Classical Geometry to Quantum Behavior correspondence in a *Virtual* Extra Dimension

ICHEP 2012 - Melbourne

Donatello Dolce

Based on:

–> Classical to quantum correspondence in a VXD D.D.; *Ann.Phys.*,on-line (2012); arXiv:1110.0316

–> Gauge interaction as periodicity modulation D.D.; *Ann.Phys.*, 327, (2012); arXiv:1110.0315

–> Compact Time and determinism: foundations D.D.; *Found.Phys.*, 41, (2011); arXiv:0903.3680

–> On the intrinsically cyclic nature of space-time in elementary particles D.D.,*JPCS*. 343 (2012); arXiv:1206.1140

–> Clockwork Quantum Universe D.D., IV Prize *FQXi* essay contest 2011; arXiv:1205.1788

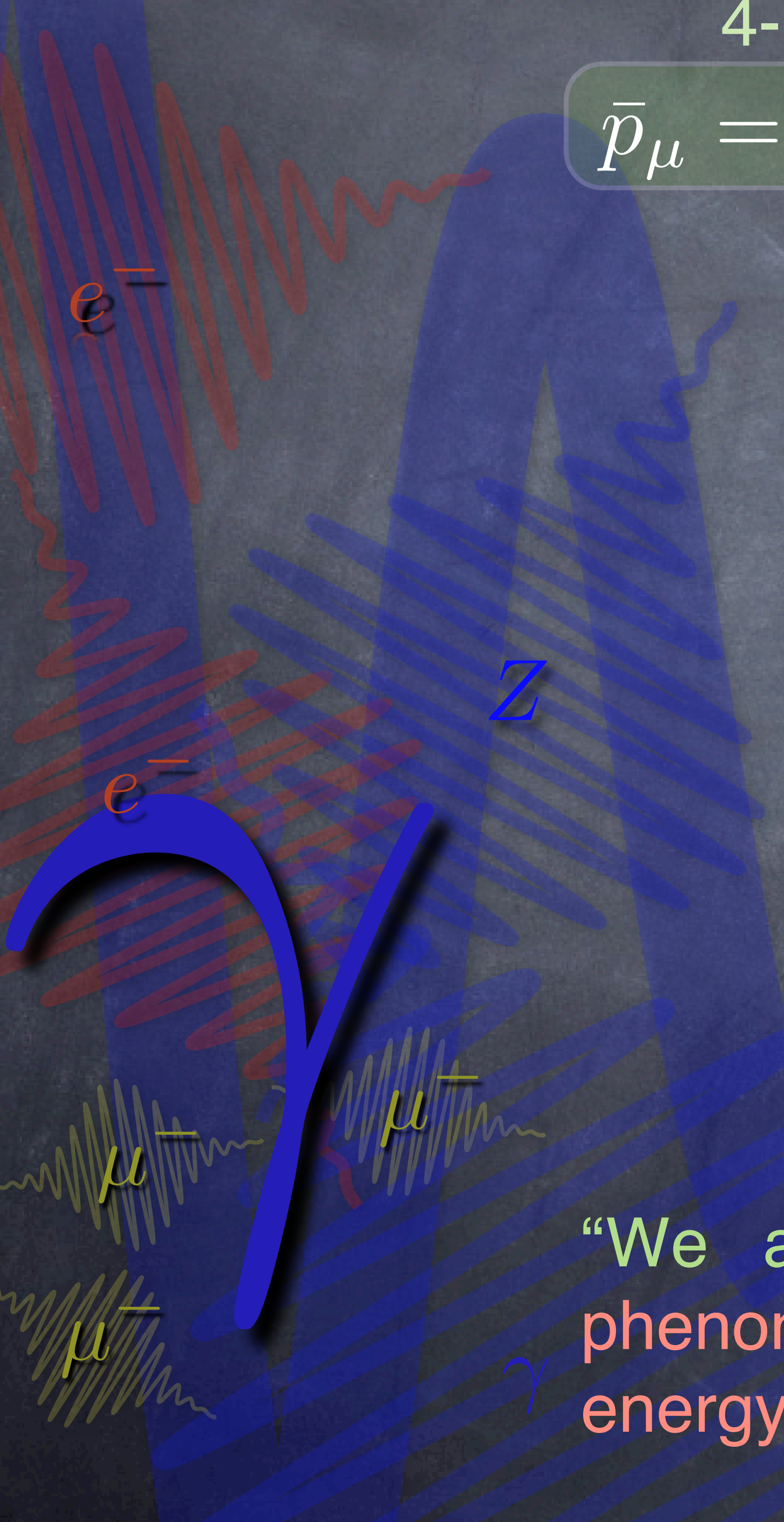
–> De Broglie deterministic dice and emergent relativistic quantum mechanics D.D.,*JPCS*. 306 (2011); arXiv:1111.3319

--> Deterministic Quantization by Dynamical Boundary Conditions D.D., AIP *Conf.Proc.* 1246 (2010); arXiv:1006.5648

–> Quantum Mechanics for Periodic Dynamics: the bosonic case D.D., AIP *Conf.Proc.* 1232 (2010); arXiv:1001.2718

Periodic phenomenon

Local retarded variations of 4-momentum in relativistic interactions can be equivalently described as local retarded modulations of 4-periodicity.



4-momentum

$$\bar{p}_\mu = \{\bar{E}/c, -\bar{\mathbf{p}}\}$$

$\hbar$ → Planck Constant

4-periodicity

$$T^\mu = \{T_t, \vec{\lambda}_x/c\}$$

rest frame

$$T_\tau \bar{M} c^2 = h$$

Lorentz → transformation

generic frame $\bar{\mathbf{p}}$

$$c\bar{p}_\mu T^\mu = h$$

Compton periodicity:

$$T_\tau = \frac{h}{\bar{M} c^2}$$

topology $\mathbb{S}^1$

"We assume the existence of a certain periodic phenomenon to be attributed to each and every isolated energy parcel [elementary particle]".

deBroglie:1924

Relativistic clocks

$$T^{\gamma}, h_{\mu\nu} \lesssim +\infty$$
$$T^{Cs-133} \lesssim 10^{-10} s$$
$$T^{e^-} \lesssim 10^{-20} s$$
$$T^{Z^0} \lesssim 10^{-25} s$$
$$T^{\mathrm{TeV}} \lesssim 10^{-27} s$$
$$\dots$$
$$T^{Planck} \sim 10^{-44} s$$

Observed Experimentally
Gouanere:2008

Every elementary particle has intrinsic space-time periodicity

HYPOTHESIS: $c\bar{p}_\mu T^\mu = h$

Every elementary particle can be imagined as a reference clock.

