Progress with applications of the stochastic LapH method to excited states

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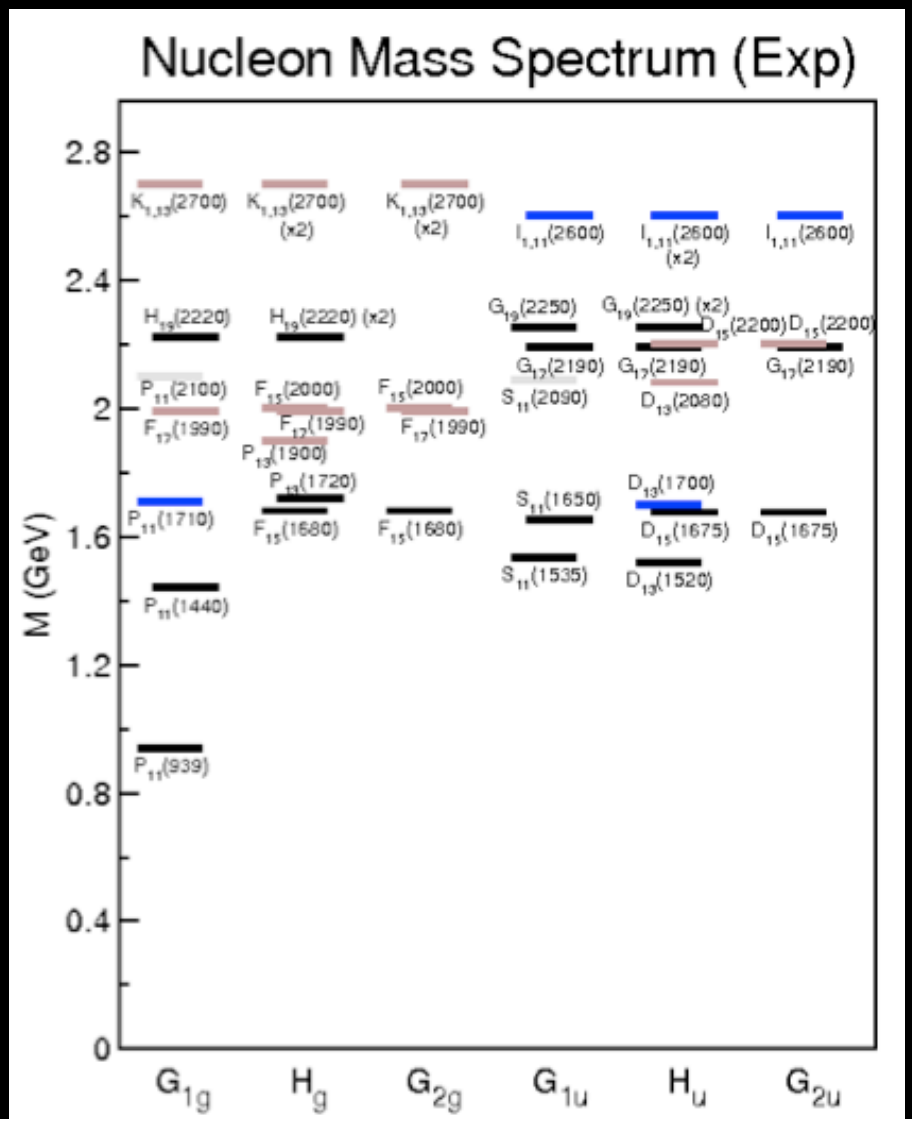
Outline

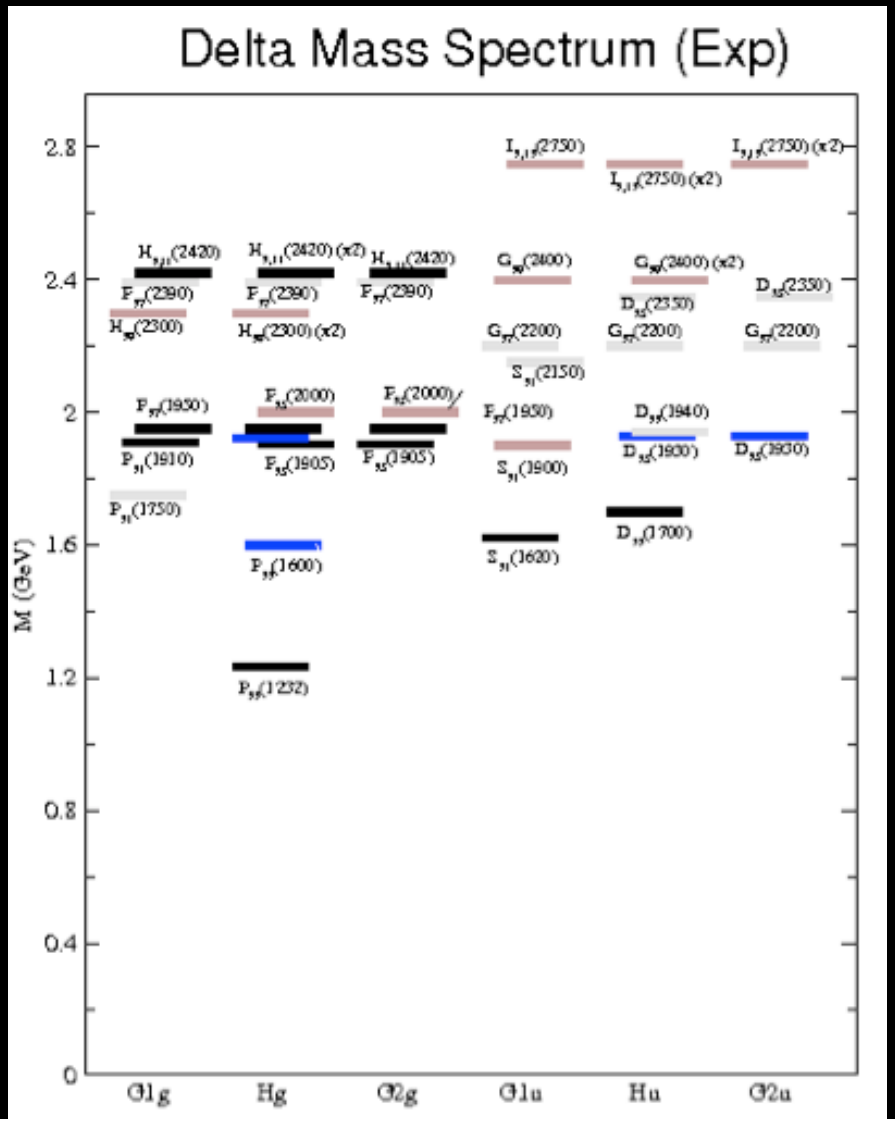
- Introduction
  goal, setup/approach
- Stochastic LapH
  - single meson correlator construction
- Two particle channel
  - Preliminary partial results for I=0,2 pi-pi
- I=1, glueball mixing, baryons
- Summary

Introduction

Goal: First principles calculation of the low-lying light hadron spectra

HadSpec Phys.Rev. D83 (2011)
PACS-CS Phys.Rev. D81 (2010)
BMW Phys.Lett. B701 (2011)
RBC,UKQCD Phys.Rev.D83 B701 (2011)
BGR Phys.Rev.D83 B701 (2011)





- We want to construct good interpolating operators that couple to the physical states in question which can be used for spectroscopy and other lattice calculations

- Require:
  - anisotropic, dynamical lattices

    Nf=2+1 anisotropic clover

    $a_s \simeq 0.12 \text{ fm} \quad a_t^{-1} \simeq 6.0 \text{ GeV} \quad \xi = 3.5$

    $m_\pi \simeq 240 \text{ MeV}, \ 390 \text{ MeV}$

    HadSpec Phys.Rev. D79, 2009
  - "large" volumes: L=1.8fm,L=2.3fm,L=2.8fm,L=3.7fm

    most recent being generated
  - good interpolating operators

    Fight noise!

    cubical symmetry, group theory (Basak etal)

    (cubical lattice in the continuum limit)

  - multi-particle channel analysis

    mixings and scattering

Anisotropic Dynamical Lattices

Edwards et al. PRD78 (2008)

$16^3 \times 128$

$m_\pi = 380$ MeV

$m_\pi L \simeq 3.8$

small and heavy but useful for pruning

$24^3 \times 128$

$m_\pi = 240$ MeV

$m_\pi L \simeq 5.7$

$m_\pi L \simeq 3.5$

$\sim$ 350 configs

$32^3 \times 256$

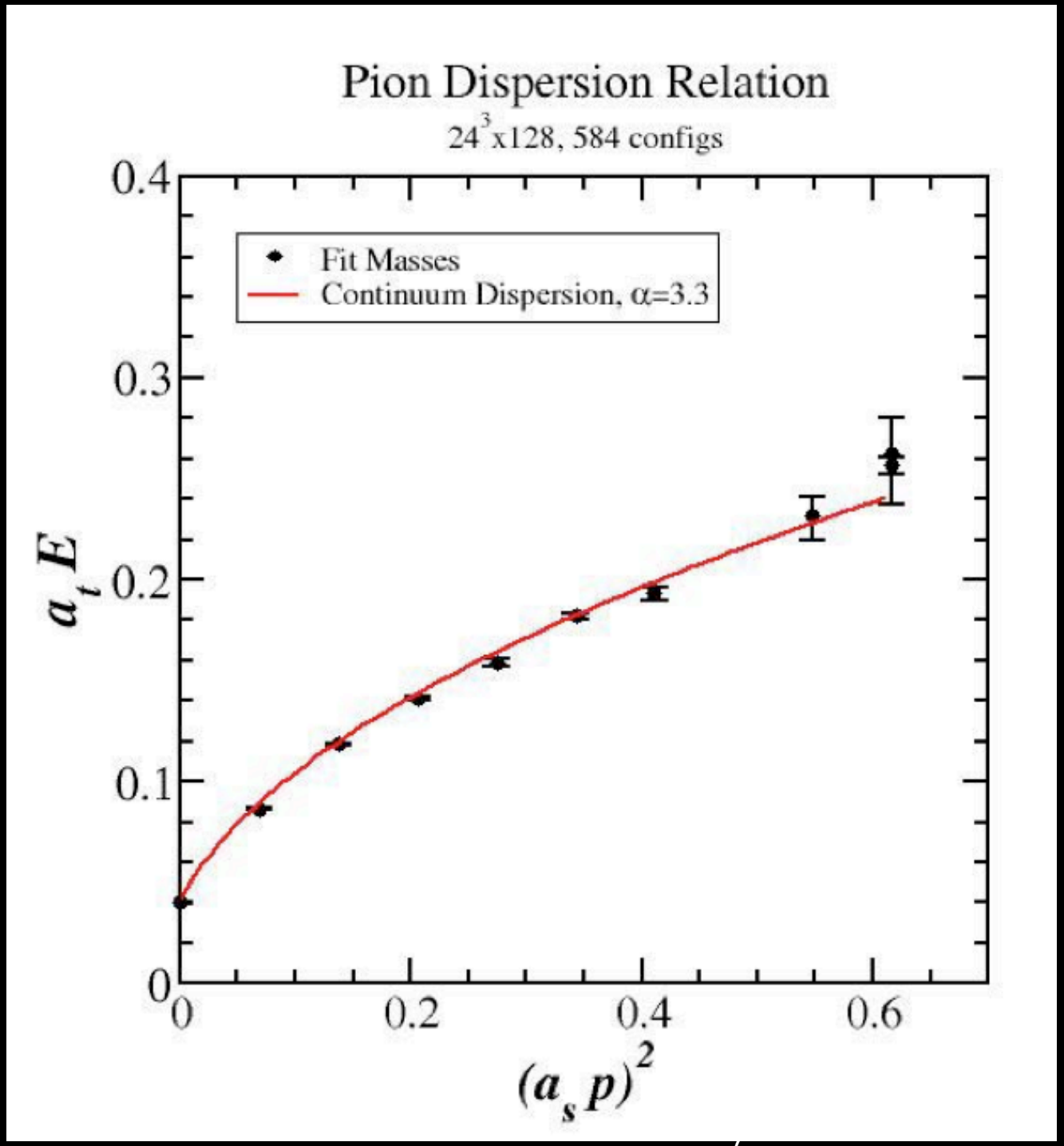
$m_\pi = 240$ MeV

$m_\pi L \simeq 4.7$

new

anisotropy = 3.459(4) Dudek etal (2011)

lightest quark mass = 3.3 (preliminary)



$$24^3 \times 128$$

$$m_\pi = 240 \text{ MeV}$$

Single hadron operators

- Smeared, covariantly-displaced "source" fields and "sink" fields
  - links: stout-smeared ... noise reduction
  - quarks: LapH smeared ... better overlap
- Separation of "source" and "sink" operators via stochastic LapH (all-to-all) quark propagators (PRD83,2011)
- Color-singlet, elemental operators which transform as one of the irreps of interest
- Group theoretical projection of the elemental operators (Basak etal PRD72, 2005)

$$H^{\Lambda\lambda F}(t) = \frac{d_\Lambda}{g_G} \sum_{R \in G} D^{(\Lambda)}_{\lambda\lambda}(R^*) U_R H^F_i(t) U^\dagger_R$$

