Predictive Monte Carlo tools for the LHC

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Lecture II

PLAN

- Basics : LO predictions and event generation
- Fixed-order calculations : from NLO to NNLO
- Exclusive predictions : Parton Shower
- Merging ME+PS
- Matching NLO with PS

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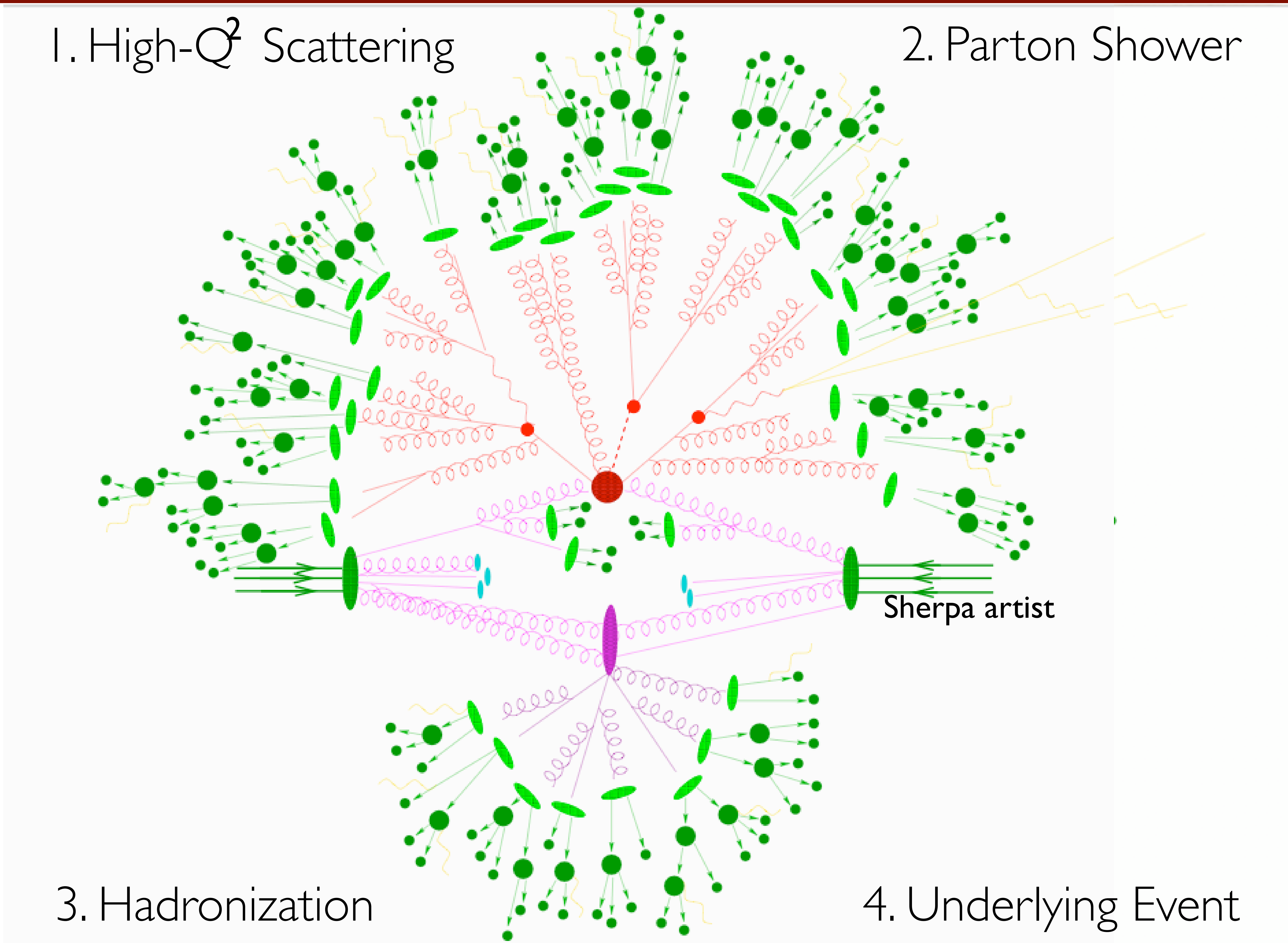
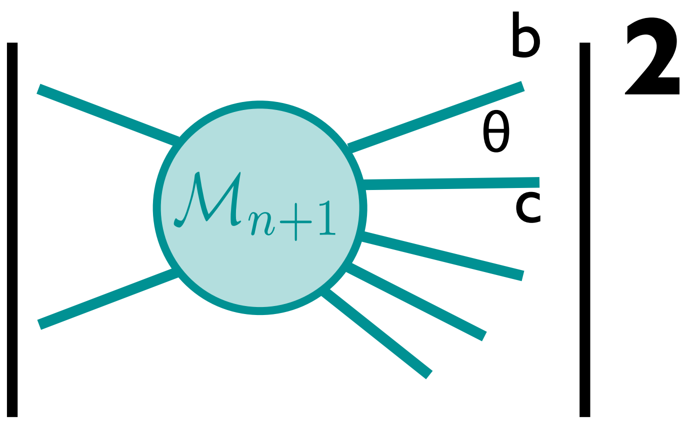
Today


1. High-Q^2 Scattering
2. Parton Shower
3. Hadronization
4. Underlying Event
Sherpa artist

Parton shower

- We need to be able to describe an arbitrarily number of parton branchings, i.e. we need to 'dress' partons with radiation
- This effect should be unitary: the inclusive cross section shouldn't change when extra radiation is added
- Remember that parton-level cross sections for a hard process are inclusive in anything else.
  E.g. for LO Drell-Yan production **all** radiation is included via PDFs (apart from non-perturbative power corrections)
- And finally we want to turn partons into hadrons (hadronization)....

Collinear factorization



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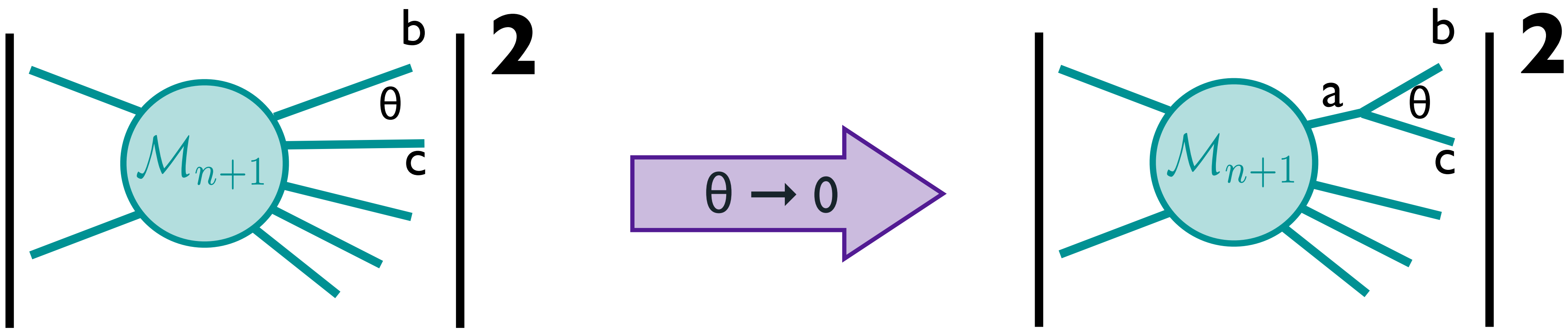
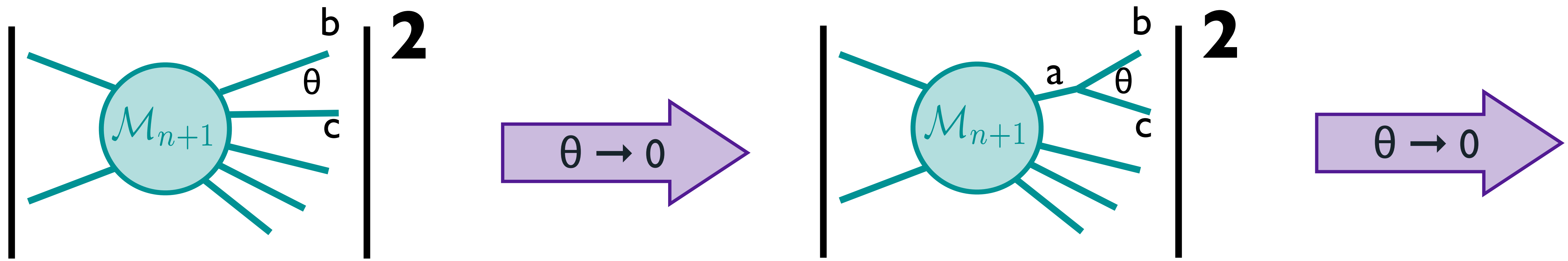


$$\left| \mathcal{M}_{n+1} \right|^2 \xrightarrow{\theta \to 0} \left| \mathcal{M}_{n+1} \right|^2$$

Collinear factorization



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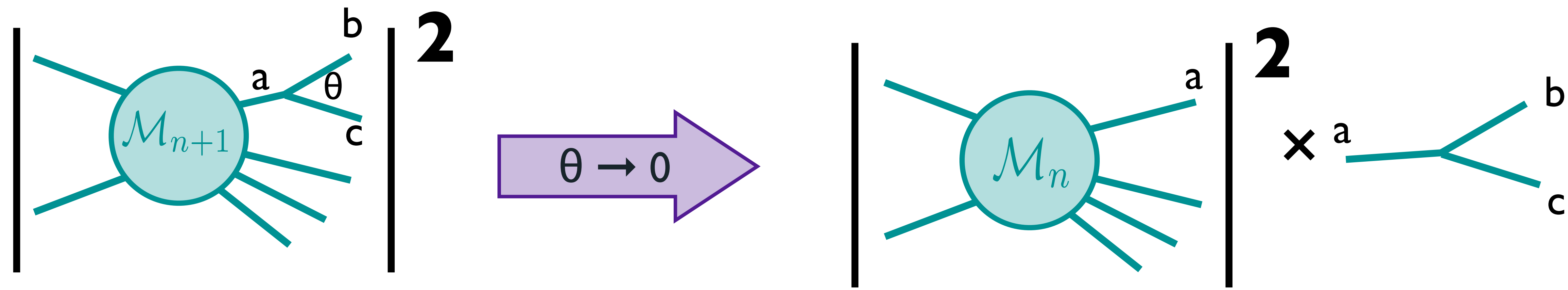

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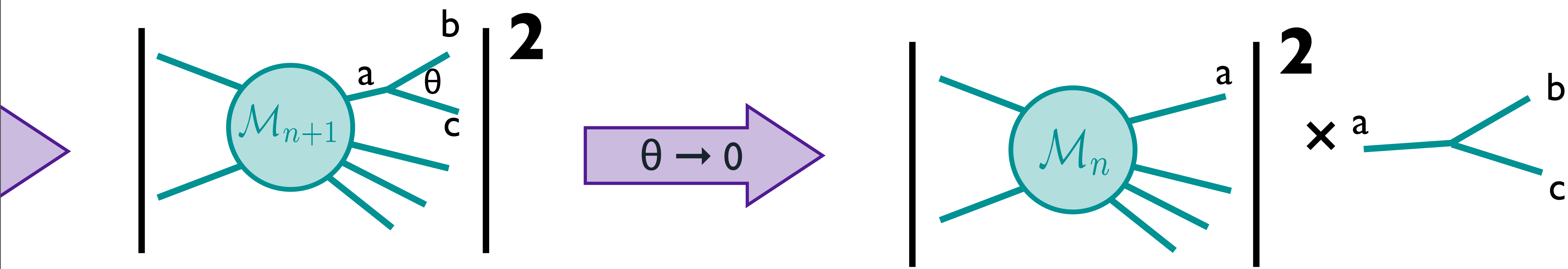
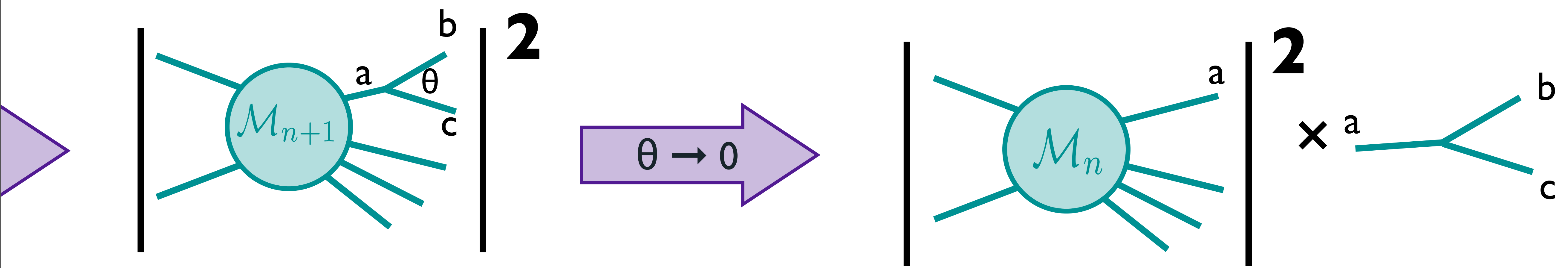


- Consider a process for which two particles are separated by a small angle θ.

Collinear factorization



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- In the limit of θ → 0 the contribution is coming from a single parent particle going on shell: therefore its branching is related to time scales which are very long with respect to the hard subprocess.

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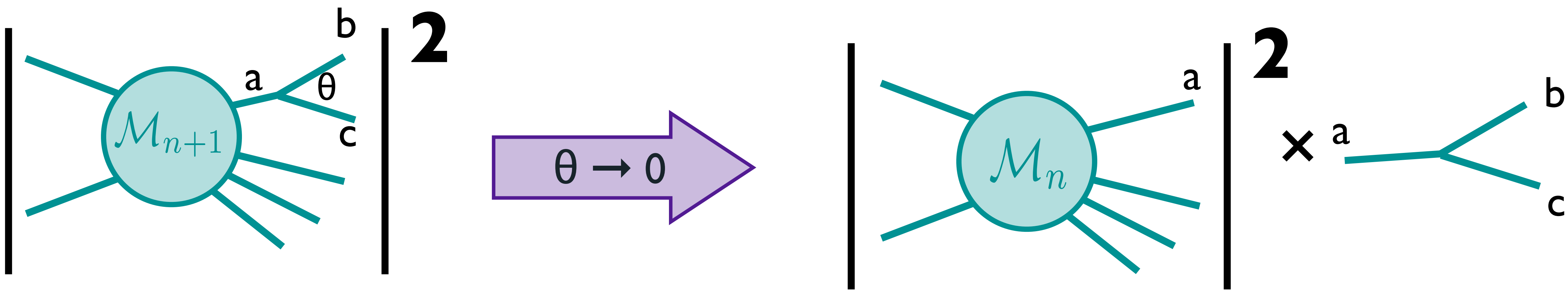

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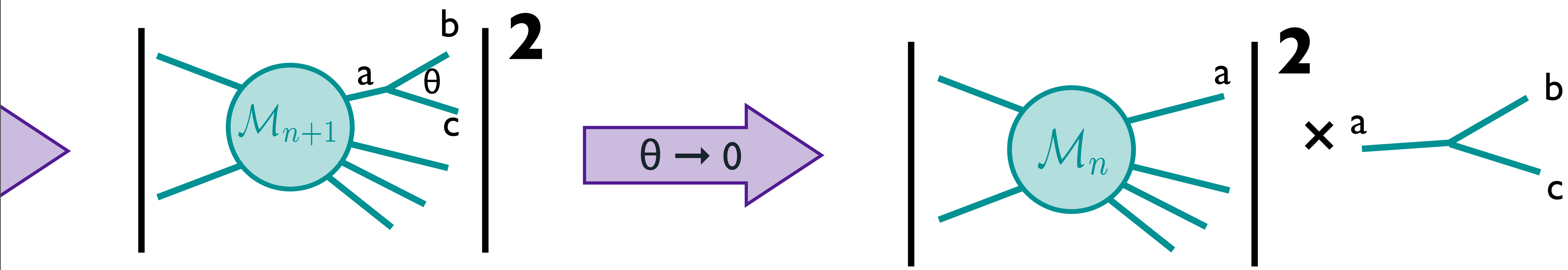

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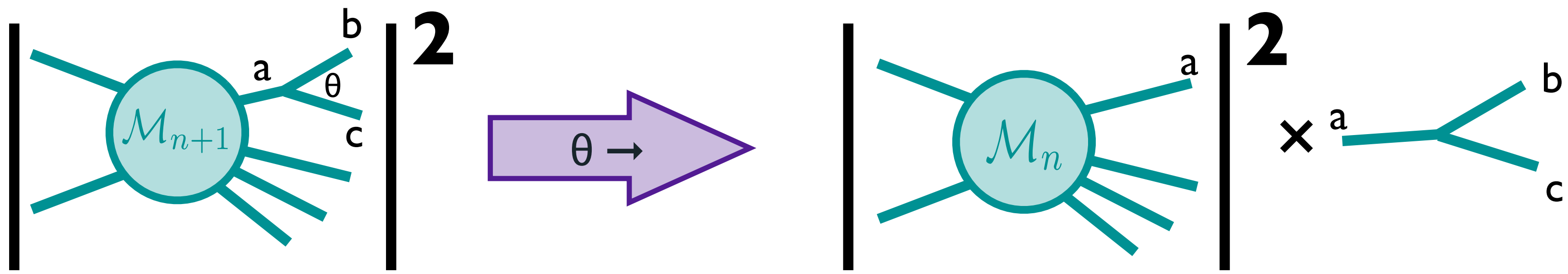
- The inclusion of such a branching cannot change the picture set up by the hard process: the whole emission process must be writable in this limit as the simpler one times a branching probability.
- The first task of Monte Carlo physics is to make this statement quantitative.

Collinear factorization



- The process factorizes in the collinear limit. This procedure it universal!

$$|\mathcal{M}_{n+1}|^2 d\Phi_{n+1} \simeq |\mathcal{M}_n|^2 d\Phi_n \frac{dt}{t} dz \frac{d\phi}{2\pi} \frac{\alpha_S}{2\pi} P_{a\to bc}(z)$$

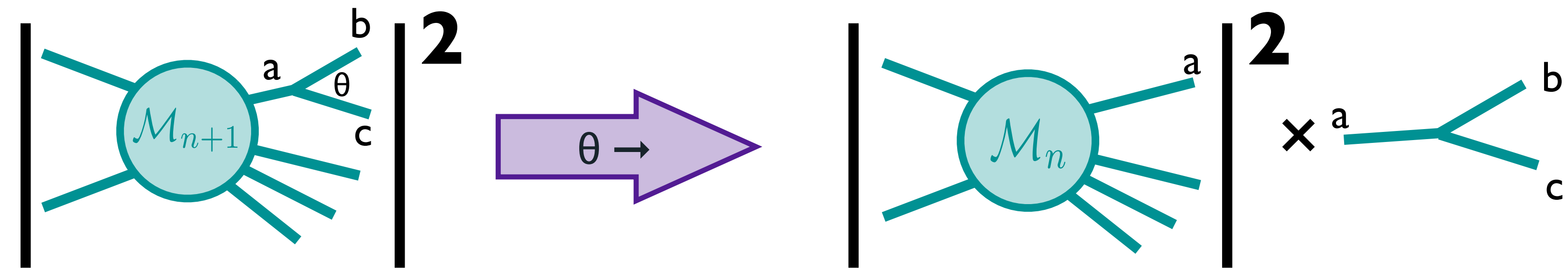
- Notice that what has been roughly called 'branching probability' is actually a singular factor, so one will need to make sense precisely of this definition.

- At the leading contribution to the (n+1)-body cross section the Altarelli-Parisi splitting kernels are defined as:

$$P_{g\to qq}(z) = T_R\left[z^2 + (1-z)^2\right], \qquad P_{g\to gg}(z) = C_A\left[z(1-z) + \frac{z}{1-z} + \frac{1-z}{z}\right],$$

$$P_{q\to qg}(z) = C_F\left[\frac{1+z^2}{1-z}\right], \qquad P_{q\to gq}(z) = C_F\left[\frac{1+(1-z)^2}{z}\right].$$

Collinear factorization



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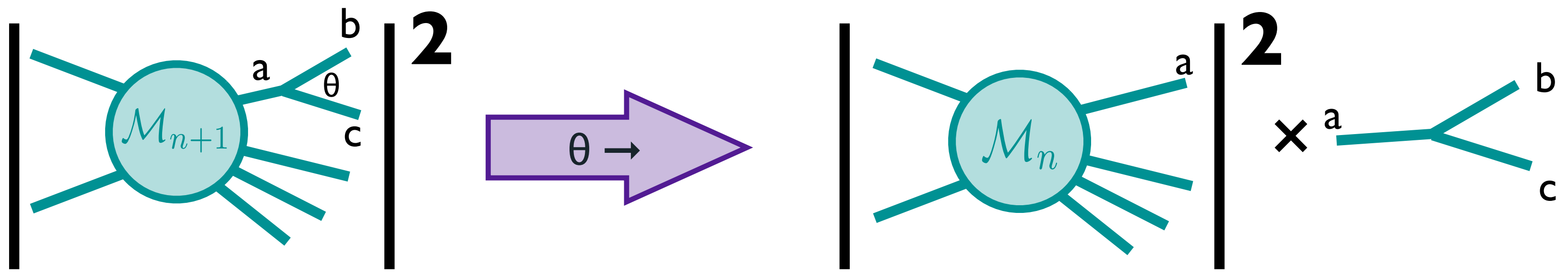
- **t** can be called the 'evolution variable' (will become clearer later): it can be the virtuality $\mathbf{m^2}$ of particle a or its $\mathbf{p_T}^2$ or $\mathbf{E^2\theta^2}$ ...
  - It represents the hardness of the branching and tends to 0 in the collinear limit.

- Indeed in the collinear limit one has: so that the factorization takes place for all these definitions:

$$m^2 \simeq z(1-z)\theta^2 E_a^2$$
$$p_T^2 \simeq z m^2$$

$$d\theta^2/\theta^2 = dm^2/m^2 = dp_T^2/p_T^2$$

Collinear factorization



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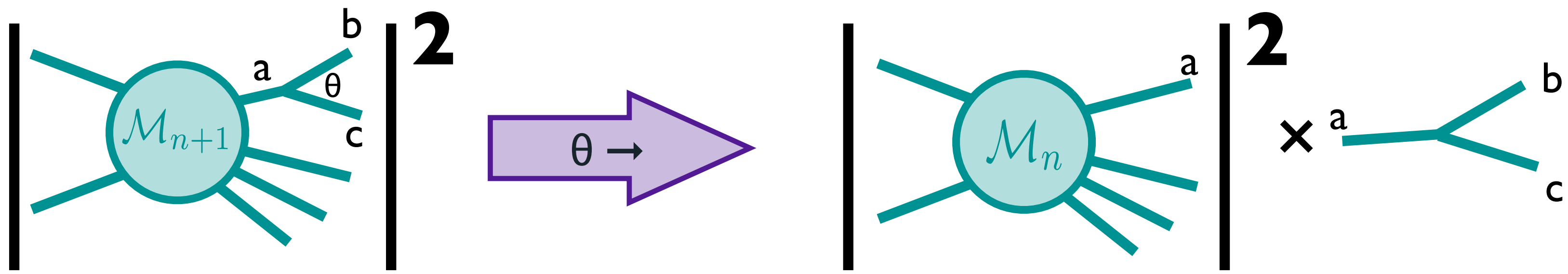
- **z** is the "energy variable": it is defined to be the energy fraction taken by parton **b** from parton **a.** It represents the energy sharing between **b** and **c** and tends to 1 in the soft limit (parton c going soft)
- **Φ** is the azimuthal angle. It can be chosen to be the angle between the polarization of a and the plane of the branching.

Collinear factorization



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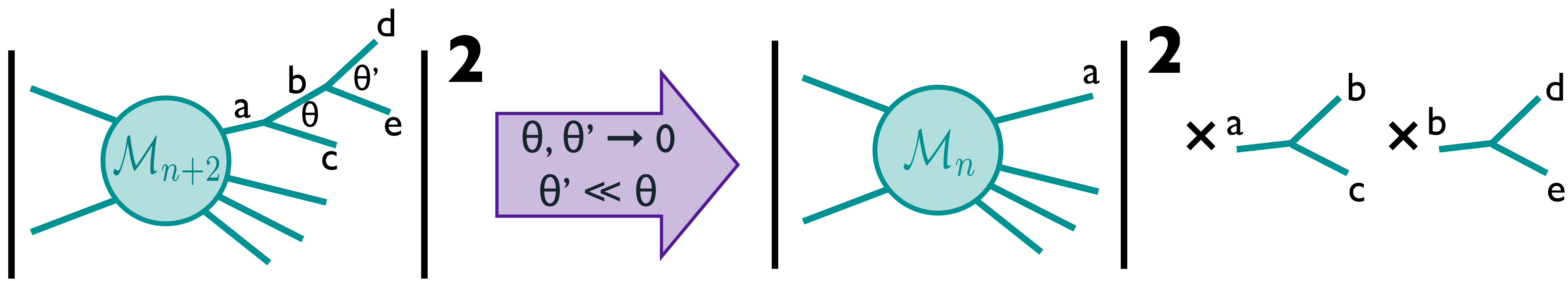
- This is an amplitude squared: naively one would maybe expect **1/t²** dependence. Why is the square not there?
  - It's due to angular-momentum conservation. E.g., take the splitting **q ⟶ qg**: helicity is conserved for the quarks, so the final state spin differs by one unity with respect to the initial one. The scattering happens in a p-wave (orbital angular momentum equal to one), so there is a suppression factor as **t ⟶ 0**.
  - In fact, a factor **1/t** is always cancelled in an explicit computation

Multiple emission

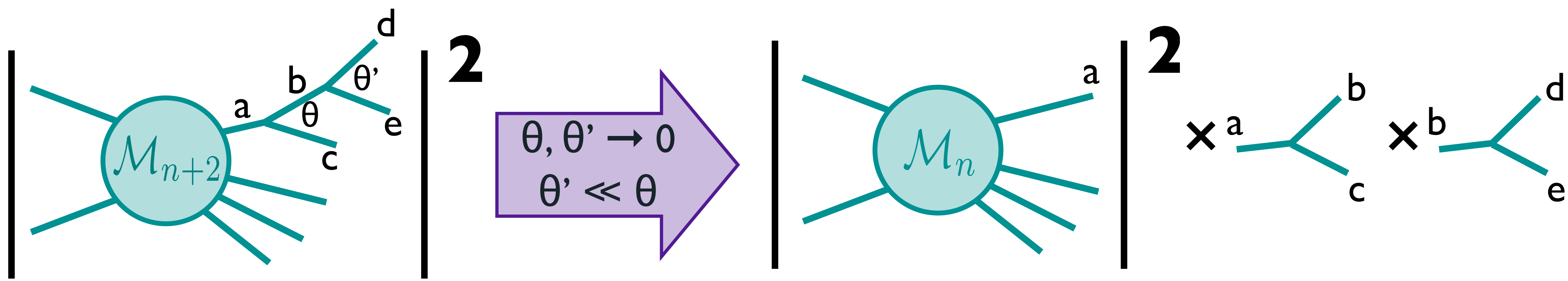


- Now consider $\mathbf{M_{n+1}}$ as the new core process and use the recipe we used for the first emission in order to get the dominant contribution to the (n+2)-body cross section: add a new branching at angle much smaller than the previous one:

$$|\mathcal{M}_{n+2}|^2 d\Phi_{n+2} \simeq |\mathcal{M}_n|^2 d\Phi_n \frac{dt}{t} dz \frac{d\phi}{2\pi} \frac{\alpha_S}{2\pi} P_{a\to bc}(z)$$
$$\times \frac{dt'}{t'} dz' \frac{d\phi'}{2\pi} \frac{\alpha_S}{2\pi} P_{b\to de}(z')$$

- This can be done for an arbitrary number of emissions. The recipe to get the leading collinear singularity is thus cast in the form of an iterative sequence of emissions whose probability does not depend on the past history of the system: a ‘Markov chain’. No interference!!!

Multiple emission



- The dominant contribution comes from the region where the subsequently emitted partons satisfy the strong ordering requirement: θ » θ’ » θ’’...
  For the rate for multiple emission we get

$$\sigma_{n+k} \propto \alpha_S^k \int_{Q_0^2}^{Q^2} \frac{dt}{t} \int_{Q_0^2}^{t} \frac{dt'}{t'} \dots \int_{Q_0^2}^{t^{(k-2)}} \frac{dt^{(k-1)}}{t^{(k-1)}} \propto \sigma_n \left(\frac{\alpha_S}{2\pi}\right)^k \log^k(Q^2/Q_0^2)$$

  where Q is a typical hard scale and Q_0 is a small infrared cutoff that separates perturbative from non perturbative regimes.

- Each power of α_s comes with a logarithm. The logarithm can be easily large, and therefore it can lead to a breakdown of perturbation theory.

Absence of interference

- The collinear factorization picture gives a branching sequence for a given leg starting from the hard subprocess all the way down to the non-perturbative region.
- Suppose you want to describe two such histories from two different legs: these two legs are treated in a completely uncorrelated way. And even within the same history, subsequent emissions are uncorrelated.
- The collinear picture completely misses the possible interference effects between the various legs. The extreme simplicity comes at the price of quantum inaccuracy.
- Nevertheless, the collinear picture captures the leading contributions: it gives an excellent description of an arbitrary number of (collinear) emissions:
    - it is a "resummed computation"
    - it bridges the gap between fixed-order perturbation theory and the non-perturbative hadronization.

Sudakov form factor

The differential probability for the branching **a ⟶ bc** between scales **t** and **t+dt** knowing that no emission occurred before:

$$dp(t) = \sum_{bc} \frac{dt}{t} \int dz \frac{d\phi}{2\pi} \frac{\alpha_S}{2\pi} P_{a\to bc}(z)$$

The probability that a parton does NOT split between the scales **t** and **t+dt** is given by **1-dp(t).**

Probability that particle a does not emit between scales **Q²** and **t**

$$\Delta(Q^2,t) = \prod_k \left[1 - \sum_{bc} \frac{dt_k}{t_k} \int dz \frac{d\phi}{2\pi} \frac{\alpha_S}{2\pi} P_{a\to bc}(z)\right] =$$

$$\exp\left[-\sum_{bc} \int_t^{Q^2} \frac{dt'}{t'} dz \frac{d\phi}{2\pi} \frac{\alpha_S}{2\pi} P_{a\to bc}(z)\right] = \exp\left[-\int_t^{Q^2} dp(t')\right]$$

- **Δ(Q²,t)** is the Sudakov form factor
  - Property: **Δ(A,B) = Δ(A,C) Δ(C,B)**

Parton shower

- The Sudakov form factor is the heart of the parton shower. It gives the probability that a parton does not branch between two scales
- Using this no-emission probability the branching tree of a parton is generated.
- Define **dP_k** as the probability for k ordered splittings from leg a at given scales

$$\begin{aligned} dP_1(t_1) &= \Delta(Q^2,t_1)\; dp(t_1)\Delta(t_1,Q_0^2), \\ dP_2(t_1,t_2) &= \Delta(Q^2,t_1)\; dp(t_1)\; \Delta(t_1,t_2)\; dp(t_2)\; \Delta(t_2,Q_0^2)\Theta(t_1-t_2), \\ \ldots &= \ldots \\ dP_k(t_1,\ldots,t_k) &= \Delta(Q^2,Q_0^2)\prod_{l=1}^{k} dp(t_l)\Theta(t_{l-1}-t_l) \end{aligned}$$

- **Q_0^2** is the hadronization scale (~1 GeV). Below this scale we do not trust the perturbative description for parton splitting anymore.
- This is what is implemented in a parton shower, taking the scales for the splitting **t_i** randomly (but weighted according to the no-emission probability).

