 0.3\ {\rm GeV/cm}^3$.
- Maxwellian distribution of halo velocities for all χ_j.
- Local circular velocity $v_0 \approx 220$ km/s, galactic escape velocity $v_e \approx 540$ km/s.
- Woods-Saxon form factor.
- Spin-independent (SI) scattering dominates.
- Isospin conservation: $f_{pj} = f_{nj}$.
- Local DM abundance $\propto$ global DM abundance: $\rho_j^{\rm loc}/\rho_{\rm tot}^{\rm loc} \approx \Omega_j/\Omega_{\rm tot}$.

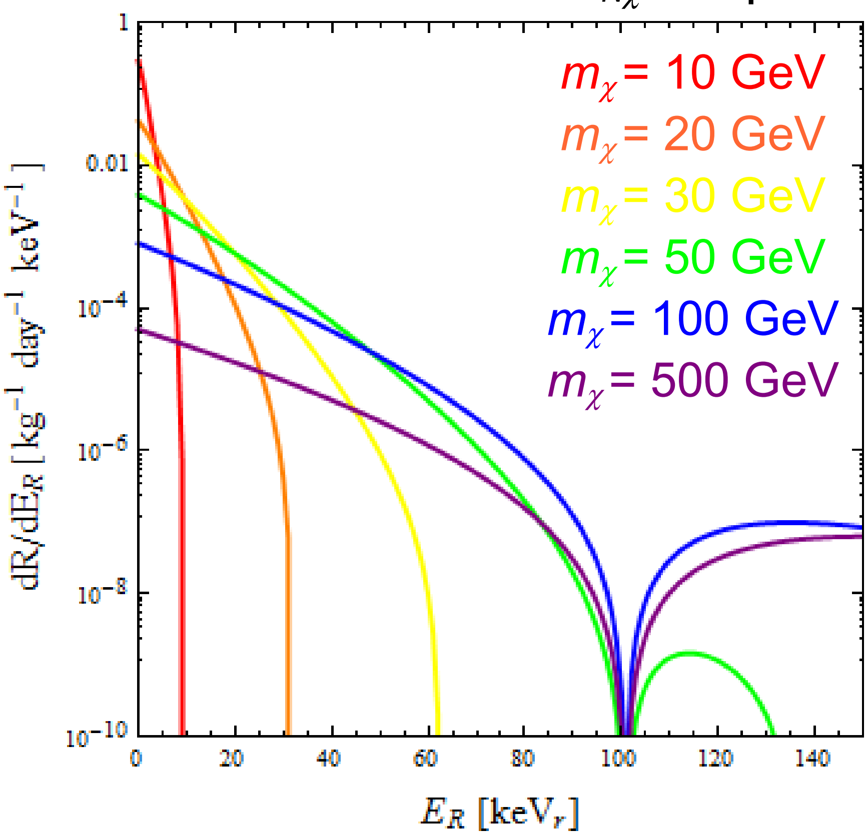
Departures from this standard picture (isospin violation, non-standard velocity distributions, etc.) can have important experimental consequences.

Here, we examine the consequences of replacing a traditional DM candidate with a DDM ensemble, with all other things held fixed.

Recoil-Energy Spectra: Traditional DM

- Let's begin by reviewing the result for the spin-independent scattering of a traditional DM candidate χ off a an atomic nucleus N with mass m_N.
- Recoil rate exponentially suppressed for $E_R \gtrsim 2m_\chi^2 m_N v_0^2/(m_\chi+m_N)^2$

Target material: Xe
Normalization: $\sigma_{N\chi}$ = 1 pb



Form-factor effect

Two Mass Regimes:

Low-mass regime: $m_\chi \lesssim$ 20 - 30 GeV

Spectrum sharply peaked at low E_R due to velocity distribution. Shape quite sensitive to m_χ.

High-mass regime: $m_\chi \gtrsim$ 20 - 30 GeV

Broad spectrum. Shape not particularly sensitive to m_χ.

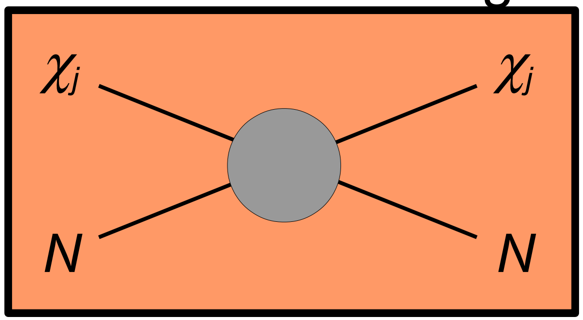
DDM Ensembles and Particle Physics

- Cross-sections depend on effective couplings between the χ_j and nuclei.
  - Both **elastic and inelastic scattering** can in principle contribute significantly to the total SI scattering rate for a DDM ensemble.
  - In this talk, I'll focus on elastic scattering: $\chi_j N \to \chi_j N$.
  - For concreteness, I'll focus on the case where the couplings between the χ_j and nucleons scale like:

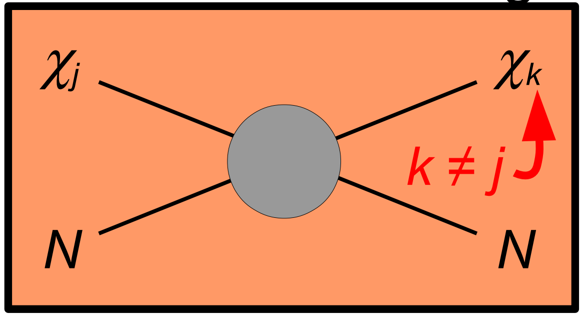
$$f_{nj} = f_{n0}\left(\frac{m_j}{m_0}\right)^{\beta} \quad \Rightarrow \quad \sigma_{nj}^{(\mathrm{SI})} = \frac{4\mu_{nj}^2}{\pi} f_{nj}^2$$

  - However, note that inelastic scattering has special significance within the DDM framework:

Elastic Scattering



Inelastic Scattering



- Possibility of **downscattering** ($m_k < m_j$) as well as upscattering ($m_k > m_j$) within a DDM ensemble.
- Scattering rates for $\chi_j N \to \chi_k N$ place lower bounds on rates for decays of the form $\chi_j \to \chi_k$ + [SM fields] and hence bounds **on the lifetimes** of the χ_j.