Einstein relativistic clock

"by a clock we understand anything characterized by a phenomenon passing periodically through identical phases, [...] all that happens in a given period is identical with all that happens in an arbitrary period. Einstein:1910

Time unit [SI]

A "*second*" is the duration of 9,192,631,770 periods of the radiation corresponding to [...] the Cs 133 atom.

Vibrating cyclic-string

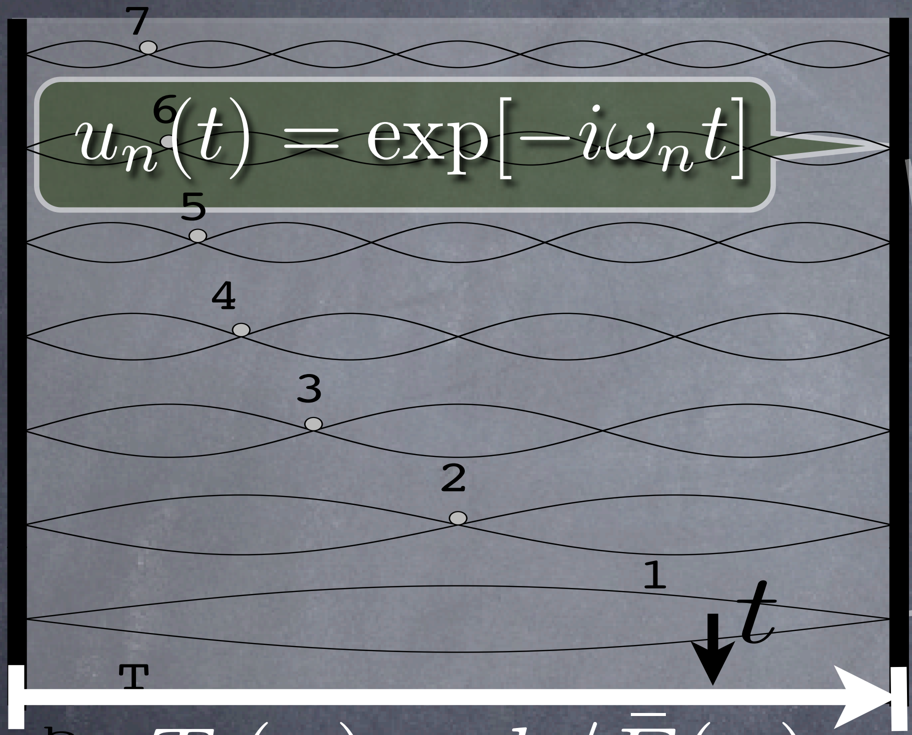
$\bar{p}_\mu = \{\bar{E}/c, -\bar{\mathbf{p}}\}$ $T^\mu = \{T_t, \vec{\lambda}_x/c\}$

intrinsic periodicity

$c\bar{p}_\mu T^\mu = h$ topology $\mathbb{S}^1$

"particle in a box"

$$\Phi(\mathbf{x}, t) = \sum_n A_n a_n(\bar{p}) \phi_n(\mathbf{x}) u_n(t)$$



$$u_n(t) = \exp[-i\omega_n t]$$

$$T_t(\bar{\mathbf{p}}) = h/\bar{E}(\bar{\mathbf{p}})$$

Energy quantization

$$\bar{E}_n(\bar{\mathbf{p}}) = n\hbar\bar{\omega}(\bar{\mathbf{p}}) = n\bar{E}(\bar{\mathbf{p}}) = n\frac{h}{T_t(\bar{\mathbf{p}})}$$

$$\omega_n(\bar{\mathbf{p}}) = n\bar{\omega}(\bar{\mathbf{p}})$$

Einstein:1910

Physical information contained in a single period

Compact space-time dimensions

with Periodic Boundary Conditions

$$\oint_0^{T^\mu} d^4x \bar{\mathcal{L}}(\partial_\mu \bar{\Phi}, \bar{\Phi})$$

String vibrating with four periodicity T^μ

$$\Phi(x^\mu) = \Phi(x^\mu + T^\mu)$$

Covariance

$$c\bar{p}_\mu T^\mu = h \rightarrow c\bar{p}'_\mu T'^\mu = h$$

Lorentz transformation of reference frame $\quad x^\mu \rightarrow x'^\mu = x^a \Lambda_a^\mu$

⇕

Transformation of 4-momentum $\quad \bar{p}_\mu \rightarrow \bar{p}'_\mu = \Lambda^\nu{}_\mu \bar{p}_\nu$

⇕ $\hbar$

Transformation of (tangent) 4-periodicity $\quad T^\mu \rightarrow T'^\mu \sim T^a \Lambda_a^\mu$