Unitarity

$$dP_k(t_1, ..., t_k) \quad = \quad \Delta(Q^2, Q_0^2) \prod_{l=1}^{k} dp(t_l)\Theta(t_{l-1} - t_l)$$

- The parton shower has to be unitary (the sum over all branching trees should be 1). We can explicitly show this by integrating the probability for **k** splittings:

$$P_k \equiv \int dP_k(t_1, ..., t_k) = \Delta(Q^2, Q_0^2) \frac{1}{k!} \left[\int_{Q_0^2}^{Q^2} dp(t) \right]^k, \quad \forall k = 0, 1, ...$$

- Summing over all number of emissions

$$\sum_{k=0}^{\infty} P_k = \Delta(Q^2, Q_0^2) \sum_{k=0}^{\infty} \frac{1}{k!} \left[\int_{Q_0^2}^{Q^2} dp(t) \right]^k = \Delta(Q^2, Q_0^2) \exp \left[\int_{Q_0^2}^{Q^2} dp(t) \right] = 1$$

- Hence, the total probability is conserved

Cancellation of singularities

- We have shown that the showers is unitary. However, how are the IR divergences cancelled explicitly? Let's show this for the first emission: Consider the contributions from (exactly) 0 and 1 emissions from leg a:

$$\frac{d\sigma}{\sigma_n} = \Delta(Q^2, Q_0^2) + \Delta(Q^2, Q_0^2) \sum_{bc} dz \frac{dt}{t} \frac{d\phi}{2\pi} \frac{\alpha_S}{2\pi} P_{a\to bc}(z)$$

- Expanding to first order in α_s gives

$$\frac{d\sigma}{\sigma_n} \simeq 1 - \sum_{bc} \int_{Q_0^2}^{Q^2} \frac{dt'}{t'} dz \frac{d\phi}{2\pi} \frac{\alpha_S}{2\pi} P_{a\to bc}(z) + \sum_{bc} dz \frac{dt}{t} \frac{d\phi}{2\pi} \frac{\alpha_S}{2\pi} P_{a\to bc}(z)$$

- Same structure of the two latter terms, with opposite signs: cancellation of divergences between the approximate virtual and approximate real emission cross sections.

- The probabilistic interpretation of the shower ensures that infrared divergences will cancel for each emission. The cancellation of infinities comes simply out as the basic statement that probabilities are conserved

ARGUMENT OF α_S

- Each choice of argument for α_S is equally acceptable at the leading-logarithmic accuracy. However, there is a choice that allows one to resum certain classes of subleading logarithms.

- The higher order corrections to the partons splittings imply that the AP splitting kernels should be modified: $P_{a\to bc}(z) \longrightarrow P_{a\to bc}(z) + \alpha_s P'_{a\to bc}(z)$

For $g \longrightarrow gg$ branchings $P'_{a\to bc}(z)$ diverges as $-b_0 \log[z(1-z)]\ P_{a\to bc}(z)$
(just z or 1-z if quark is present)

- Recall the one-loop running of the strong coupling:

$$\alpha_S(Q^2) = \frac{\alpha_S(\mu^2)}{1+\alpha_S(\mu^2) b_0 \log\frac{Q^2}{\mu^2}} \sim \alpha_S(\mu^2)\left(1-\alpha_S(\mu^2) b_0 \log\frac{Q^2}{\mu^2}\right)$$

- We can therefore include the $P'(z)$ terms by choosing $p_T^2 \sim z(1-z)Q^2$ as argument of α_S:

$$\begin{aligned}\alpha_S(Q^2)\left(P_{a\to bc}(z)+\alpha_S(Q^2)P'_{a\to bc}\right) &= \alpha_S(Q^2)\left(1-\alpha_S(Q^2) b \log z(1-z)\right) P_{a\to bc}(z) \\ &\sim \alpha_S(z(1-z)Q^2) P_{a\to bc}(z)\end{aligned}$$

Choice of evolution parameter

$$\Delta(Q^2,t) = \exp\left[-\sum_{bc}\int_t^{Q^2}\frac{dt'}{t'}dz\frac{d\phi}{2\pi}\frac{\alpha_S}{2\pi}P_{a\to bc}(z)\right]$$

- There is a lot of freedom in the choice of evolution parameter t. It can be the virtuality m^2 of particle a or its $p_T{}^2$ or $E^2\theta^2$ ... For the collinear limit they are all equivalent
- However, in the soft limit ($z \longrightarrow 1$) they behave differently
- Can we chose it such that we get the correct soft limit?

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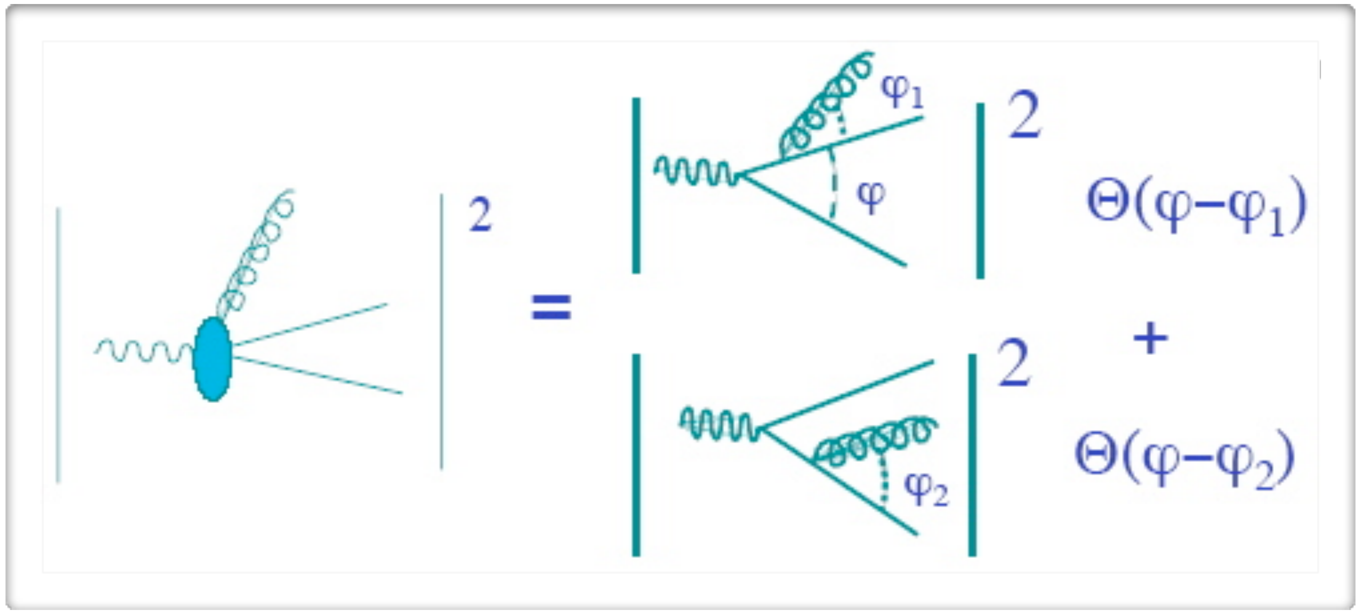
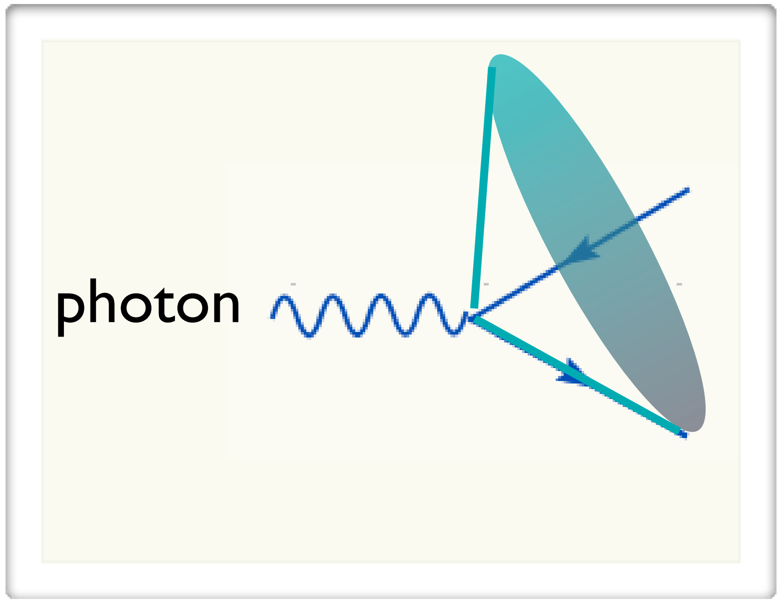
YES! It should be (proportional to) the angle θ

Angular ordering



Radiation inside cones around the orginal partons is allowed (and described by the eikonal approximation), outside the cones it is zero (after averaging over the azimuthal angle)



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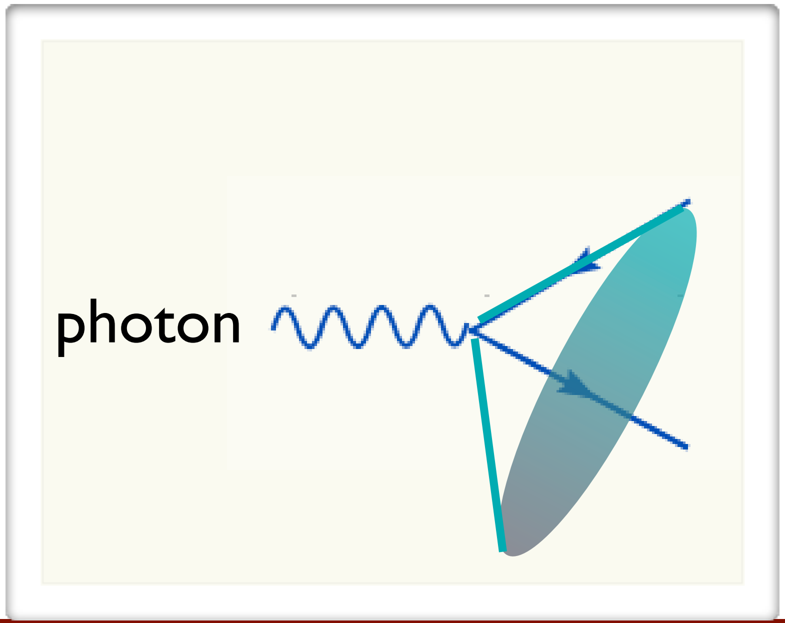
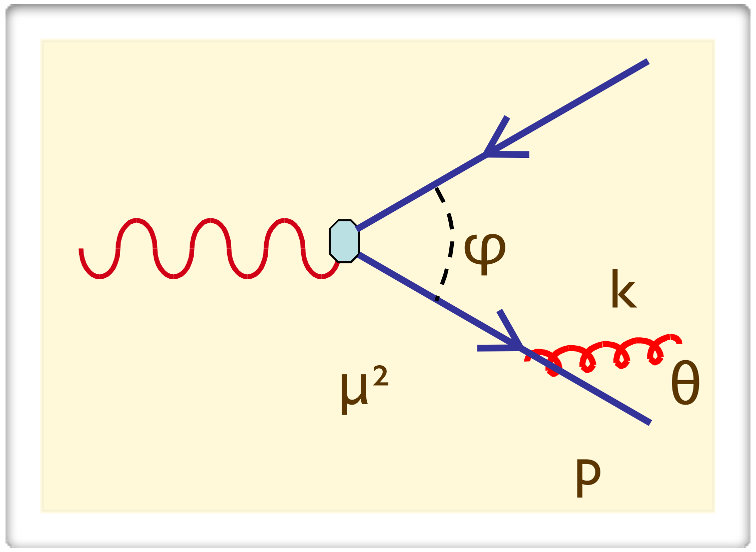


Intuitive explanation



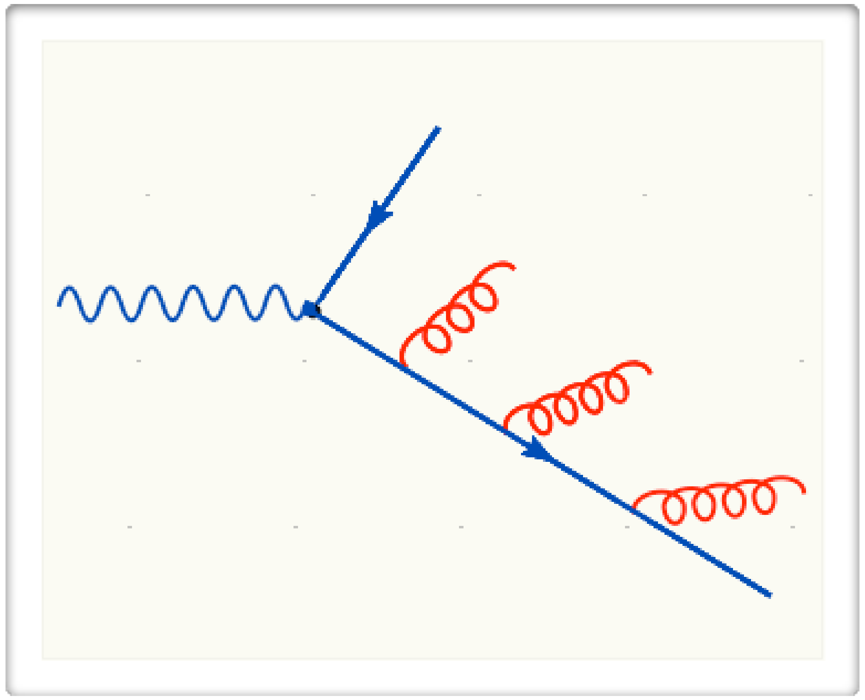
- Lifetime of the virtual intermediate state: $\tau < \gamma/\mu = E/\mu^2 = 1/(k_0\theta^2) = 1/(k_\perp\theta)$
- Distance between q and qbar after τ: $d = \varphi\tau = (\varphi/\theta)\ 1/k_\perp$

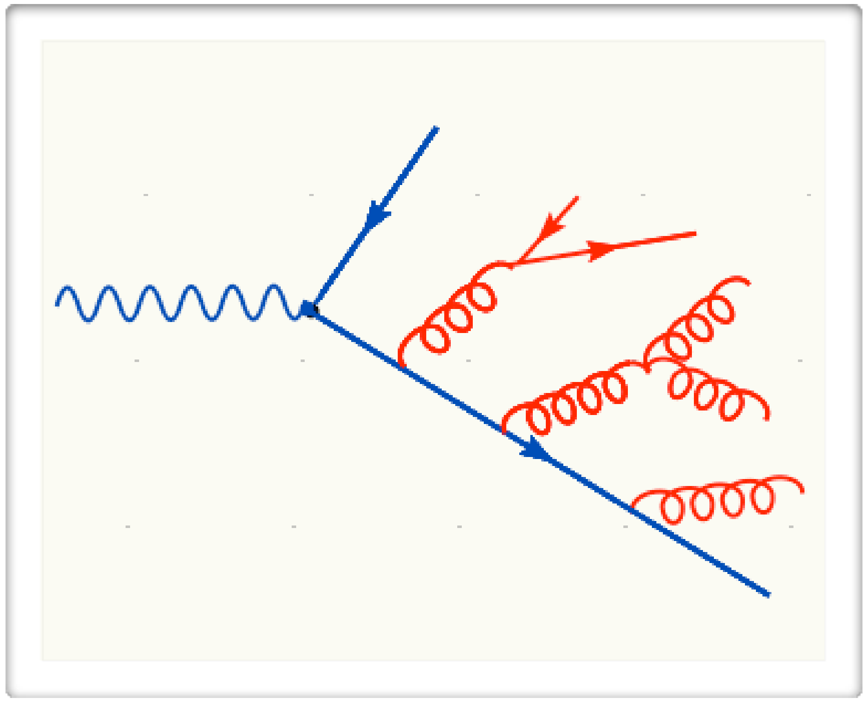
$$\mu^2 = (p+k)^2 = 2E\, k_0\, (1-\cos\theta) \sim E\, k_0\, \theta^2 \sim E\, k_\perp\, \theta$$

If the transverse wavelength of the emitted gluon is longer than the separation between q and qbar, the gluon emission is suppressed, because the q qbar system will appear as colour neutral (i.e. dipole-like emission, suppressed)

Therefore $d>1/k_\perp$, which implies $\theta < \varphi$.

ANGULAR ORDERING





- The construction can be iterated to the next emission, with the result that the emission angles keep getting smaller and smaller.
- One can generalize it to a generic parton of color charge Q_k splitting into two partons i and j, $Q_k=Q_i+Q_j$. The result is that inside the cones i and j emit as independent charges, and outside their angular-ordered cones the emission is coherent and can be treated as if it was directly from color charge Q_k.
- KEY POINT FOR THE MC!
- Angular ordering is automatically satisfied in θ ordered showers! (and easy to account for in p_T ordered showers).

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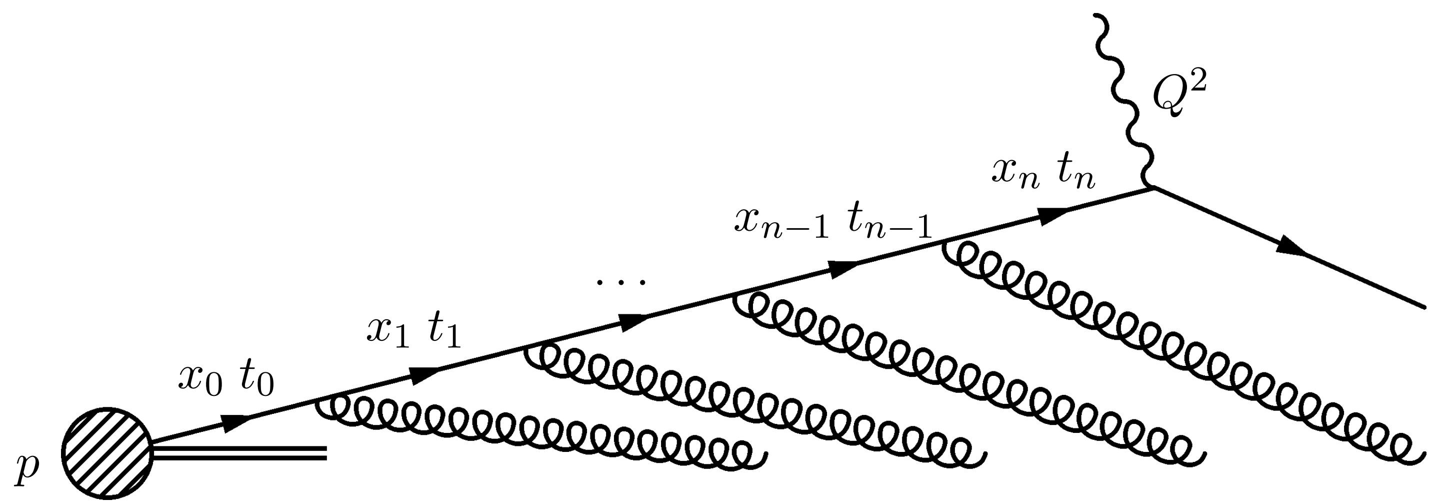
3. It is not an exclusive property of QCD (i.e., it is also present in QED) but in QCD produces very non-trivial effects, depending on how particles are color connected.

Initial-state parton splittings



- So far, we have looked at final-state (time-like) splittings. For initial state, the splitting functions are the same
- However, there is another ingredient: the parton density (or distribution) functions (PDFs). Naively: Probability to find a given parton in a hadron at a given momentum fraction **$x = p_z/P_z$** and scale **t**.

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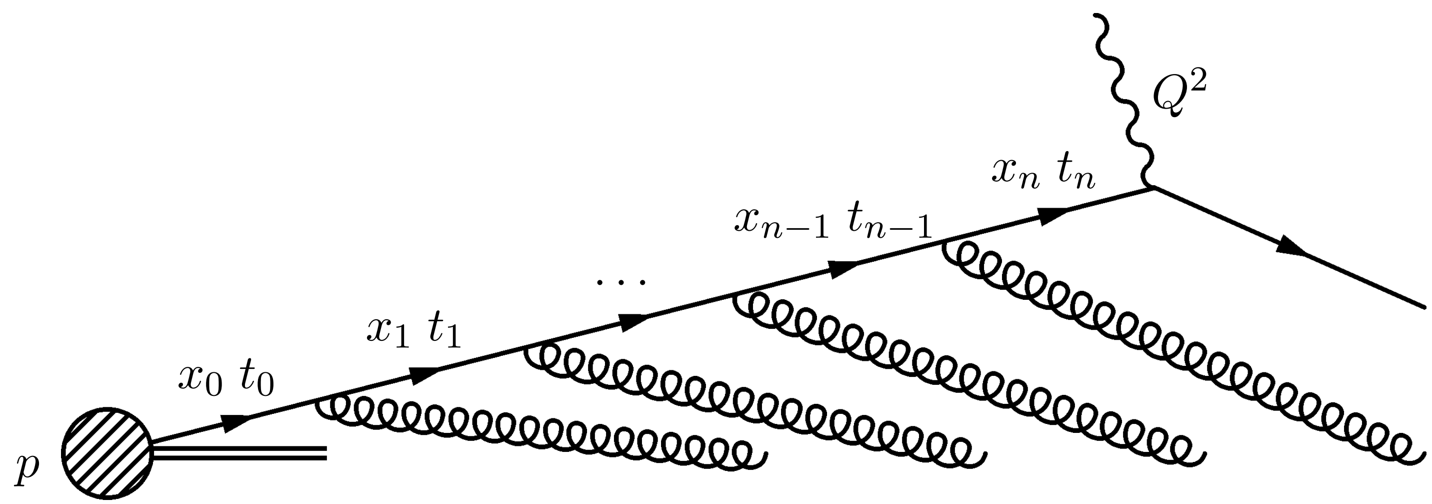

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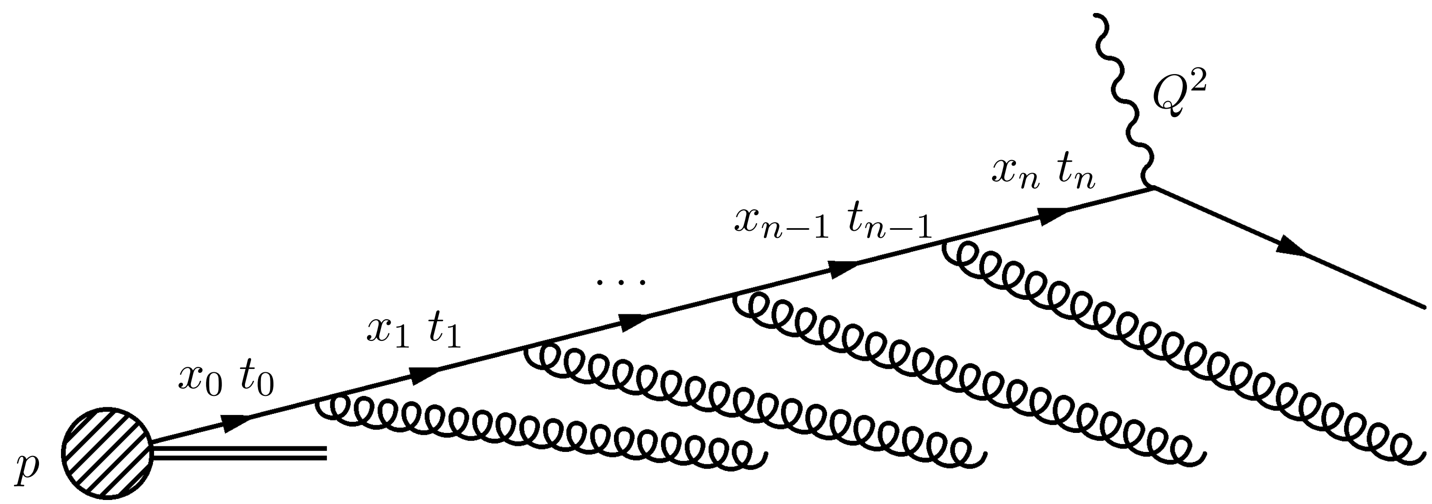

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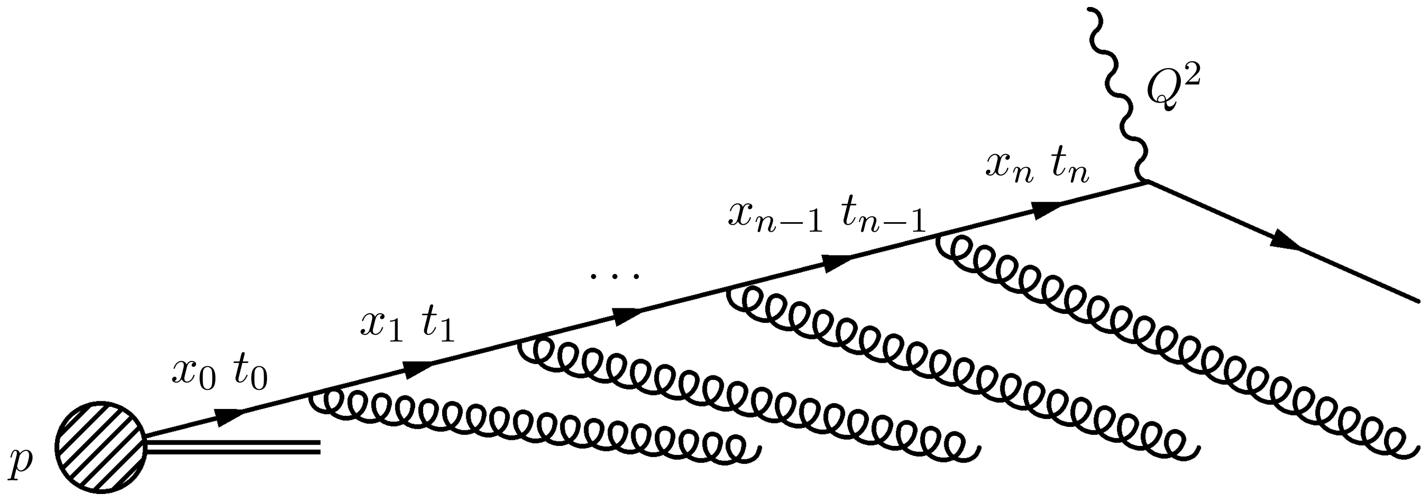
- How do the PDFs evolve with increasing **t**?

$$t\frac{\partial}{\partial t}f_i(x,t) = \int_x^1 \frac{dz}{z}\frac{\alpha_s}{2\pi}P_{ij}(z)f_j\left(\frac{x}{z},t\right) \qquad \text{DGLAP}$$

Initial-state parton splittings



- Start with a quark PDF **$f_0(x)$** at scale **t_0**. After a single parton emission, the probability to find the quark at virtuality **$t > t_0$** is

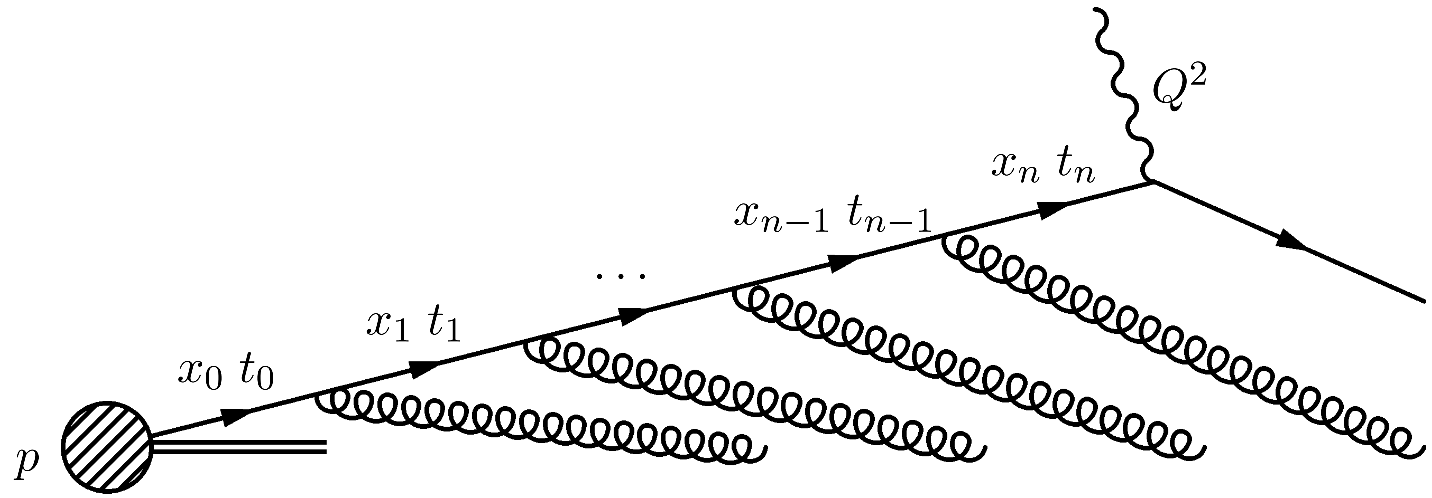
$$f(x,t) = f_0(x) + \int_{t_0}^{t} \frac{dt'}{t'} \frac{\alpha_s}{2\pi} \int_x^1 \frac{dz}{z} P(z) f_0\left(\frac{x}{z}\right)$$

- After a second emission, we have

$$f(x,t) = f_0(x) + \int_{t_0}^{t} \frac{dt'}{t'} \frac{\alpha_s}{2\pi} \int_x^1 \frac{dz}{z} P(z) \Big\{ f_0\left(\frac{x}{z}\right) + \int_{t_0}^{t'} \frac{dt''}{t''} \frac{\alpha_s}{2\pi} \int_{x/z}^1 \frac{dz'}{z'} P(z') f_0\left(\frac{x}{zz'}\right) \Big\}$$

f(x/z, t')

THE DGLAP EQUATION



- So for multiple parton splittings, we arrive at an integral-differential equation:

$$t\frac{\partial}{\partial t} f_i(x,t) = \int_x^1 \frac{dz}{z}\frac{\alpha_s}{2\pi} P_{ij}(z) f_j\left(\frac{x}{z}, t\right)$$

- This is the famous DGLAP equation (where we have taken into account the multiple parton species i, j). The boundary condition for the equation is the initial PDFs $f_{i0}(x)$ at a starting scale t_0 (around 2 GeV).

- These starting PDFs are fitted to experimental data.

Initial-state parton showers

- To simulate parton radiation from the initial state, we start with the hard scattering, and then "deconstruct" the DGLAP evolution to get back to the original hadron: backwards evolution!
  - i.e. we undo the analytic resummation and replace it with explicit partons (e.g. in Drell-Yan this gives non-zero p_T to the vector boson)
- In backwards evolution, the Sudakovs include also the PDFs -- this follows from the DGLAP equation and ensures conservation of probability:

$$\Delta_{Ii}(x,t_1,t_2) = \exp\left\{ -\int_{t_1}^{t_2} dt' \sum_j \int_x^1 \frac{dx'}{x'} \frac{\alpha_s(t')}{2\pi} P_{ij}\left(\frac{x}{x'}\right) \frac{f_i(x',t')}{f_j(x,t')} \right\}$$

This represents the probability that parton **i** will stay at the same **x** (no splittings) when evolving from $\mathbf{t_1}$ to $\mathbf{t_2}$.

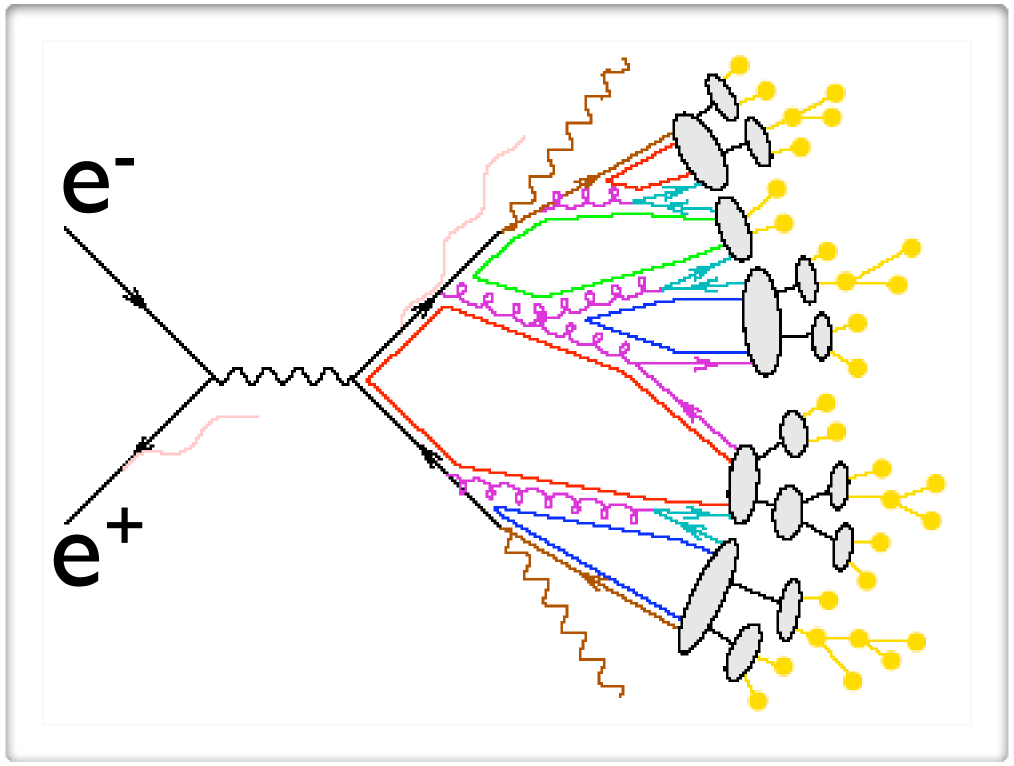
- The shower simulation is now done as in a final state shower!