Stochastic LapH Propagators

- LapH smearing (Distillation PRD80,2009)
  - smear quarks and propagate the low modes

$$\tilde{\psi}(x,t) = S(x,y;t)\psi(y,t)$$

$$= \mathbf{V}_s \mathbf{V}_s{}^\dagger \psi(y,t)$$

3D Laplacian

$\mathbf{V}_s$ matrix of the eigenvectors of ∇^2

control amount of smearing (overlap) by varying number of eigenvectors that are retained

  - 3-D eigenvector calculations vs 4-D

- Stochastic LapH: combine LapH with stochastic method
  - introduce only in the complementary space
  - use dilution projectors to partition the noise `wisely'

Stochastic LapH

Quark Propagator on a given config

Diluted Z4 noise source (in LapH subspace) → time, spin / LapH eigenvector # (Nev=112)

Foley et al. Com.Phy.Com.172 (2005)

dilution projectors $\mathcal{P}^{(i)}$
- full (F)
- interlace (I#) "skip"
- block (B#)

example: $\mathcal{P}^{(t=3)}$ picks out t=3 timeslice

full noise source

$$\varrho(x,t) = \sum_{i=1}^{N_d} \mathcal{P}^{(i)} \varrho(x,t)$$

$$= \sum_{i=1}^{N_d} \varrho^{(i)}(x,t)$$

Source: Smear and Displace

$$\tilde{\varrho}_k^{(d)} = \tilde{D}_k \mathbf{V_s} \varrho^{(d)}$$

Sink: (solution) Solve for solution, project on eigenvector and smear

$$\tilde{\varphi}_j^{(d)} = \tilde{D}_j \mathbf{V_S}\ \mathbf{V_S}^\dagger M^{-1} \tilde{\varrho}_k^{(d)}$$

Quark Propagator

$$Q(x,t;x_0,t_0) = \sum_{d=1}^{N_d} \tilde{\varphi}_j^{(d)}(x,t) \tilde{\varrho}_k^{(d)\dagger}(x_0,t_0)$$

$$\overline{Q}_{jk} = \sum_a \gamma_5 \gamma_4 \tilde{\varrho}_k^{(a)}(t_0) \tilde{\varphi}_j^{(a)\dagger}(t) \gamma_4 \gamma_5$$

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Elemental baryon (sink) operator

$$\mathcal{B}_\iota^F(t) = \sum_x e^{-ipx} \phi^F_{ABC} \epsilon_{abc} \left(\tilde{D}_i^{(d)} \tilde{\varphi}(x,t)_{Aa\alpha}\right) \left(\tilde{D}_j^{(d)} \tilde{\varphi}(x,t)_{Bb\beta}\right) \left(\tilde{D}_k^{(d)} \tilde{\varphi}(x,t)_{Cc\gamma}\right)$$

$\tilde{\varphi}$ solution to $\mathbf{M}\tilde{\varphi}(x,t) = \tilde{\varrho}(x_0, t_0)$

Elemental meson (sink) operator

$$\mathcal{M}_\iota^F(t) = \sum_x e^{-ip\cdot x} \phi^F_{AB} \delta_{ab} \left(\tilde{D}_i^{(d)} \tilde{\varrho}(x,t)\right)^\dagger_{Aa\alpha} \left(\tilde{D}_j^{(d)} \tilde{\varphi}(x,t)\right)_{Bb\beta}$$

'tilde' operators are stout-link smeared

Morningstar, Peardon PRD69 (2004)

Correlate the creation and annihilation operators

$$\langle \mathcal{B}_\iota(t) \mathcal{B}^{src}_{\iota'}(t_0) \rangle \qquad \langle \mathcal{M}_\iota(t) \mathcal{M}^{src}_{\iota'}(t_0) \rangle$$

to form the correlation matrix

Diagonalize (solve generalized eigenvalue problem)

$$\mathbf{C}(t_0)^{-1/2}\mathbf{C}(t)\mathbf{C}(t_0)^{-1/2}v = \lambda(t,t_0)v$$

The system determines the optimal linear combination of the operators for best overlap

We typically fix t_0 and choose t^* (and t_0) large where noise is still manageable
(Alpha Collaboration 2009)

Typical optimization range
$t0 = 3 \quad t^*=8$
but varies for each simulation

Lattice Technology Summary

- anisotropic, 2+1 dynamical lattices (HadSpec Coll)
  - anisotropy 3.5
    - cutoff 6 GeV but still have 3 fm lattices
  - pion masses (240 MeV, 400 MeV)
- stout link smearing (Morningstar,Peardon)
  - link noise reduction
- stochastic LapH (all-to-all) quark propagators
  - disconnected diagrams, finite momentum operators
  - source and sink separation (in correlation functions)

Lattice Technology Summary

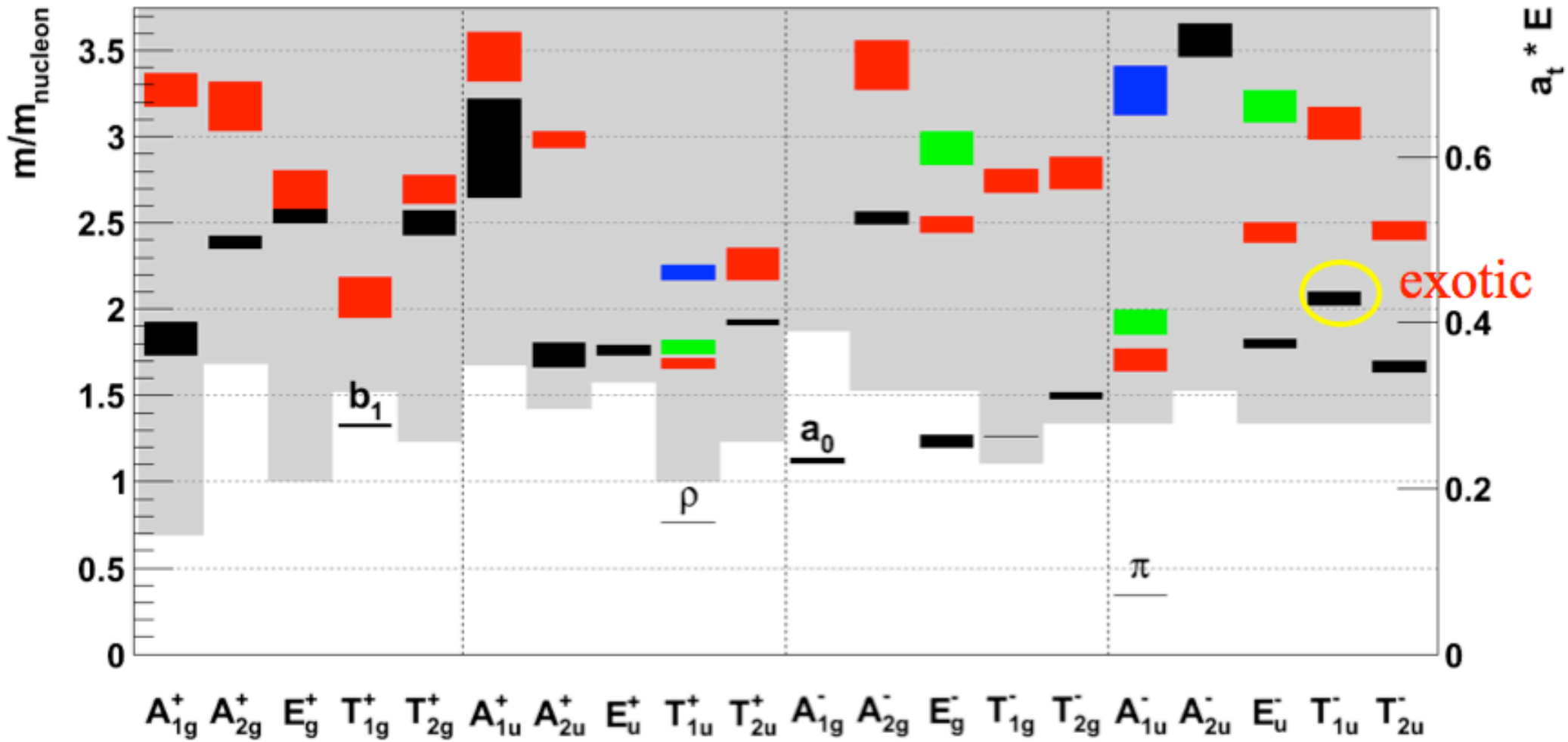
- group theory projections (cubic group)
  - cubical lattice even in the continuum limit
  - optimization of operators via generalized eigenvalue method for use in other calculations