DDM Ensembles and Cosmology

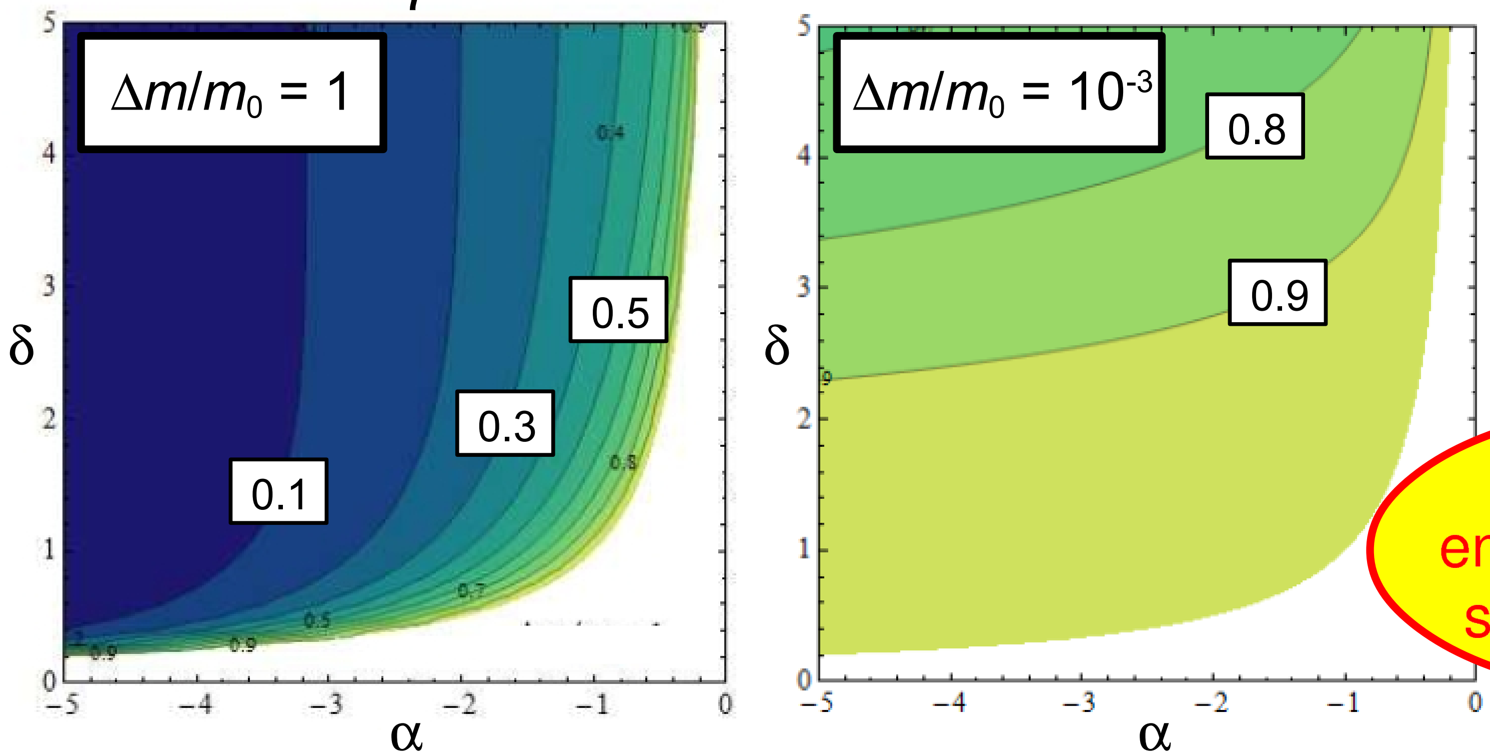
- In contrast to the collider analysis presented in the first half of this talk, direct detection involves **a cosmological population** of DM particles, and thus aspects of DDM cosmology.
- Recall that the cosmology of a given DDM ensemble is primarily characterized by two parameters: η and Ω_{tot}.
- For concreteness, consider the case where $m_j = m_0 + n^\delta \Delta m$ and the present-day abundances Ω_j scale like:

$$\Omega_{\text{tot}} = \sum_j \Omega_j$$

$$\eta = 1 - \frac{\Omega_0}{\Omega_{\text{tot}}}$$

$$\Omega_j = \Omega_0 \left(\frac{m_j}{m_0}\right)^\alpha$$

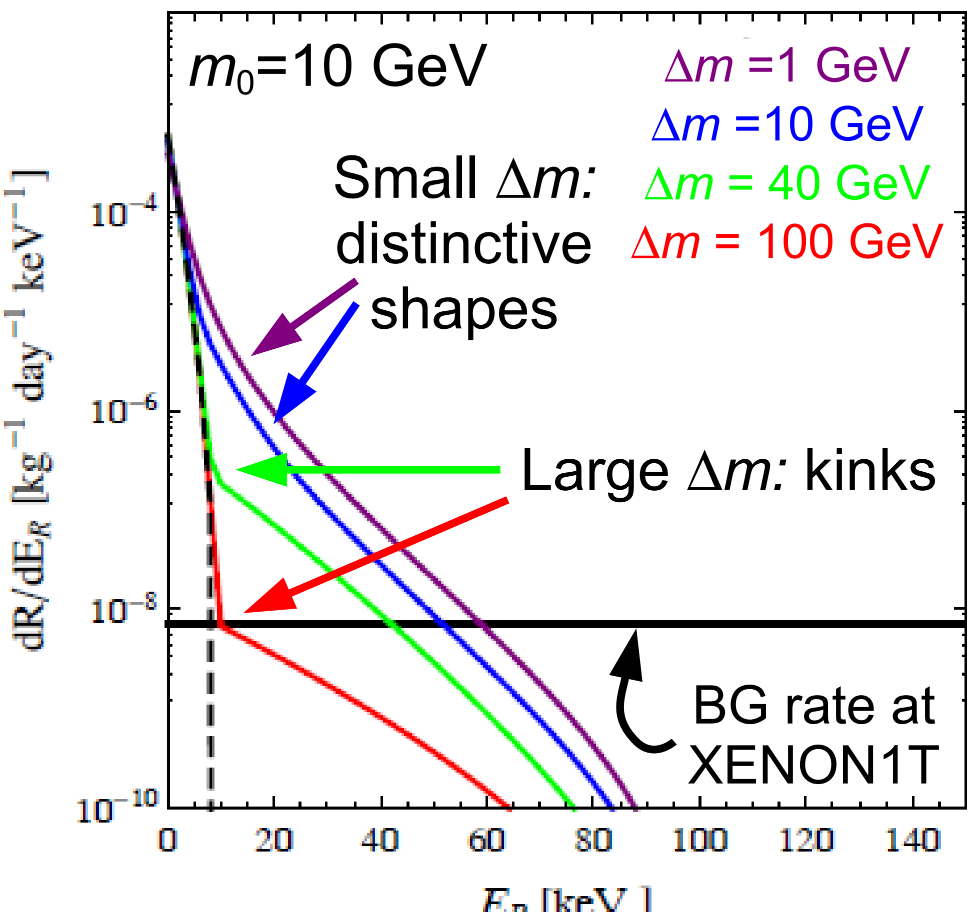
η as a function of α and δ



$\eta \sim \mathcal{O}(1)$: the full ensemble contributes significantly to Ω_{tot}.

Recoil-Energy Spectra: DDM

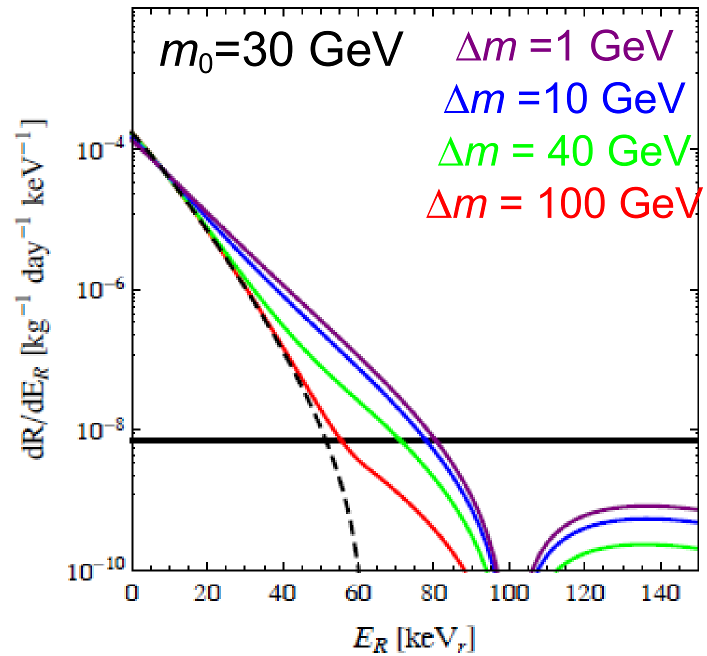
- **Distinctive features** emerge in the recoil-energy spectra of DDM models, especially when one or more of the χ_j are in the low-mass regime.
- As m_0 increases, more of the χ_j shift to the high-mass regime. Spectra increasingly resemble those of traditional DM candidates with $m_\chi \approx m_0$.