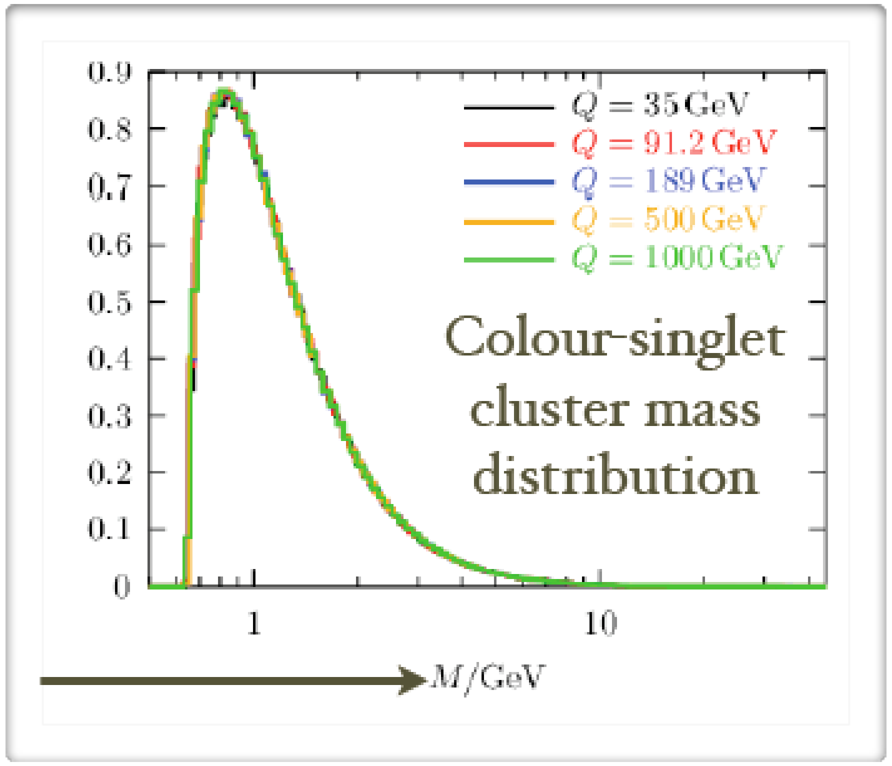
Hadronization

- The shower stops if all partons are characterized by a scale at the IR cut-off: $Q_0 \sim 1$ GeV.
- Physically, we observe hadrons, not (colored) partons.
- We need a non-perturbative model in passing from partons to colorless hadrons.
- There are two models (string and cluster), based on physical and phenomenological considerations.

Cluster model

The structure of the perturbative evolution including angular ordering, leads naturally to the clustering in phase-space of color-singlet parton pairs (preconfinement). Long-range correlations are strongly suppressed. Hadronization will only act locally, on low-mass color singlet clusters.





String model

From lattice QCD one sees that the color confinement potential of a quark-antiquark grows linearly with their distance: V(r) ~ kr, with k ~ 0.2 GeV. This is modeled with a string with uniform tension (energy per unit length) k that gets stretched between the qq pair.

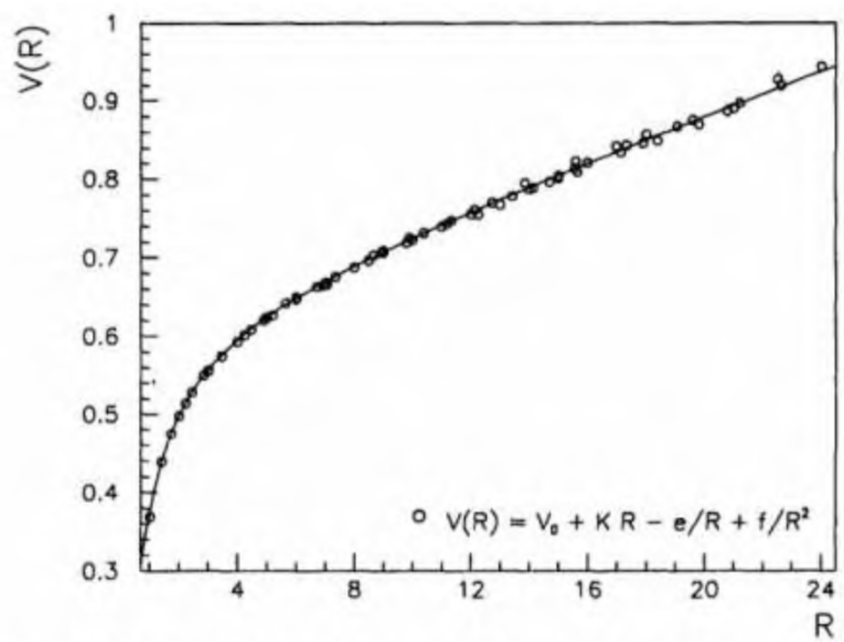
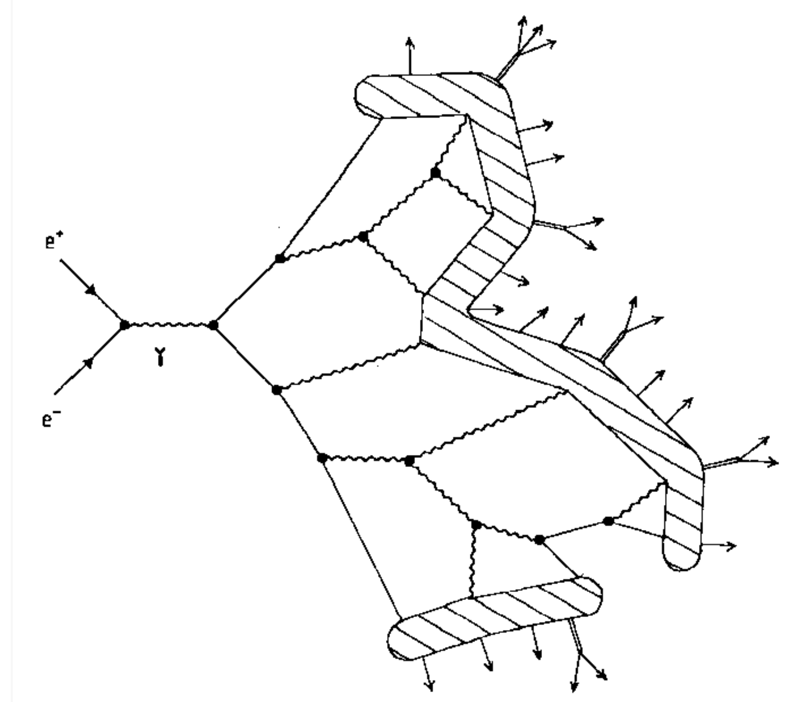
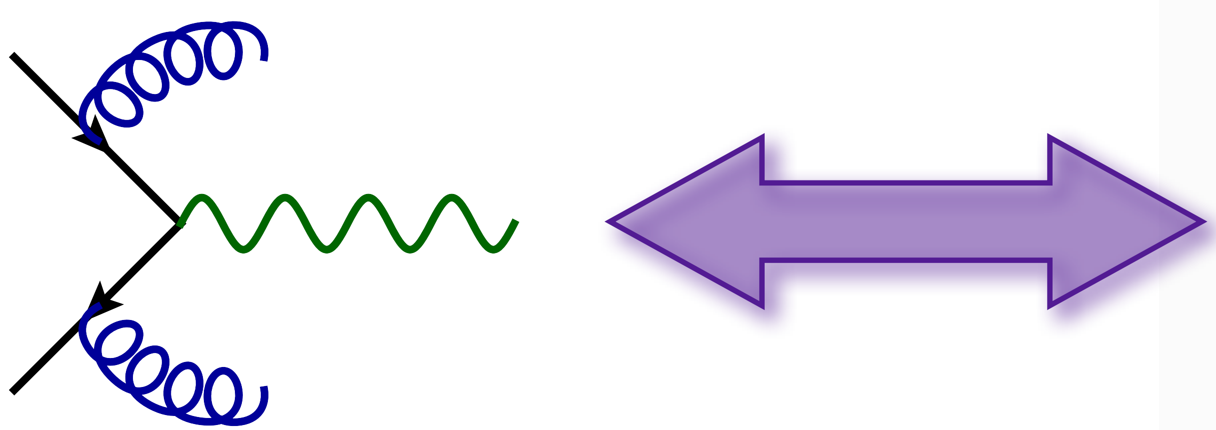


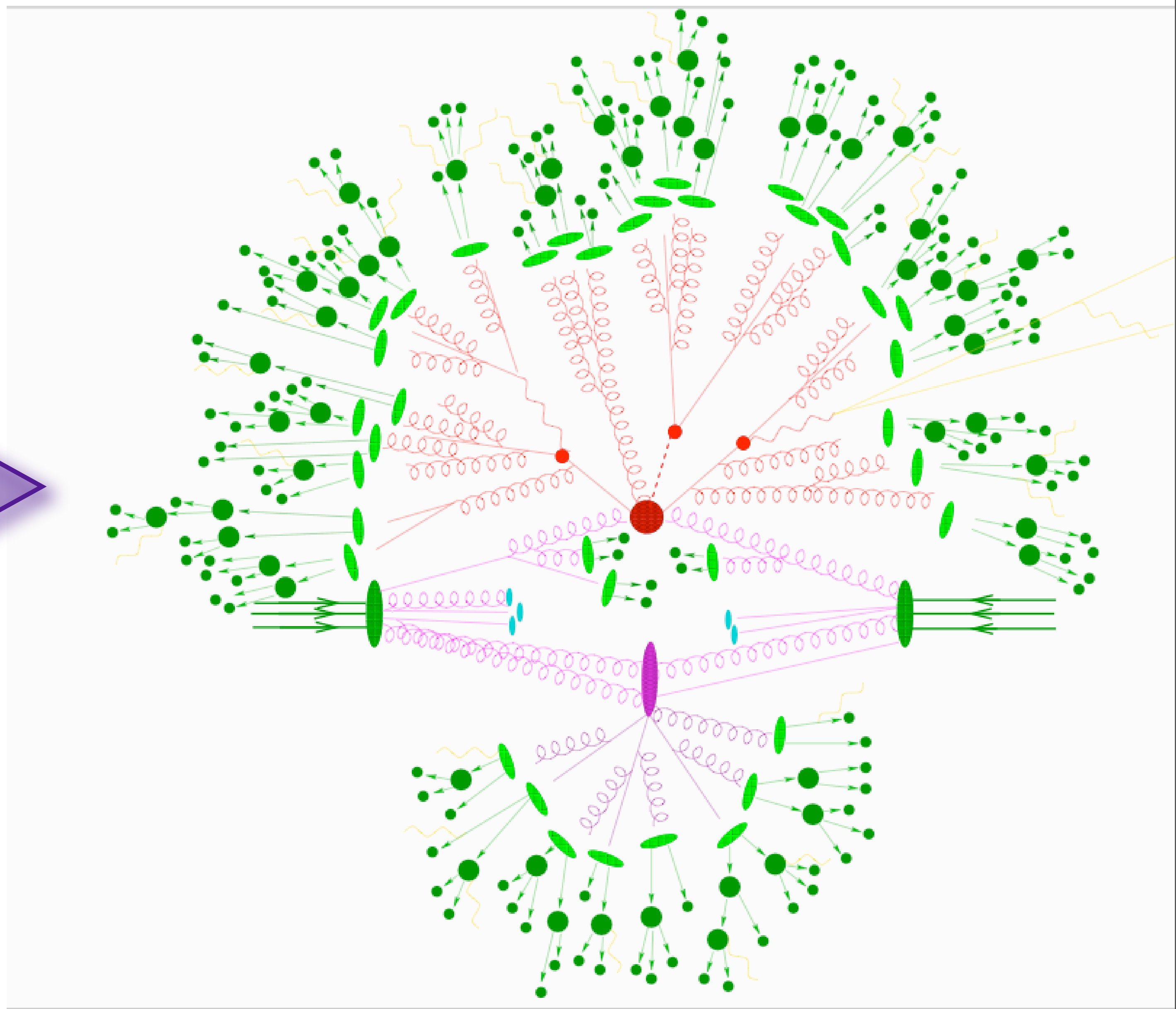
Fig. 2.9. QCD potential *vs.* R (in lattice units) from lattice QCD. Figure from ref. [23].



When quark-antiquarks are too far apart, it becomes energetically more favorable to break the string by creating a new qq pair in the middle.

Exclusive observable





A parton shower program associates one of the possible histories (and pre-histories in case of pp collisions) of an hard event in an explicit and fully detailed way, such that the sum of the probabilities of all possible histories is unity.

Parton shower MC event generators

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- General-purpose tools
- Always the first experimental choice
- Complete exclusive description of the events: hard scattering, showering & hadronization (and underlying event)
- Reliable and well-tuned tools
- Significant and intense progress in the development of new showering algorithms with the final aim to go at NLO in QCD

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Shower MC Generators: PYTHIA, HERWIG, SHERPA

Parton shower : summary

- The parton shower dresses partons with radiation. This makes the inclusive parton-level predictions (i.e. inclusive over extra radiation) completely exclusive
  - In the soft and collinear limits the partons showers are exact, but in practice they are used outside this limit as well.
  - Partons showers are universal (i.e. independent from the process)
- There is a cut-off in the shower (below which we don't trust perturbative QCD) at which a hadronization model takes over
  - Hadronization models are universal and independent from the energy of the collision

HERWIG

- All HERWIG versions implement the angular-ordering: subsequent emissions are characterized by smaller and smaller angles.

HERWIG 6: $$t = \frac{p_b \cdot p_c}{E_b E_c} \simeq 1 - \cos\theta$$

HERWIG++: $$t = \frac{(p_{b\perp})^2}{z^2(1-z)^2} = t(\theta)$$

- With angular ordering the parton shower does not populate the full phase space: empty regions of the phase space, called "dead zones", will arise.

- It may seem that the presence of dead zones is a weakness, but it is not so: they implement correctly the collinear approximation, in the sense that they constrain the shower to live uniquely in the region where it is reliable.
Matrix element corrections (MLM/CKKW matching) remove the dead-zones

- Hadronization: cluster model.

Pythia

- Choice of evolution variables for Fortran and C++ versions:

PYTHIA 6: $t = (p_b + p_c)^2 \sim z(1-z)\theta^2 E_a^2$

PYTHIA 8: $t = (p_b)^2_\perp$

- Simpler variables, but decreasing angles not guaranteed: PYTHIA rejects the events that do not respect the angular ordering. In practice equivalent to angular ordering (in particular for Pythia 8)

- Not implementing directly angular ordering, the phase space can be filled entirely (even without matrix element corrections), so one can have the so called "power shower" (use with a certain care: it uses the collinear/soft approximation for from the region where it is valid)

- Hadronization: string model.

SHERPA

- SHERPA uses a different kind of shower not based on the collinear $1 \longrightarrow 2$ branching, but on more complex $2 \longrightarrow 3$ elementary process: emission of the daughter off a color dipole

- The real emission matrix element squared is decomposed into a sum of terms $D_{ij,k}$ (dipoles) that capture the soft and collinear singularities in the limits i collinear to j, i soft (k is the spectator), and a factorization formula is deduced in the leading color approximation:

$$D_{ij,k} \rightarrow B \frac{\alpha_S}{p_i \cdot p_j} K_{ij,k}$$

- The shower is developed from a Sudakov form factor

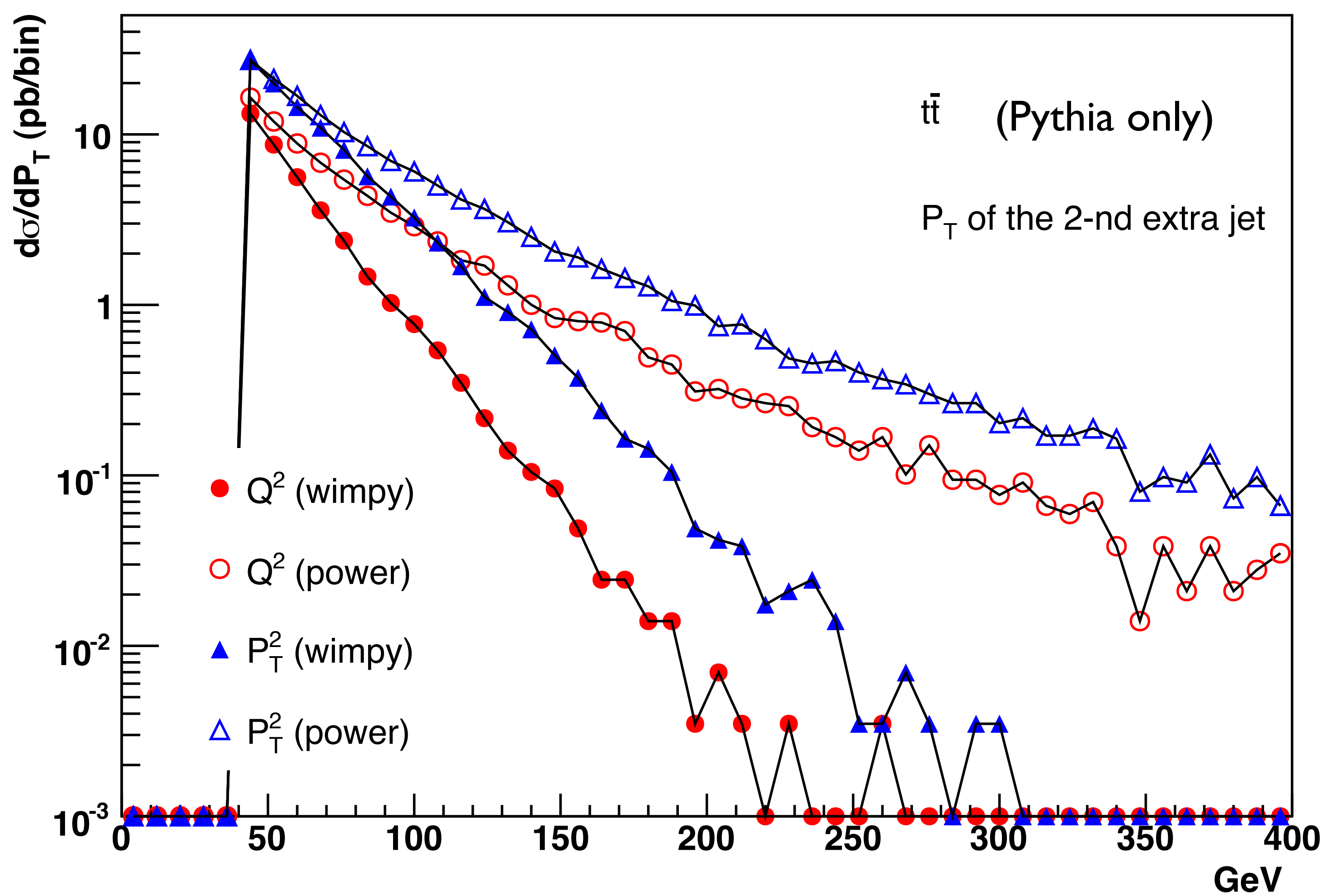
$$\Delta = \exp\left(-\int \frac{dt}{t} \int dz\, \alpha_S\, K_{ij,k}\right)$$

- It treats correctly the soft gluon emission off a color dipole, so angular ordering is built in.

- Hadronization: cluster model (default) and string model

Shower starting scale

Varying the shower starting scale ('wimpy' or 'power') and the evolution parameter ('Q^2' or 'p_T^2') a whole range of predictions can be made:



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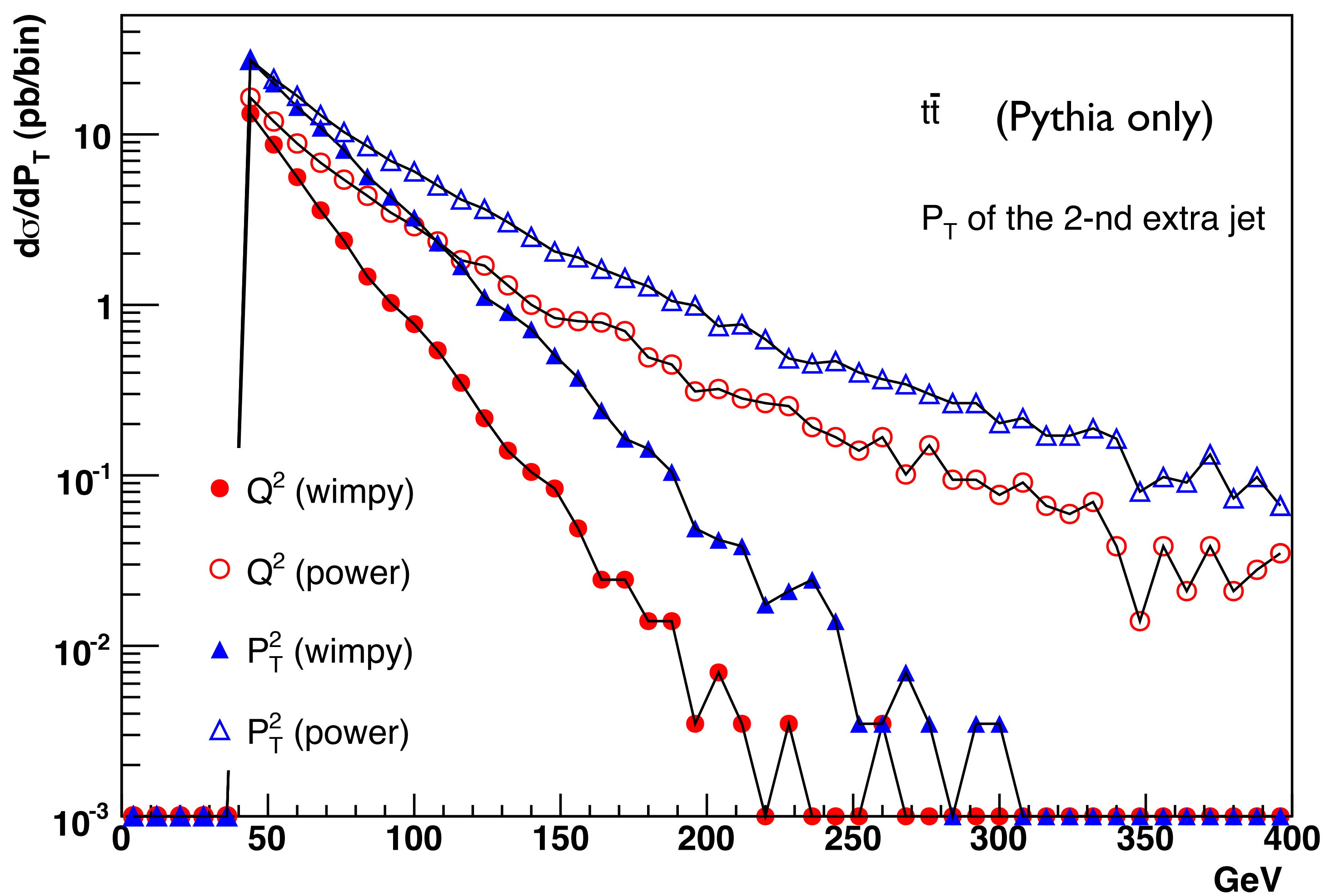

Ideal to describe the data: one can tune the parameters and fit it!
But is this really what we want...Does it work for other procs?

Predictive MC's

- There are better ways to describe hard radiation: matrix elements!
- There are two ways to improve a Parton Shower Monte Carlo event generator with matrix elements:
  - ME+PS merging: Include matrix elements with more final state partons to describe hard, well-separated radiation better
  - NLO+PS matching: Include full NLO corrections to the matrix elements to reduce theoretical uncertainties in the matrix elements. The real-emission matrix elements will describe the hard radiation

Merging ME+PS

Matrix elements vs. Parton showers

Matrix elements vs. Parton showers

ME

1. Fixed order calculation
2. Computationally expensive
3. Limited number of particles
4. Valid when partons are hard and well separated
5. Quantum interference correct
6. Needed for multi-jet description

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Approaches are complementary: merge them!

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Approaches are complementary: merge them!

Difficulty: avoid double counting, ensure smooth distributions

Goal for ME/PS merging

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- Regularization of matrix element divergence

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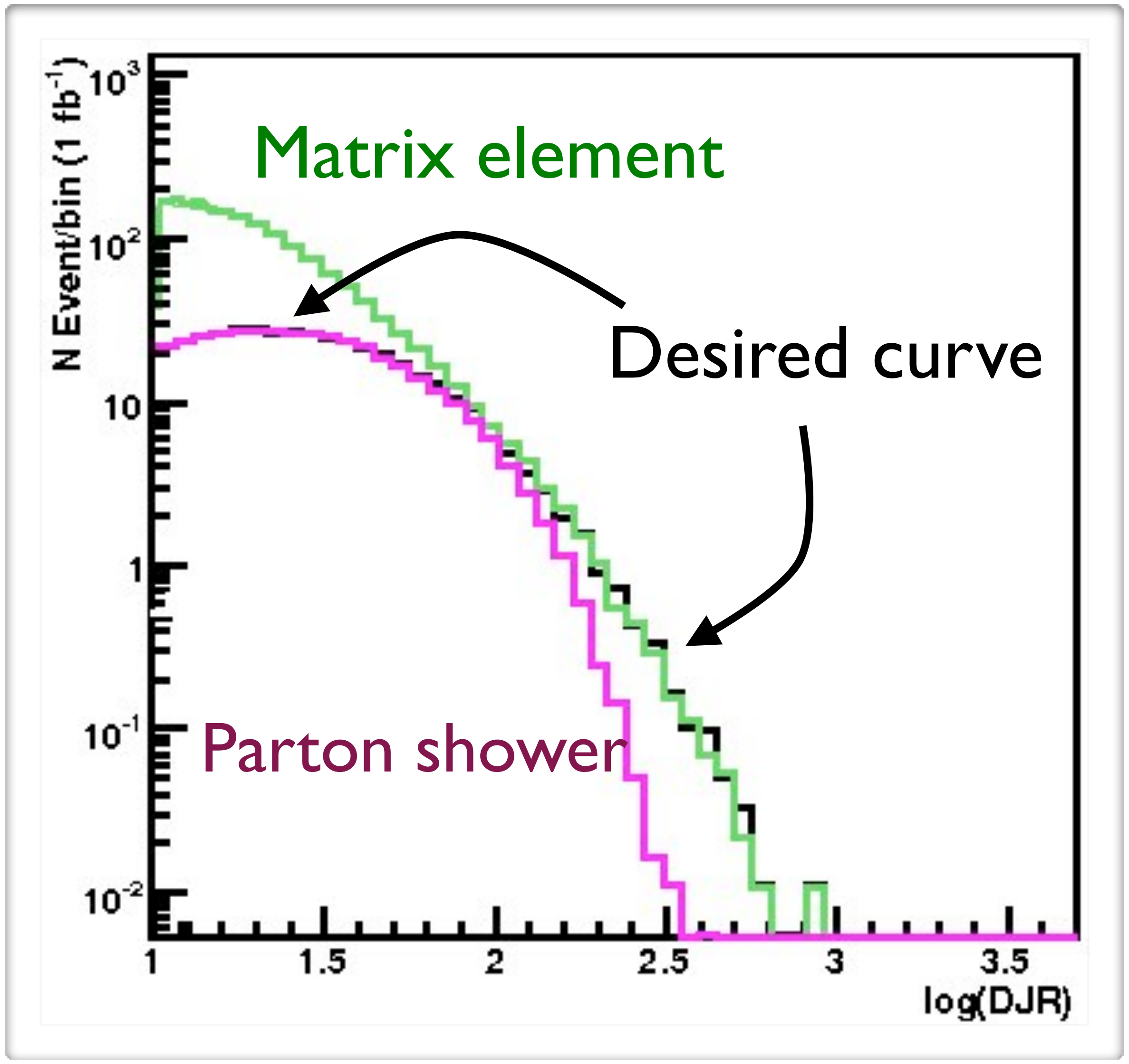
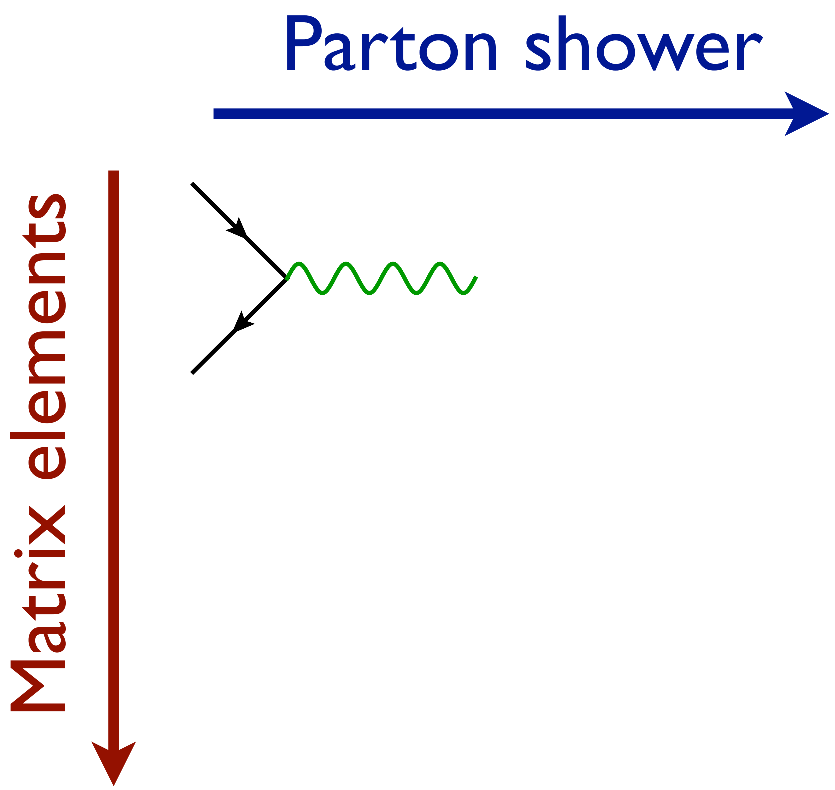


2nd QCD radiation jet in top pair production at the LHC

Possible double counting



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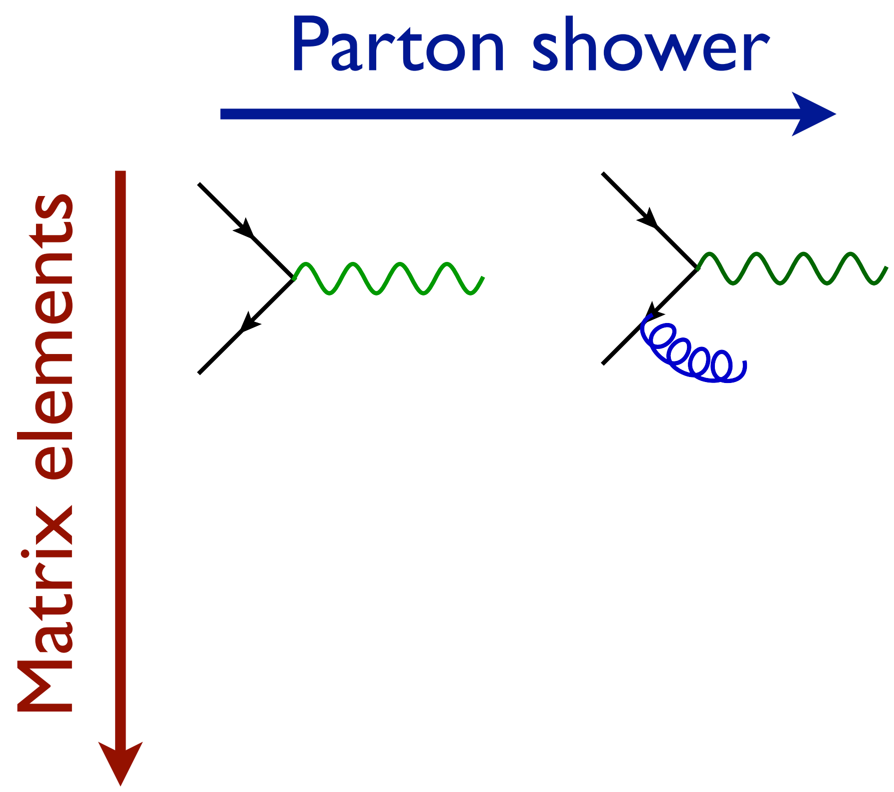