Mesons

Isovector spectrum presented at Lattice '11

$24^3 \times 128$ m_π =380 MeV 170 cfgs

shaded region needs multi-particle analysis



Multi-particle analysis requires meson operators with finite momentum

Injecting momentum is simple with all-to-all propagators

recall

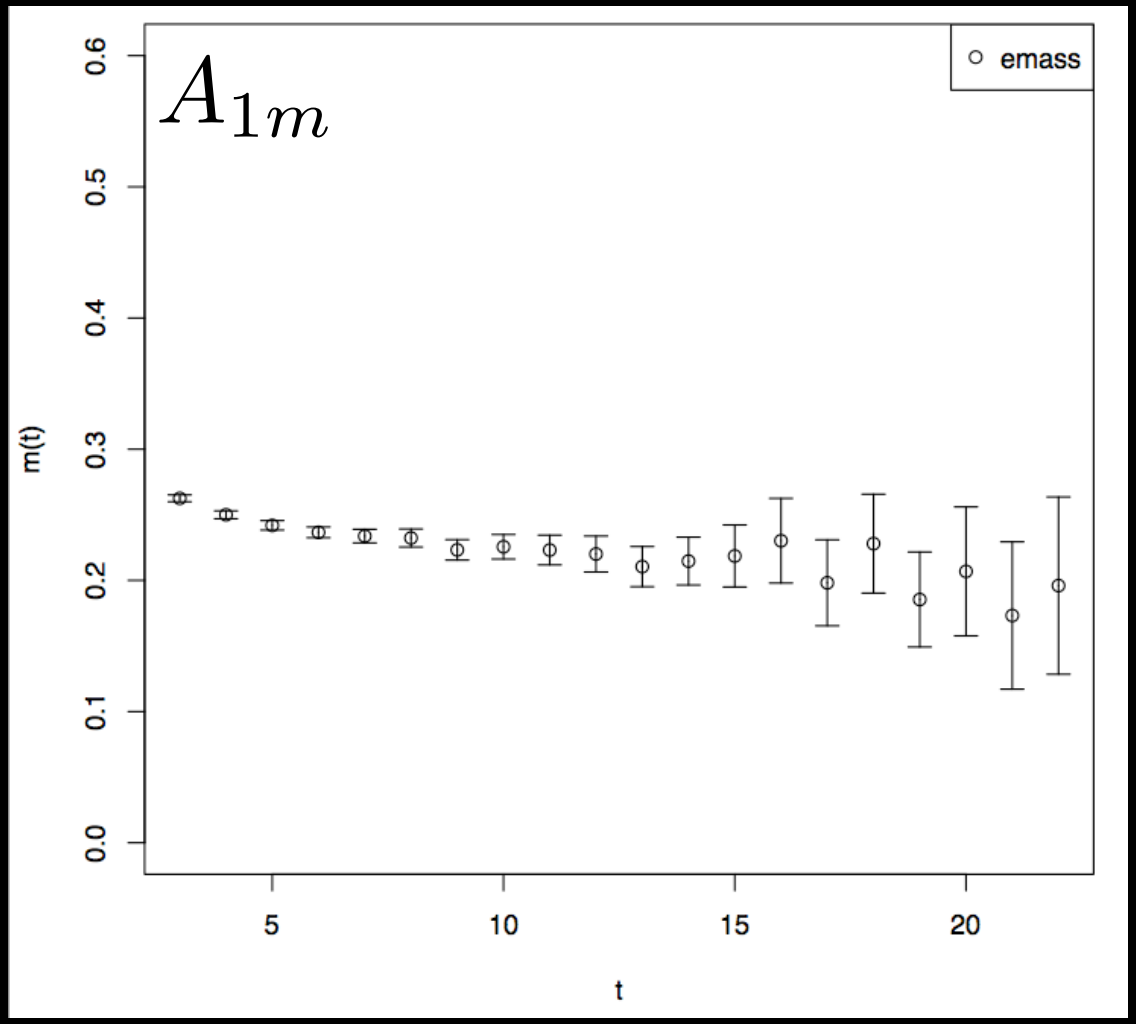
$$\mathcal{B}_\iota^F(t) = \sum_x e^{-ipx} \phi^F_{ABC} \epsilon_{abc} \left(\tilde{D}_i^{(d)} \tilde{\varphi}(x,t)_{Aa\alpha}\right) \left(\tilde{D}_j^{(d)} \tilde{\varphi}(x,t)_{Bb\beta}\right) \left(\tilde{D}_k^{(d)} \tilde{\varphi}(x,t)_{Cc\gamma}\right)$$

$$\mathcal{M}_\iota^F(t) = \sum_x e^{-ip\cdot x} \phi^F_{AB} \delta_{ab} \left(\tilde{D}_i^{(d)} \tilde{\varrho}(x,t)\right)^\dagger_{Aa\alpha} \left(\tilde{D}_j^{(d)} \tilde{\varphi}(x,t)\right)_{Bb\beta}$$

Effective masses

$$m_{\text{eff}} = -\ln \frac{C(t+1)}{C(t)} \to E$$

Example of an isovector with momentum (0,1,1)

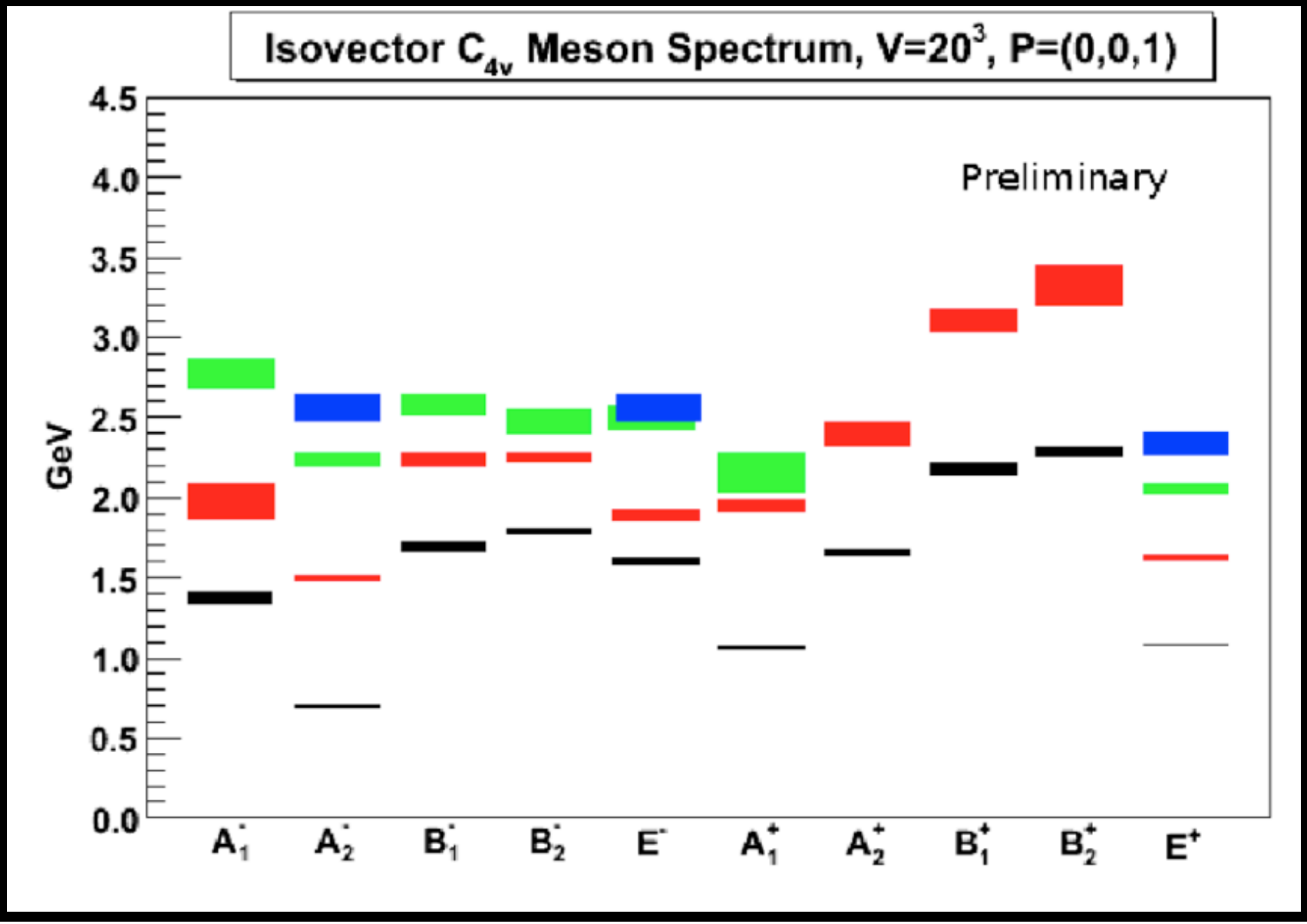


$24^3 \times 128$

$m_\pi = 240$ MeV

Moving Mesons

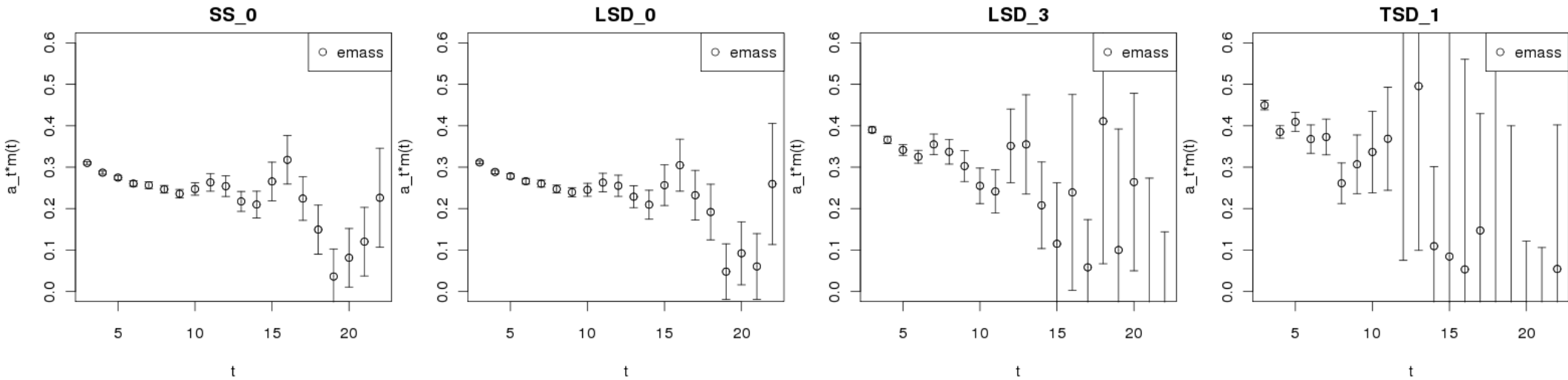
$$20^3 \times 128 \; m_\pi = 380 \text{ MeV} \; p = (0,0,1)$$