⇕

Transformation of the boundary

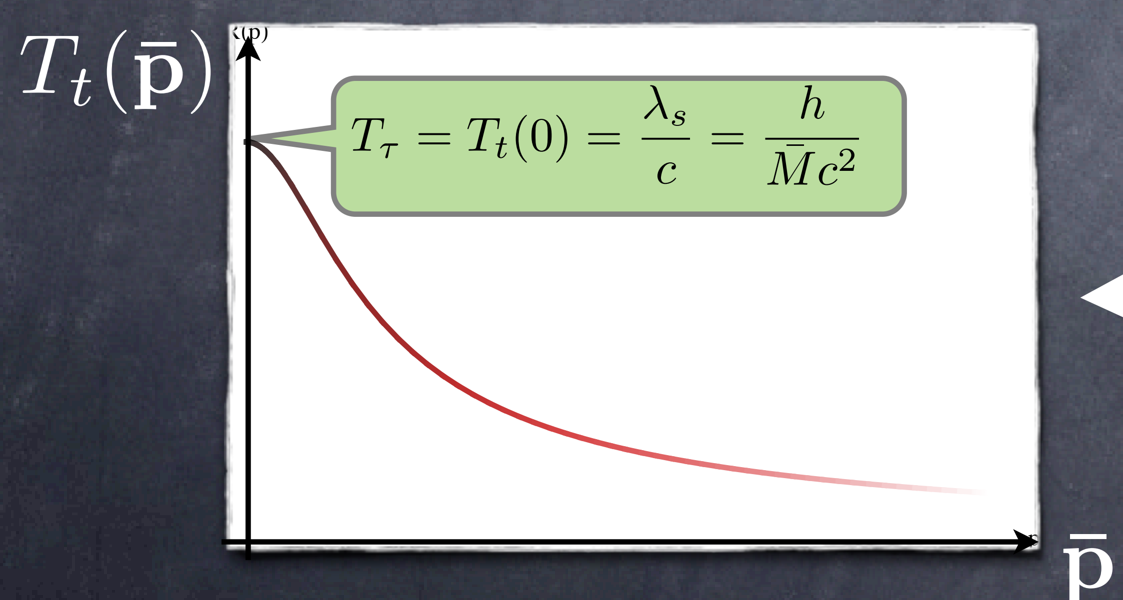
$$\oint_0^{T^\mu} d^4x \bar{\mathcal{L}}(\partial_\mu \bar{\Phi}, \bar{\Phi}) \longrightarrow \oint_0^{T'^\mu} d^4x' \bar{\mathcal{L}}(\partial_\mu \bar{\Phi}', \bar{\Phi}')$$

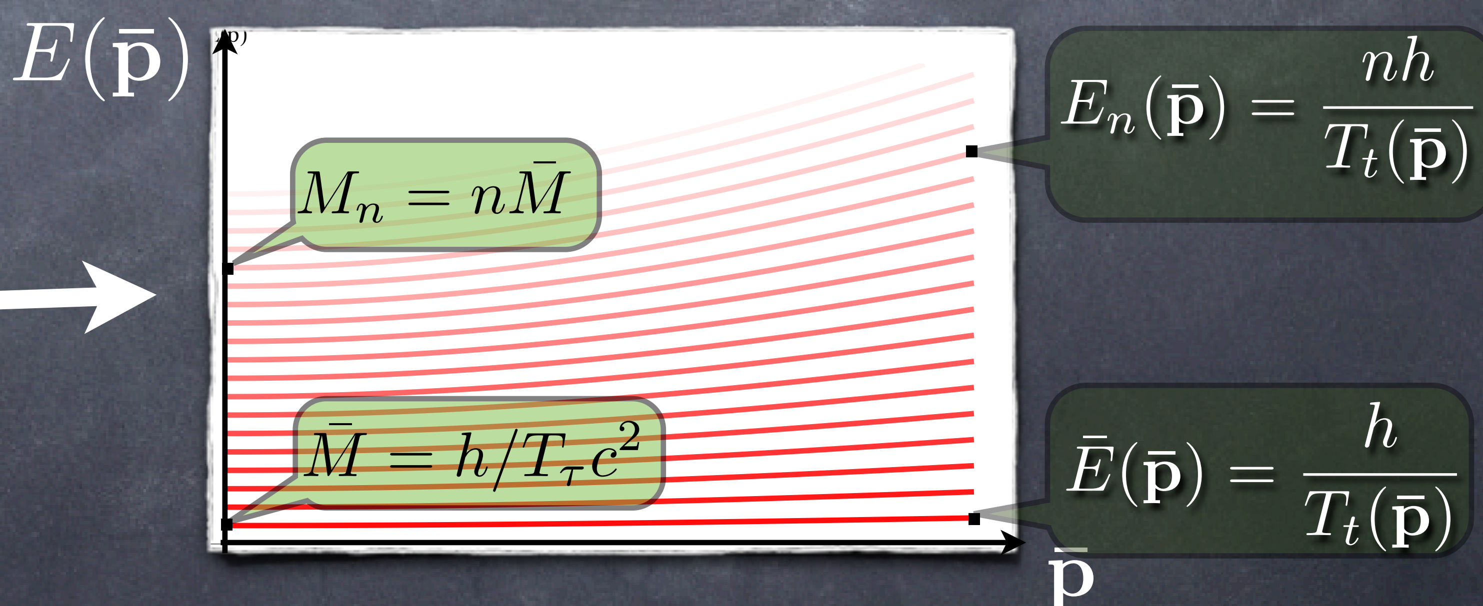
T^μ is a contravariant, tangent four-vector.

$$\bar{M}^2 c^2 = \bar{p}_\mu \bar{p}^\mu \quad \overset{\hbar}{\longleftrightarrow} \quad \frac{1}{T_\tau^2} = \frac{1}{T_\mu}\frac{1}{T^\mu}$$

relativistic modulation of periodicity

spectrum dispersion relation





Energy spectrum of second quantization:

(after normal ordering)

$$E_n(\bar{\mathbf{p}}) = nE_n(\bar{\mathbf{p}}) = n\sqrt{\bar{M}^2c^4 + \bar{\mathbf{p}}^2 c^2}$$

Interactions as Boundary Geometrodynamics

Generic interaction

free case | interaction

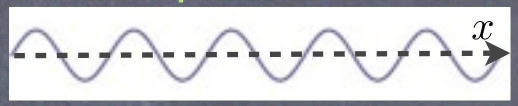
variation of 4-momentum $\quad \bar{p}_\mu \rightarrow \bar{p}'_\mu(x) = e^a_{\ \mu}(x)\bar{p}_a$

$\Updownarrow \hbar$

modulation of 4-periodicity $\quad T^\mu \rightarrow T'^\mu(x) \sim T^a e_a^{\ \mu}(x)$