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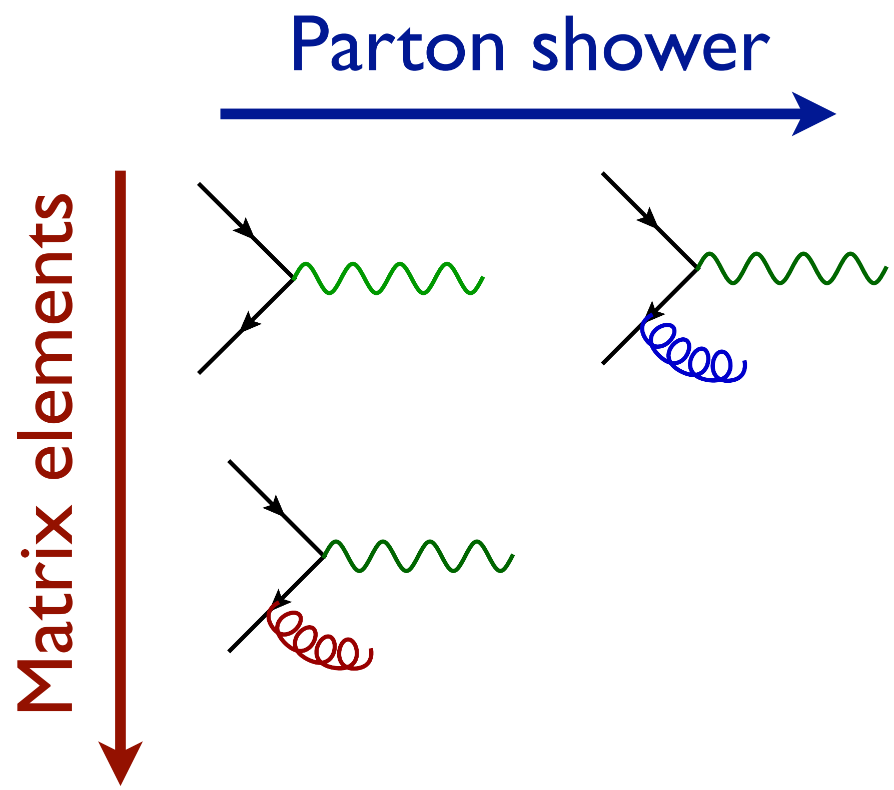

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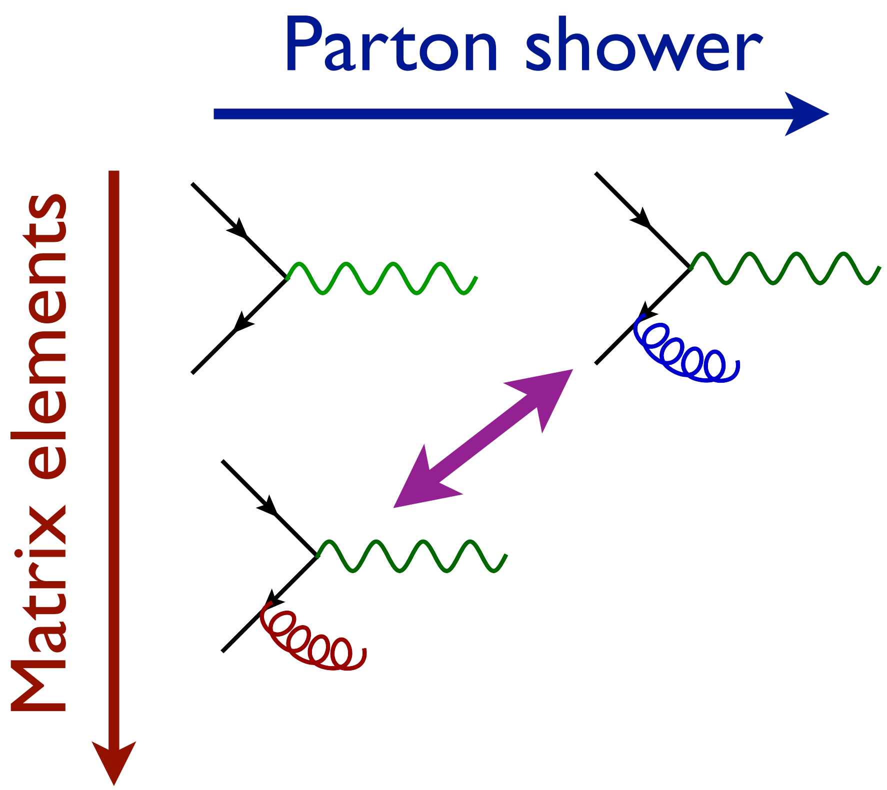

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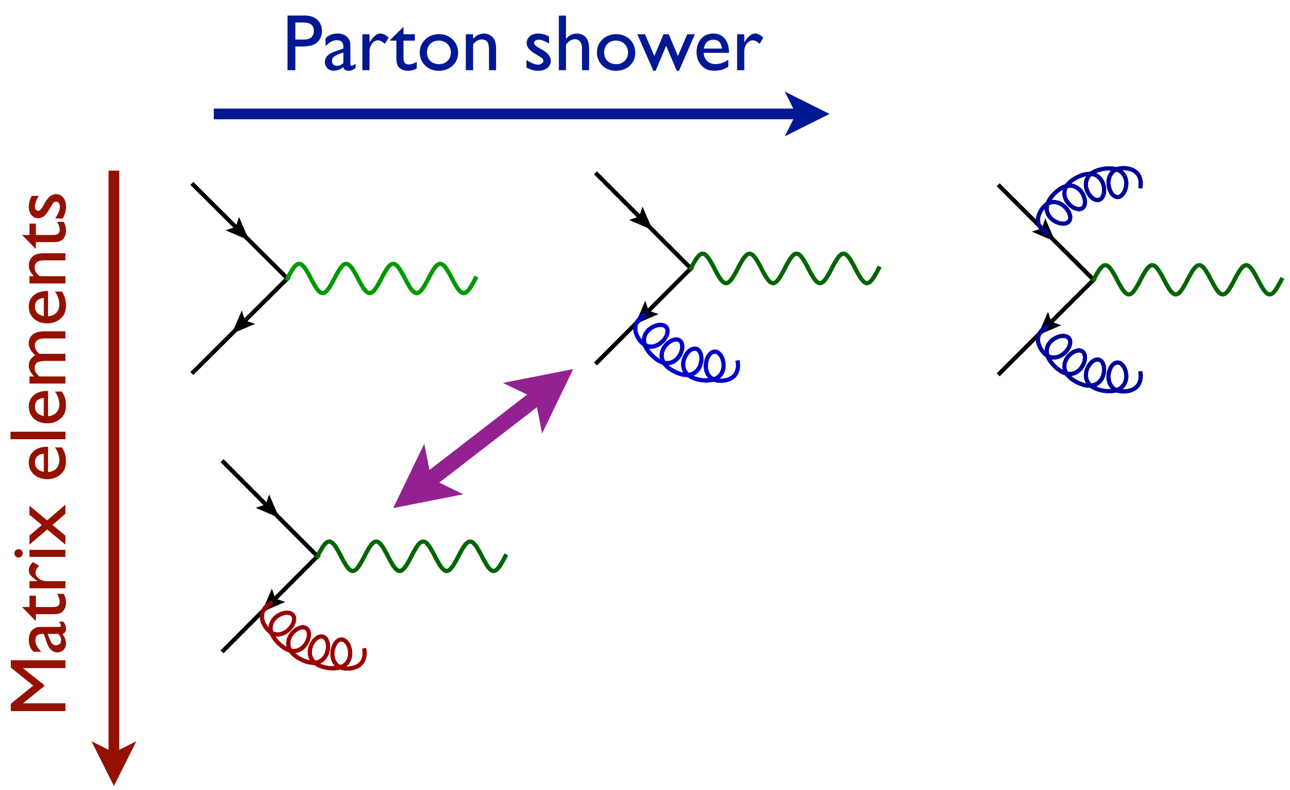

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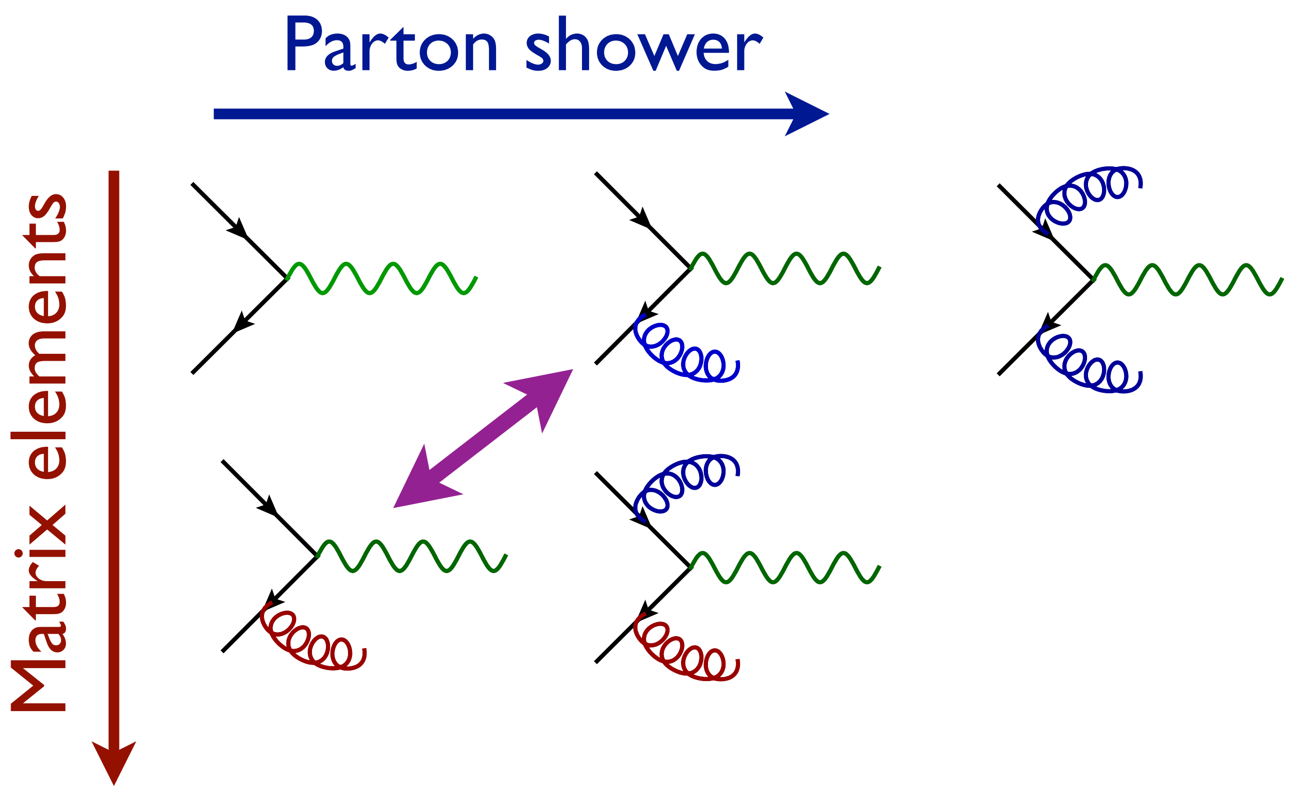

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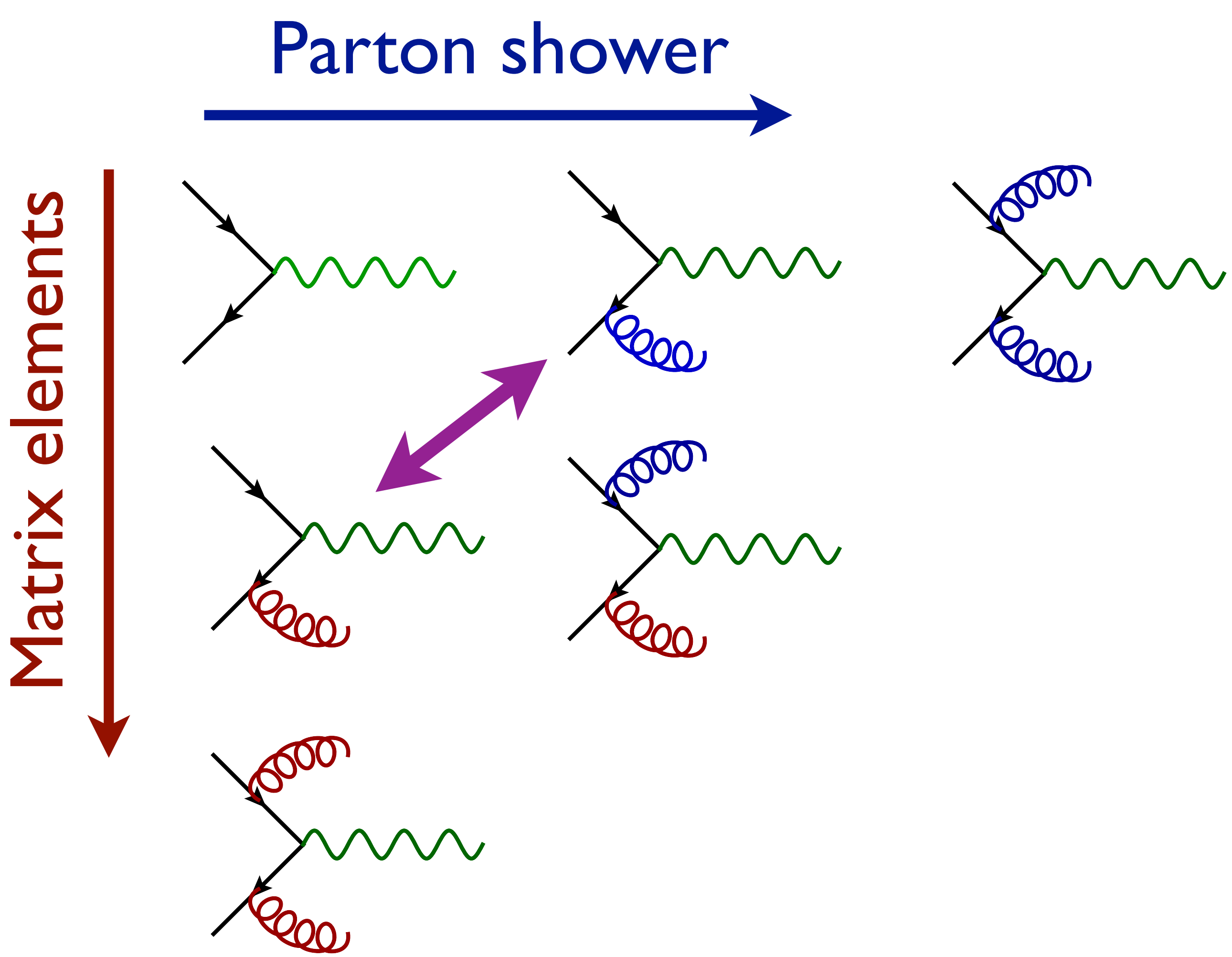

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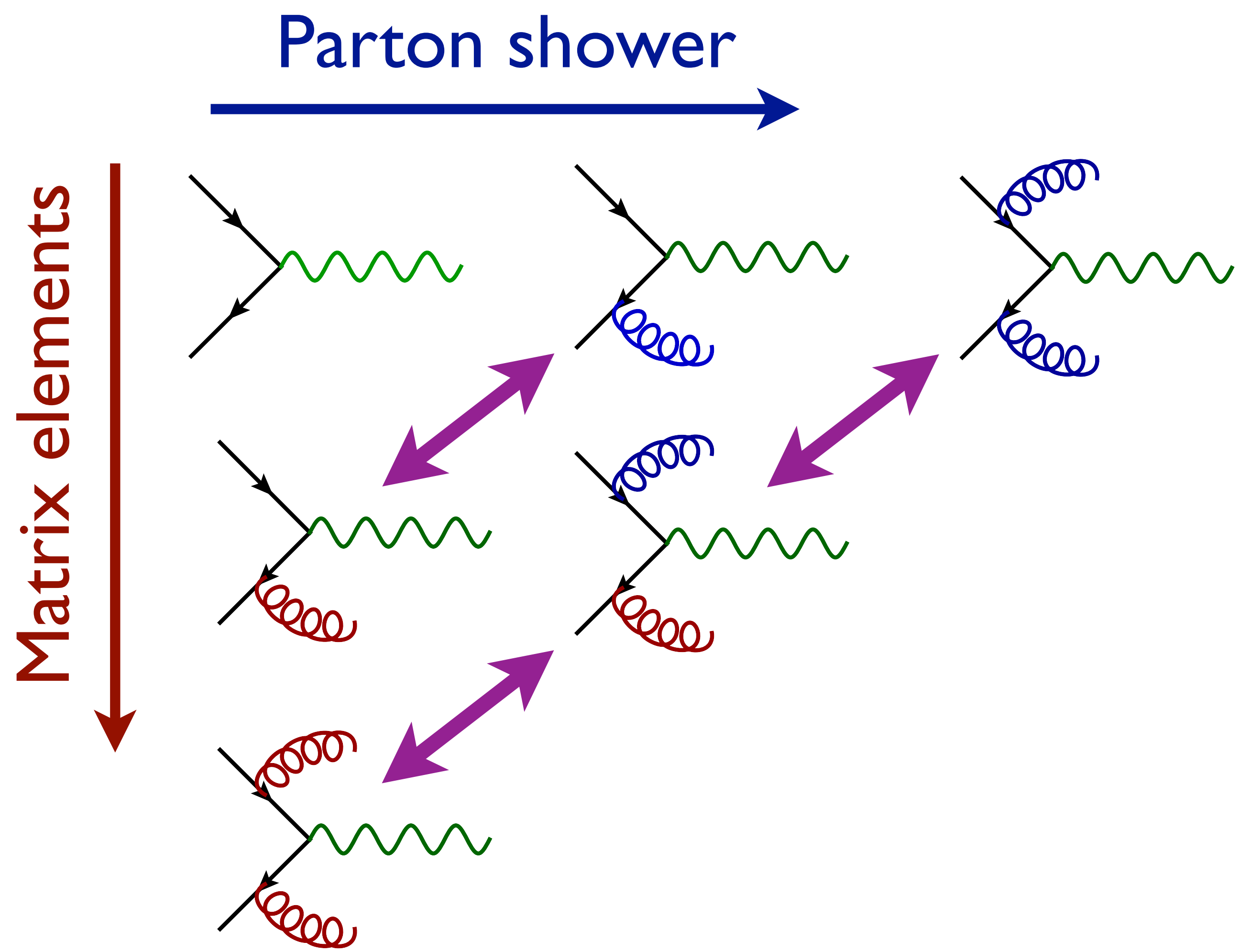

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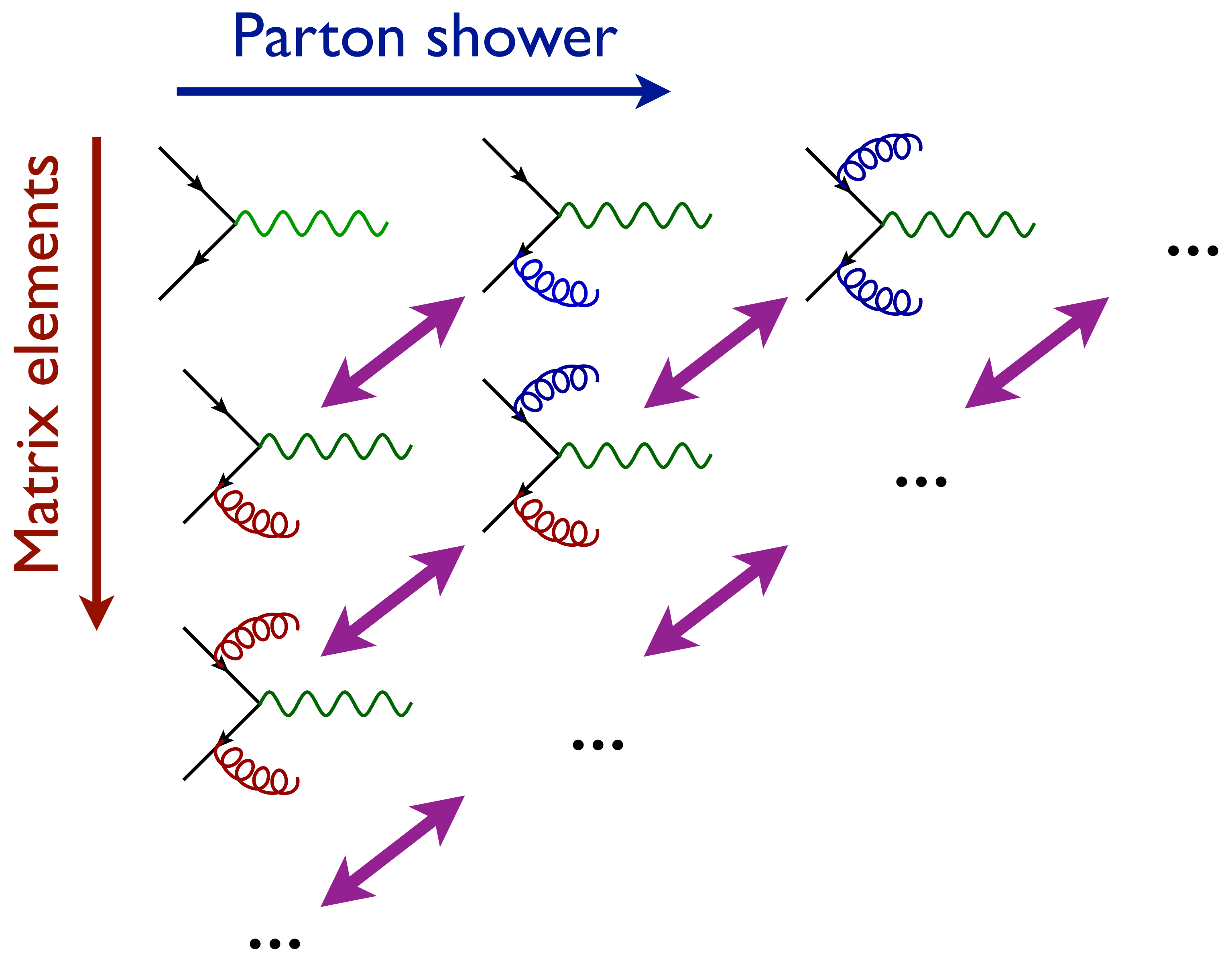

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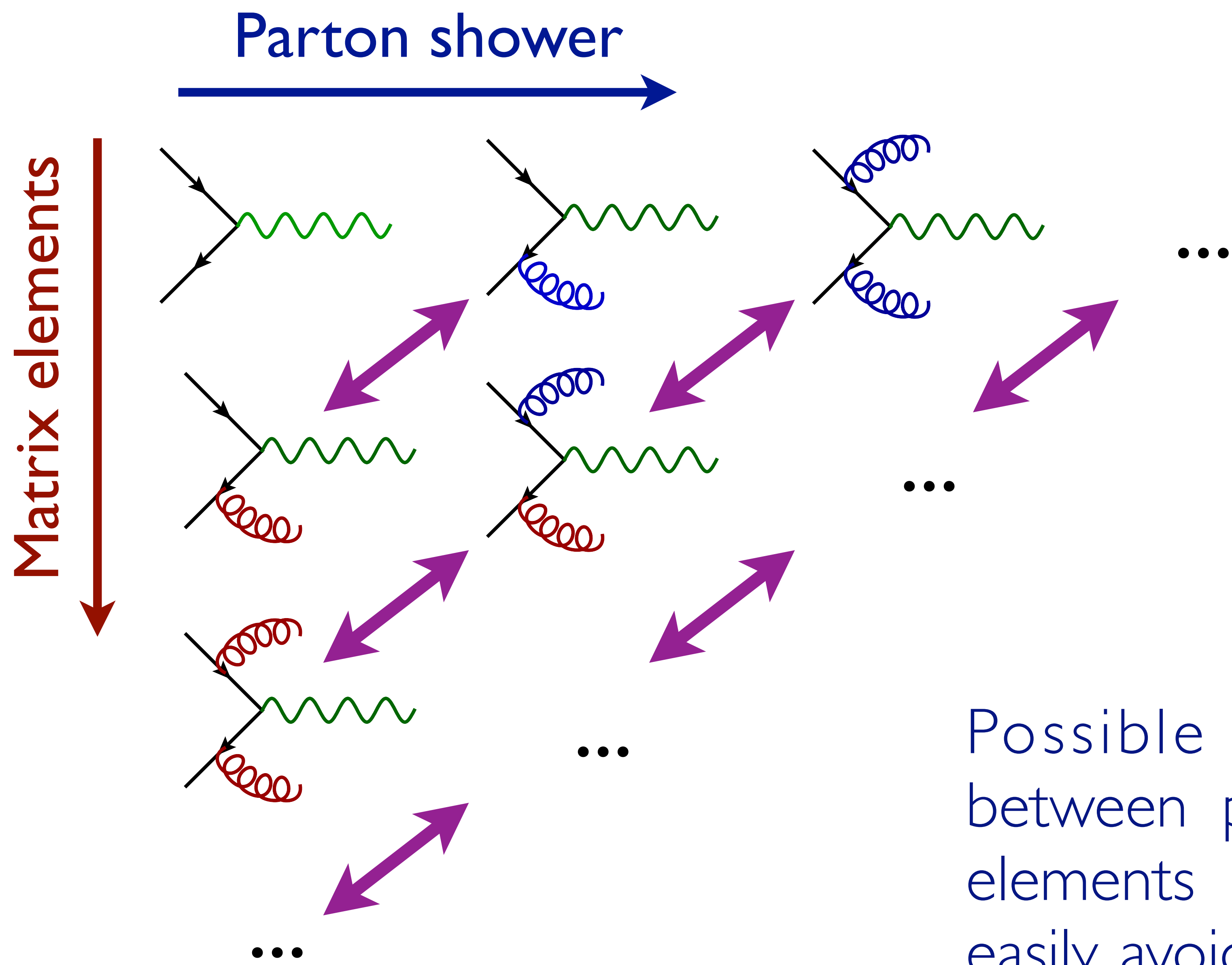

Possible double counting between partons from matrix elements and parton shower easily avoided by applying a cut in phase space

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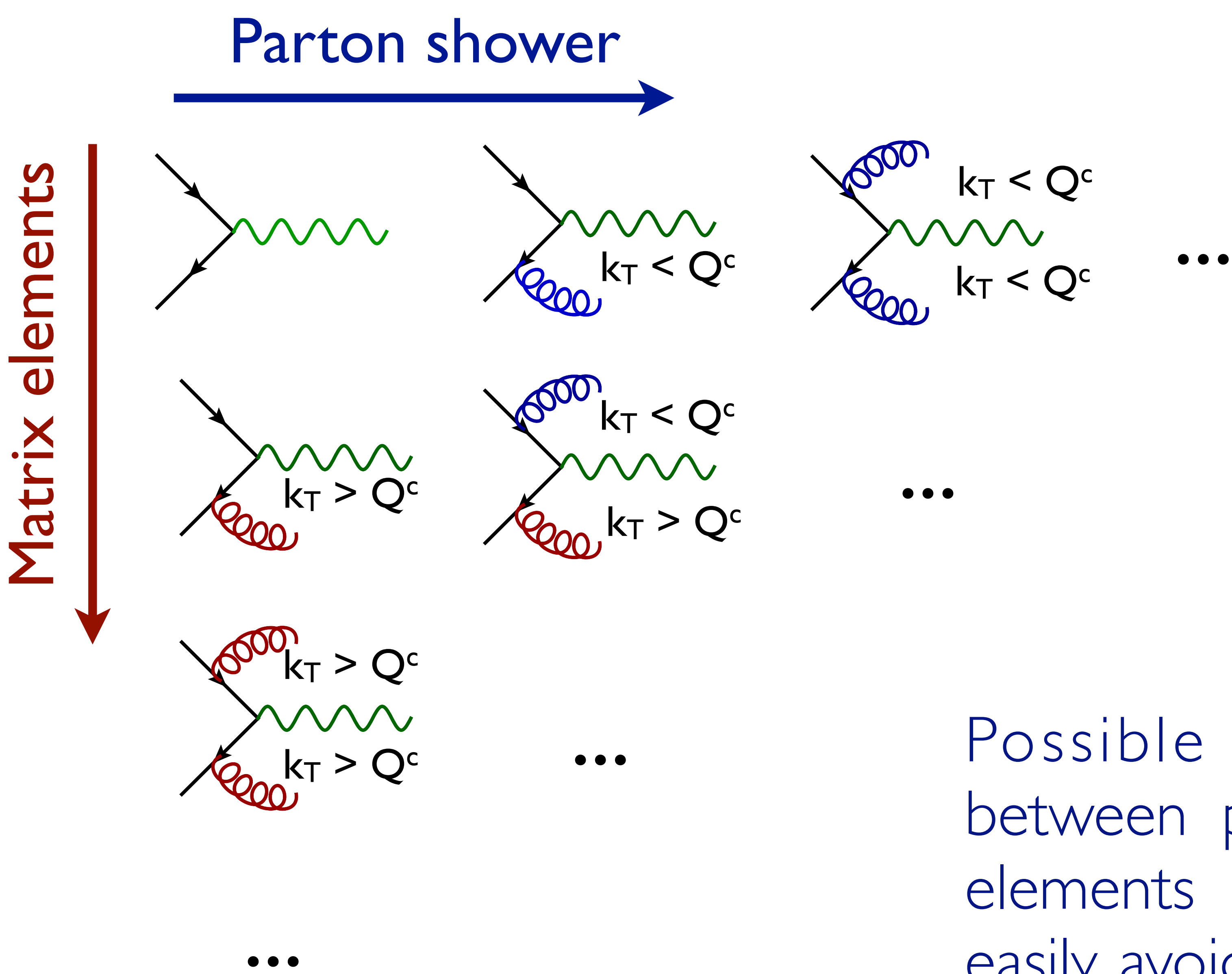


Possible double counting between partons from matrix elements and parton shower easily avoided by applying a cut in phase space

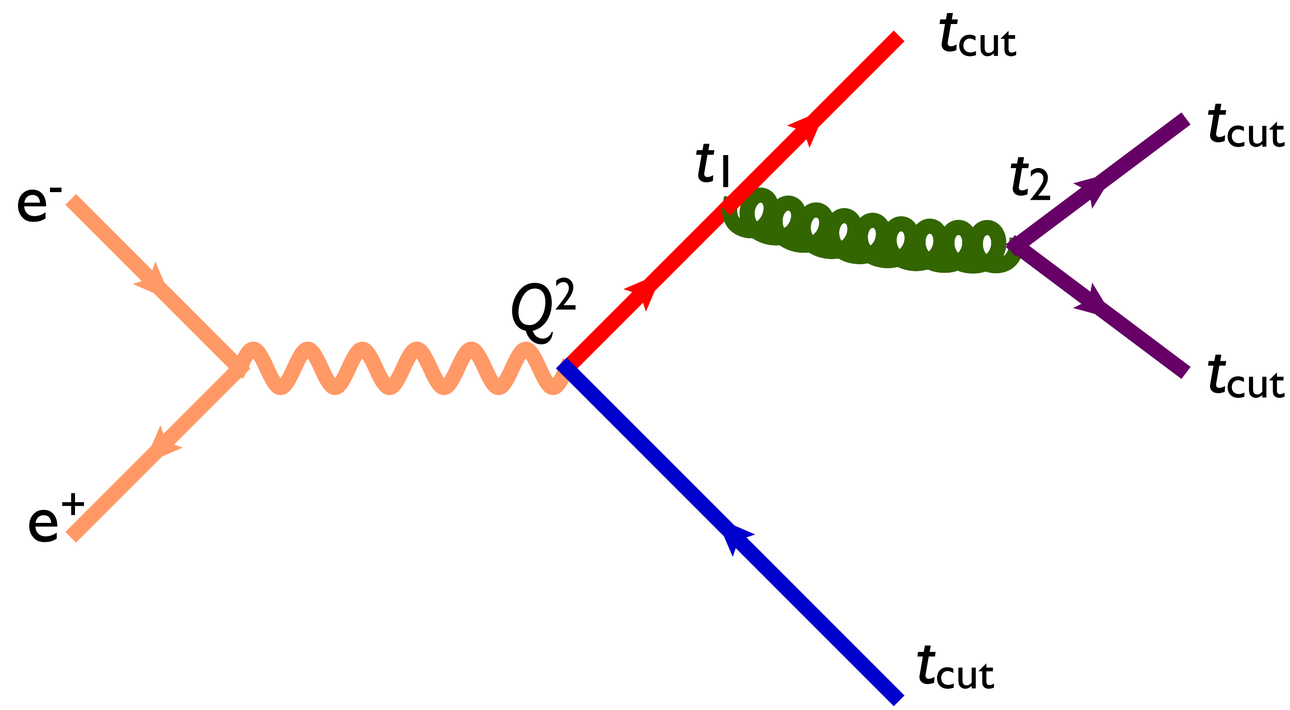
Merging ME with PS

- So double counting no problem, but what about getting smooth distributions that are independent of the precise value of Q^c?