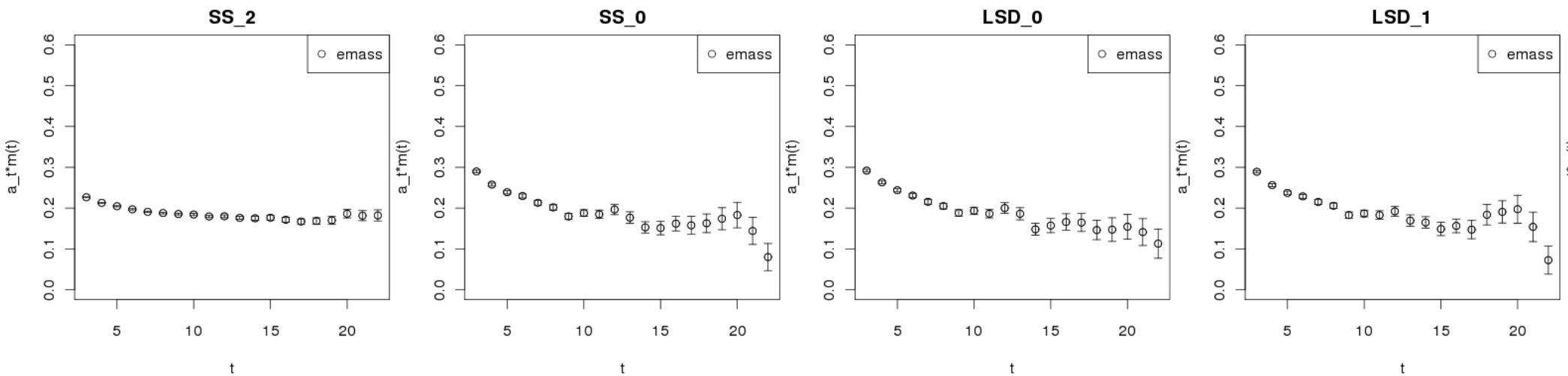
$24^3 \times 128$

$m_\pi = 240$ MeV

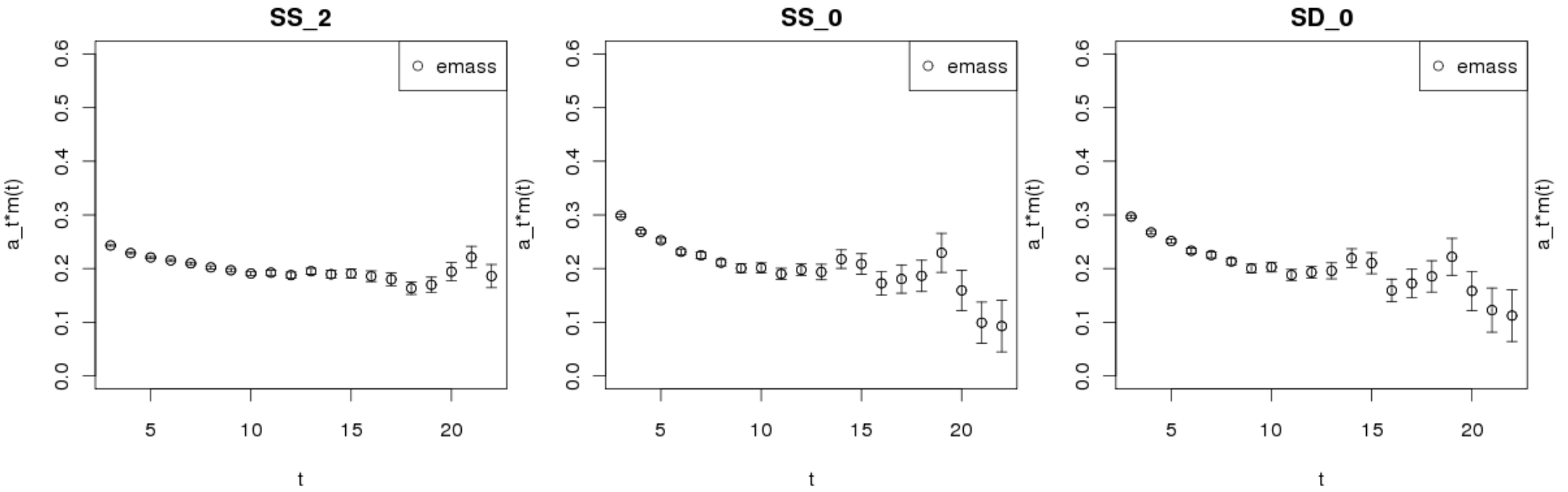
A1m <0,0,2> ← momentum



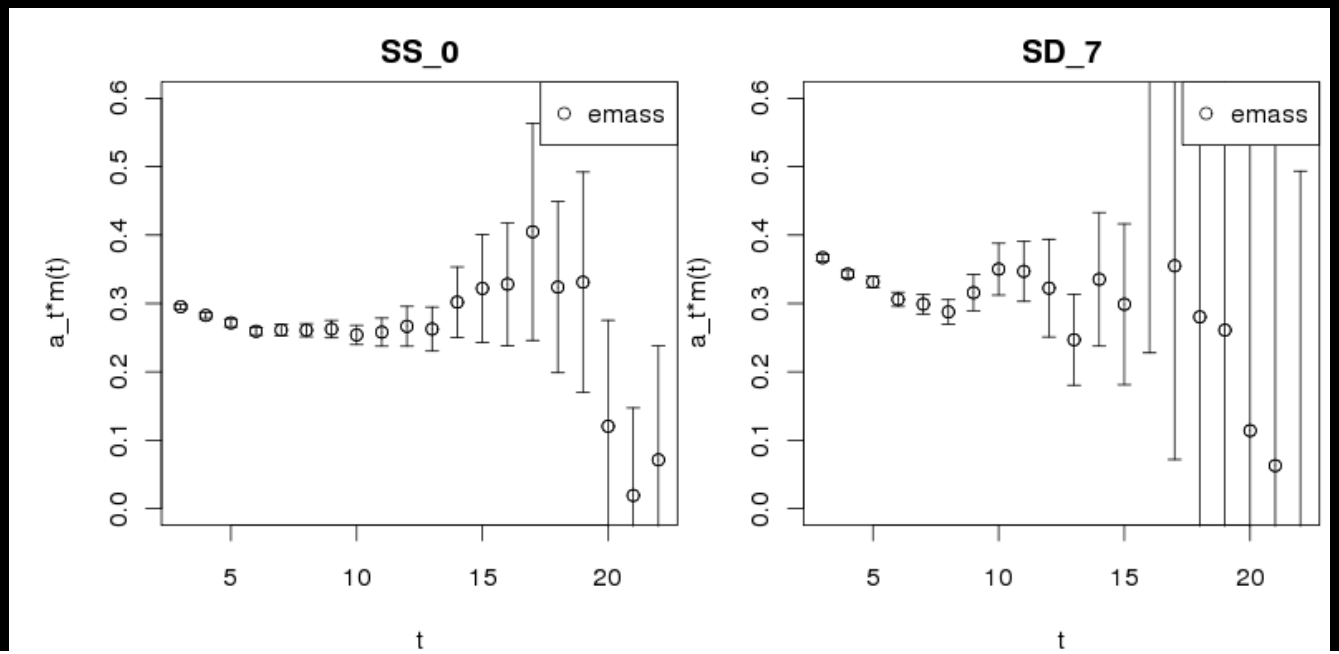
A1p <0,1,1>



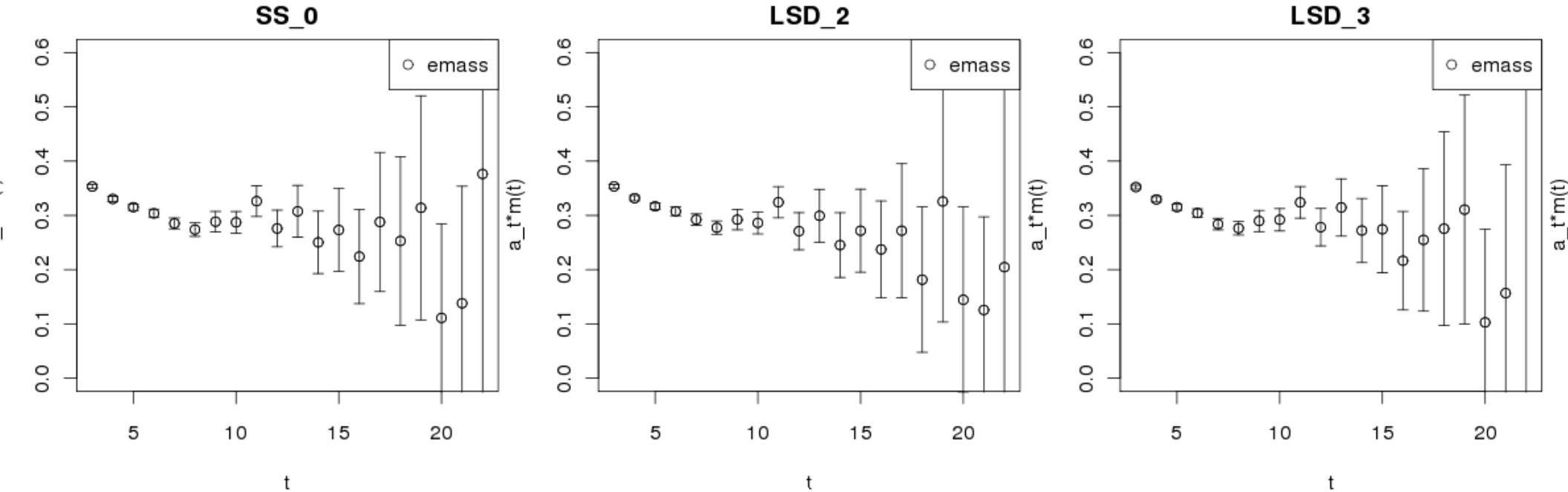
A1p <1,1,1>



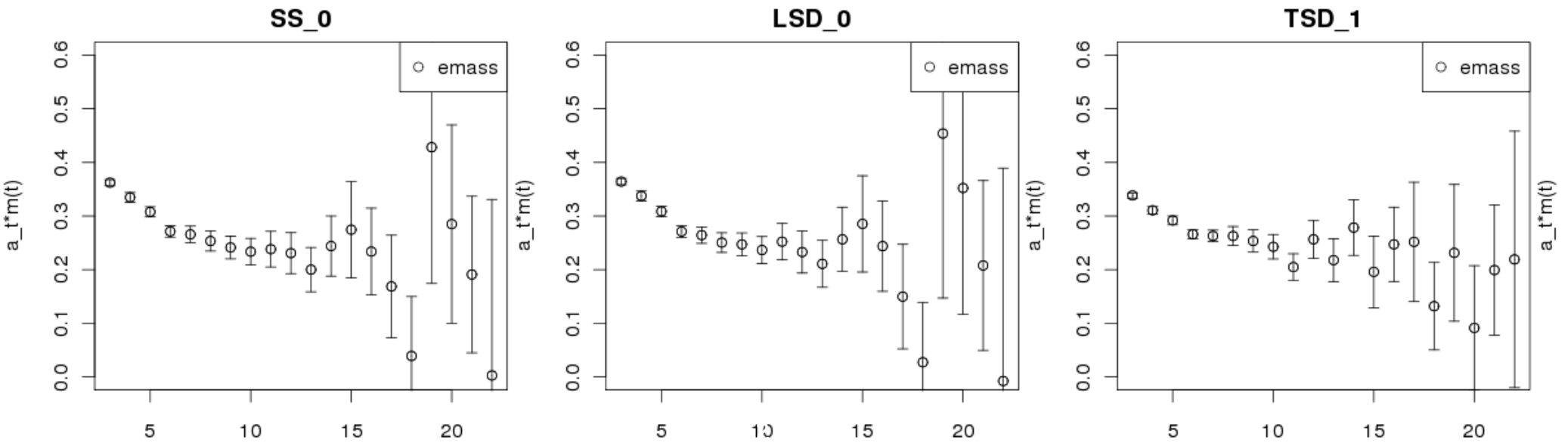
A1m <1,1,1>



B2p <1,1,1>



A2m <0,2,2> and more …

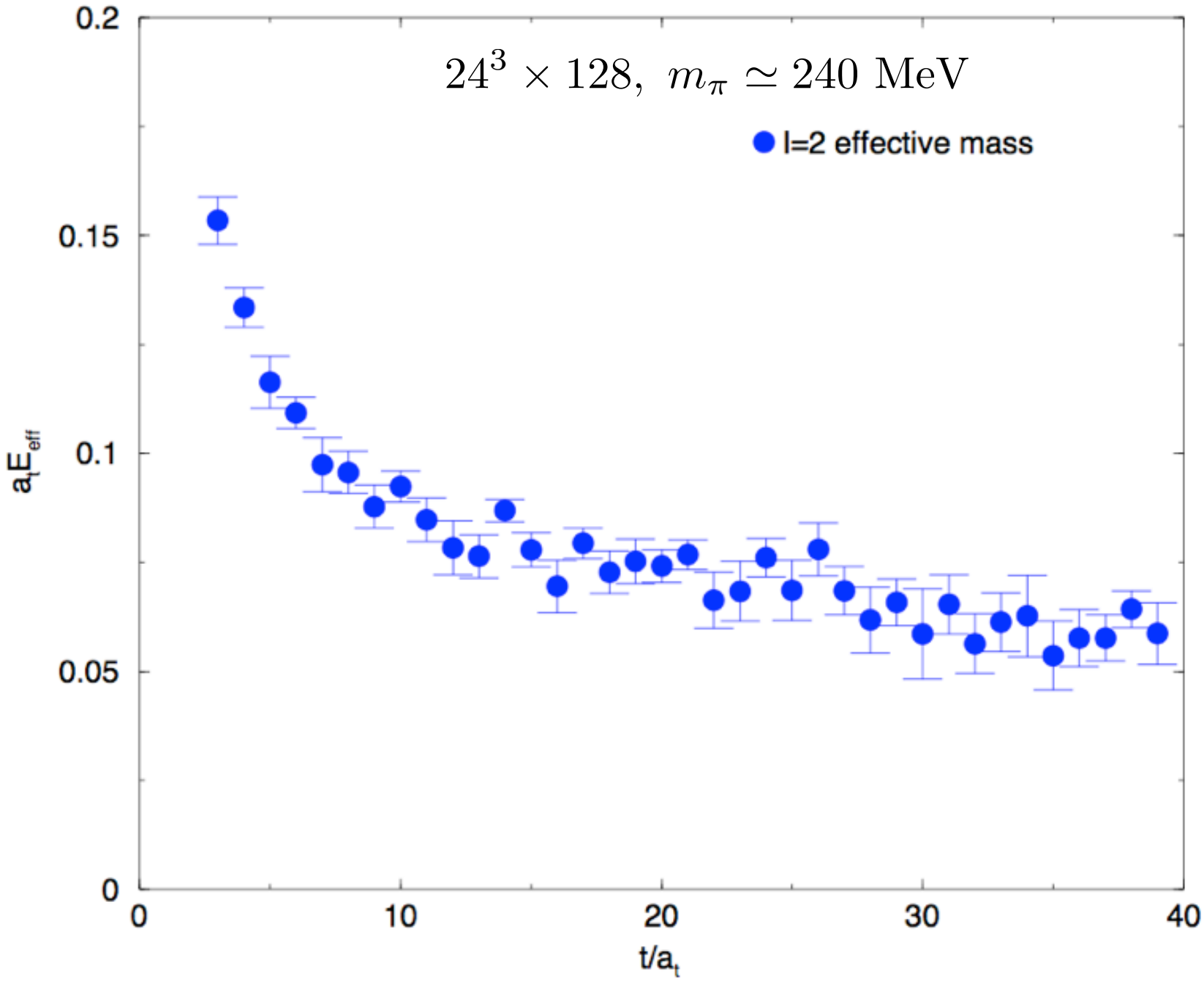


Multi-particle operators constructed from simple elemental operators with finite momentum