$\Updownarrow$

persistent $\rightarrow$ modulated



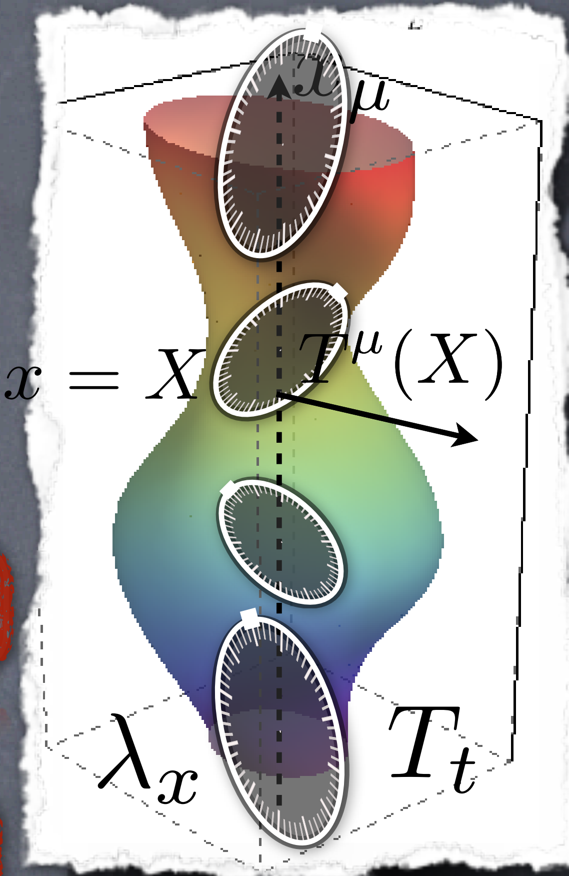
deformation of the metric $\quad \eta_{\mu\nu} \rightarrow g_{\mu\nu}(x) = e^a_{\ \mu}(x) e_\nu^{\ b}(x)\eta_{ab}$

$\Updownarrow$

deformation of the boundary

$$\oint_0^{T^\mu} d^4x \mathcal{L}(\partial_\mu\Phi, \Phi) \rightarrow \oint_0^{T^a e_a^\mu(X)} d^4x \sqrt{-g}\mathcal{L}(e_a^{\ \mu}\partial_\mu\Phi', \Phi')$$

Holography: kinematics encoded in the boundary.



Linearized gravity

$$V_N(r) = -GM_\odot/r \ll 1$$

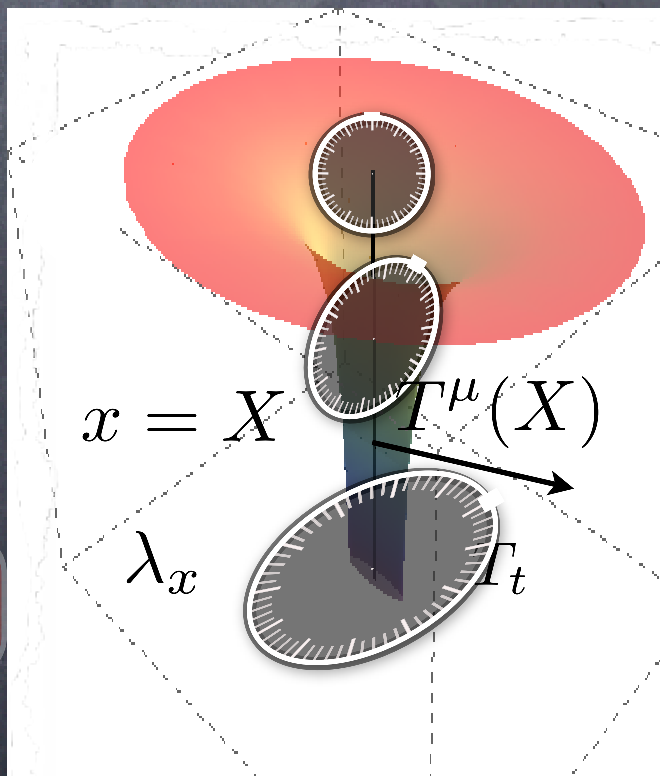
Redshift $\quad E' \sim E(1 - \frac{GM_\odot}{r})$; $\mathbf{p}' \sim \mathbf{p}(1 + \frac{GM_\odot}{r})$

$\Updownarrow \hbar$

Time dilatation $\quad T' \sim T(1 + \frac{GM_\odot}{r})$; $\lambda'_x \sim \lambda_x(1 - \frac{GM_\odot}{r})$

$\Updownarrow$

Schwarzschild $\quad ds^2 \sim \left(1 - \frac{2GM_\odot}{r}\right)dt^2 - \left(1 + \frac{2GM_\odot}{r}\right)d\mathbf{x}^2$



"What is fixed at the boundaries of GR?" York,84

Canonical QM,

in every interaction point (locally)

$$\Phi(\mathbf{x},t) = \sum_n A_n a_n(\bar{p})\phi_n(\mathbf{x})u_n(t)$$

$V_x = N\lambda_x$

$N \in \mathbb{Z}.$

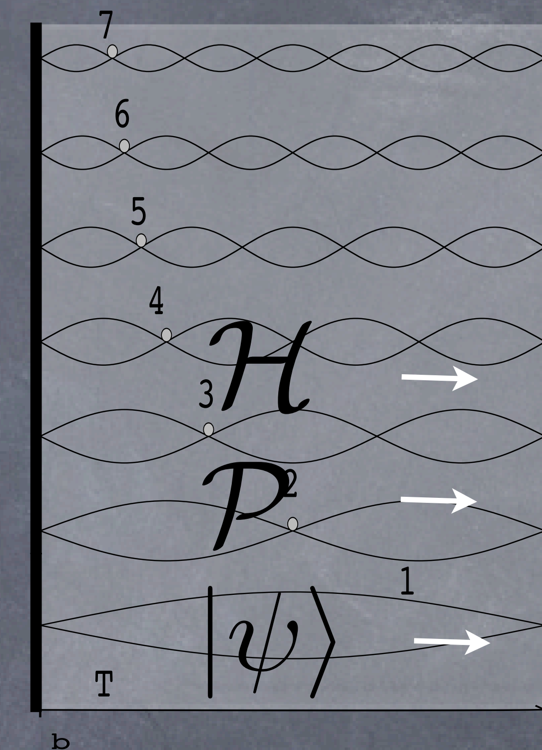
Inner Product: $$\langle\phi|\chi\rangle = \int_0^{V_x} \frac{d\mathbf{x}^3}{V_x}\phi(\mathbf{x})\chi(\mathbf{x})$$