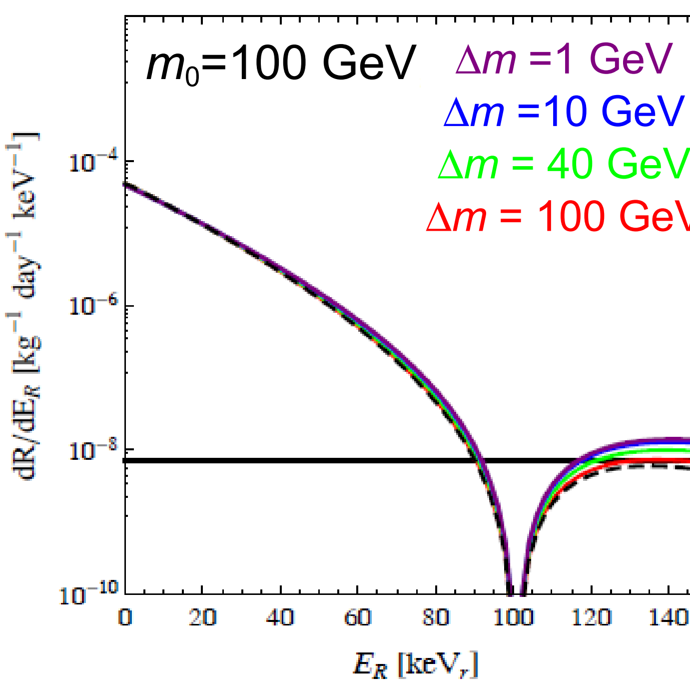


α = -1.5
β = -1
δ = 1

Xe target

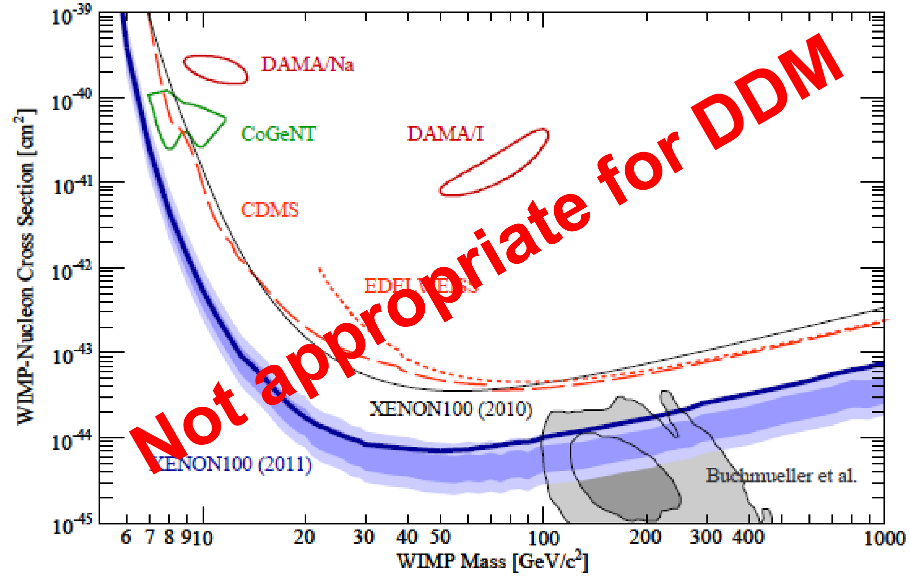
Rate normalized to that of χ with $\sigma_\chi^{(SI)}$=10^{-9} pb



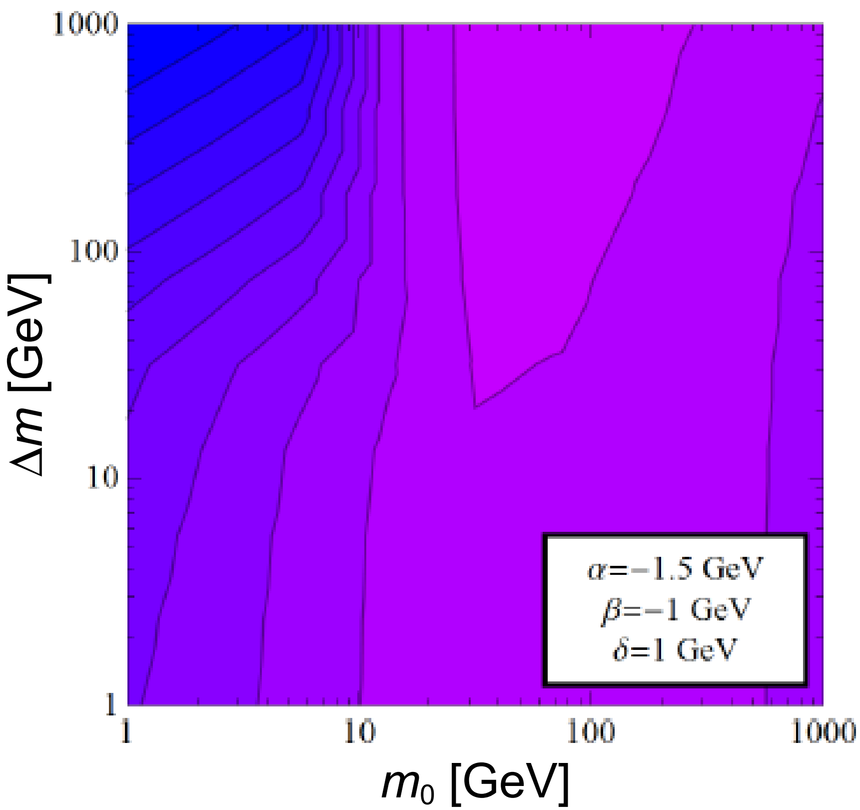


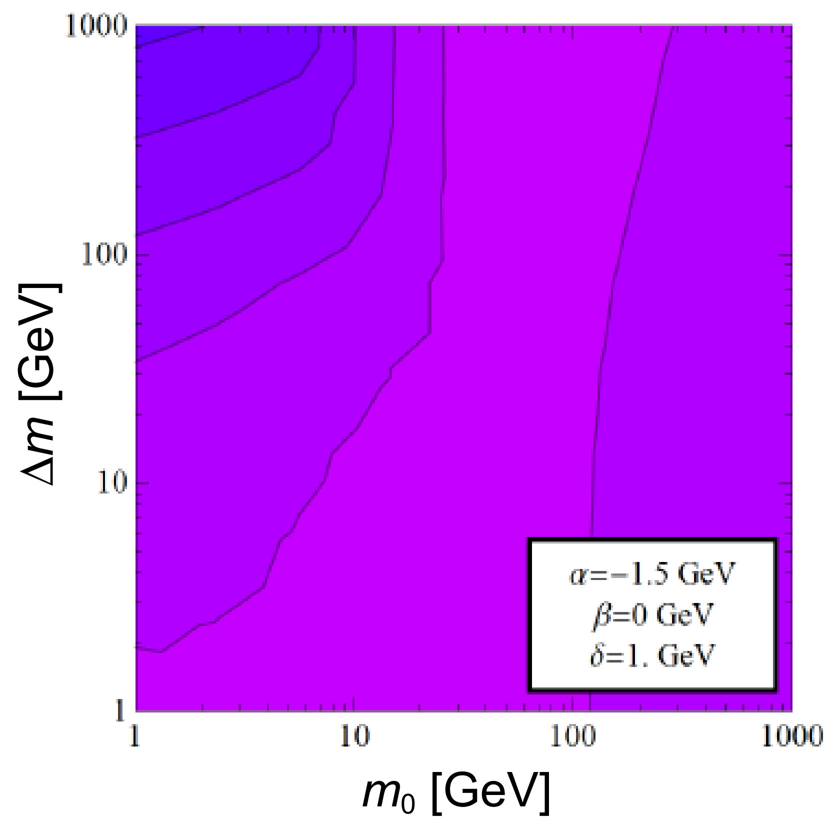
Constraining Ensembles:

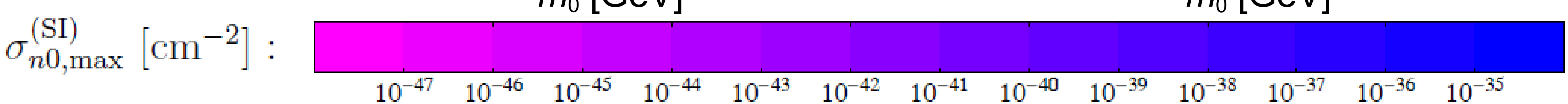
- Experimental limits constrain DDM models just as they constrain traditional DM models.
- A DDM ensemble has no well-defined mass or interaction cross-section: limits *cannot* be phrased as bounds on m_χ and $\sigma_\chi^{(SI)}$.
- Most stringent limits from XENON100 data.



Bounds on χ_0 $\sigma_{n0}^{(SI)}$ in DDM models:







How well can we distinguish a departure from the standard picture of DM due to the presence of a DDM ensemble on the basis of direct-detection data?

Consider the case in which a *particular* experiment, characterized by certain attributes including...

Target material(s)	Fiducial Volume	Signal acceptance
Detection method	Data-collection time	Recoil-energy window

…reports a statistically significant excess in the number of signal events.

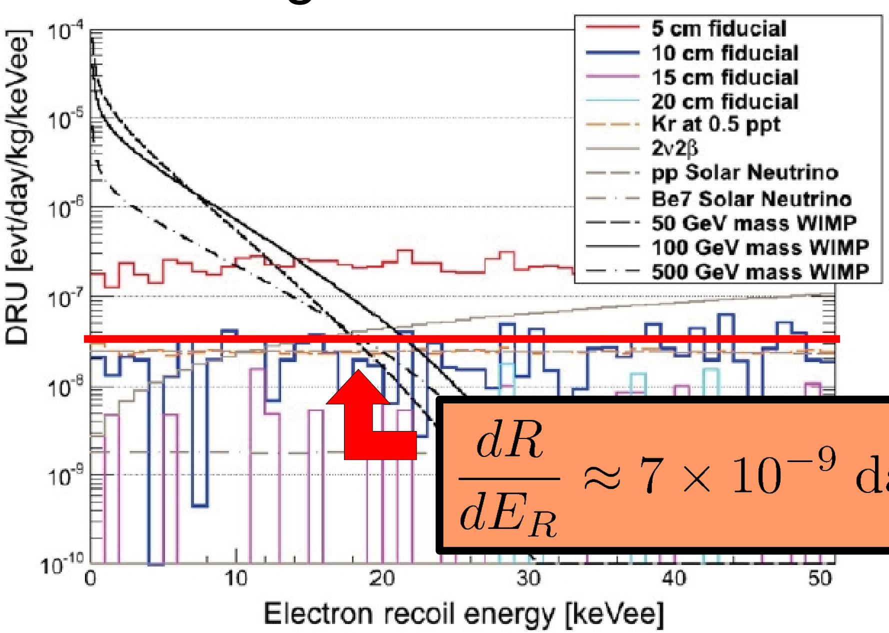
The Procedure (much like in our collider analysis):

- Compare the recoil-energy spectrum for a given DDM ensemble to those of traditonal DM candidates which yield the **same total event rate** at a given detector.
- Survey over traditional DM candidates with different m_χ and define a χ^2 statistic for each m_χ to quantify the degree to which the corrsponding recoil-energy spectrum differs from that associated with the DDM ensemble.
- The minimum $\chi^2_{\rm min}$ of these quantifies the degree to which the DDM model can be distinguished from traditional DM candidates, under standard astrophysical assumptions.

As an example, consider a detector with similar attributes to those anticipated for the next generation of noble-liquid experiments (XENON1T, LUX, PANDA-X, et al.). In particular, we take:

- Liquid-xenon target
- Fiducial volume ~ 1000 kg
- Five live years of operation.
- Energy resolution similar to XENON100
- Acceptance window: 8.4 keV < E_R < 44.6 keV