- Below cutoff, distribution is given by PS
  - need to make ME look like PS near cutoff

- Let's take another look at the PS!

Merging ME with PS



- How does the PS generate the configuration above (i.e. starting from e^+e^- -> qqbar events)?
- Probability for the splitting at t_1 is given by

$$(\Delta_q(Q^2,t_1))^2 \frac{\alpha_s(t_1)}{2\pi} P_{gq}(z)$$

and for the whole tree (remember $\Delta(A,B) = \Delta(A,C)\,\Delta(C,B)$)

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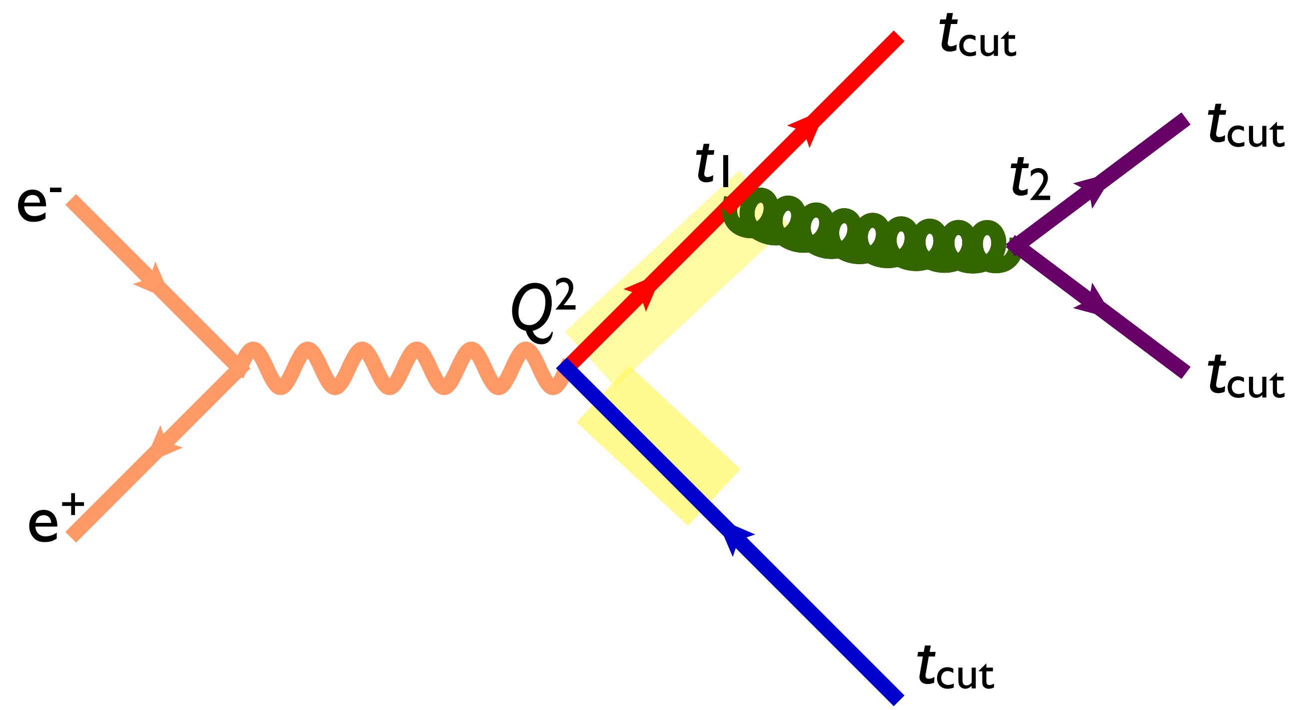

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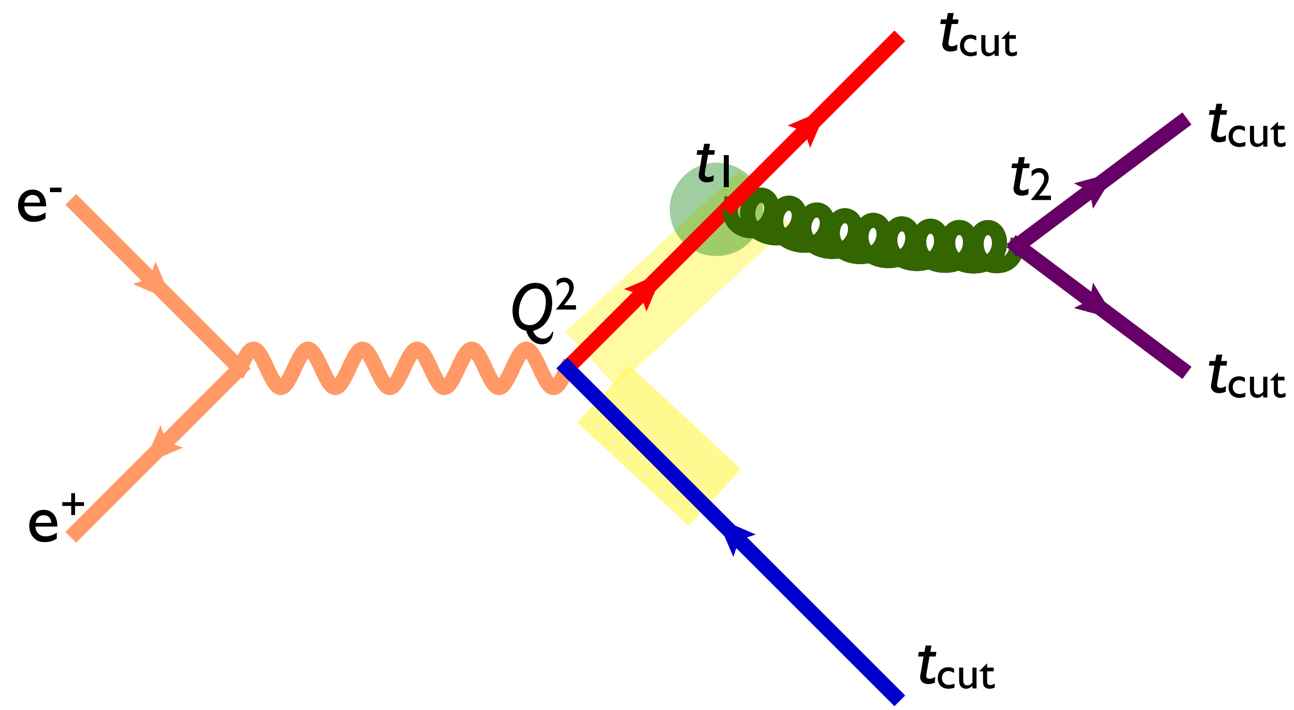

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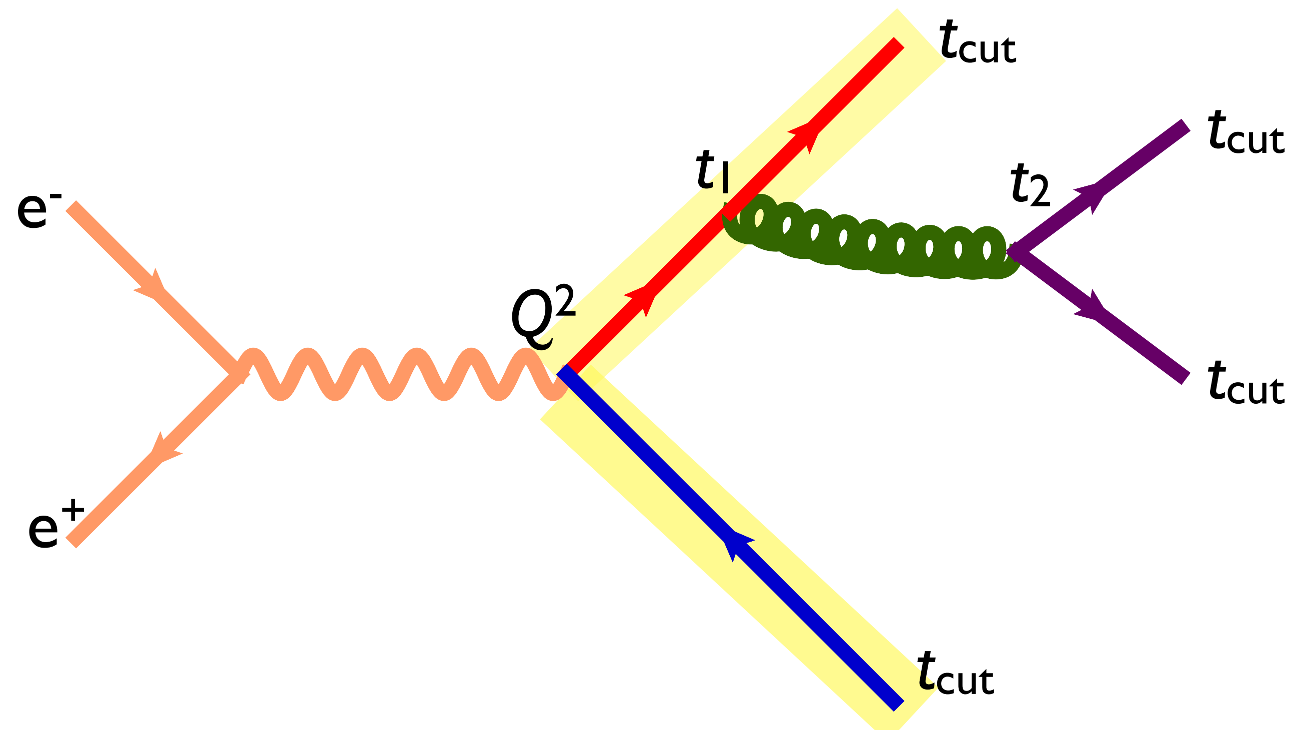

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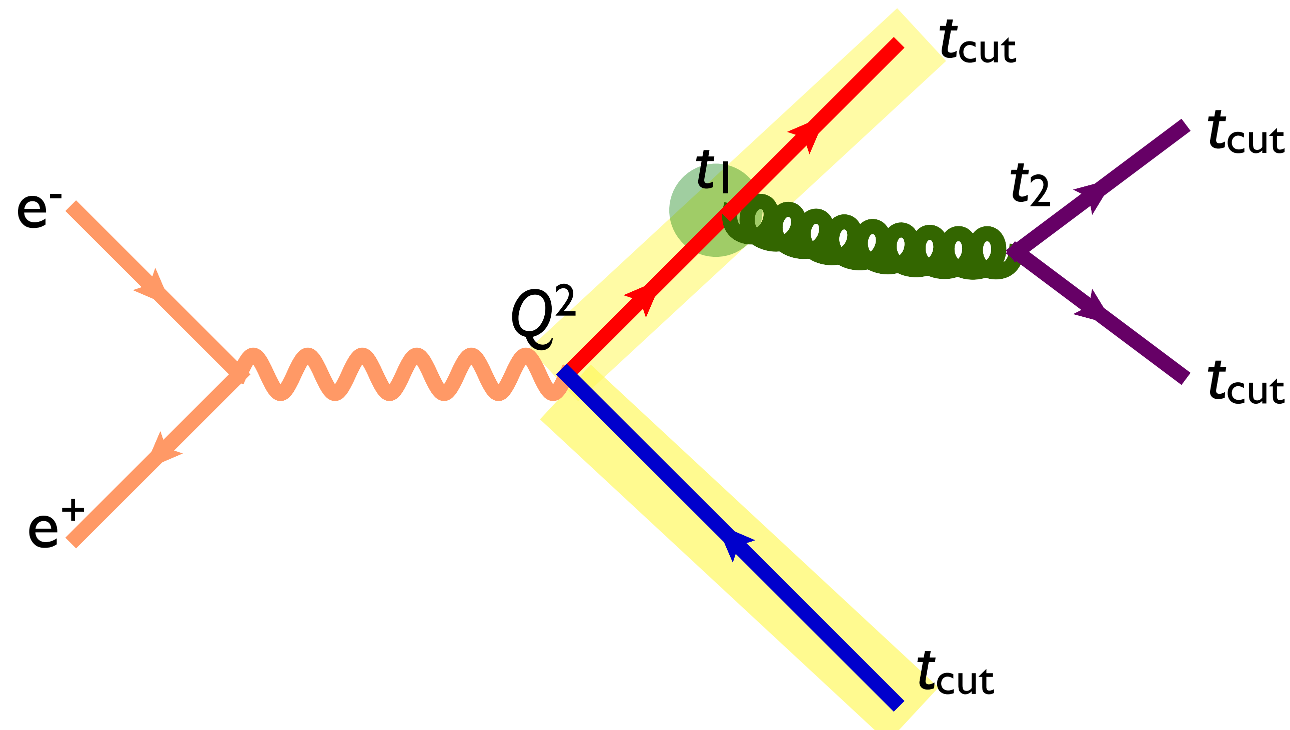

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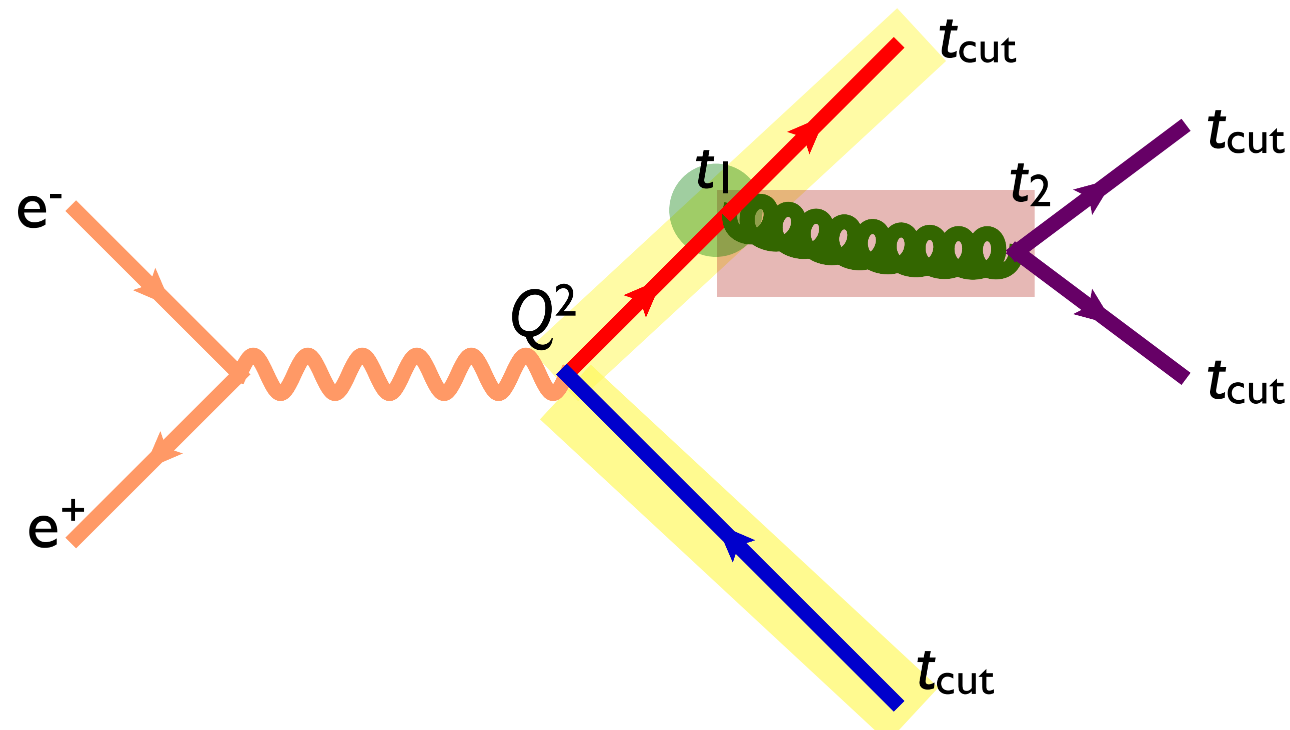

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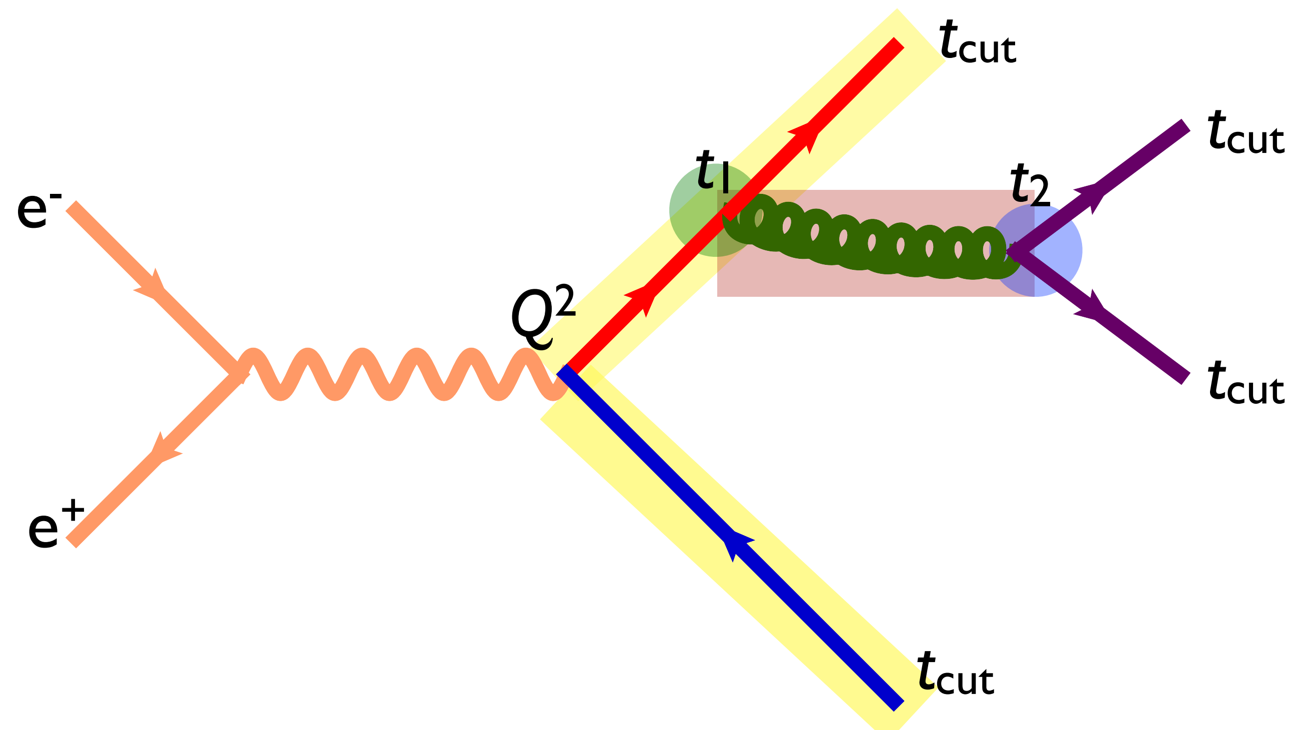

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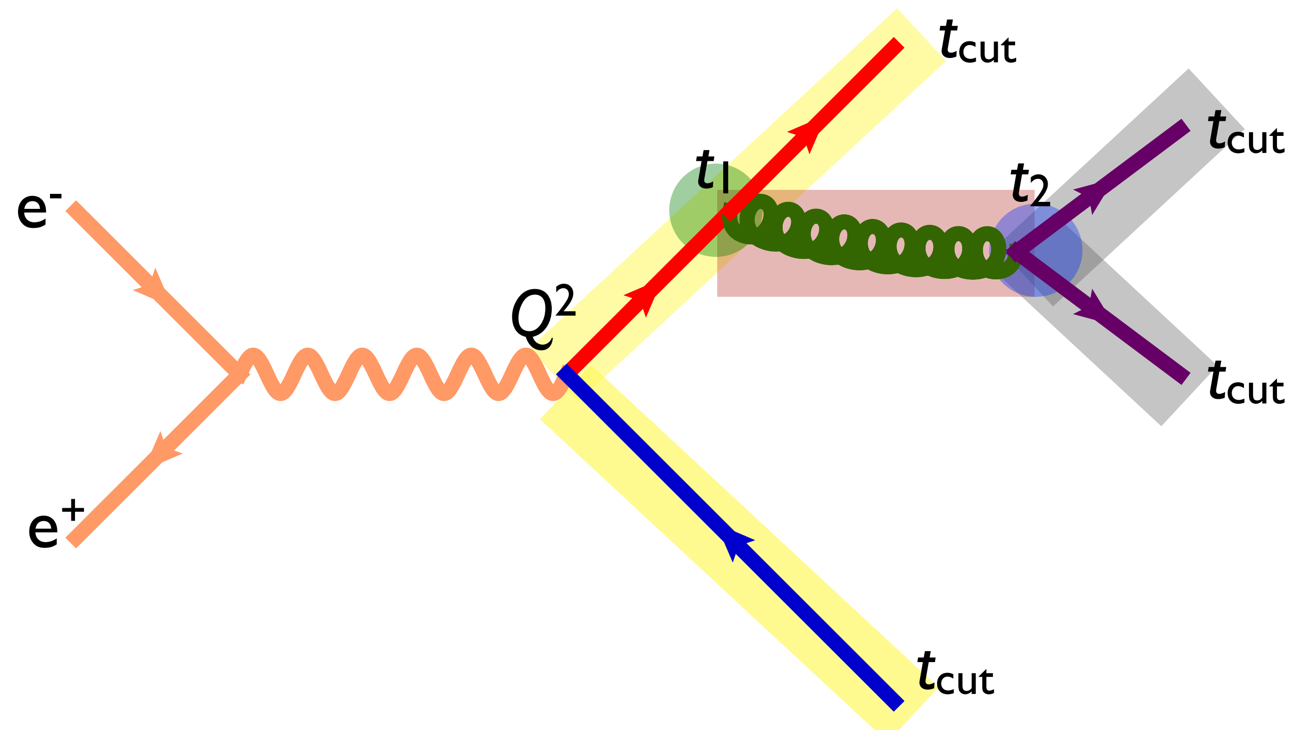

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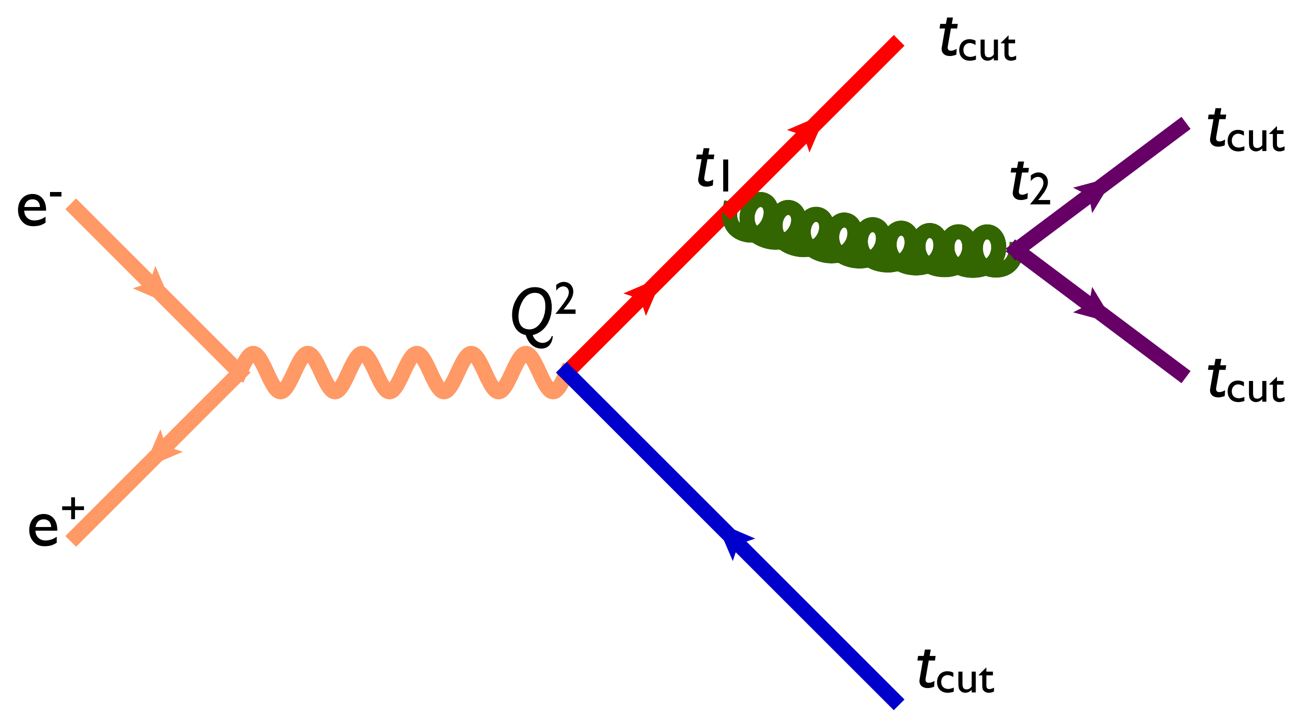

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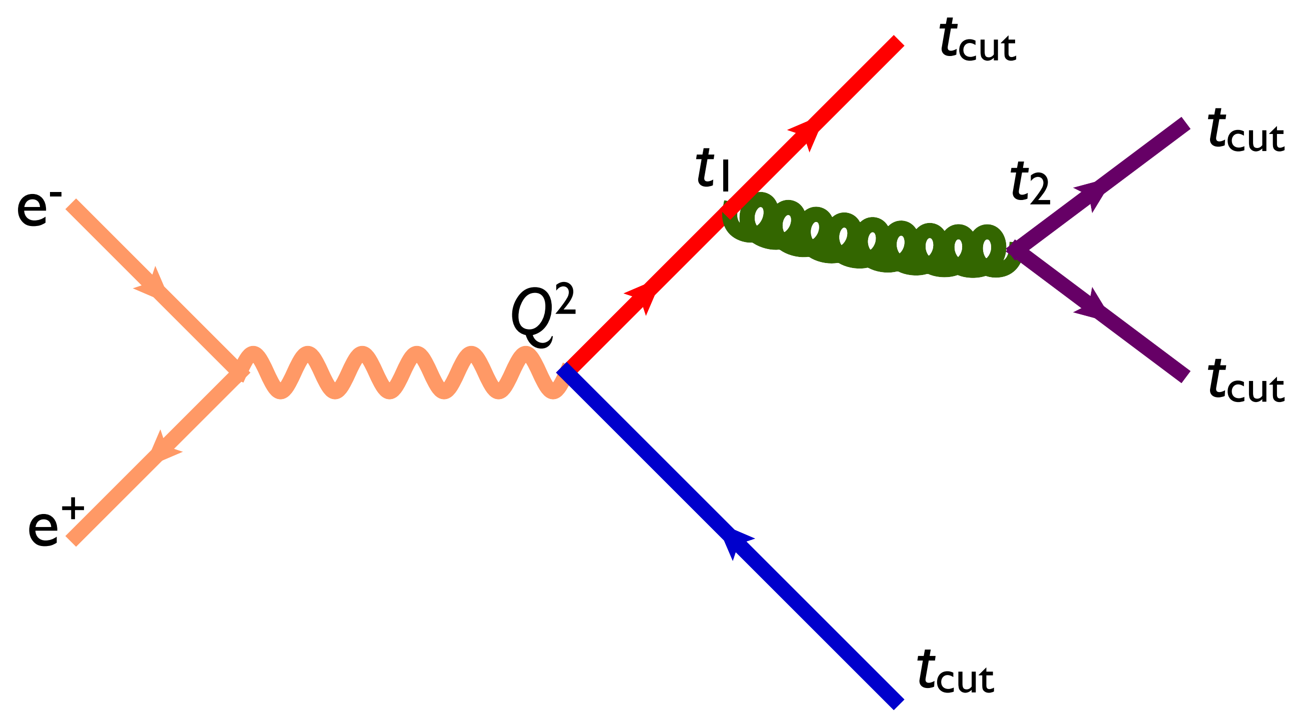

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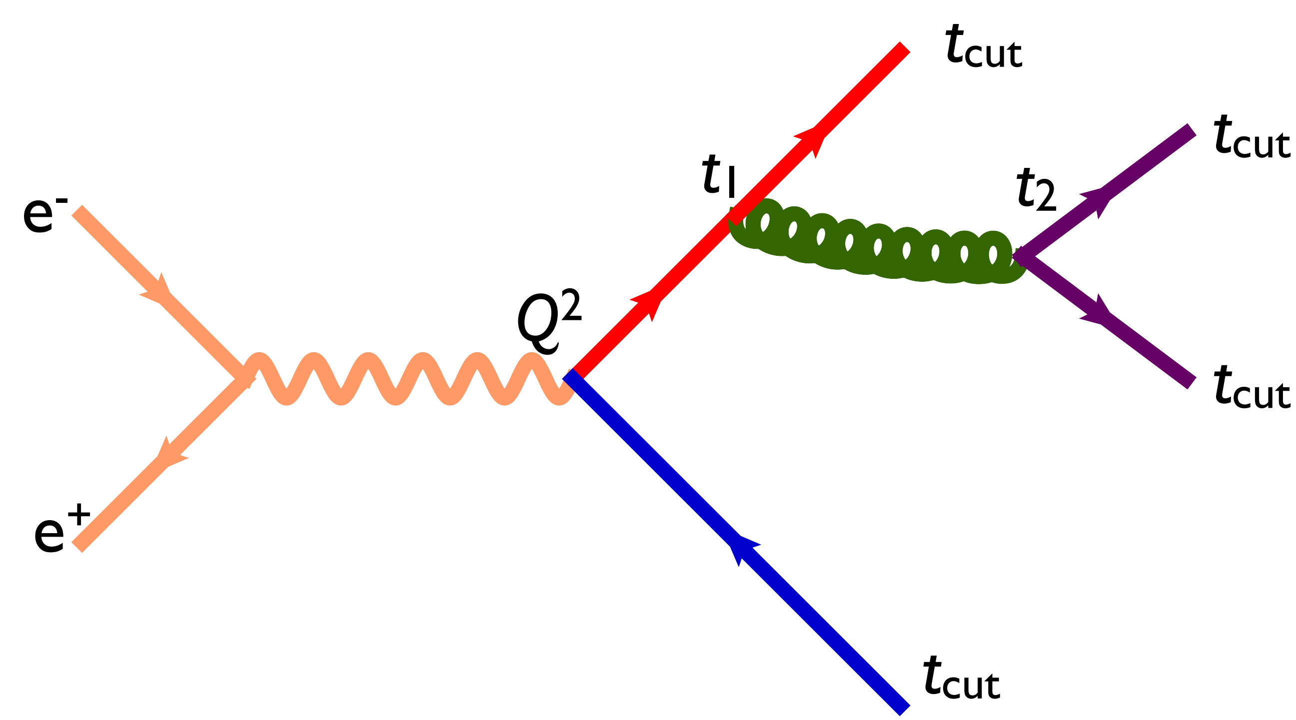

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Sudakov suppression due to disallowing additional radiation above the scale t_{cut}

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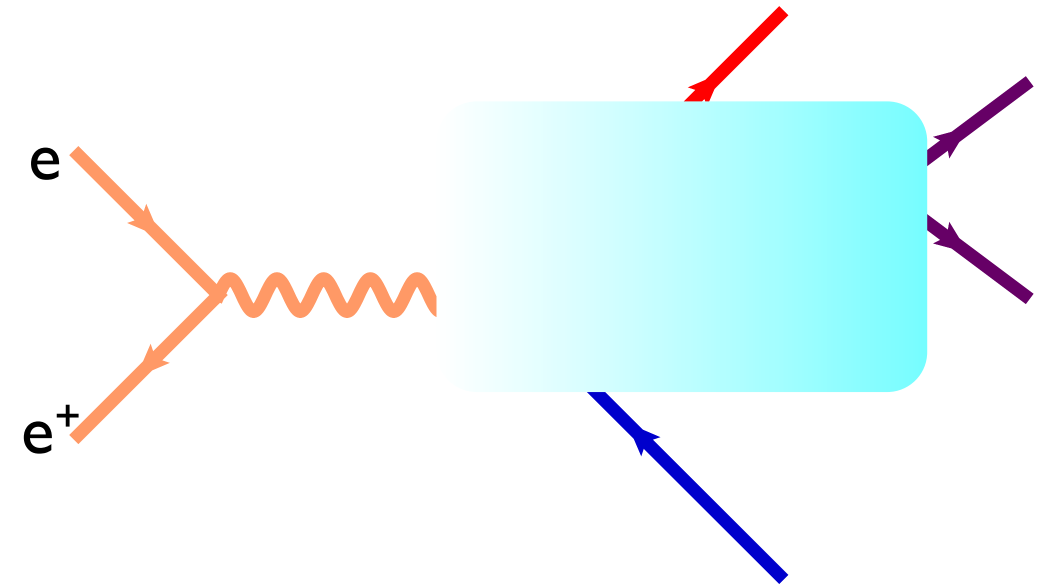

$$|\mathcal{M}|^2(\hat{s}, p_3, p_4, \ldots)$$

To get an equivalent treatment of the corresponding matrix element, do as follows:

1. Cluster the event using some clustering algorithm
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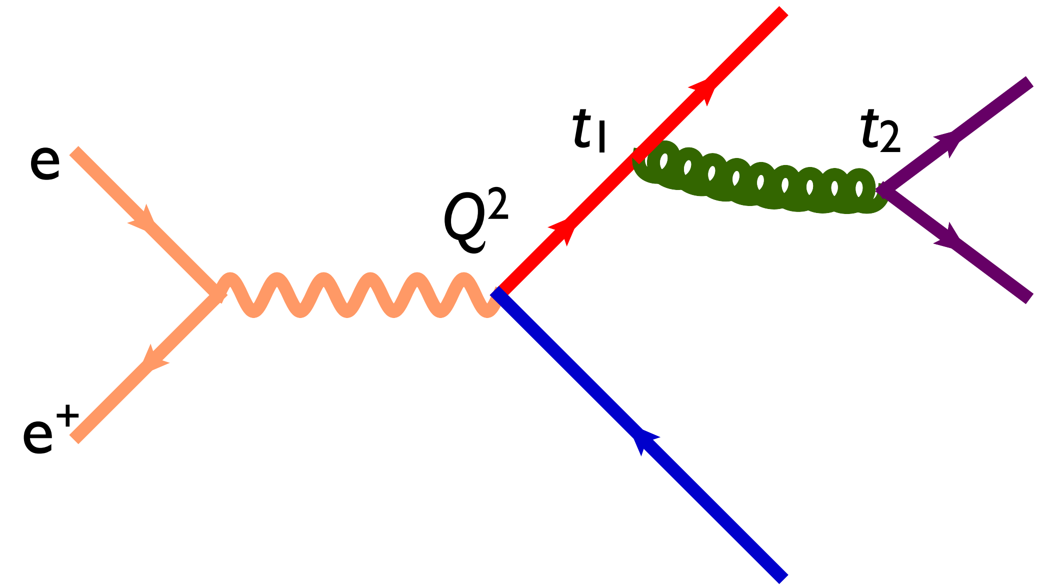

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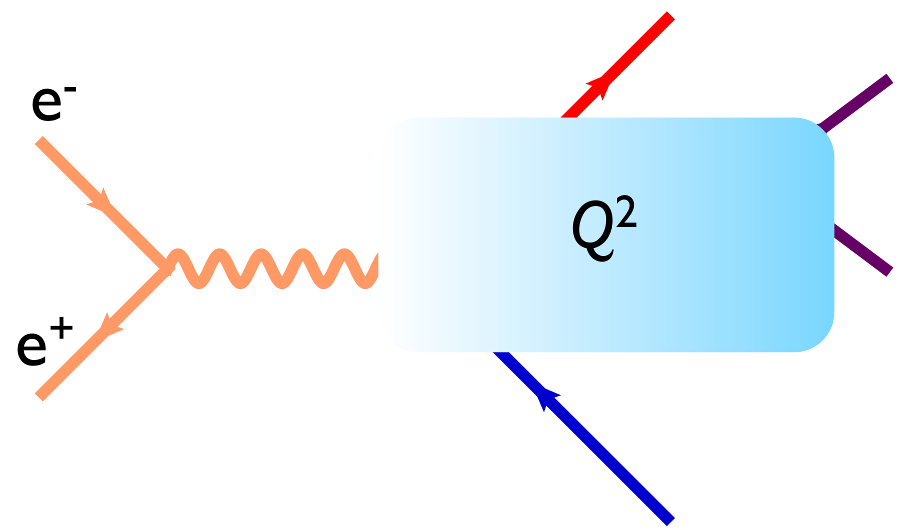
[M.L. Mangano, 2002, 2006]
[J. Alwall et al 2007, 2008]

- The simplest way to do the Sudakov suppression is to run the shower on the event, starting from **t_0**!