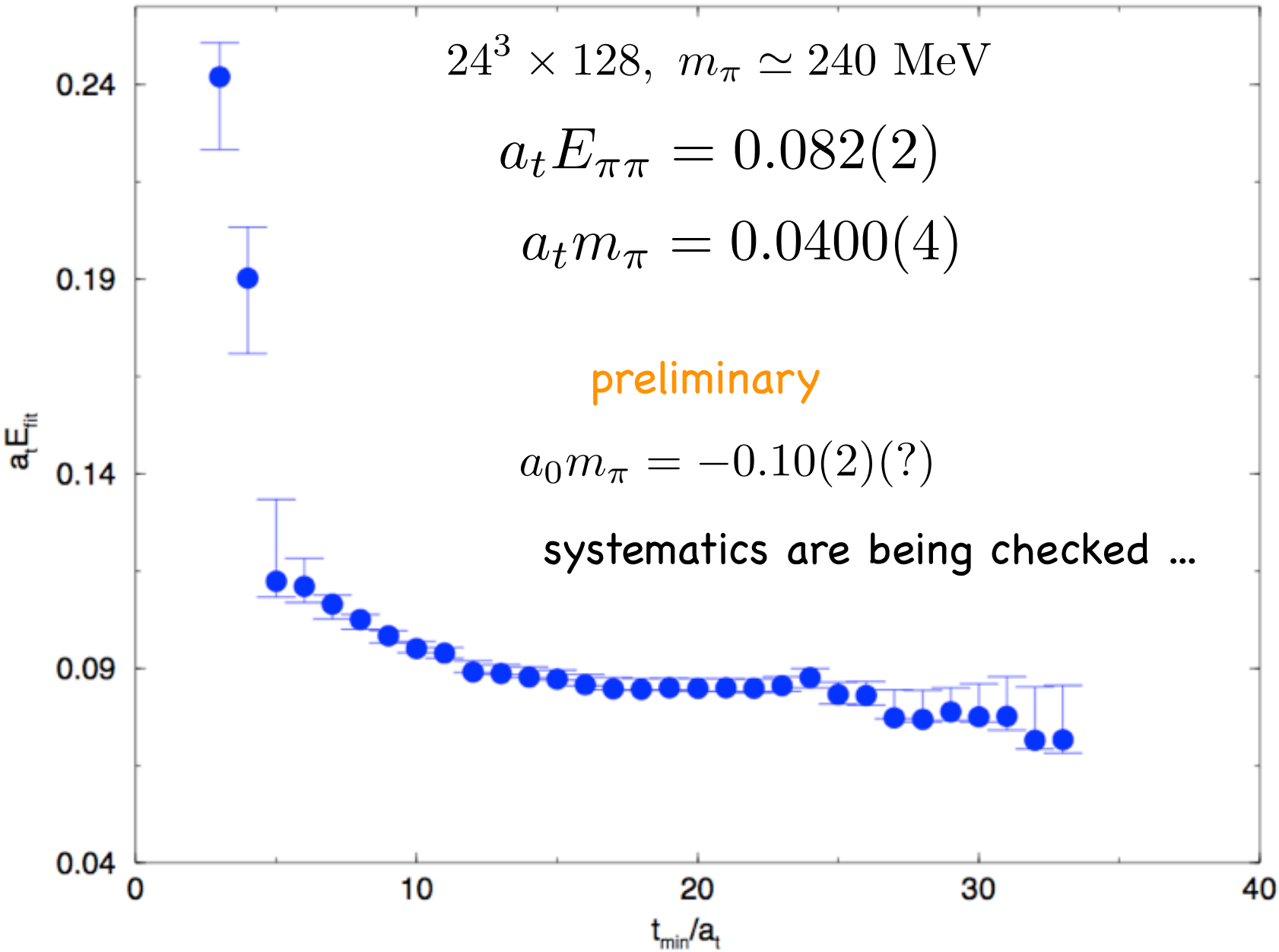
"Simplest" two-particle state

$\longrightarrow$ two-pion state

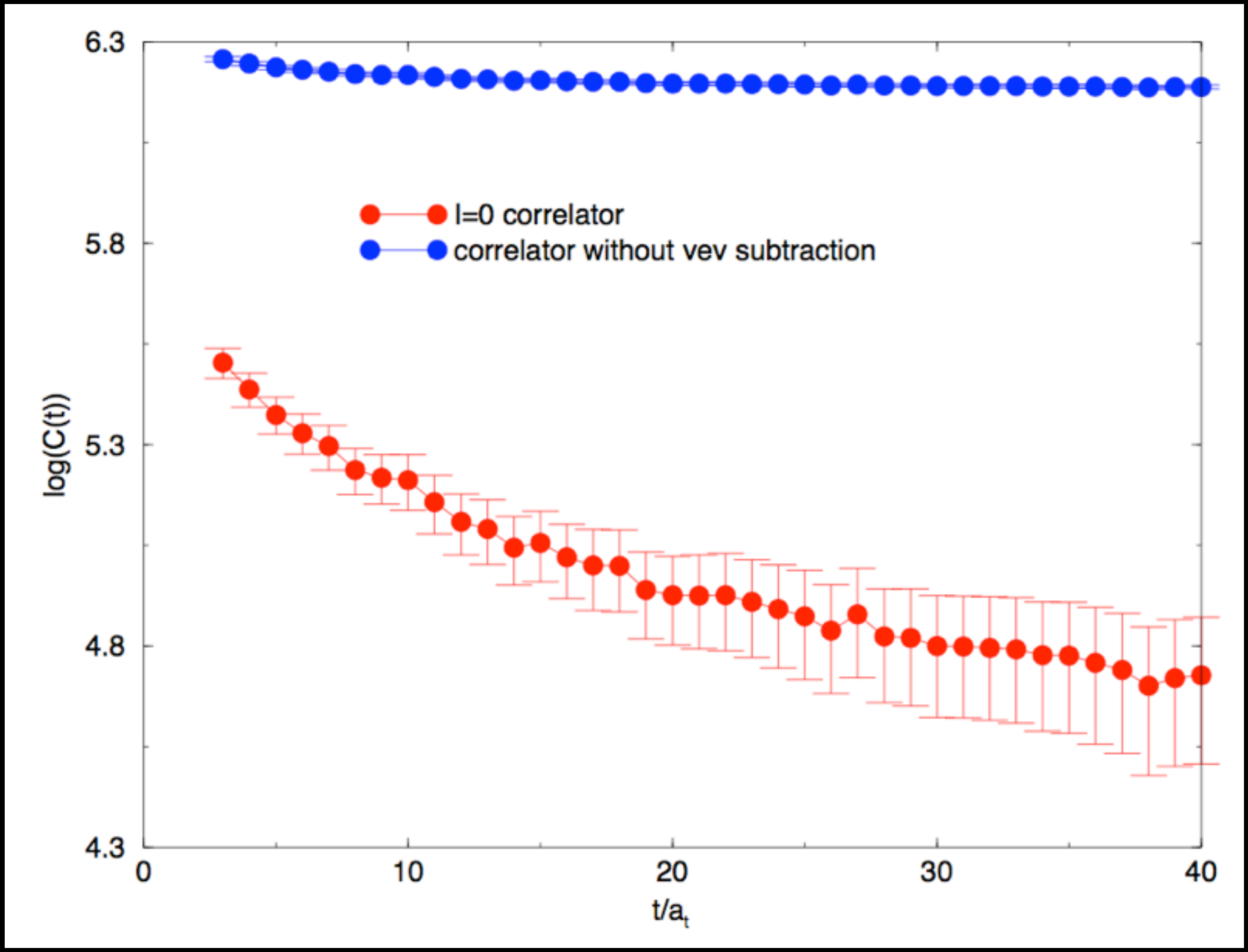
I=2 Effective Mass



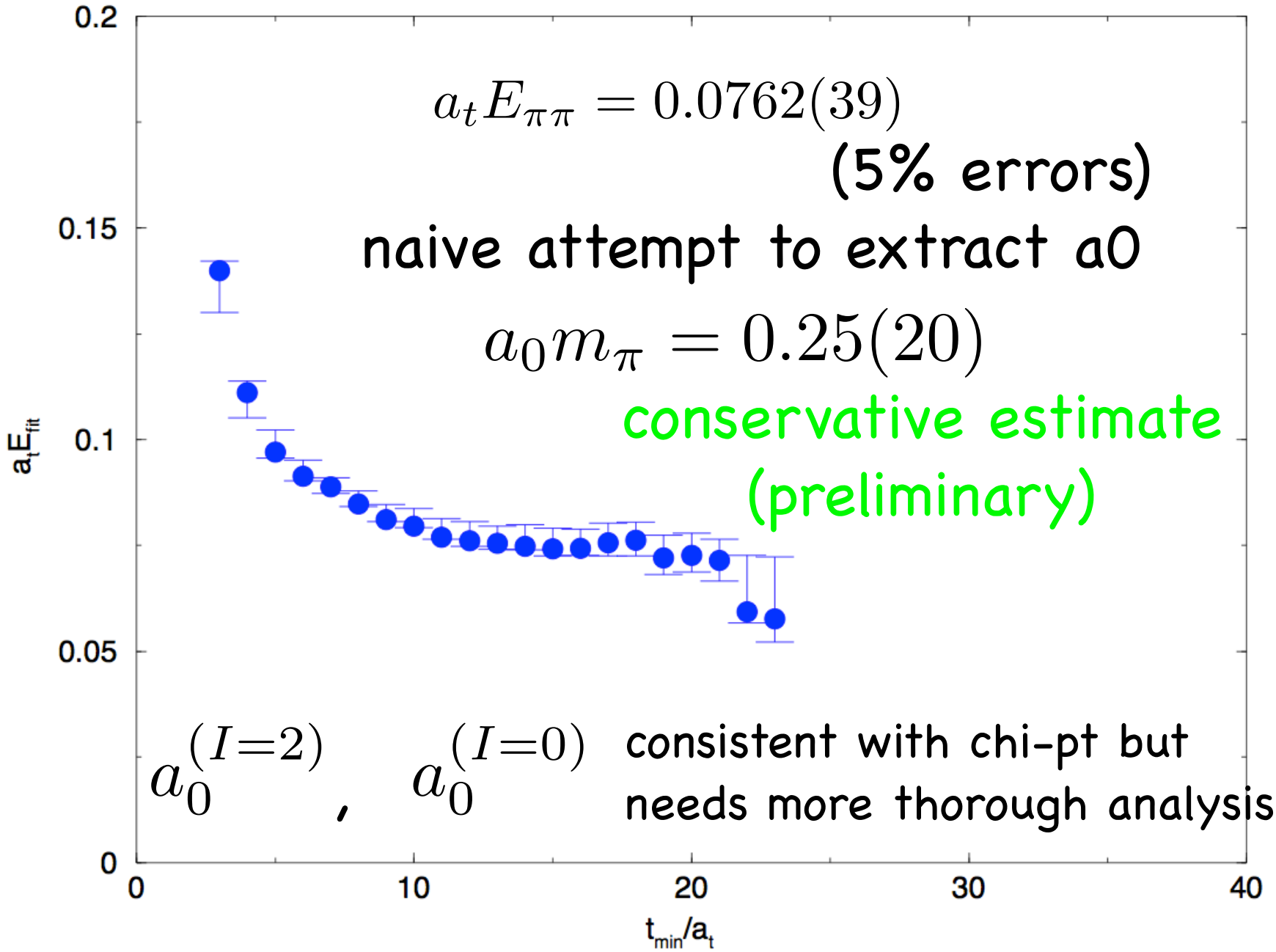
I=2 tmin plot


$24^3 \times 128,\ m_\pi \simeq 240$ MeV
$a_t E_{\pi\pi} = 0.082(2)$
$a_t m_\pi = 0.0400(4)$
preliminary
$a_0 m_\pi = -0.10(2)(?)$
systematics are being checked ...
0.24
0.19
0.14
0.09
0.04
$a_t E_{fit}$
0
10
20
30
40
t_{min}/a_t