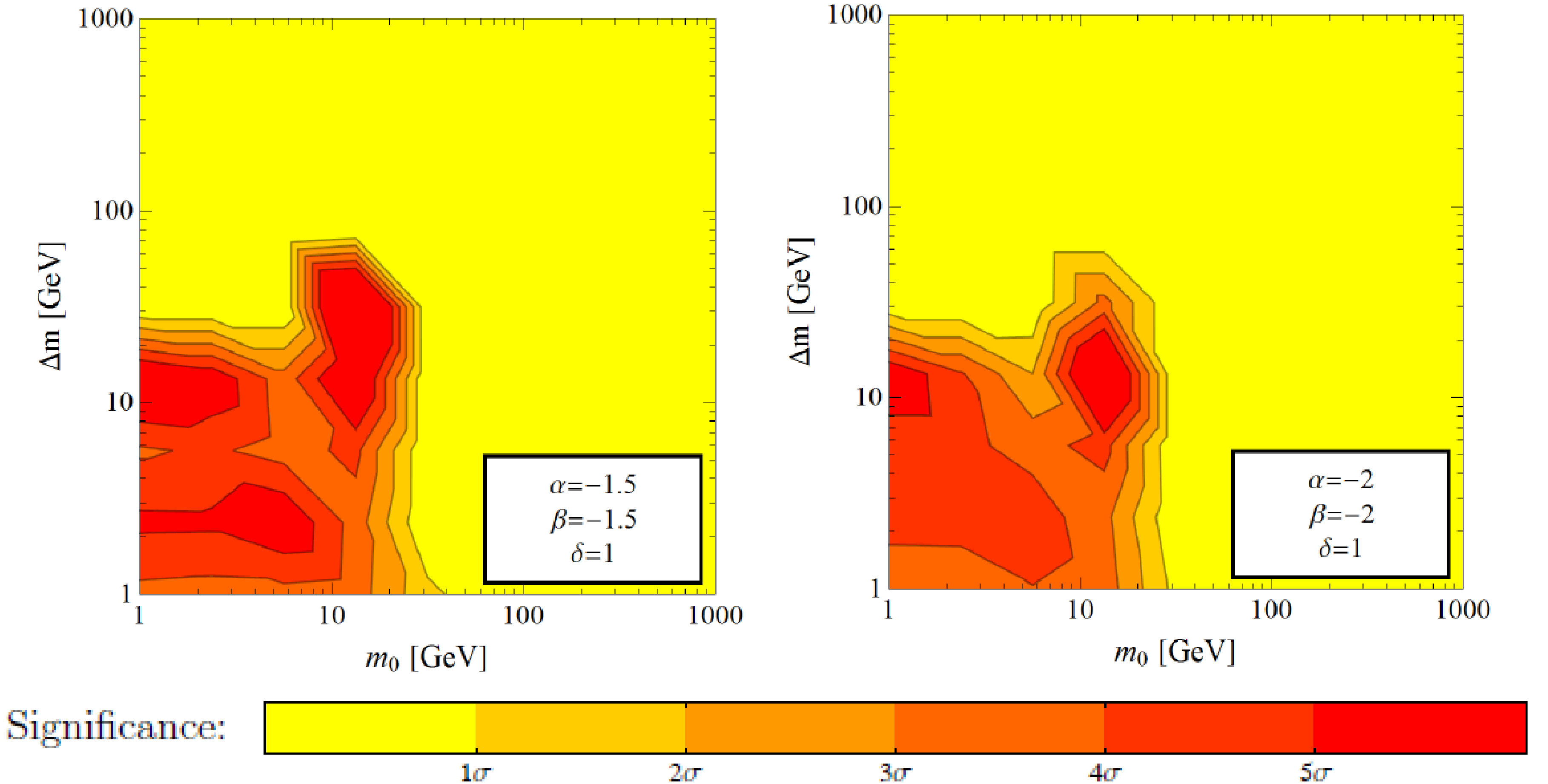
Background Contribution



$$\frac{dR}{dE_R} \approx 7 \times 10^{-9}\ \mathrm{day}^{-1}\mathrm{kg}^{-1}\mathrm{keV}_{\mathrm{nr}}^{-1}$$

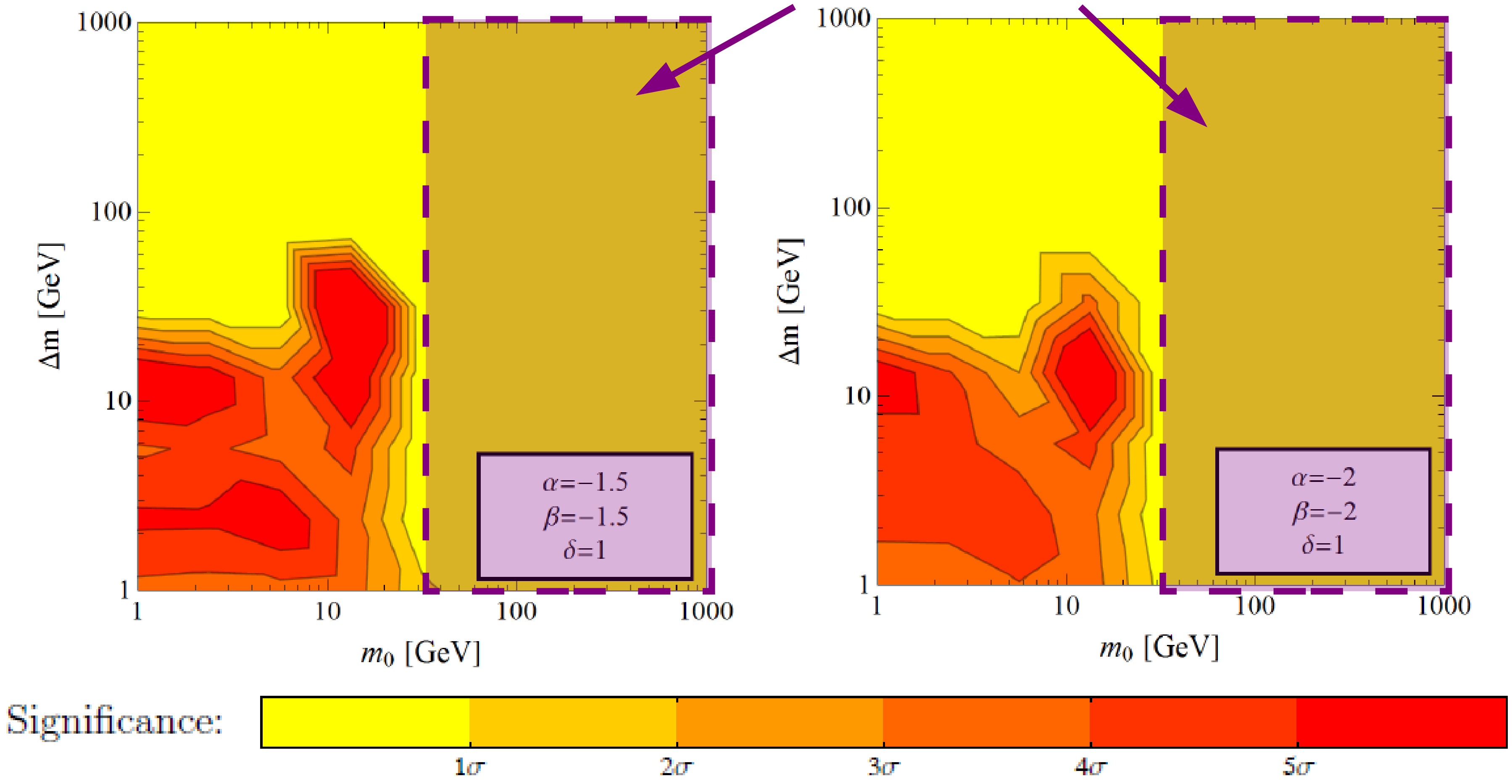
- N_e ~ 1000 total signal events observed (consistent with most stringent current limits from XENON100).
- Background dR/dE_R spectrum essentially flat

Distinguishing DDM Ensembles: Results



Distinguishing DDM Ensembles: Results

All χ_n in high-mass regime: little difference between their dR/dE_R contributions