Hilbert Eigenstates: $$\langle\mathbf{x}|\phi_n\rangle = \phi_n(\mathbf{x})/\sqrt{V_x}$$

Hamiltonian Operator: $$\mathcal{H}\,|\phi_n\rangle = E_n\,|\phi_n\rangle$$

Momentum Operator: $$\mathcal{P}\,|\phi_n\rangle = \mathbf{p}_n\,|\phi_n\rangle$$

Generic Hilbert State: $$|\psi\rangle = \sum_n c_n\,|\phi_n\rangle$$



$\mathcal{H} \rightarrow E_n$

$\mathcal{P} \rightarrow \mathbf{p}_n$

$|\psi\rangle \rightarrow |\phi_n\rangle$

Bulk EoMs: $$(\partial_t^2 + \omega_n^2)\phi_n(\mathbf{x},t) = 0 \longrightarrow i\hbar\partial_t\phi_n(\mathbf{x},t) = E_n\phi_n(\mathbf{x},t)$$

Carena:'03

Schrödinger equation: $$i\hbar\partial_t\,|\psi(t)\rangle = \mathcal{H}\,|\psi(t)\rangle$$

Unitary time evolution: $$\hat{U}(dt) = e^{-\frac{i}{\hbar}\mathcal{H}(t)dt}$$ $$\hat{U}(t'';t') = \prod_{m=0}^{N-1}\hat{U}(t'+t_{m+1};t'+t_m-\epsilon)$$

Commutation relation

Expectation Value: $$\partial_x\mathcal{F}(x)$$ $$\langle\chi_f|\hbar\partial_x\mathcal{F}(x)|\phi_i\rangle = i\langle\chi_f|\mathcal{P}\mathcal{F}(x) - \mathcal{F}(x)\mathcal{P}|\phi_i\rangle$$

Commutation relations: $$[\mathcal{F}(x),\mathcal{P}] = i\hbar\partial_x\mathcal{F}(x)\;;\quad \mathcal{F}(x)\equiv x \longrightarrow [x,\mathcal{P}] = i\hbar$$

Feynamn:'42

Implicit!

Feynman Path Integral

Complete set of eigenfunctions in Hilbert space, with unitary time evolution:

$$\mathcal{Z} = \int_{V_x} \mathcal{D}x e^{\frac{i}{\hbar}\mathcal{S}'[t_f, t_i]}$$

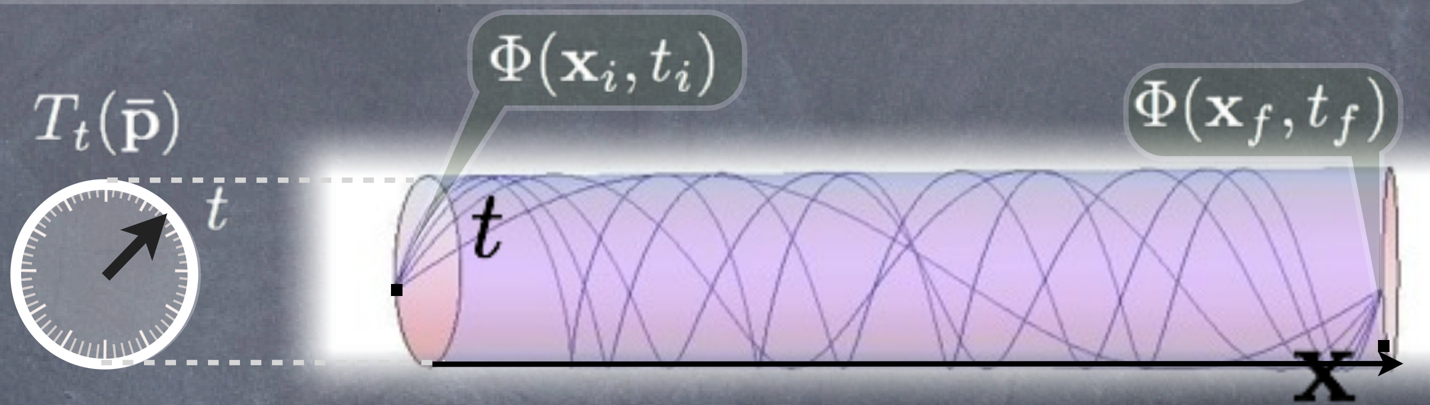
$V_x = N\lambda_x$ bigger (or infinite) than the interaction region

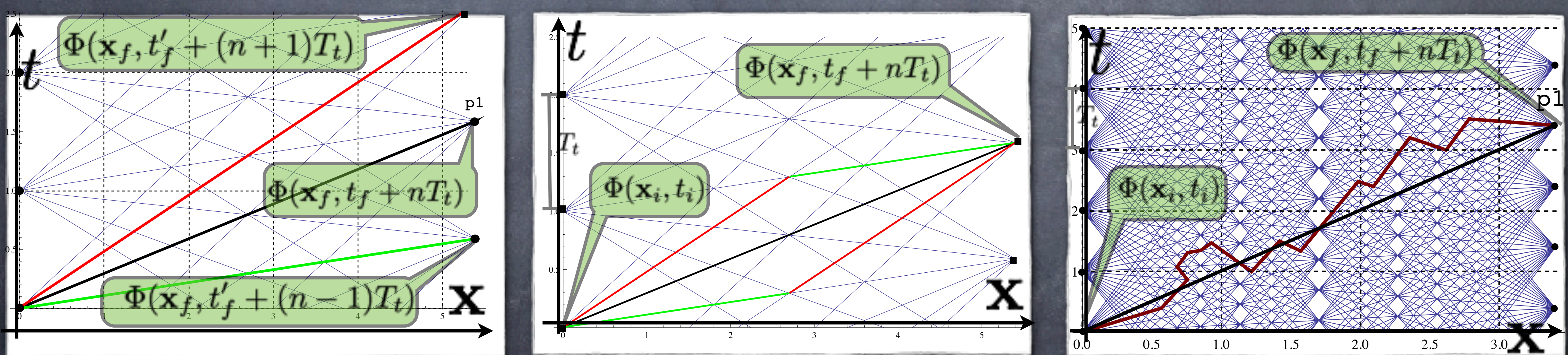
$$\mathcal{S}'(t_b, t_a) \equiv \int_{t_a}^{t_b} dt \mathcal{L}'(x,t)$$