I=0 pi-pi correlator



I=0 tmin plot



$$a_t E_{\pi\pi} = 0.0762(39)$$

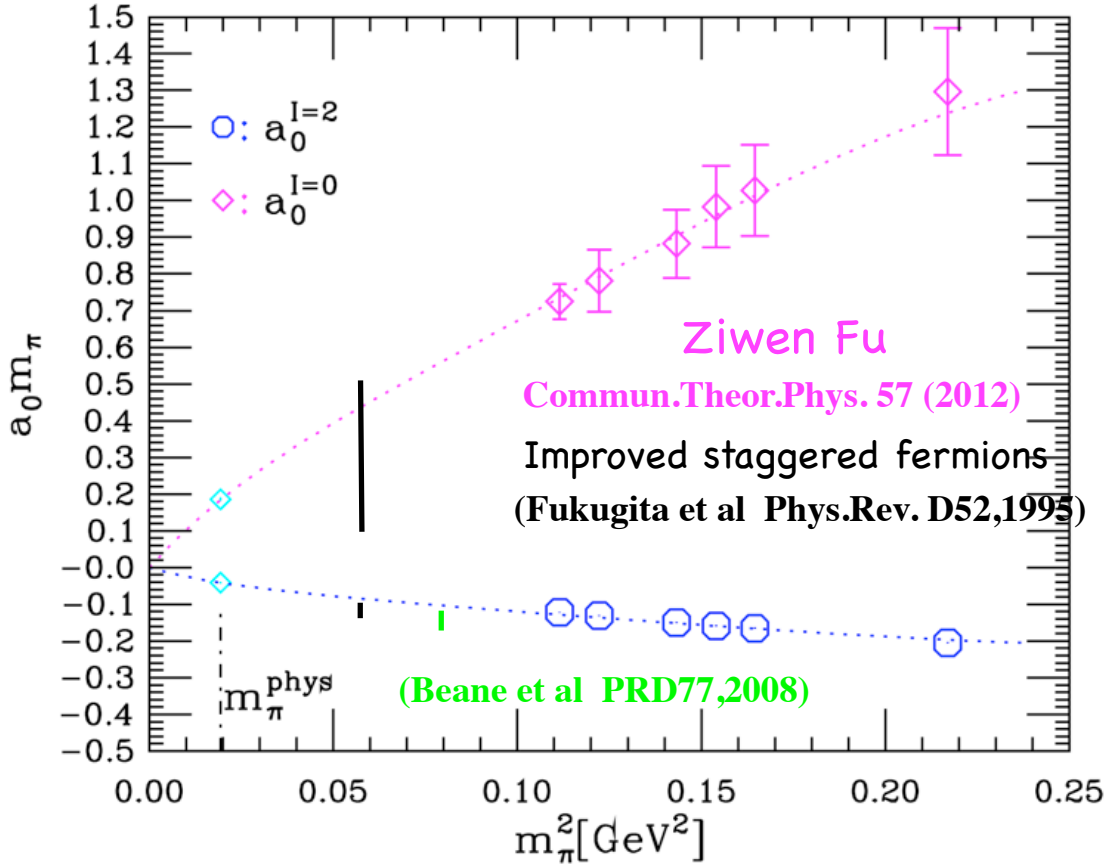
(5% errors)

naive attempt to extract a0

$$a_0 m_\pi = 0.25(20)$$

conservative estimate (preliminary)

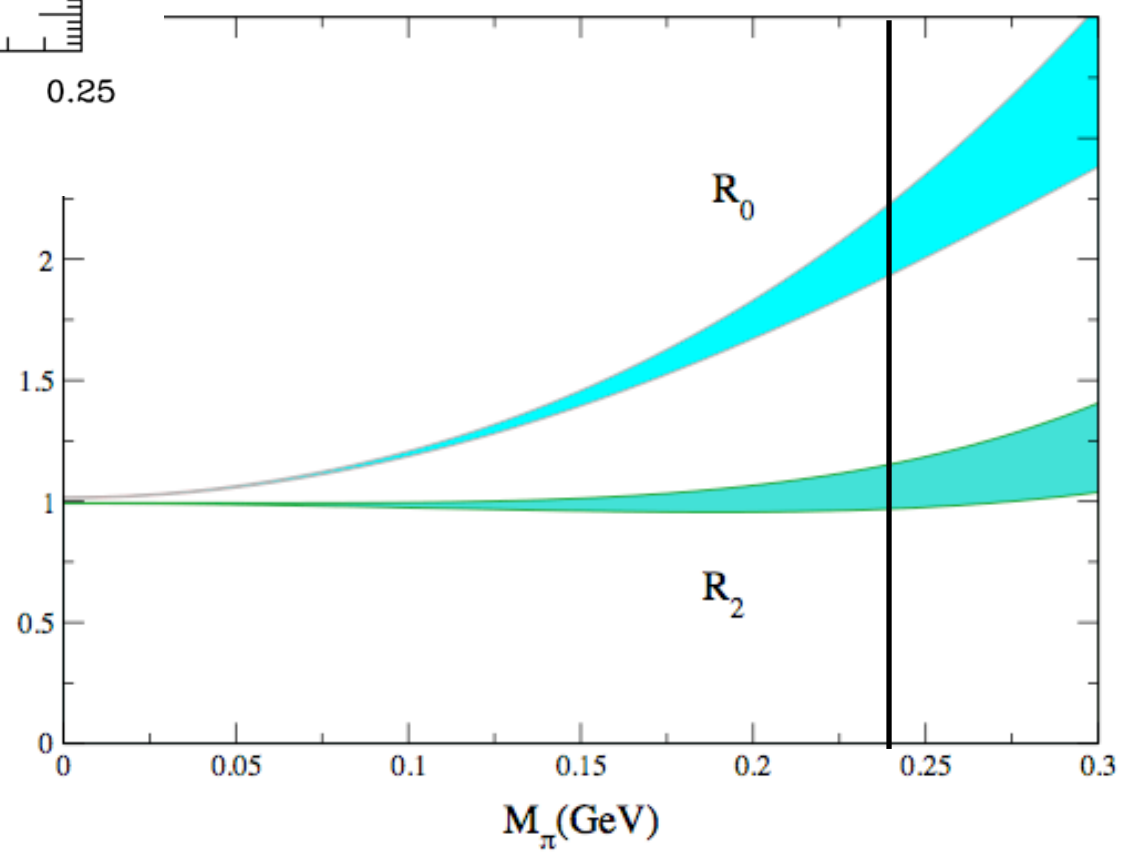
$a_0^{(I=2)}$, $a_0^{(I=0)}$ consistent with chi-pt but needs more thorough analysis



Fully correlated analysis underway with the new value of the renormalized anisotropy …

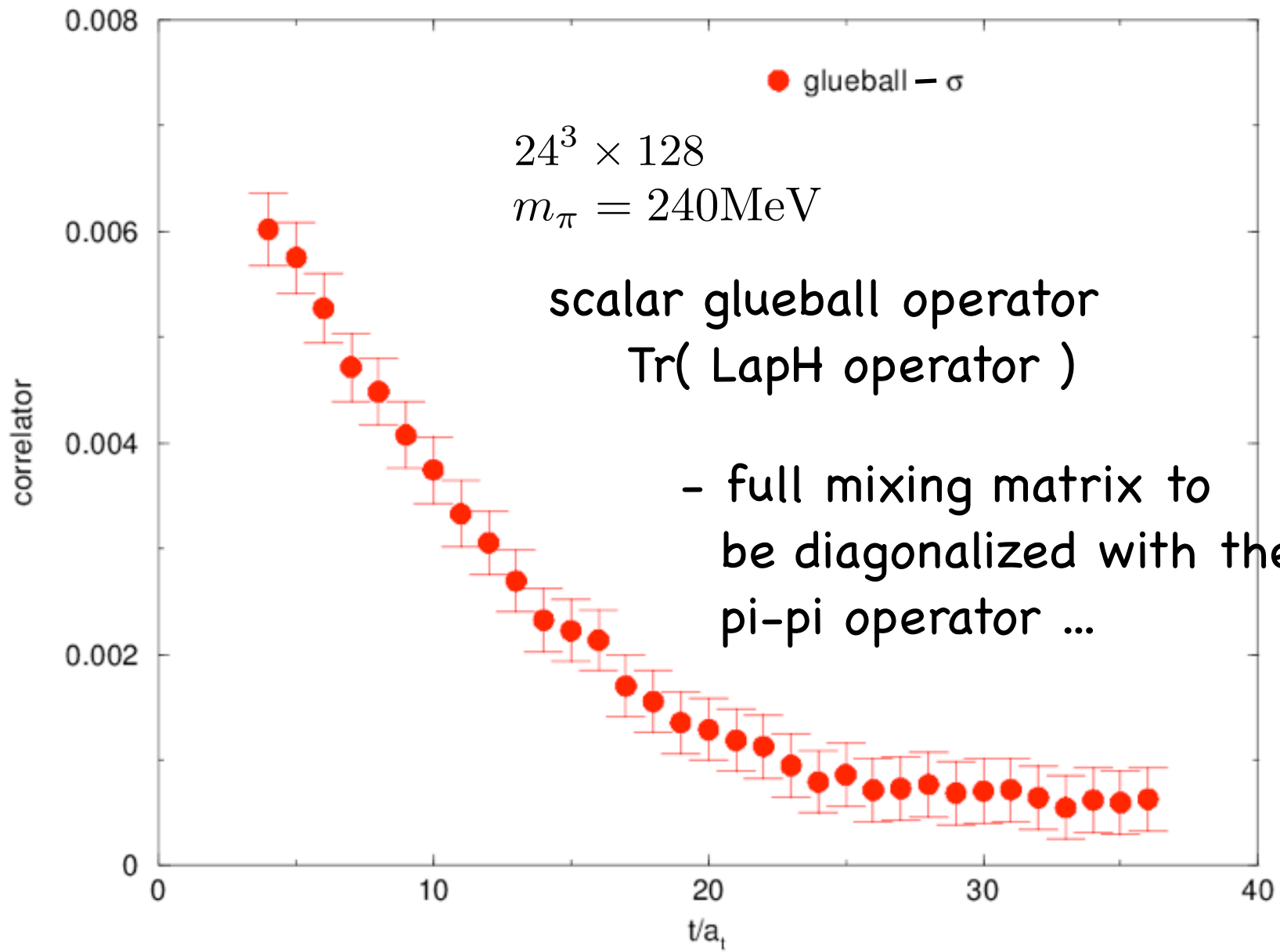
$$a_0^{(I=0)} = \frac{7m_\pi^2}{32\pi f_\pi^2} R_0$$