Distinguishing DDM Ensembles: Results

χ_0 contributes mostly at $E_R < E_R^{min}$, all other χ_j in high-mass regime

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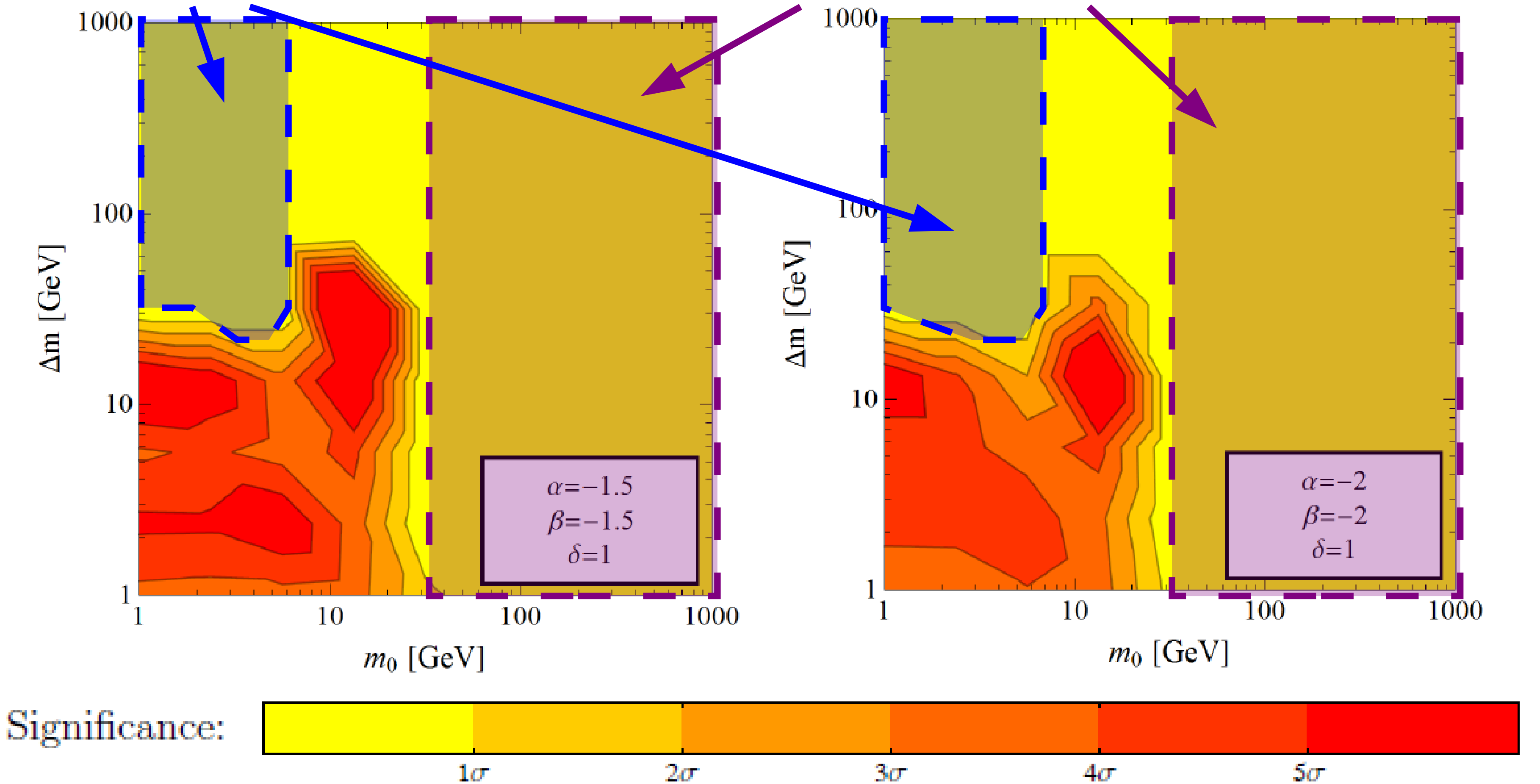

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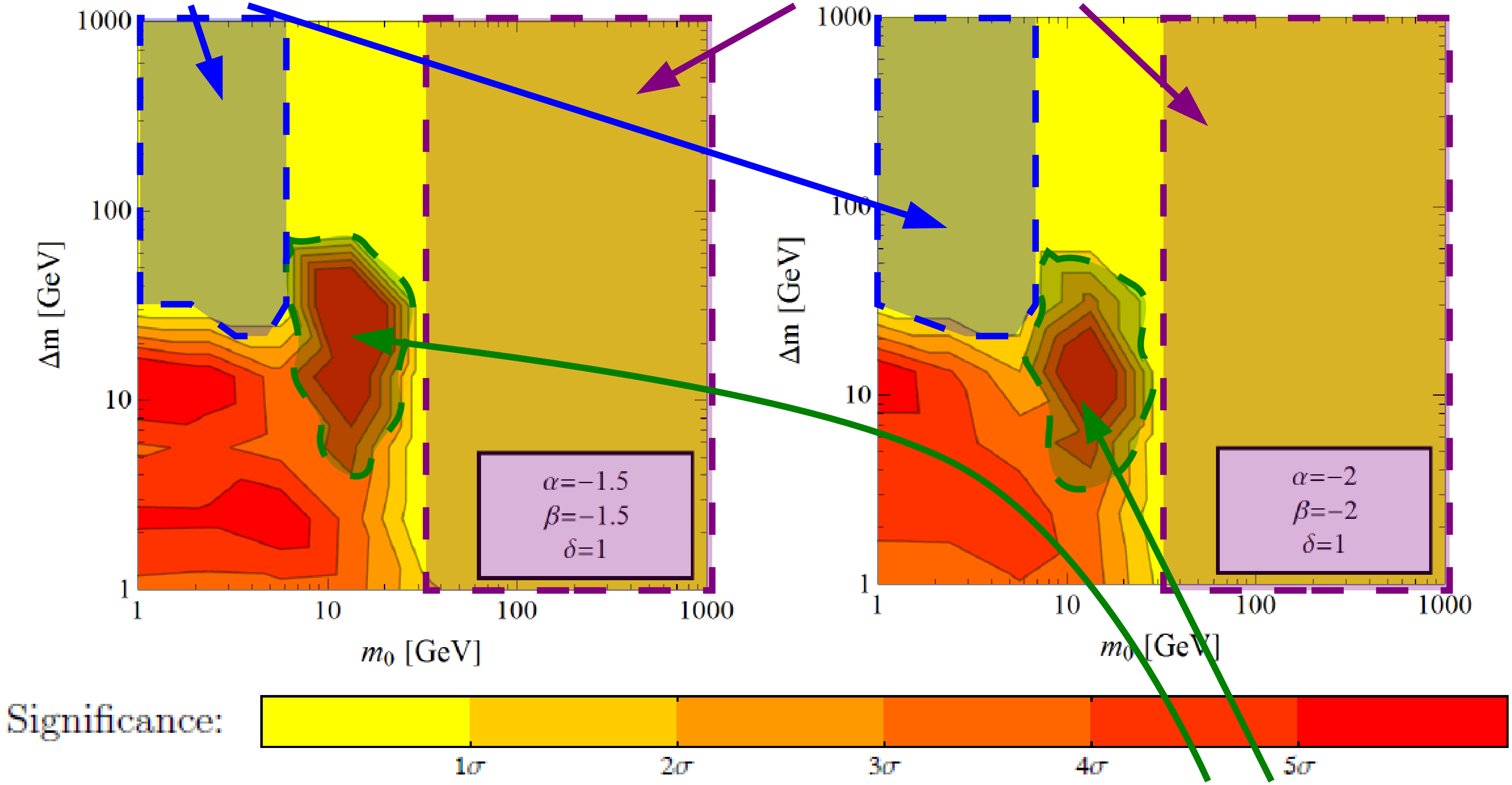

χ_0 in low-mass regime, all χ_j with $j \geq 1$ in high-mass regime: kink in dR/dE_R spectrum