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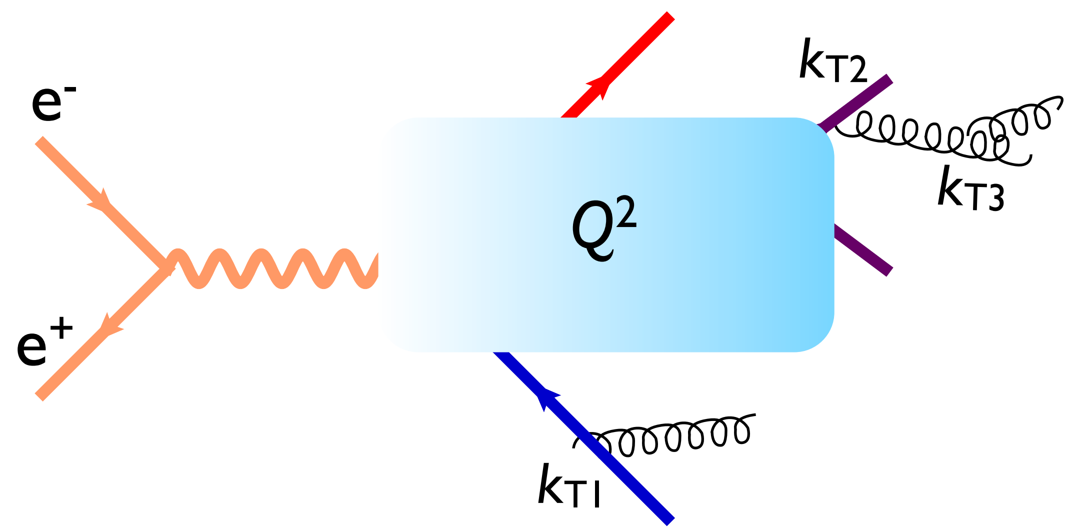

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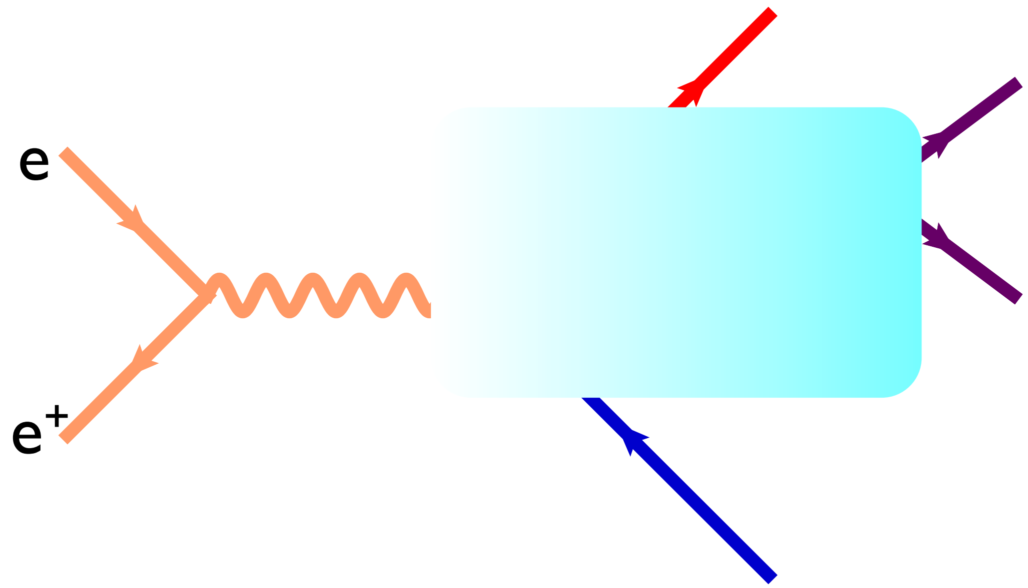

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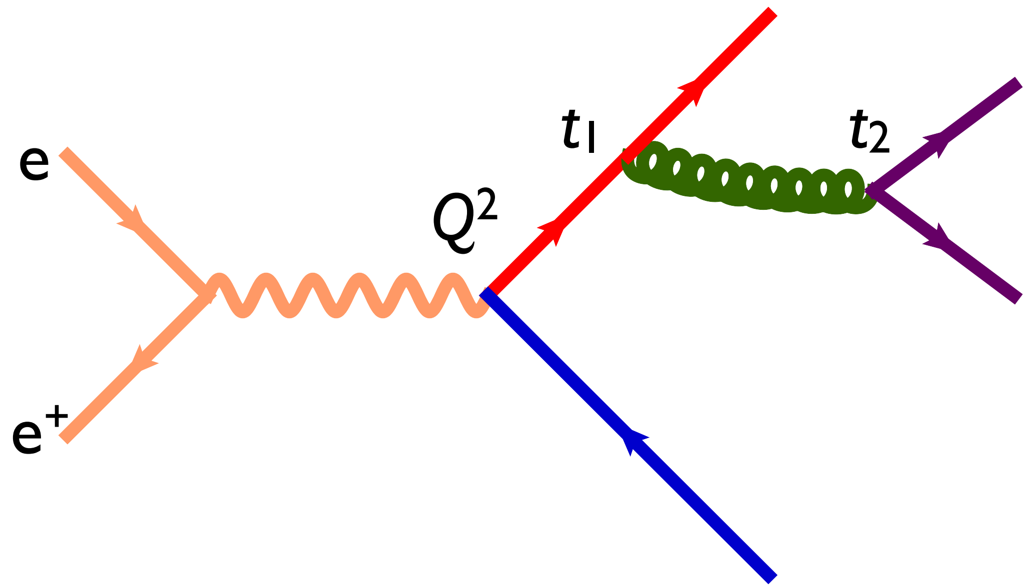

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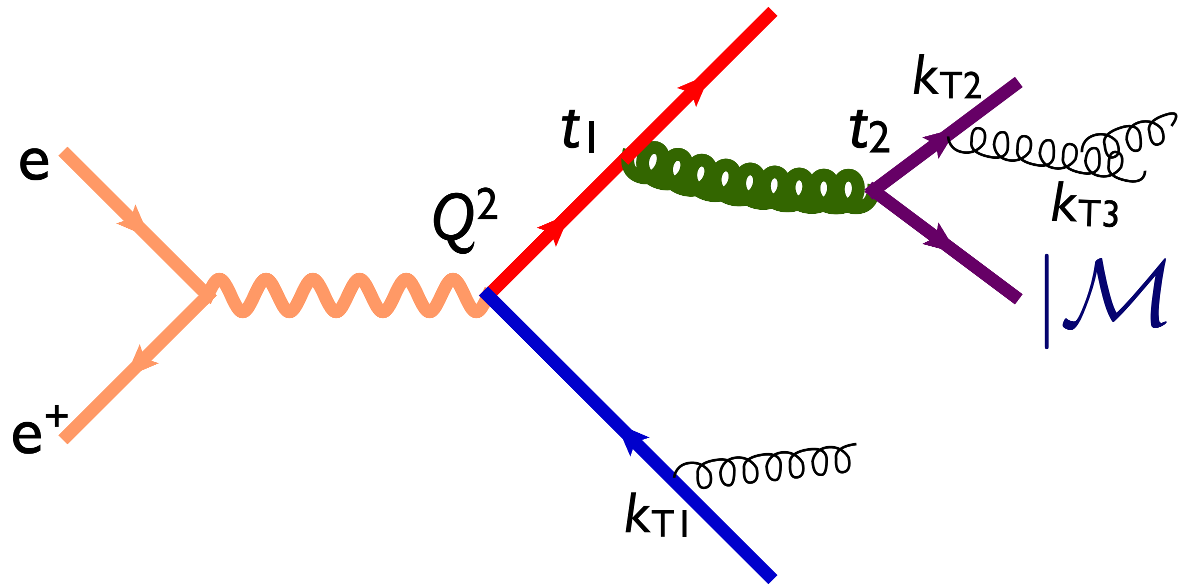

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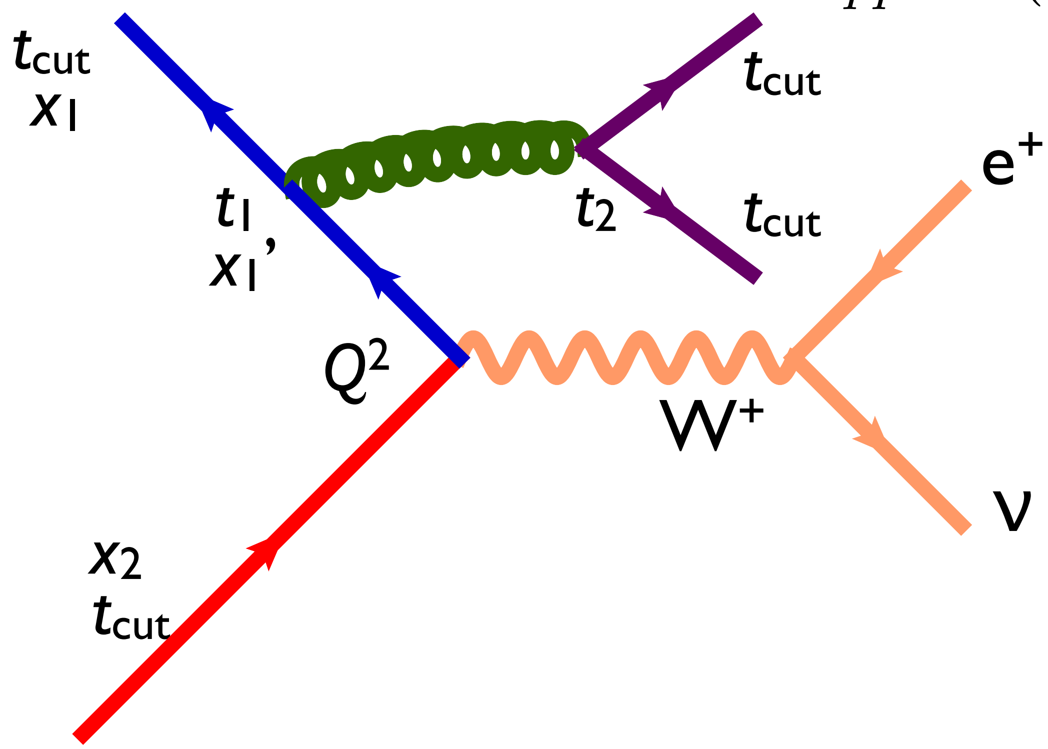

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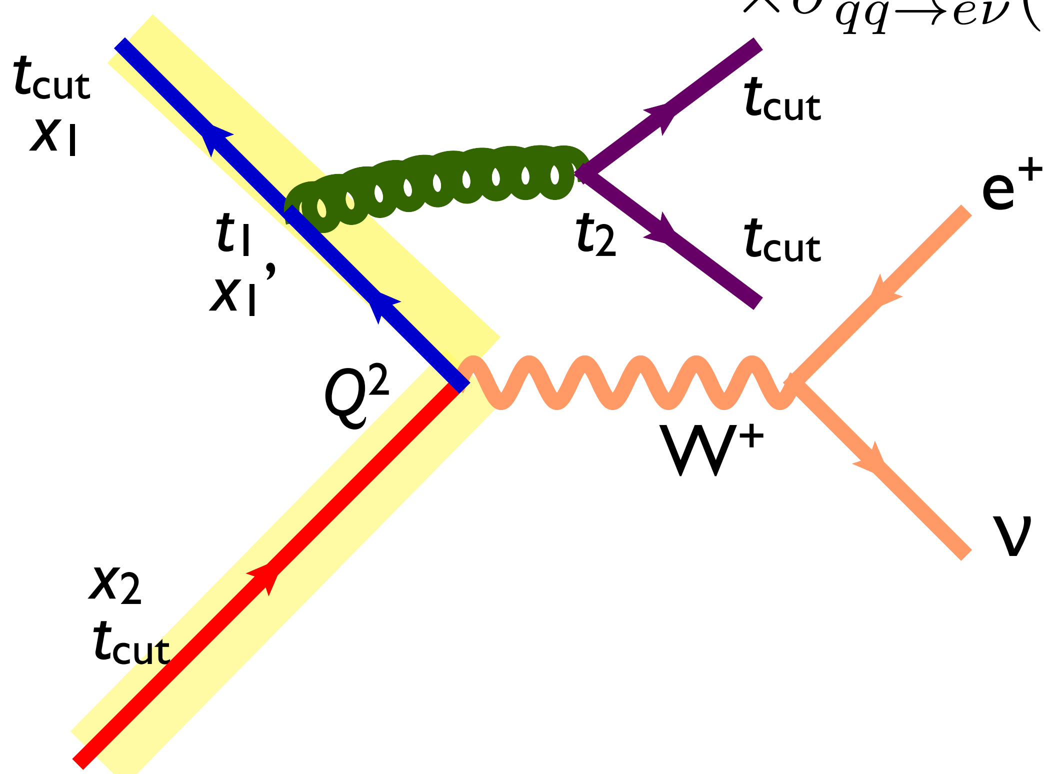

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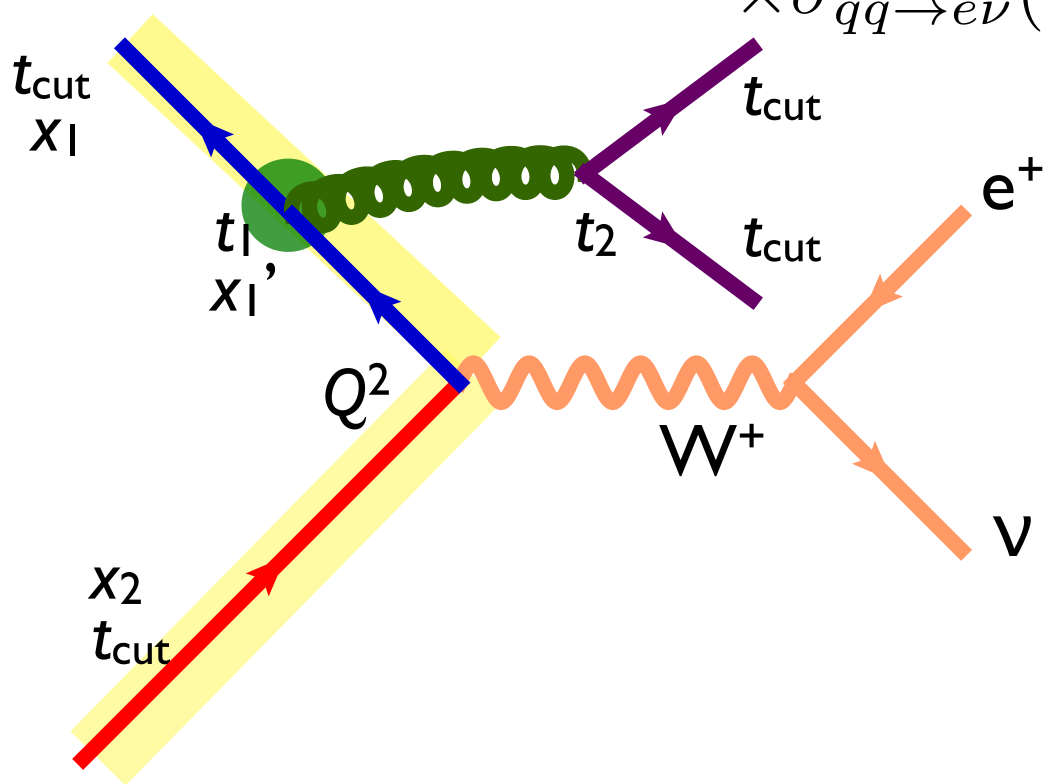

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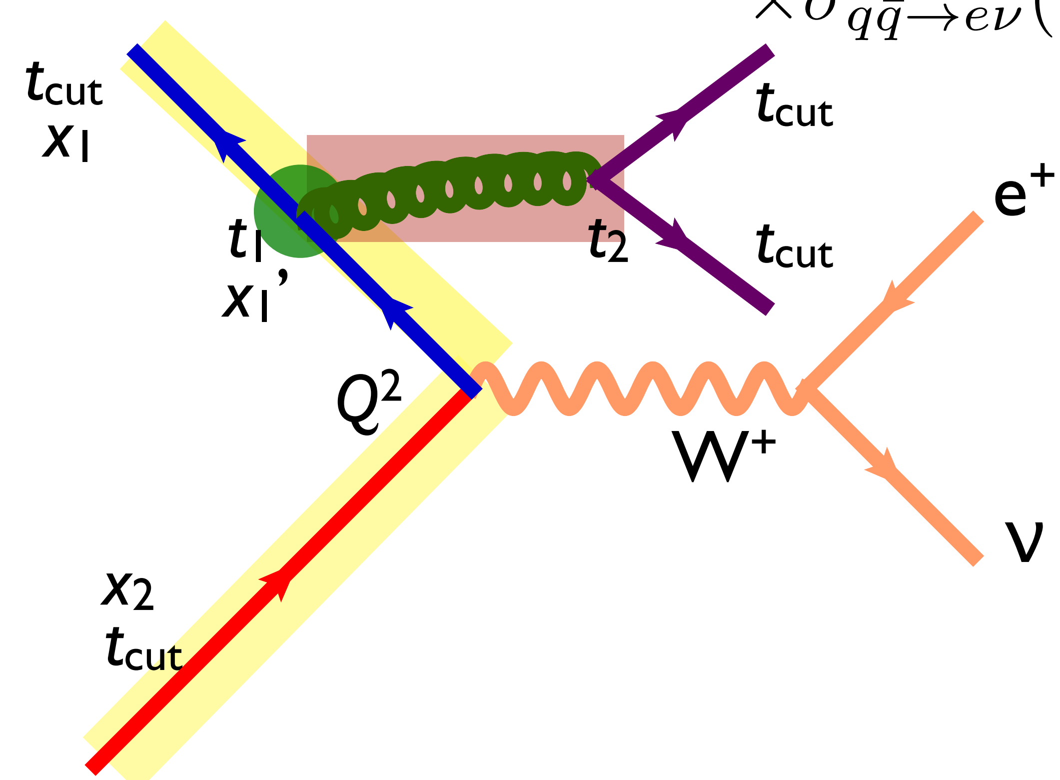

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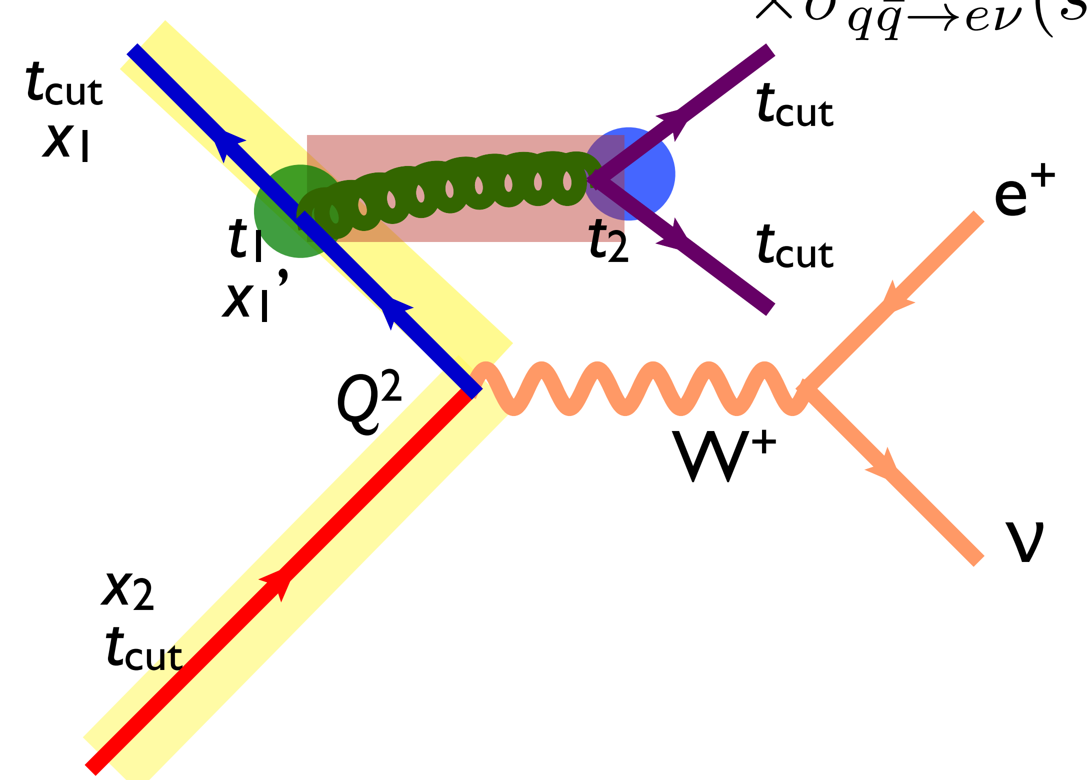

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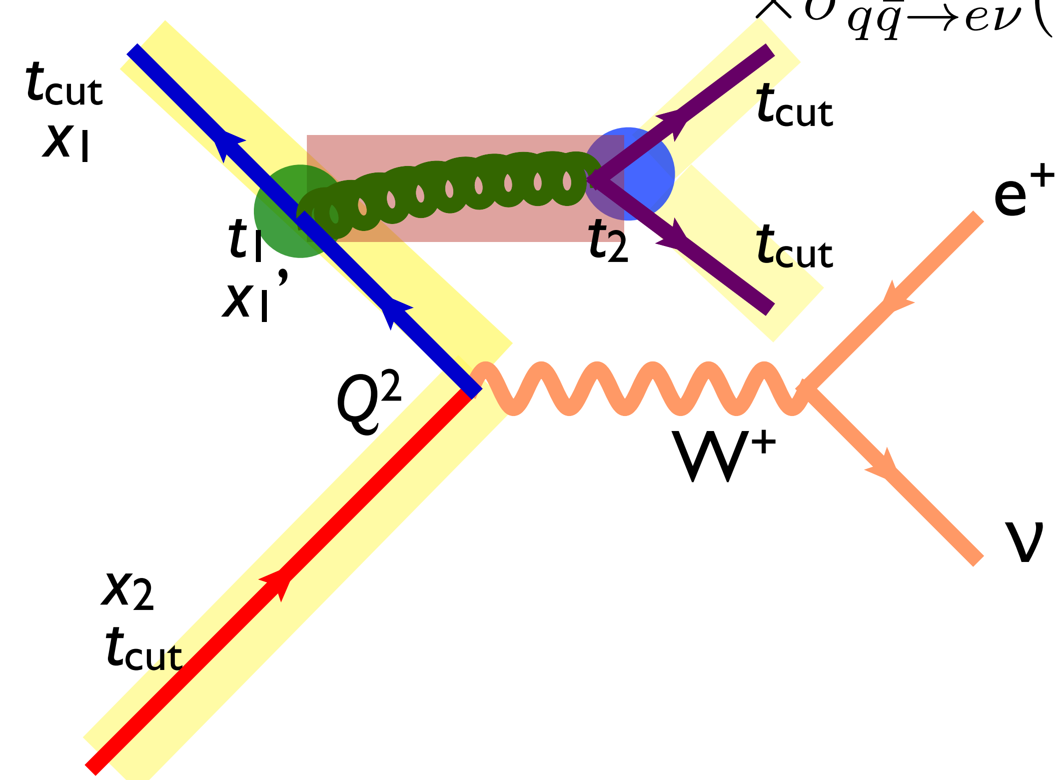

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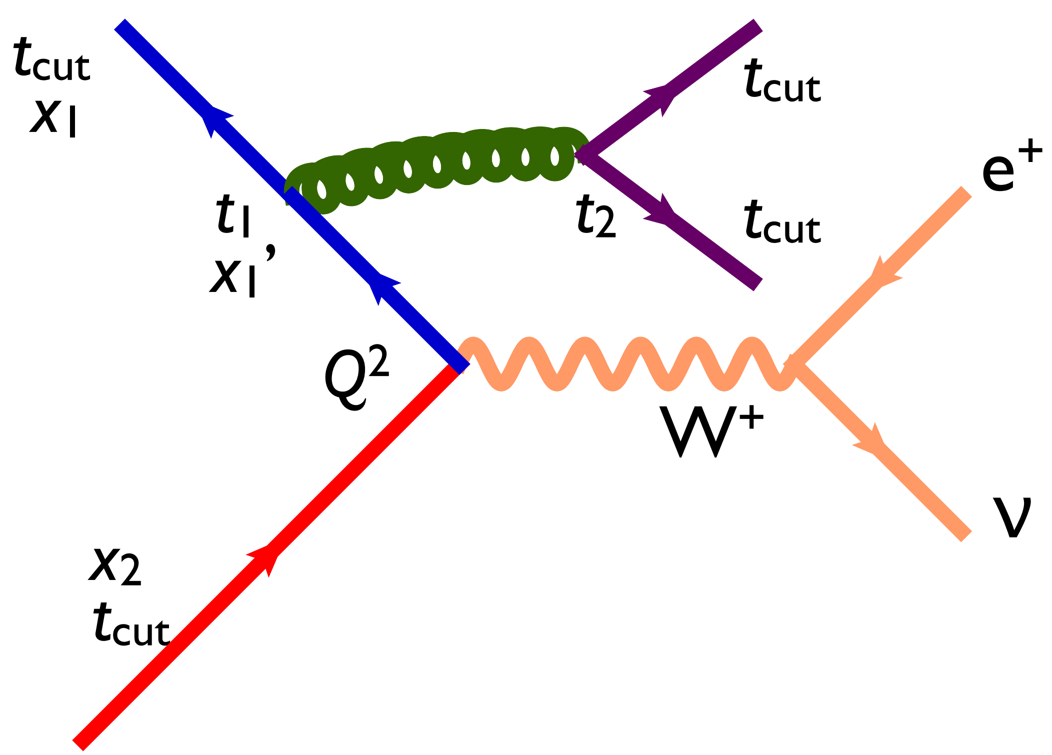

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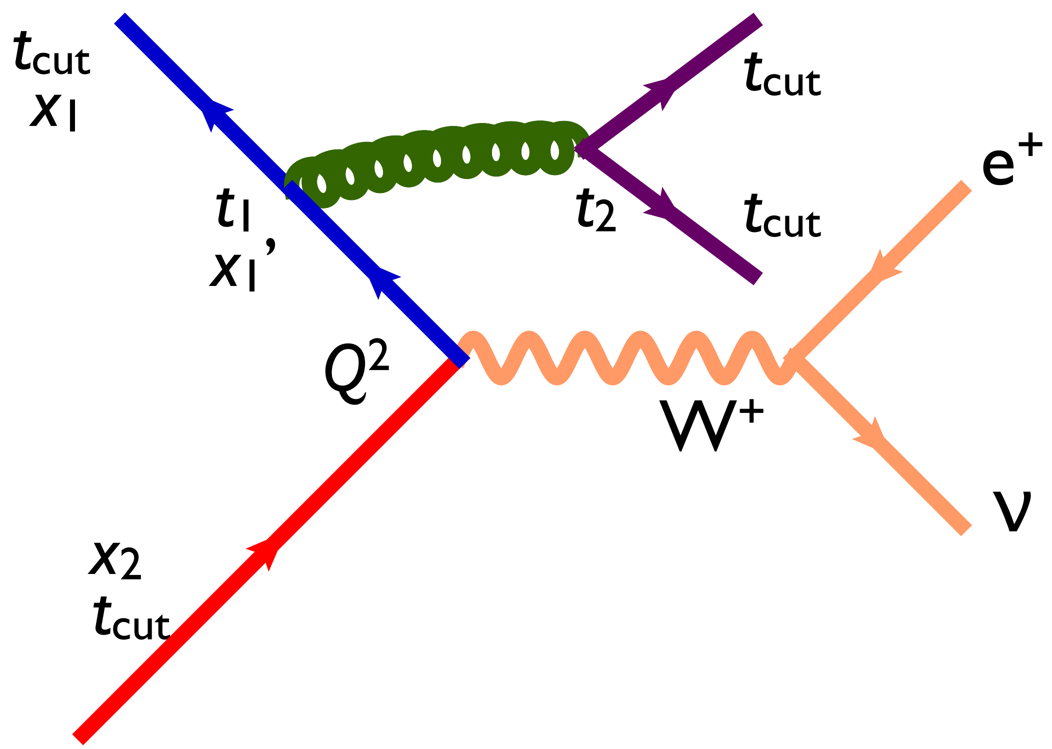