$$a_0^{(I=2)} = -\frac{m_\pi^2}{16\pi f_\pi^2} R_2$$



Other channels: 0++

scalar glueball mixing

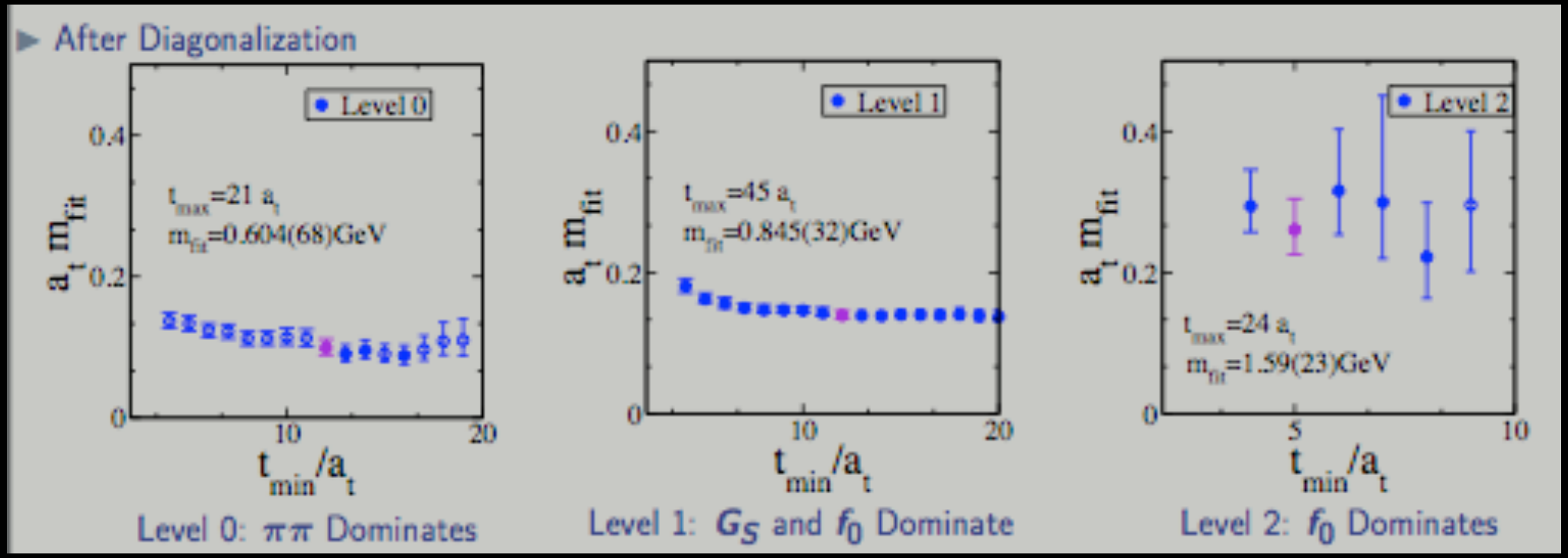


scalar glueball operator
Tr(LapH operator)

- full mixing matrix to be diagonalized with the pi-pi operator ...

0++

16^3 lattice with 380 MeV pions



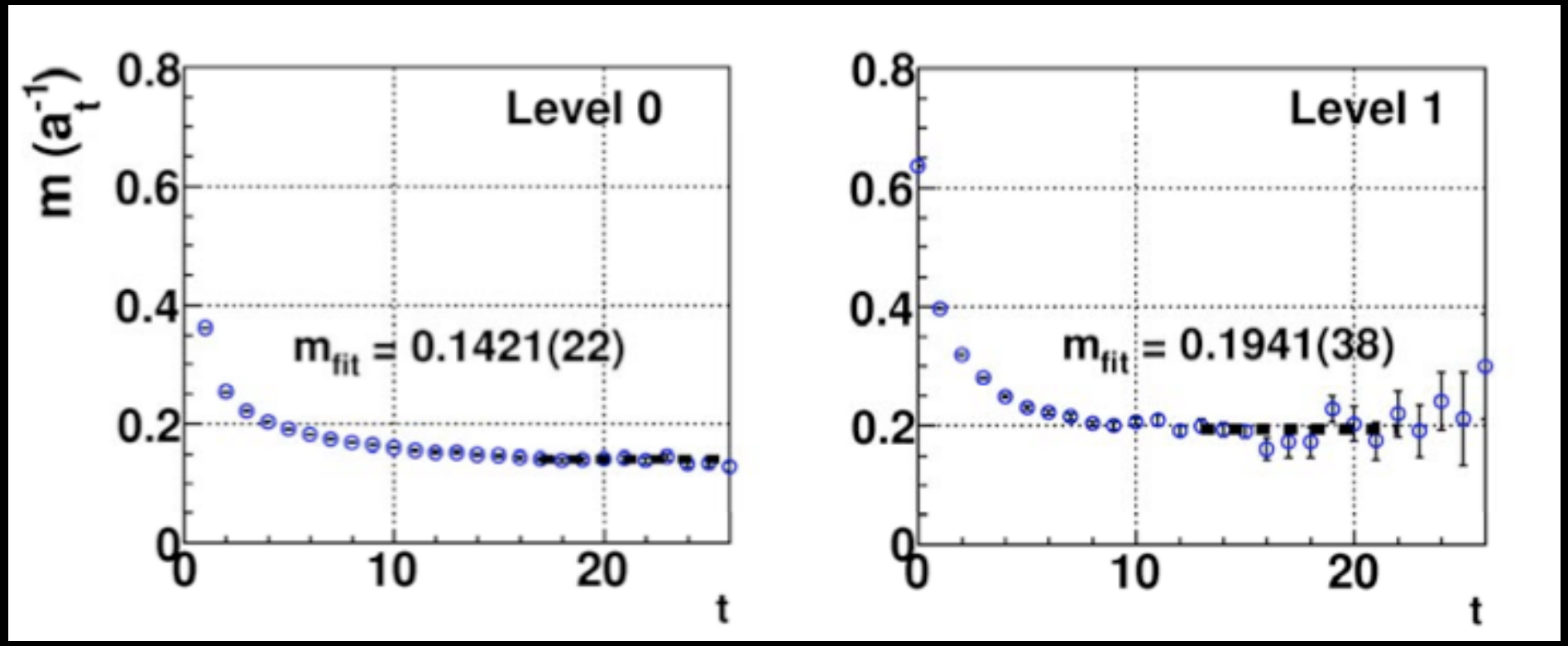
Ricky Wong
(Lattice2012)

similar analysis on the 24^3 to follow ...

1--

$$\rho \to \pi\pi$$

diagonalize correlation matrix with single rho operator and pi-pi operator



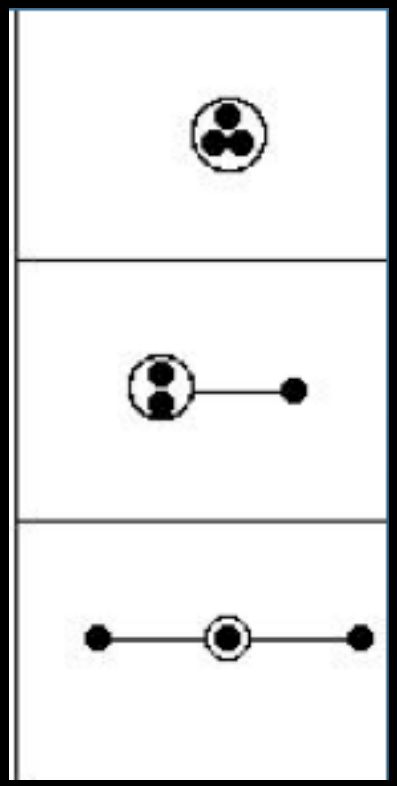
16^3 lattice with 380 MeV pions

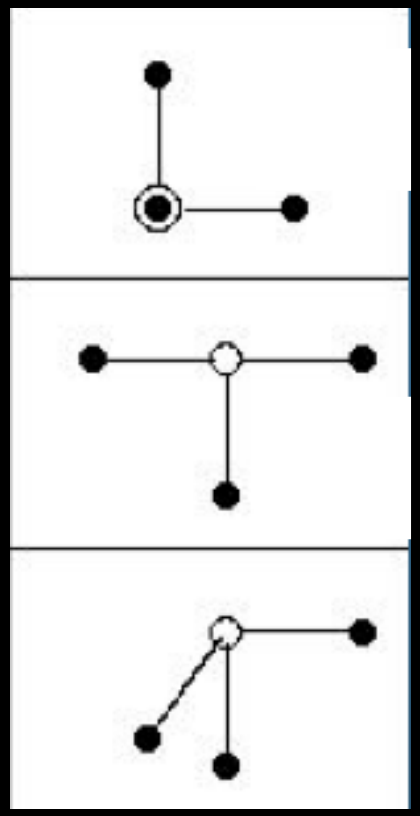
Similar analysis on the 24^3 lattice with 240 MeV pions being done ...

Baryons

Pruning of the single baryon operator done on the 16^3 volume and heavy pions 380 MeV