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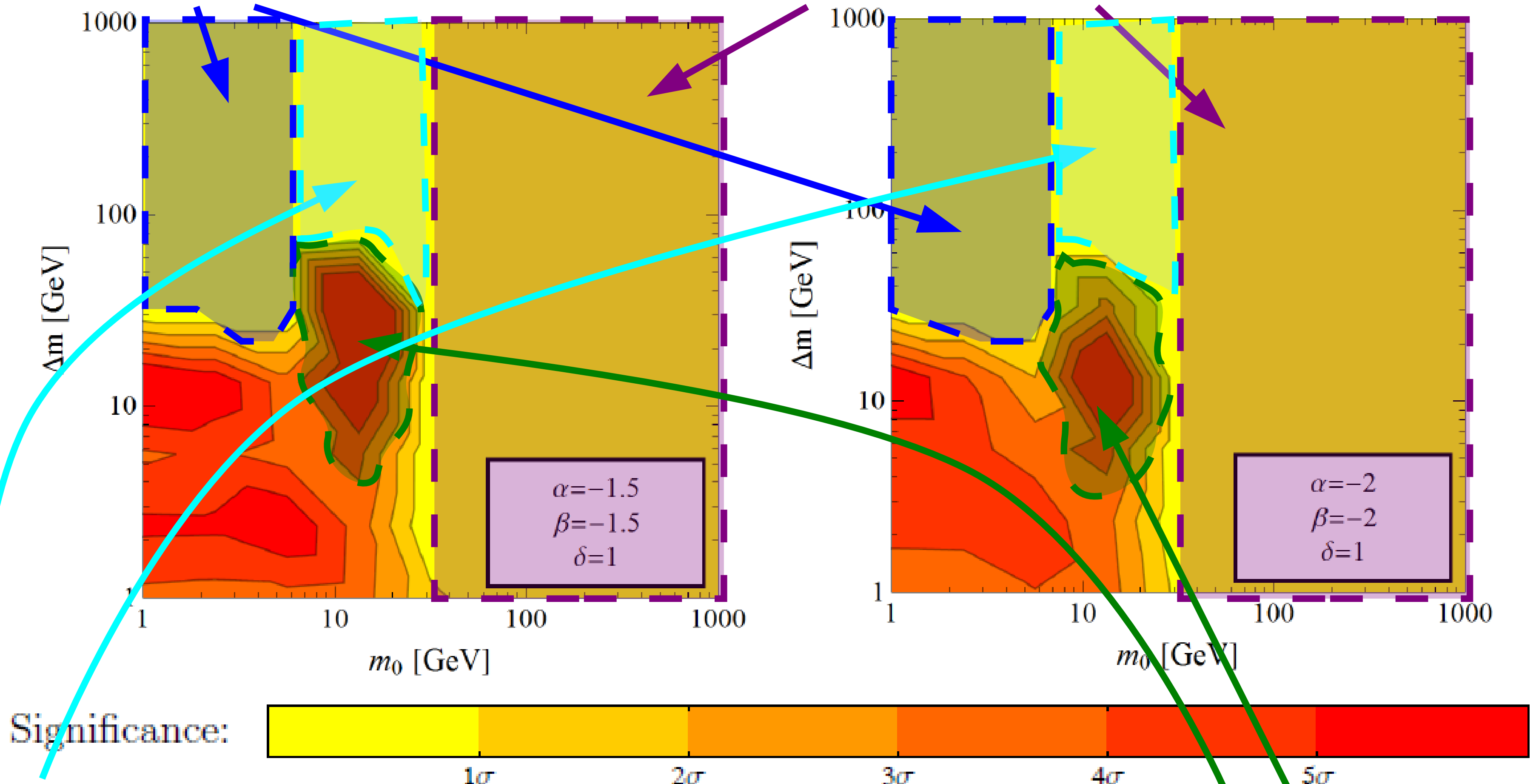

Only χ_0 contributes perceptible to overall rate: looks like regular low-mass DM

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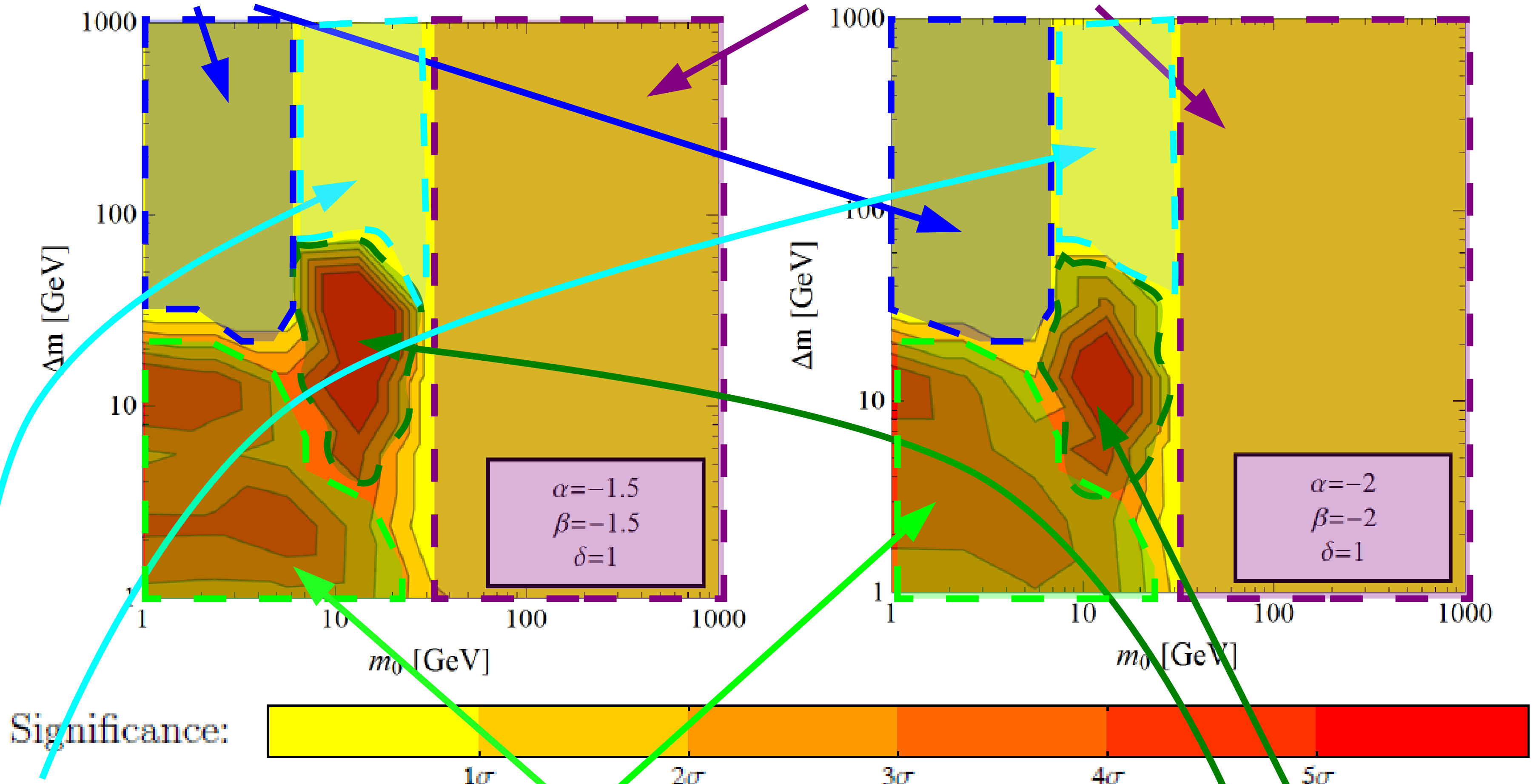

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Multiple χ_j in low-mass region: distinctive dR/dE_R spectra

χ_0 in low-mass regime, all χ_j with $j \geq 1$ in high-mass regime: kink in dR/dE_R spectrum

The upshot:

In a variety of situations, it should be possible to distinguish characteristic features to which DDM ensembles give rise at the next generation of direct-detection experiments.

- The best prospects are obtained in cases where multiple χ_j are in the low-mass regime: $m_j \lesssim 30$ GeV.
- A 5σ significance of differentiation is also possible in cases in which only χ_0 is in the low-mass regime and a kink in the spectrum can be resolved.

CAUTION Discrepancies in recoil-energy spectra from standard expectations can arise due to several other factors as well (complicated halo-velocity distribution, velocity-dependent interactions, etc.). Care should be taken in interpreting such discrepancies in the context of any particular model.

However,

By comparing/correlating signals from **multiple experiments** it should be possible to distinguish between a DDM interpretation and many of these alternative possibilities.