$$\mathcal{L}'(x_m, t_m) \equiv \mathcal{P}\dot{x}_m - \mathcal{H}'(x_m, t_m)$$

Set of classical paths with different winding number:

topology $\mathbb{S}^1$





Feynamn:'42

Composition of periodic paths → variation around a give path

Path Integral as interference of classical paths!

--> Gauge interaction as periodicity modulation D.D.; Ann.Phys., 327, (2012); arXiv:1110.0315

--> Compact Time and determinism: foundations D.D.; Found.Phys., 41, (2011); arXiv:0903.3680

Gauge Interaction

(Weyl's and Kaluza's original proposal!)

--> Gauge interaction as periodicity modulation

D.D.; Ann.Phys., 327, (2012); arXiv:1110.0315

$$\omega_\mu^a(x)\bar{p}_a \equiv \bar{A}_\mu(x)$$

$\omega(x) \in U(1)$

Min substitution

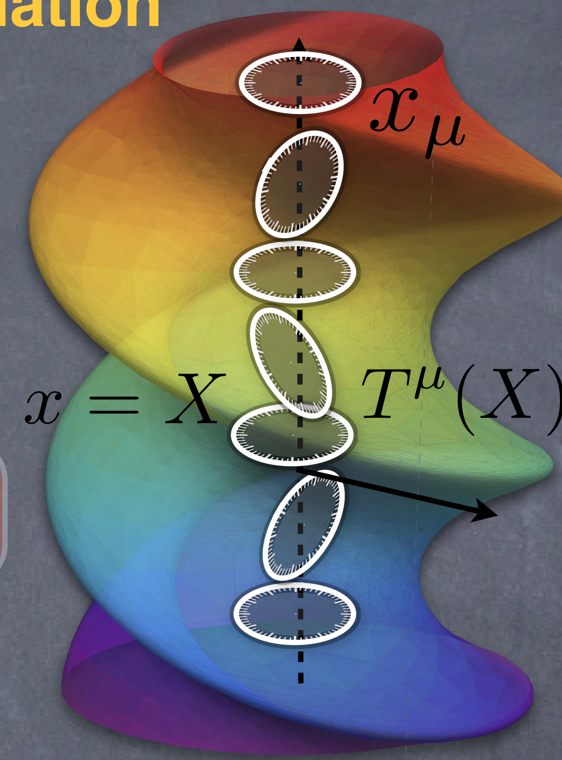
$$\bar{p}_\mu \longrightarrow \bar{p}'_\mu(x) \sim \bar{p}_\mu - e\bar{A}_\mu(x)$$

$\hbar \updownarrow$

Local Isometry

(~zitterbewegung)

$$T^\mu \longrightarrow T'^\mu(x) \sim T^\mu + e\omega_a^\mu(x)T^a$$



Local transformation of space-time induces internal transformation of the solution

Gauge transformation

$$\delta\bar{\Phi}(x) \sim ie\bar{A}_\nu(x)\bar{\Phi}(x)\delta x^\nu$$

$\Rightarrow$ FPI for scalar QED

$$\mathcal{Z} = \int_{V_x} \mathcal{D}x e^{\frac{i}{\hbar}\mathcal{S}_{QED}[t_f,t_i]}$$

Dirac Quantization condition

$$e^{-\frac{i}{h}\oint dx\cdot eA_n(x)} = e^{-i2\pi n} \Rightarrow \oint dx \cdot eA_n(x) = ge = hn$$

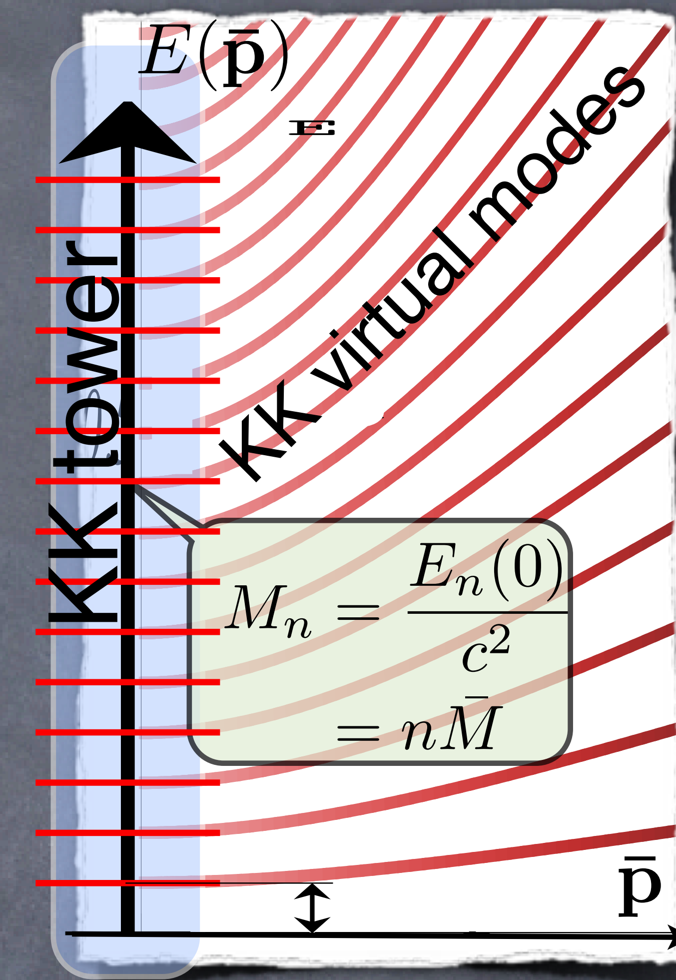
Virtual Extra Dimension (VXD)