Basic building blocks

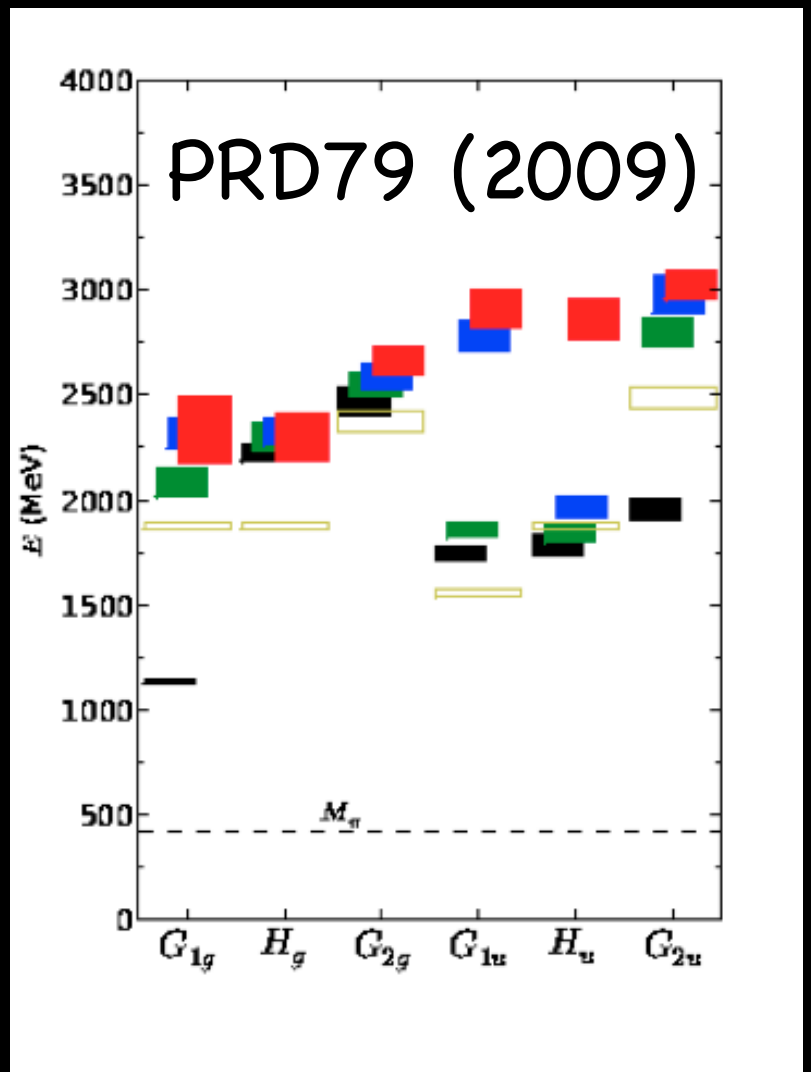




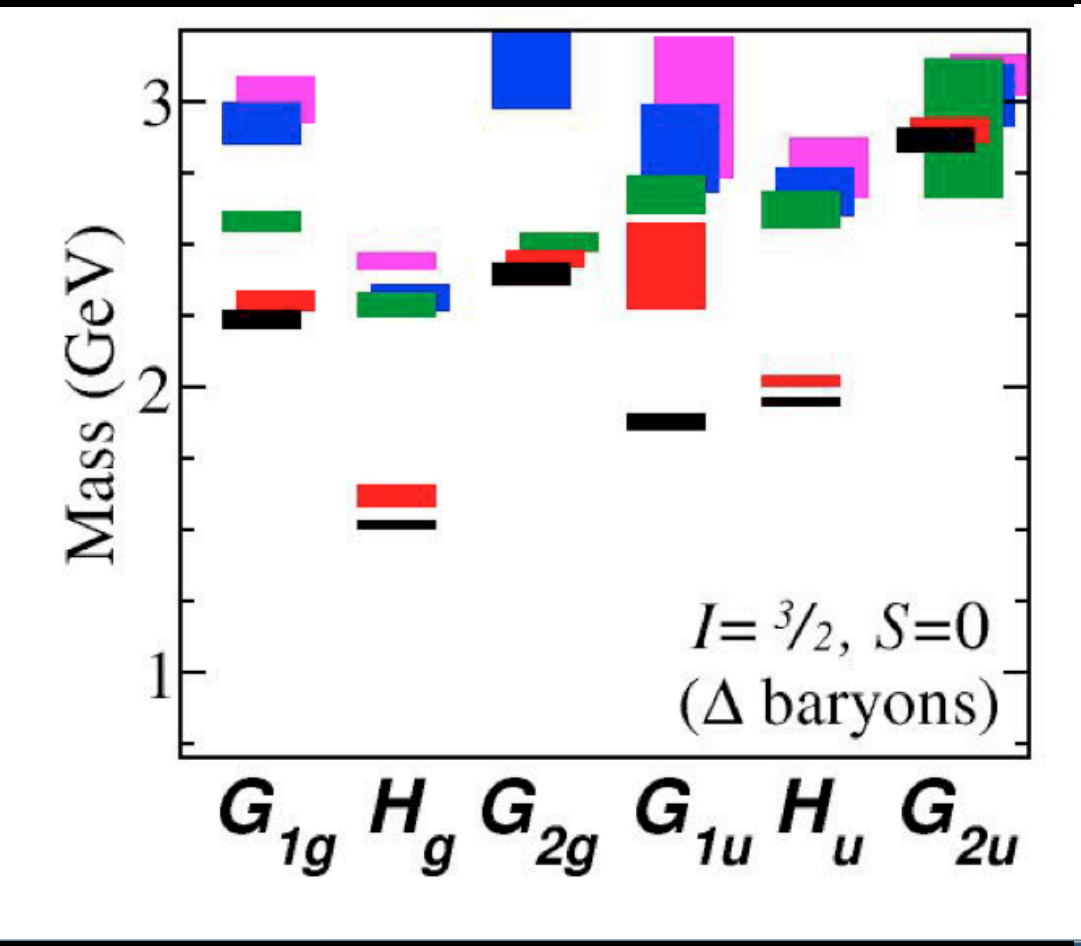
Group theoretical projections

Basak et al. PRD72 (2005)

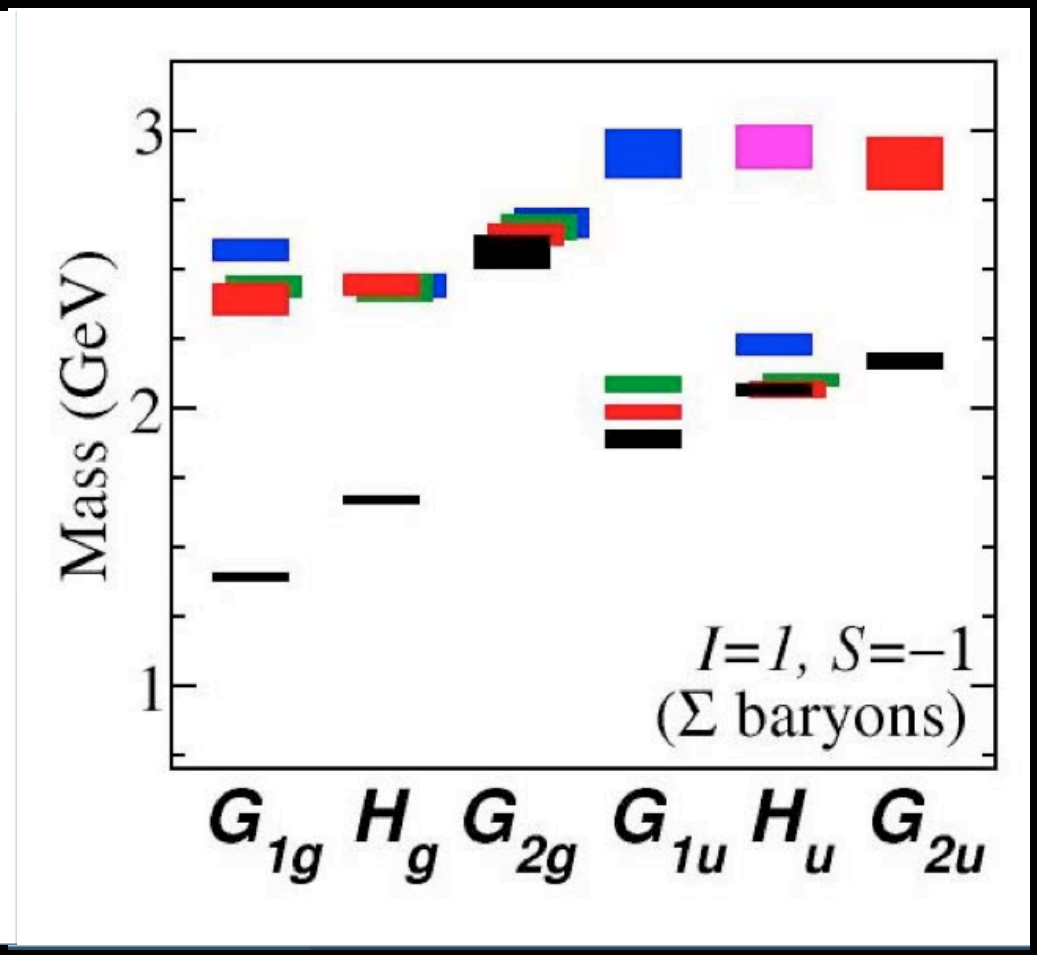
Nucleons done on 24^3



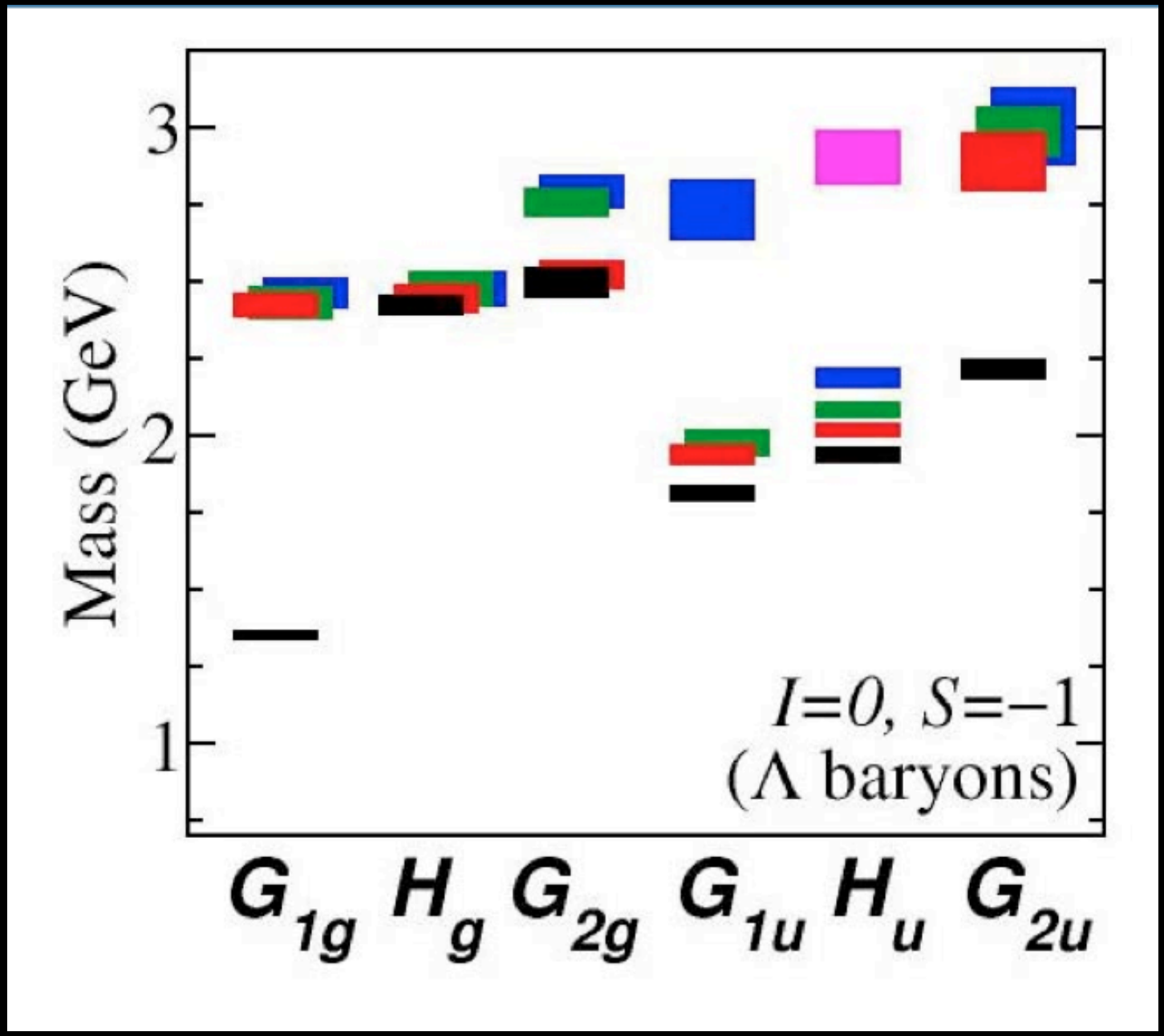
Delta baryons
(16^3, 380MeV)



Sigma baryons
(16^3, 380MeV)



Lambda baryons
(16^3, 380MeV)



Conclusions

- Steady progress towards the dynamical simulation of the low-lying hadron spectrum
  - LapH eigenvectors and solutions computed on the 24^3, 32^3 lattices at the 240 MeV mass
  - renormalized anisotropy being finalized
  - single particle meson operators and correlation functions computed
    - isovector operators, finite momentum operators computed (spectrum computed on small lattices)
    - coefficients to construct two-particle states done

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- two-particle correlation functions computed
  - I=0, 2 final analysis is being finalized with more careful look at finite size effects and operator overlaps
  - I=1 coming soon
- glueball mixings on the large lattices to be completed with sigma, pi-pi.

- Baryons (Nucleon/Delta) to be done on the 24^3 lattices
  - operator pruning done for Nucleons, Delta, Sigma, Lambda on 16^3 lattices with 380 MeV pions
- inversions on the 32^3 lattices at the 240 MeV pion masses are done so that the spectrum can be computed on our (3.6fm) box

We are almost there to finally compute the optimized hadron lattice operators of the low-lying hadron spectrum
(at mpi=240 MeV)
Phenomenology with these "good" operators will be the next step ...