Summary

- Dynamical dark matter (DDM) is a new framework for addressing the dark-matter question in which stability is replaced by a **balancing between lifetimes and abundances** across a vast **ensemble** of particles χ_n which collectively account for Ω_{CDM}.
- DDM scenarios give rise to a variety of **distinctive experimental signatures** which can be used to differentiate DDM ensembles from traditional DM candidates.
- DDM ensembles can give rise to distinctive features in the **kinematic distributions** of SM fields produced in conjunction with the χ_n via the decays of other heavy particles.
- DDM ensembles can also leave imprints on the **recoil-energy spectra** observed at direct-detection experiments.

Other possibilities? Indirect detection?

Indeed, the full range of phenomenological consequences of the DDM framework is just beginning to be explored!

Extra Slides

Dark Matter: The Conventional Wisdom

In most dark-matter models, the dark sector consists of one stable dark-matter candidate χ (or a few such particles). Such a dark-matter candidate must therefore...

- account for essentially the entire dark-matter relic abundance observed by WMAP: $\Omega_\chi \approx \Omega_{CDM} \approx 0.23$.
- Respect observational limits on the decays of long lived relics (from BBN, CMB data, the diffuse XRB, etc.) which require that χ to be *extremely* stable:

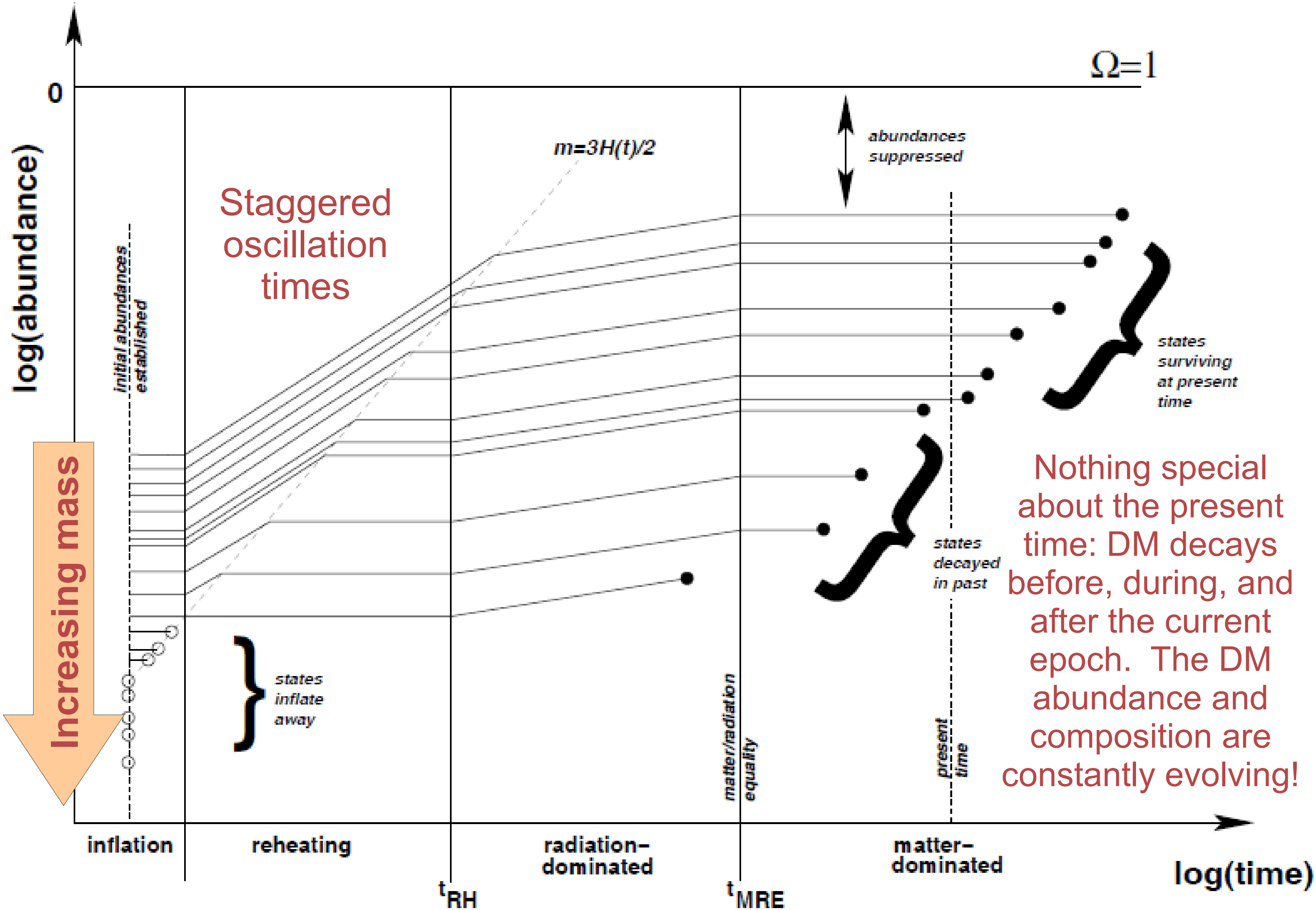
$$\boxed{\tau_\chi \gtrsim 10^{26}\ s}$$

(Age of universe: only ~10^{17} s)

Consequences

- Such "hyperstability" is the **only** way in which a single DM candidate can satisfy the competing constraints on its abundance and lifetime.
- The resulting theory is essentially "frozen in time": Ω_{CDM} changes only due to Hubble expansion, etc.

Dynamical Dark Matter: the Big Picture



Characterizing DDM Ensembles

- The cosmology of DDM models is principally described in terms of three fundamental (**time-dependent**) quantities:

1. Total relic abundance:

$$\Omega_{\rm tot}(t) = \sum_{i=0}^{N} \Omega_i(t)$$

2. Distribution of that abundance:

(One useful measure)

$$\eta(t) \equiv 1 - \frac{\Omega_0}{\Omega_{\rm tot}}$$

where

$$\Omega_0 \equiv \max\{\Omega_i\}$$

The interpretation:

$$0 \leq \eta \leq 1 \quad \begin{cases} \eta = 0 \\ \eta > 0 \end{cases}$$

$\eta = 0$ ⇒ One dominant component (standard picture)

$\eta > 0$: Quantifies depature from traditional DM

3. Effective equation of state:

$$\boxed{p = w_{\rm eff}\,\rho_{\rm tot}}$$

Not always $w = 0$!

$$w_{\rm eff}(t) = -\left(\frac{1}{3H}\frac{d\rho_{\rm tot}}{dt} + 1\right)$$

Characterizing DDM Ensembles

- Unlike traditional dark-matter candidates, a DDM ensemble has no well-defined mass, decay width, or set of scattering cross-sections.
- The natural parameters which describe such a dark-matter candidate are those which describe **the internal structure of the ensemble** itself and describe how quantities such as the constituent-particle masses, abundances, decay widths, and cross-sections scale with respect to one another across the ensemble as a whole.

For example:

$$\Omega(\Gamma) = A\,(\Gamma/\Gamma_0)^{\alpha}$$

$$n_\Gamma(\Gamma) = B(\Gamma/\Gamma_0)^{\beta}$$

Density of states per unit width Γ

The properties of the ensemble are naturally expressed in terms of the coefficients A and B and the **scaling exponents** α and β.

e.g., if we take: $\Omega_i(t) \approx \Omega_i \Theta(\tau_i - t)$

$$\sum_i \to \int n_\tau(\tau) d\tau \quad \text{with} \quad n_\tau = \Gamma^2 n_\Gamma$$

We obtain the general result: $$\frac{d\Omega_{\rm tot}(t)}{dt} \approx -\sum_i \Omega_i \delta(\tau_i - t) \approx -AB\Gamma_0^2(\Gamma_0 t)^{-\alpha-\beta-2}$$