Massless 5D theory: $dS^2 = dx_\mu dx^\mu - ds^2 \equiv 0$

Virtual Extra Dimension: $\Downarrow$ $s = c\tau$

Vibrating cyclic string in 4D: $ds^2 = dx_\mu dx^\mu$

$$E_n(\bar{\mathbf{p}}) = nE_n(\bar{\mathbf{p}}) = n\sqrt{\bar{M}^2c^4 + \bar{\mathbf{p}}^2c^2}$$



Holography, Light-Front Quantization, ..

Vary, et.al 0911.2929, Brodsky 0807.2484, 1103.1100

Interactions in VXD

(massless approximation)

$$g_{MN}(s) = \begin{pmatrix} g_{\mu\nu}(s) & 0 \\ 0 & 1 \end{pmatrix}$$

AdS/CFT

$$\mathcal{Z} = \int_{V_x} \mathcal{D}x e^{\frac{i}{\hbar}\mathcal{S}'(s,s')} \leftrightarrow e^{\frac{i}{\hbar}\mathcal{S}^{VXD}(s,s')}$$

In AdS/CFT:

“any classical configuration in XD [...] has a dual interpretation in [quantum] 4D SCFT” Gherghetta:2011

“quantum phenomena [...] are encoded in classical geometry” Witten:1998

Basic AdS/QCD

Satz:'08; Magas:'03

Quark-Gluon plasma freeze out

↕ Bjorken Hydrodynamical model

Exponential 4-momentum decay

↕ $\hbar$

Exponential 4-periodicity dilatation

↕

Virtual AdS metric

$$e^a_\mu(s) \simeq \delta^a_\mu e^{-ks}$$

$$\bar{p}'_\mu(s) \simeq e^{-ks}\bar{p}_\mu$$

$$T'^\mu(s) \simeq e^{ks}T^\mu$$

$$dS^2 \simeq e^{-2ks}dx_\mu dx^\mu - ds^2 \equiv 0$$

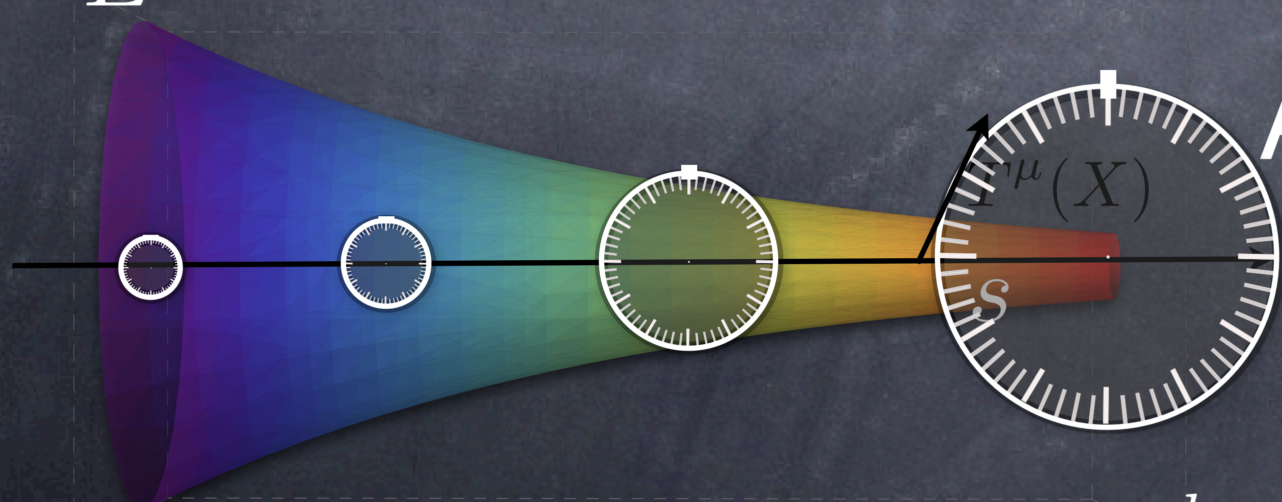
Time periodicity - Conformal parameter

$$T_t(s) = \frac{h}{\bar{E}(s)} \equiv z(s) = \frac{e^{ks}}{ck}$$

Asymptotic freedom

$$\frac{1}{e^2_{eff}(q)} \simeq \frac{1}{e^2} - \frac{N_c}{12\pi^2}\log\frac{q}{\Lambda}$$

E^{UV} $\qquad$ E^{IR}

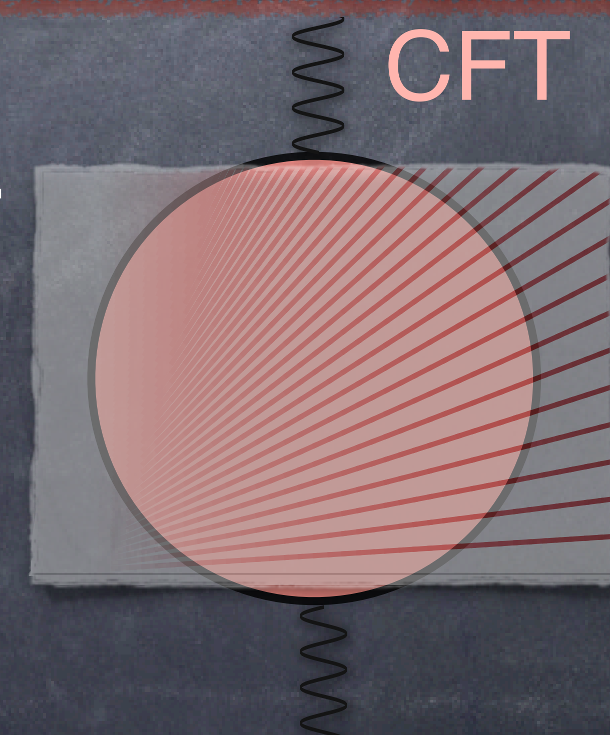


$$T_t^{UV} = \frac{h}{\Lambda} = \frac{e^{ks^{UV}}}{ck} \qquad T_t^{IR} = \frac{h}{\mu} = \frac{e^{ks^{IR}}}{ck}$$