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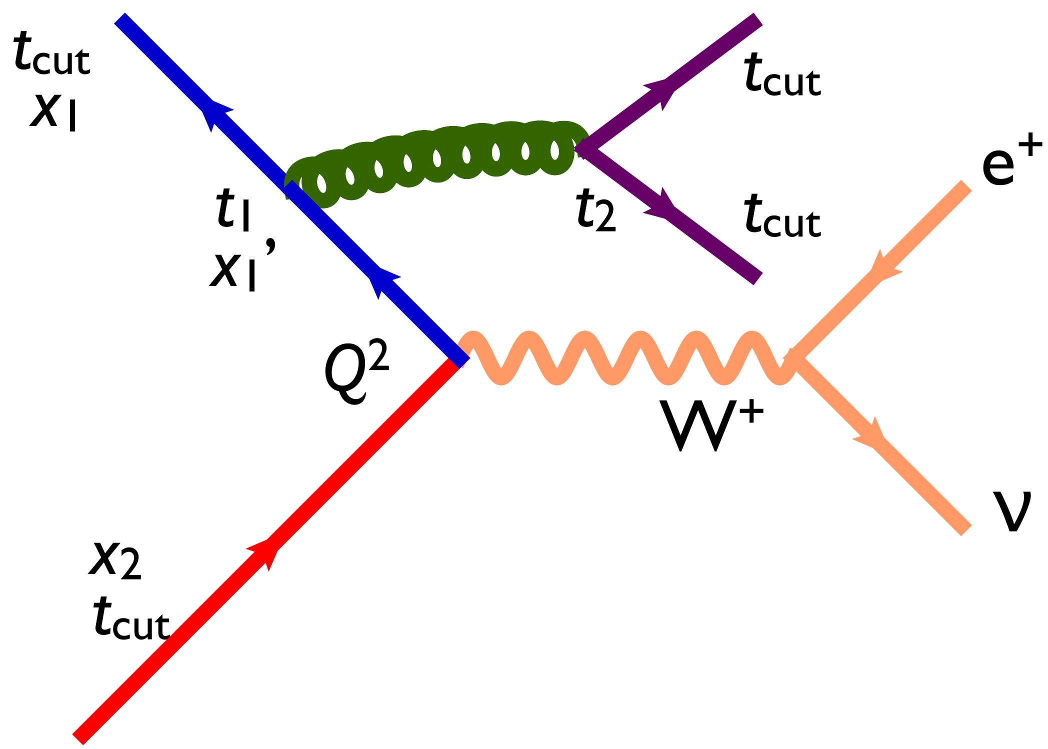

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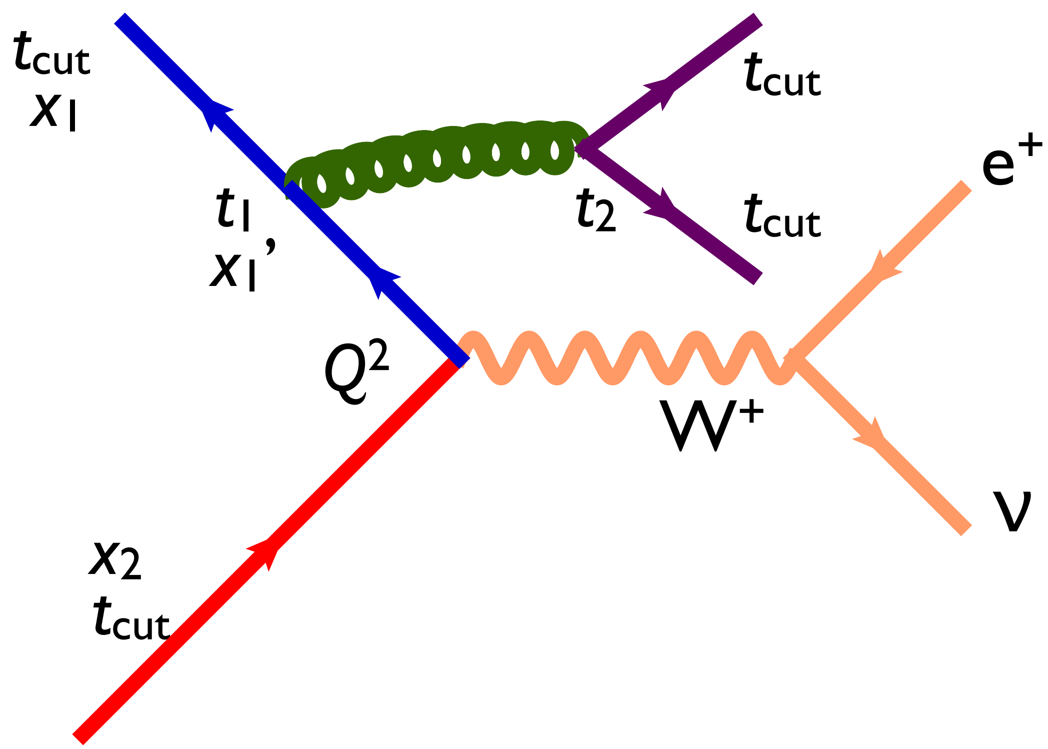

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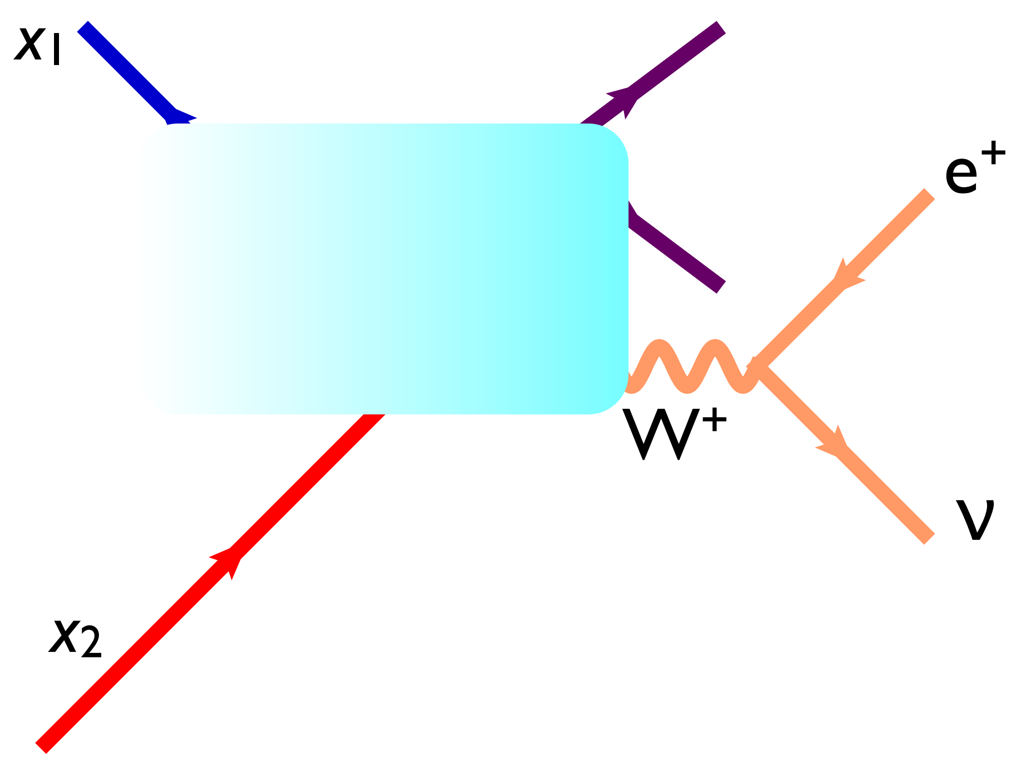

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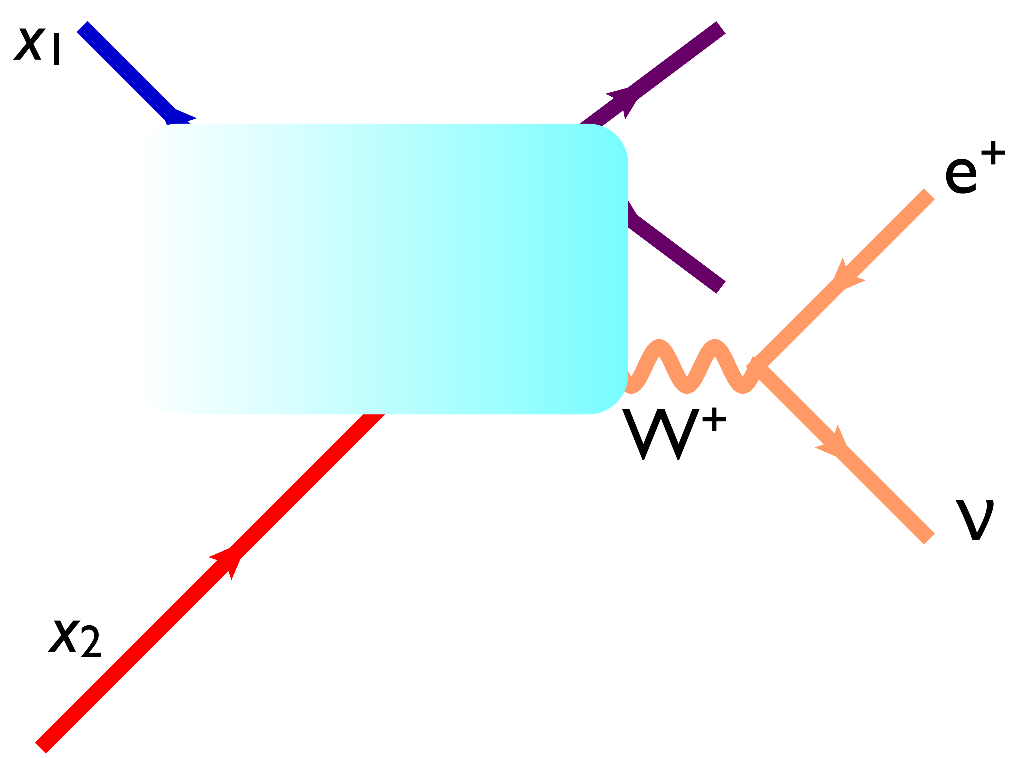

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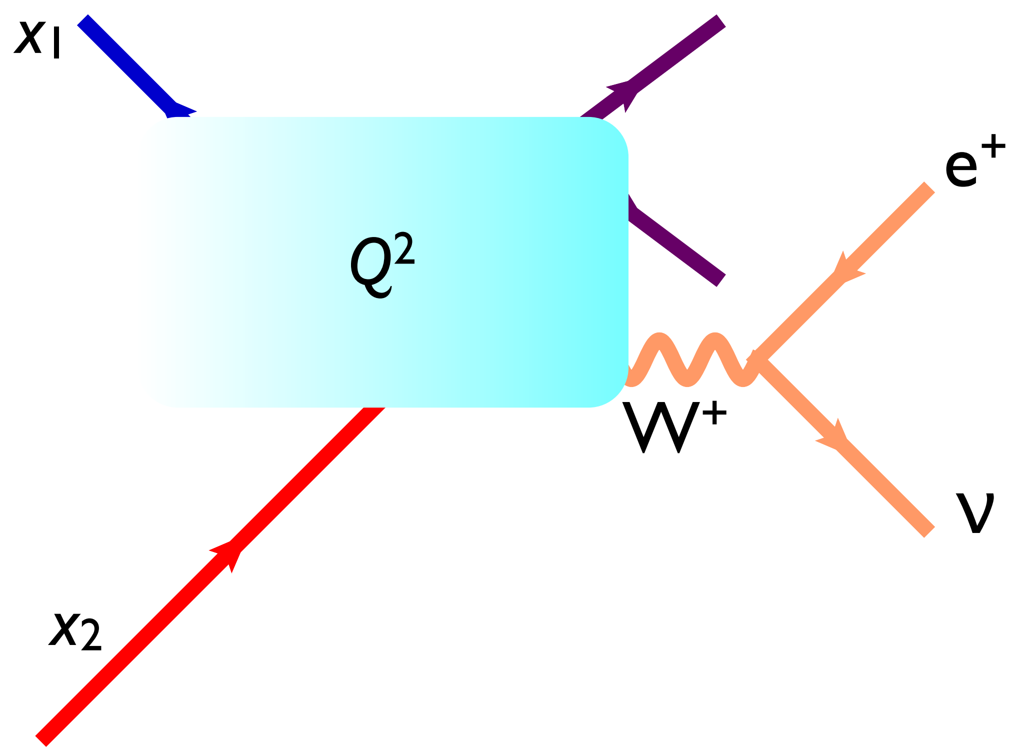

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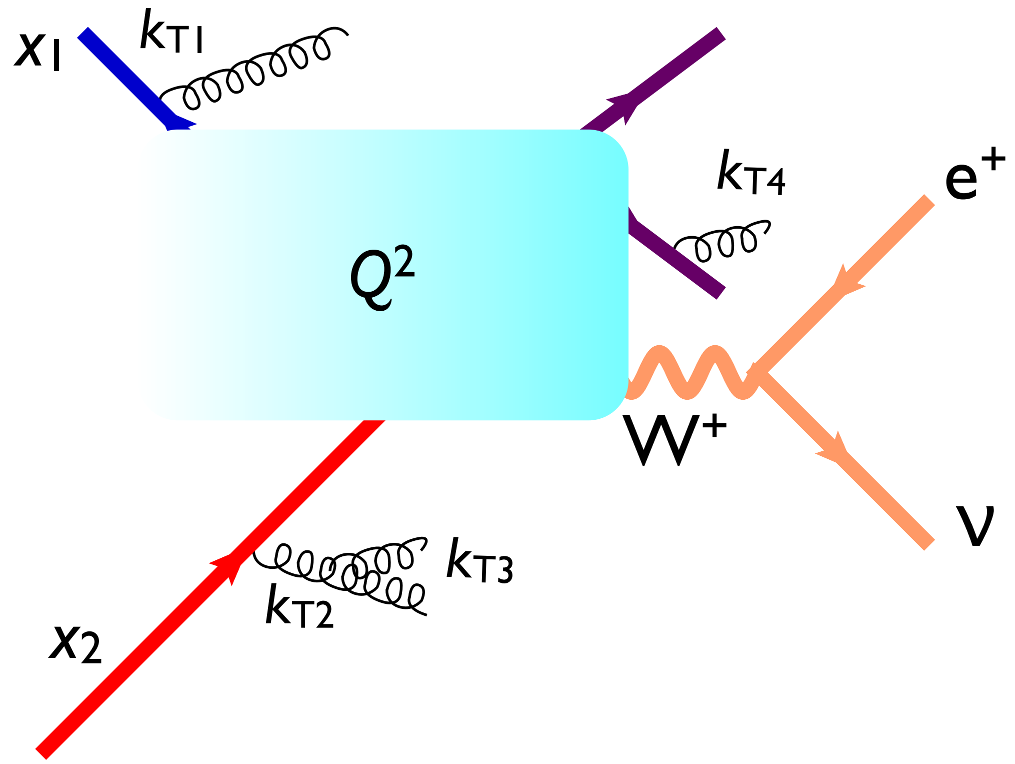

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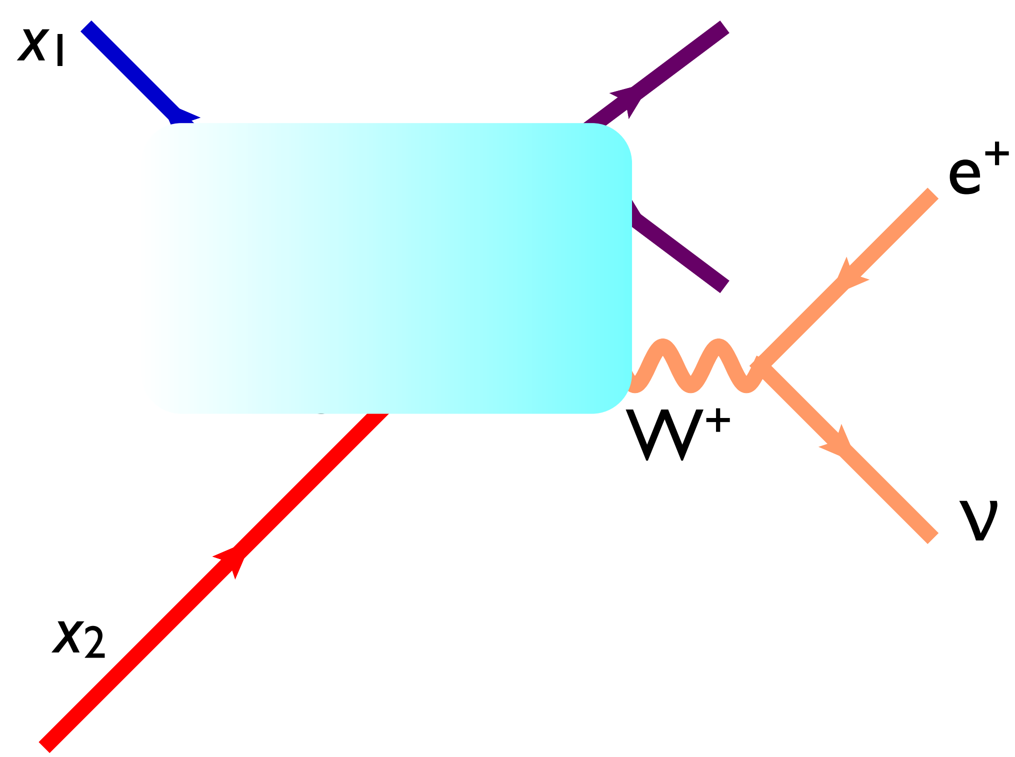

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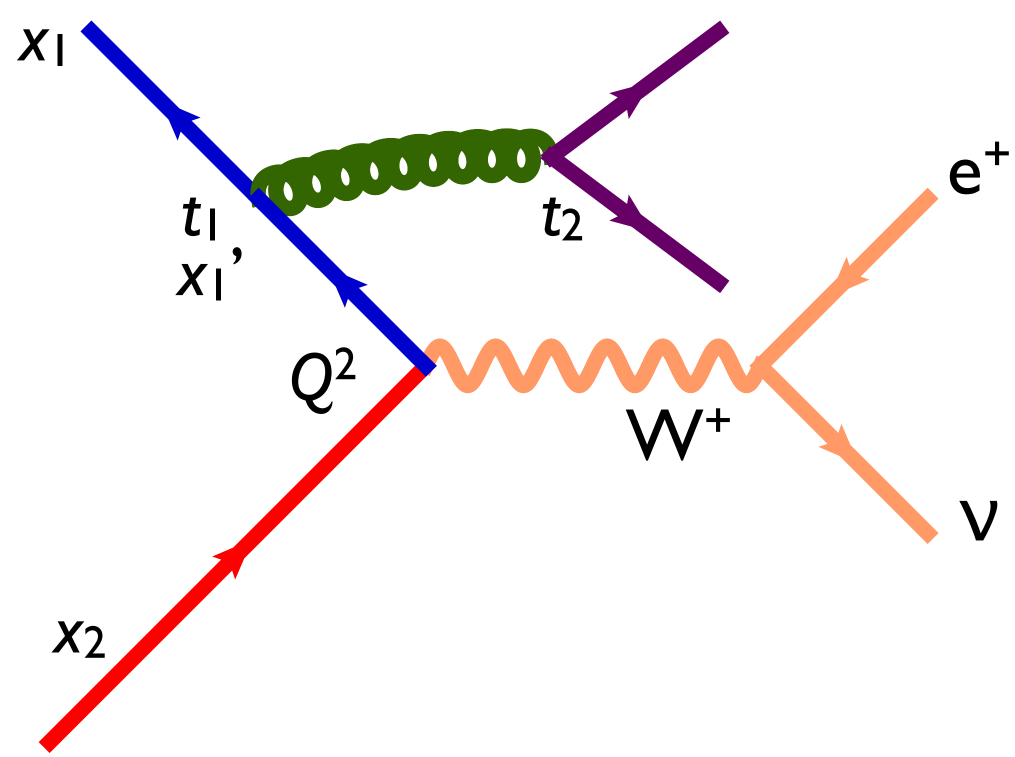

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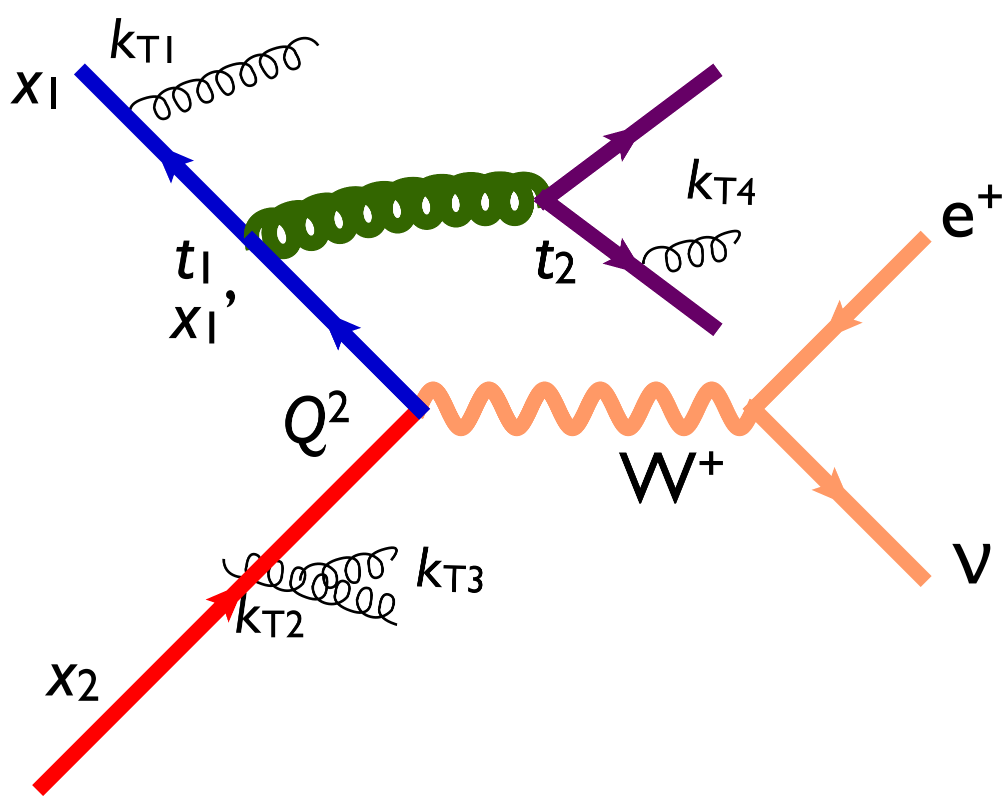


Matching schemes in existing codes

- AlpGen: MLM (cone)
- MadGraph: MLM (cone, k_T, shower-k_T)
- Sherpa: CKKW

MATCHING SCHEMES "FREEDOM"

- We have a number of choices to make in the above procedure. The most important are:

    1. The clustering scheme used to determine the parton shower history of the ME event
    2. What to use for the scale Q^2 (factorization scale)
    3. How to divide the phase space between parton showers and matrix elements

Cluster schemes

1. The clustering scheme used inside MadGraph and Sherpa to determine the parton shower history is the Durham k_T scheme. For e^+e^-:

$$k^2_{Tij} = 2\min(E_i^2, E_j^2)(1-\cos\theta_{ij})$$

and for hadron collisions, the minimum of:

$$k_{Ti\mathrm{beam}} = m_i^2 + p_{Ti}^2 = (E_i + p_{zi})(E_i - p_{zi})$$

and

$$k^2_{Tij} = \min(p^2_{Ti}, p^2_{Tj})R_{ij}$$

with

$$R_{ij} = 2[\cosh(y_i - y_j) - \cos(\phi_i - \phi_j)] \simeq (\Delta y)^2 + (\Delta\phi)^2$$

Find the smallest k_{Tij} (or $k_{Ti\mathrm{beam}}$), combine partons i and j (or i and the beam), and continue until you reach a 2 → 2 (or 2 → 1) scattering.

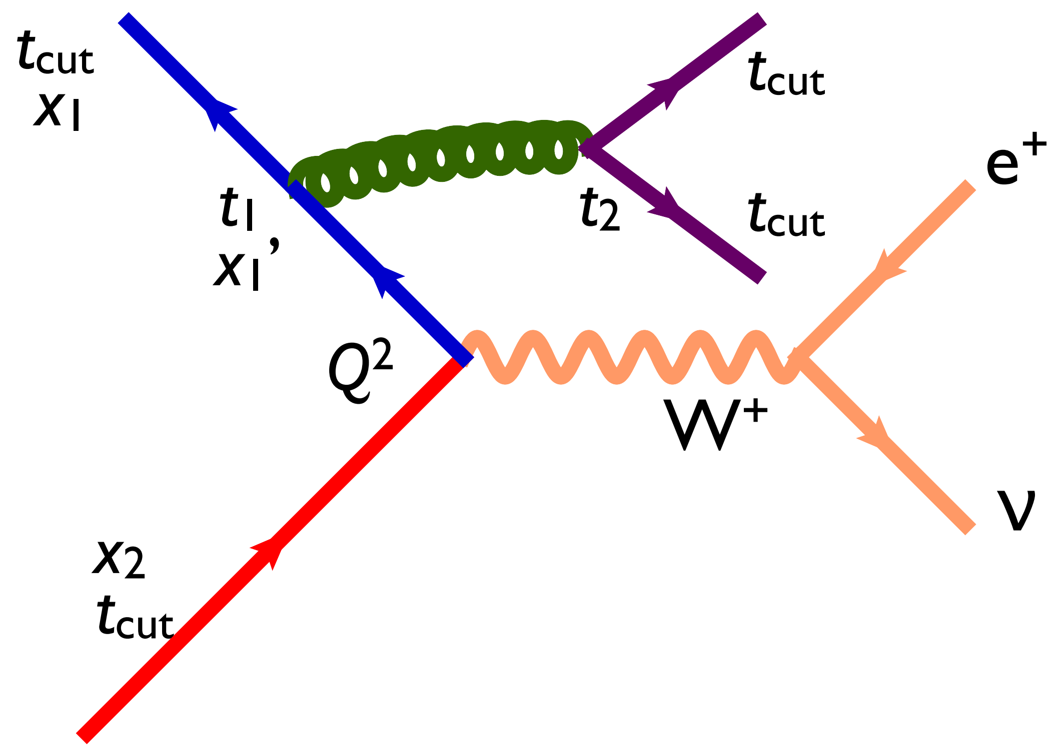
2. In AlpGen a more naive cone algorithm is used.

Cluster schemes

- Cannot use the standard k_T clustering:
  - MadGraph and Sherpa only allow clustering according to valid diagrams in the process. This means that, e.g., two quarks or quark-antiquark of different flavor are never clustered, and the clustering always gives a physically allowed parton shower history.
  - If there is an on-shell propagator in the diagram (e.g. a top quark), only clustering according to diagrams with this propagator is allowed.

Hard scale

2. The clustering provides a convenient choice for factorization scale Q^2:



Cluster back to the 2 → 2 (here qq → W-g) system, and use the W boson transverse mass in that system.

Phase-space division

Phase-space division

3. How to divide the phase space between PS and ME:
   This is where the schemes really differ:

   AlpGen: MLM Cone
   MadGraph: MLM Cone, k_T or shower-k_T
   Sherpa: CKKW

Phase-space division

3. How to divide the phase space between PS and ME:
   This is where the schemes really differ:

   AlpGen: MLM Cone
   MadGraph: MLM Cone, k_T or shower-k_T
   Sherpa: CKKW

   a. Cone jet MLM scheme (better suited for angular ordered showers, i.e. herwig, but works for all showers):
      - Use cuts in p_T (p_T^{ME})and ΔR between partons in ME
      - Cluster events after parton shower using a cone jet algorithm with the same ΔR and $p_T^{match} > p_T^{ME}$
      - Keep event if all jets are matched to ME partons (i.e., all ME partons are within ΔR of a jet)

Phase-space division

3. How to divide the phase space between PS and ME:
   b. k_T-jet MLM scheme (better suited for k_T ordered showers, i.e. pythia, but works for all showers):
   - Use cut in the Durham k_T in ME
   - Cluster events after parton shower using the same k_T clustering algorithm into k_T jets with $k_T^{match} > k_T^{ME}$
   - Keep event if all jets are matched to ME partons (i.e., all partons are within k_T^{match} to a jet)

   c. Shower-k_T scheme (works only with pythia, i.e. k_T ordered shower):
   - Use cut in the Durham k_T in ME
   - After parton shower, get information from the PS generator about the k_T^{PS} of the hardest shower emission
   - Keep event if $k_T^{PS} < k_T^{match}$

Phase-space division

3. How to divide the phase space between PS and ME:
   d. CKKW Scheme (Need special veto'ed shower):
      - Use cut in the Durham k_T in ME (k_T^{match})
      - Because the Durham k_T is not the same as the evolution parameter of the shower, we might miss contributions, therefore
      - Start the shower at the original scale, and after each emission, check the value of t_i:
      - if $t_i > k_T^{match}$ veto that emission, i.e. continue the shower as if that emission never happened

Summary of matching algorithms

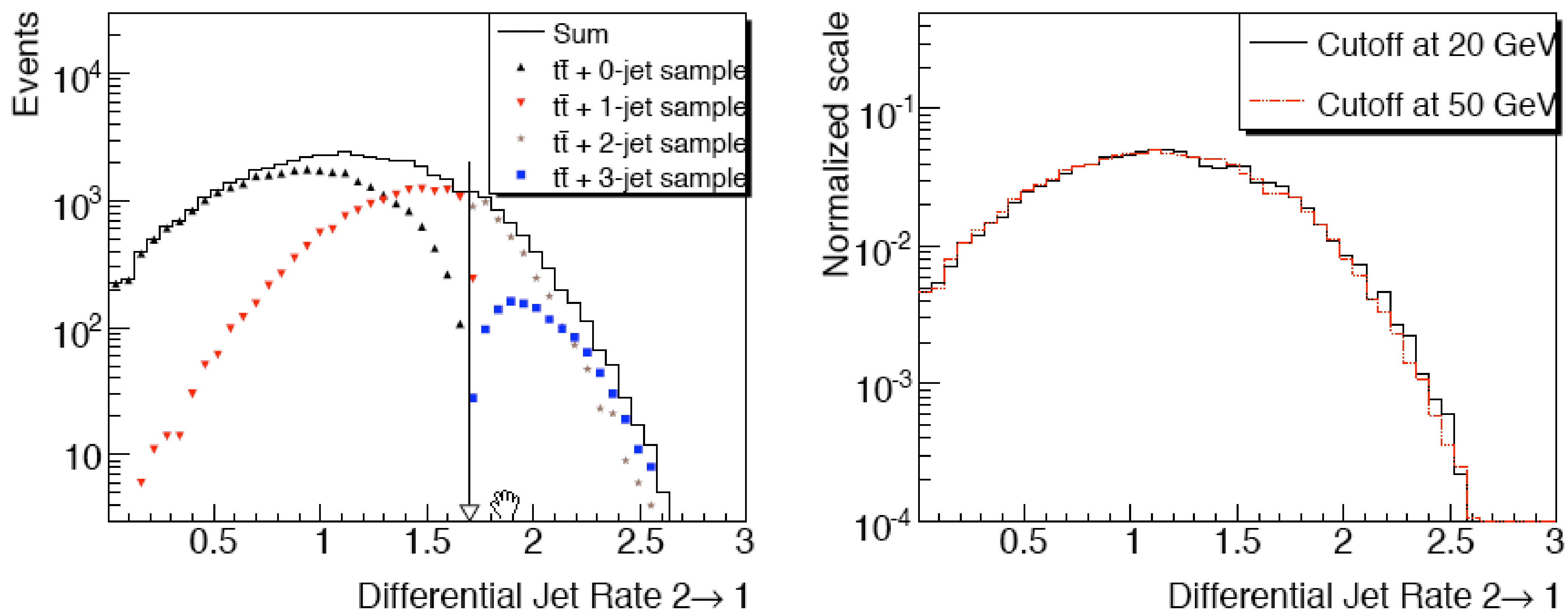
1. Generate ME events (with different parton multiplicities) using parton-level cuts ($p_T^{ME}/\Delta R$ or k_T^{ME})
2. Cluster each event and reweight α_s and PDFs based on the scales in the clustering vertices
3. Run the parton shower with starting scale $Q^2 = m_T^2$.