AdS/CFT

$\simeq$

CFT



Massless spectrum

Basic AdS/QCD

Satz:'08; Magas:'03

Quark-Gluon plasma freeze out

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Virtual AdS metric

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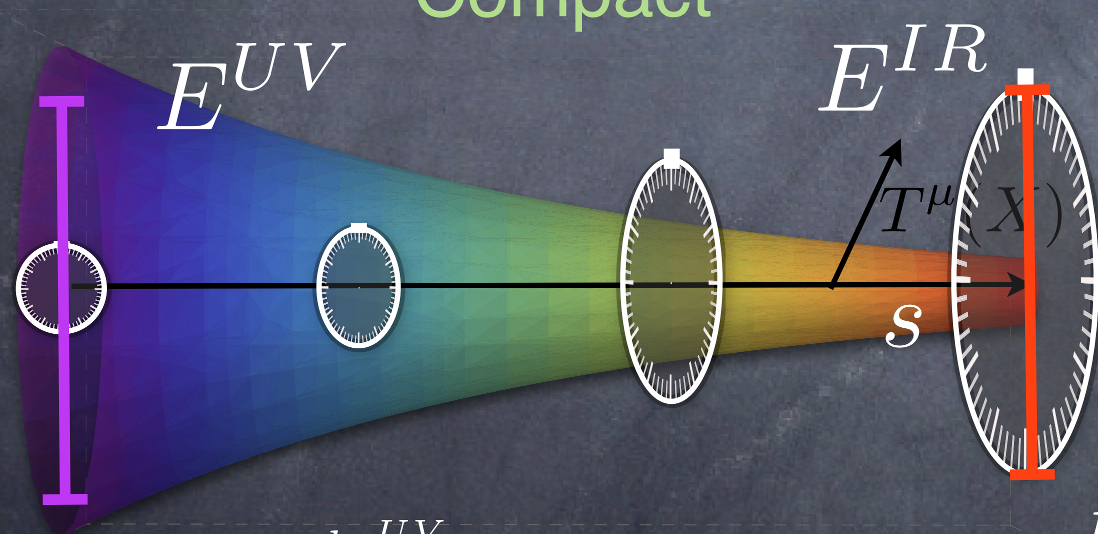
$$dS^2 \simeq e^{-2ks}dx_\mu dx^\mu - ds^2 \equiv 0$$

$$T_t(s) = \frac{h}{\bar{E}(s)} \equiv z(s) = \frac{e^{ks}}{ck}$$

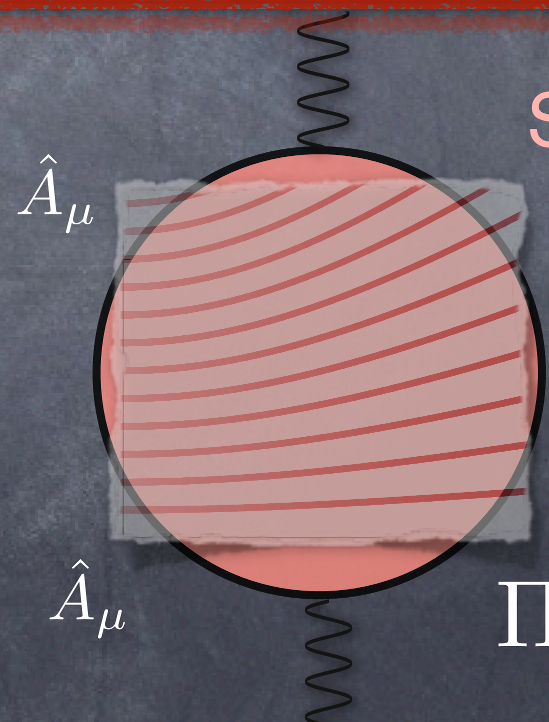
Compact



$$T_t^{UV} = \frac{h}{\Lambda} = \frac{e^{ks^{UV}}}{ck}$$

$$T_t^{IR} = \frac{h}{\mu} = \frac{e^{ks^{IR}}}{ck}$$

AdS/QCD $\simeq$



SCFT

Mass spectrum

$\Pi_{VV}(q^2)$

$$\Pi_{Holo}(q^2) \sim \Pi_{VV}(q^2)$$

WSR, hadron masses and couplings, QCD spectral functions

Massive dispersion relation -> deformation of the conformal virtual metric (dilaton, Soft-wall, ...)

Georgi:2007, Stephanov:2007, Cacciapaglia:2008
Pomarol:2000, Arkani-Hamed:2000, Son:2003, Erlich:2006
Gherghetta:2011

Conclusions

Interpretation of elementary particles in terms of relativistic modulations of 4-periodicity

$$c\bar{p}_\mu T^\mu = h$$

[de Broglie:1923]

↓

Full consistence with Special and General Relativity
Correspondence with ordinary Relativistic Quantum Mechanics (QFT)
Geometrodynamical description of interactions (including gauge interaction)
Dualism with extra dimensional theories and
Unconventional interpretation of AdS/CFT phenomenology
...

See more results in my papers...

--> **Classical to quantum correspondence in a VXD** D.D.; *Ann.Phys.*,on-line (2012); arXiv:1110.0316
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--> **Quantum Mechanics for Periodic Dynamics: the bosonic case** D.D., *AIP Conf.Proc.* 1232 (2010); arXiv:1001.2718