Summary of matching algorithm

4. a) For MLM: Check that the number of jets after parton shower is the same as ME partons, and that all jets after parton shower are matched to the ME partons (using one of the schemes in the last slides) at a scale Q^{match}. If yes, keep the event. If no, reject the event. Q^{match} is called the *matching scale*.

   b) For CKKW: Reweight the matrix elements with the Sudakovs related to the “most-likely parton shower history”. Start the shower at the at the scale Q^2, but veto emissions which are already taken care of by the matrix elements.

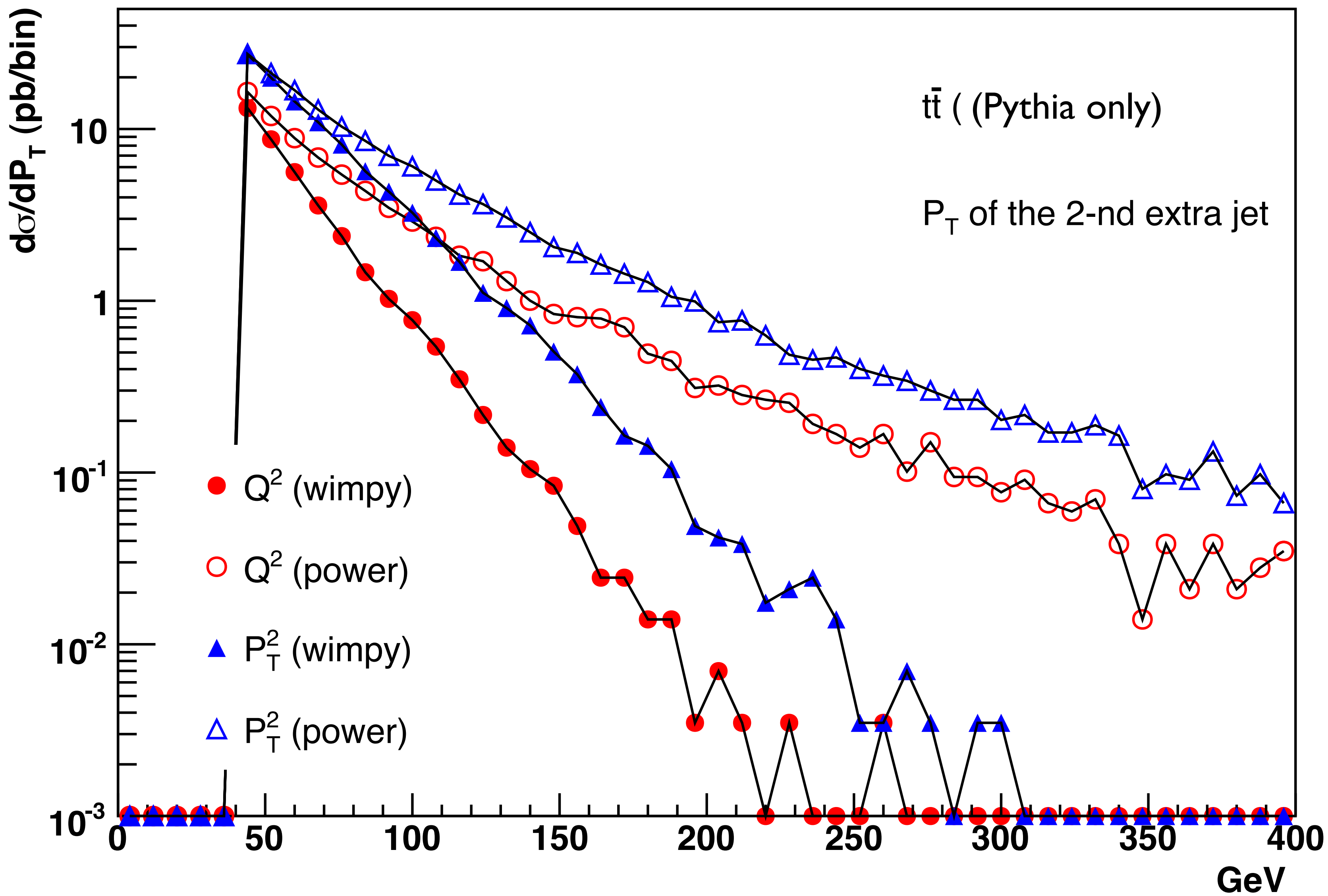
Sanity checks: differential jet rates



Jet rates are independent of and smooth at the cutoff scale

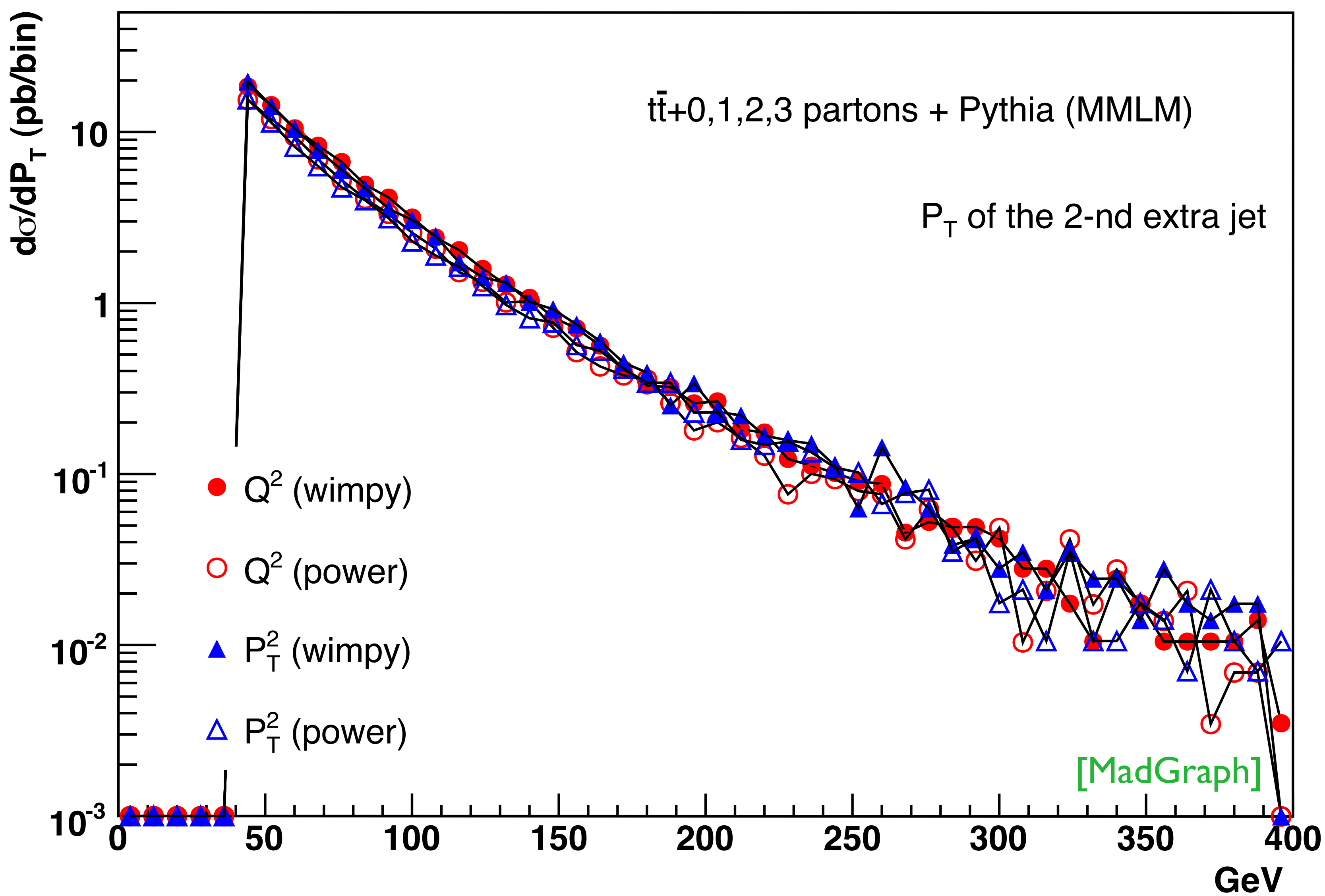
PS alone vs. Matched sample

In the soft-collinear approximation of Parton Shower MCs, parameters are used to tune the result ⇒ Large variation in results (small prediction power)

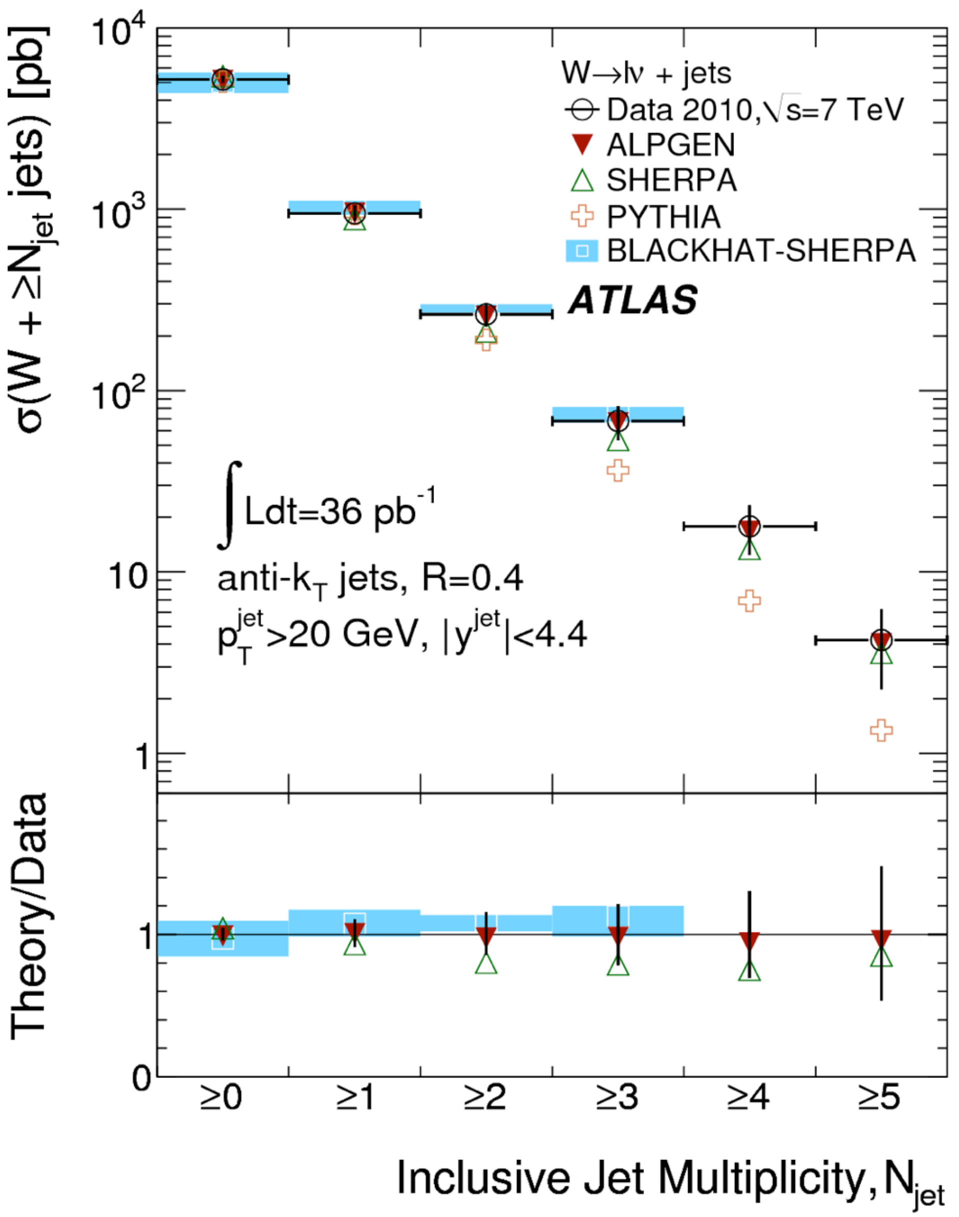


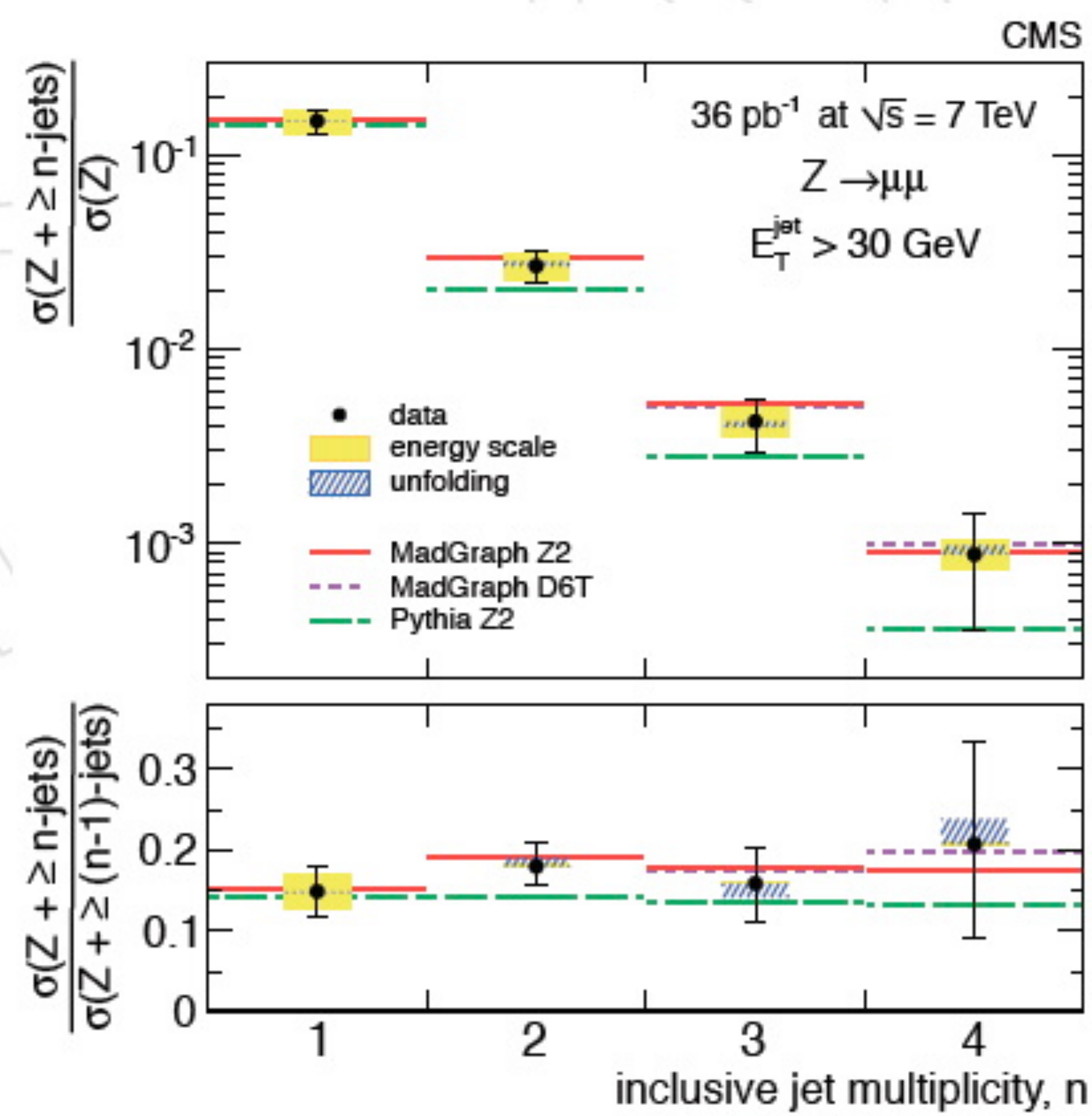
PS alone vs. Matched sample

In a matched sample these differences are irrelevant since the behavior at high pt is dominated by the matrix element.



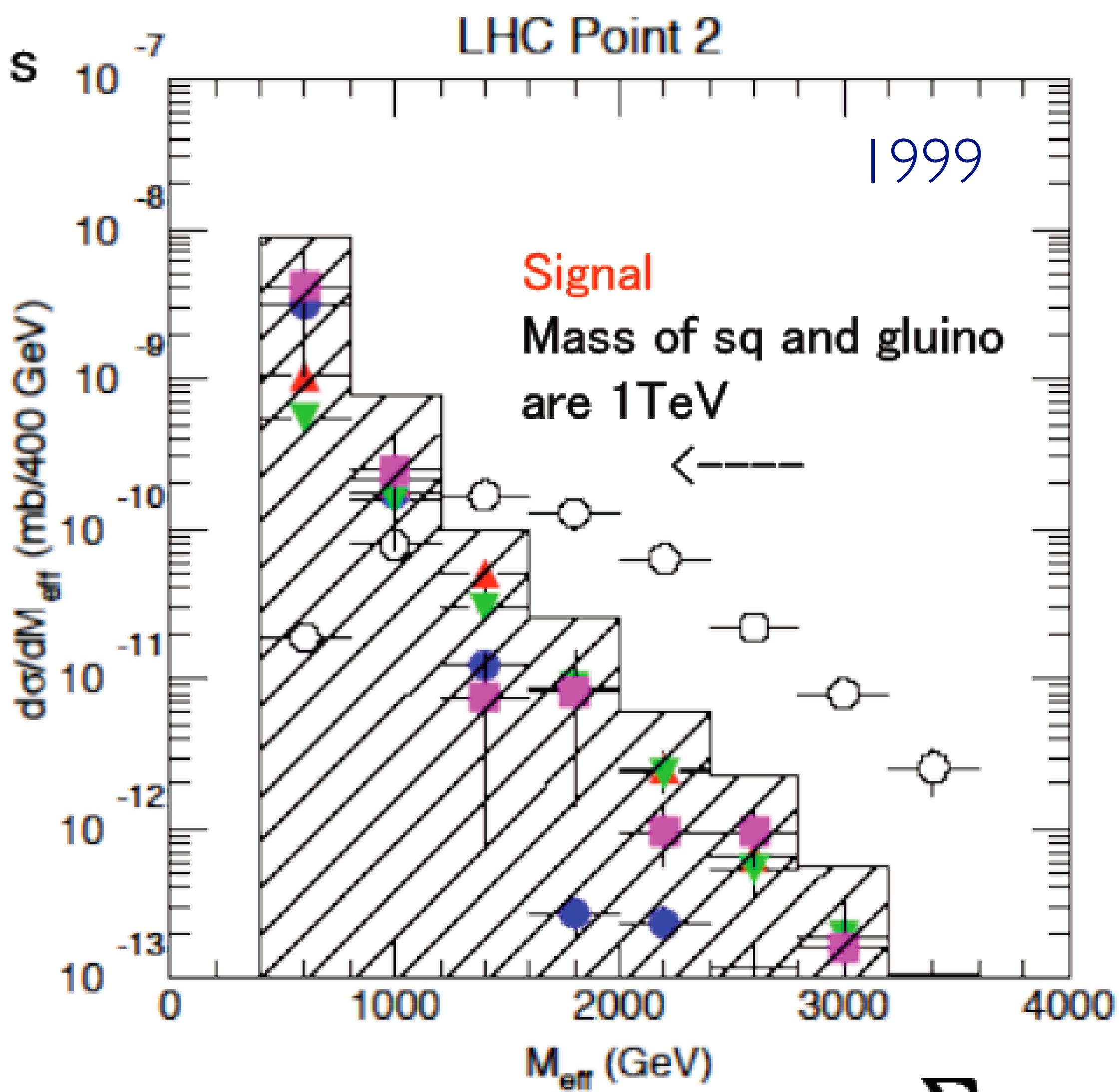
TH/EXP comparison at the LHC



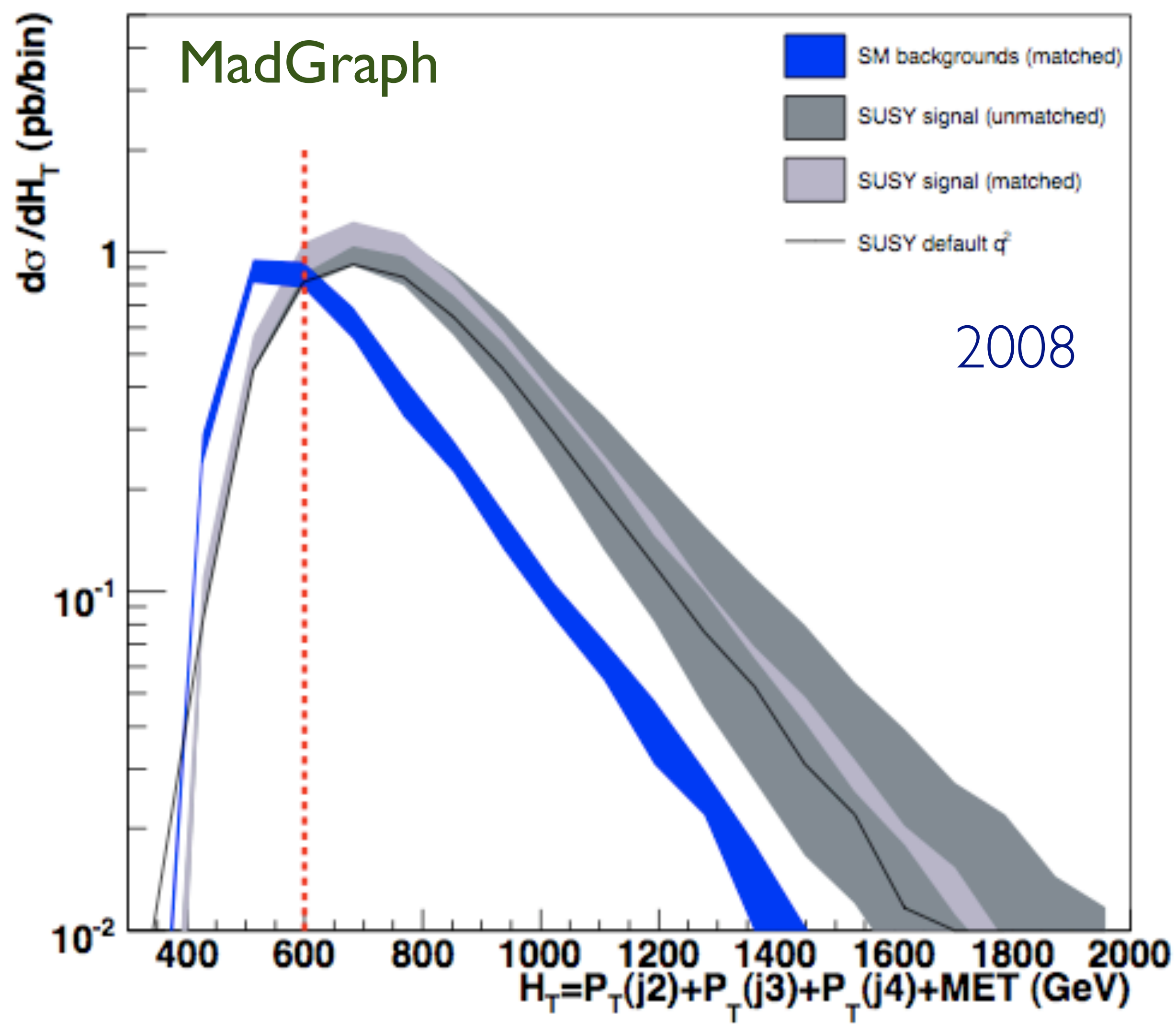


Bonus: Even rates in outstanding agreement with data and NLO

SUSY matched samples



$$= \not{E}_T + \sum P_T$$

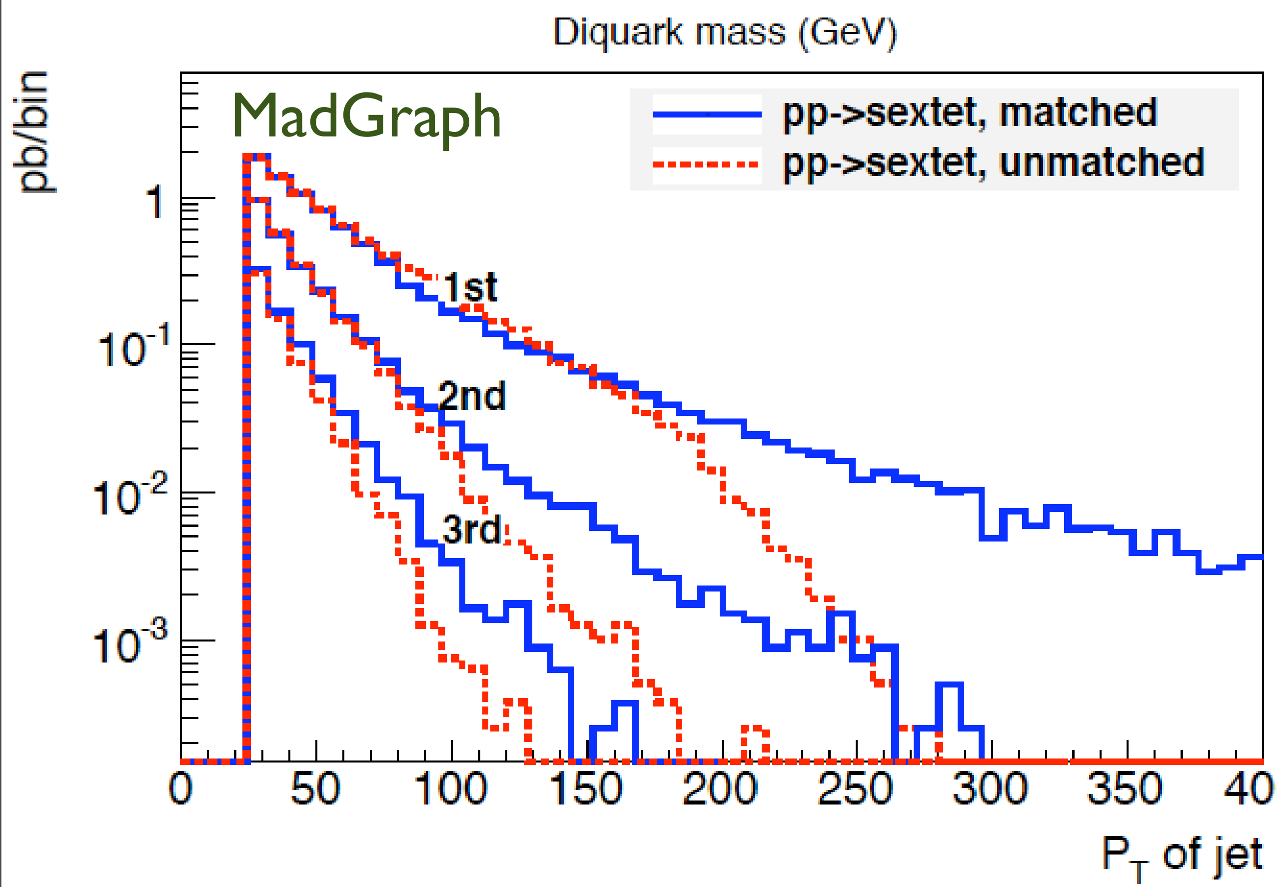


Both signal and background matched!

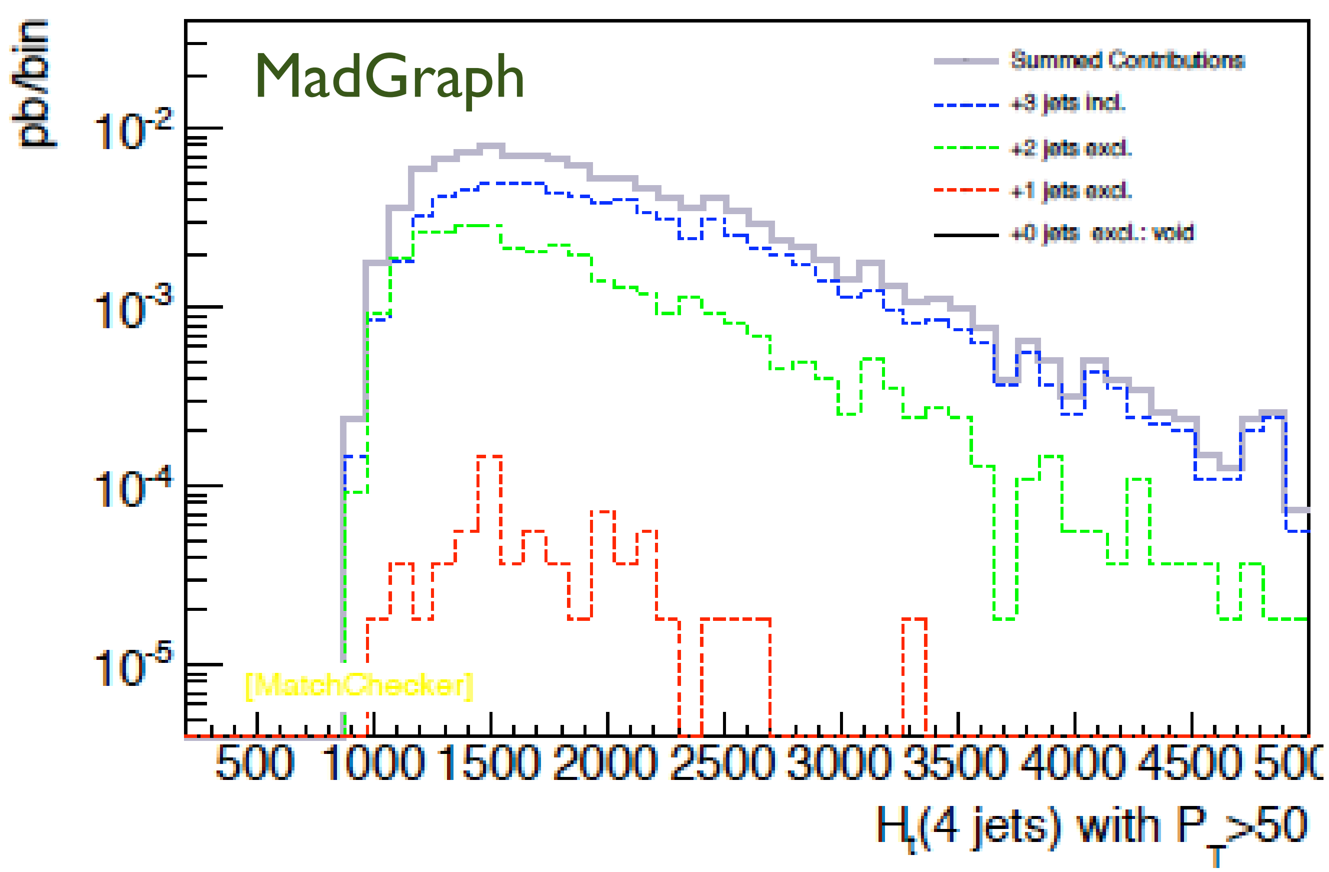
Sizable reduction of the uncertainties and simulation consistency .

Example: BSM multijet final states

pp→X6 +jets



pp→Graviton (ADD&RS) +jets



New Physics models can be easily included in Matrix Element generators via FeynRules and results automatically for multi-jet inclusive final state obtained at the same level of accuracy that for the SM.

Summary of ME/PS merging

- Merging matrix elements of various multiplicities with parton showers improves the predictive power of the parton shower outside the collinear/soft regions.
  - These matched samples give excellent prescription of the data (except for the total normalization).
- There is a dependence on the parameters responsible for the cut in phase-space (i.e. the matching scale).
- By letting the matrix elements mimic what the parton shower does in the collinear/soft regions (PDF/alpha$_s$ reweighting and including the Sudakov suppression) the dependence is greatly reduced.
- In practice, one should check explicitly that this is the case by plotting differential jet-rate plots for a couple of values for the matching scale.

Credits

To organize this presentation I have benefited from lectures (and actual slides), talks and discussions with many people.
In particular:

- Mike Seymour (MC basics)
- Claude Duhr (FeynRules)
- Johan Alwall (ME+PS merging)
- Rikkert Frederix, Paolo Torrielli (NLO+PS)
- Stefano Frixione, Michelangelo Mangano, Paolo Nason (for QCD, PS, LO, NLO, and more...)
- ....

Whom I all warmly thank!!