The heavy quark potential and jet quenching parameter in a Super Yang-Mills Plasma

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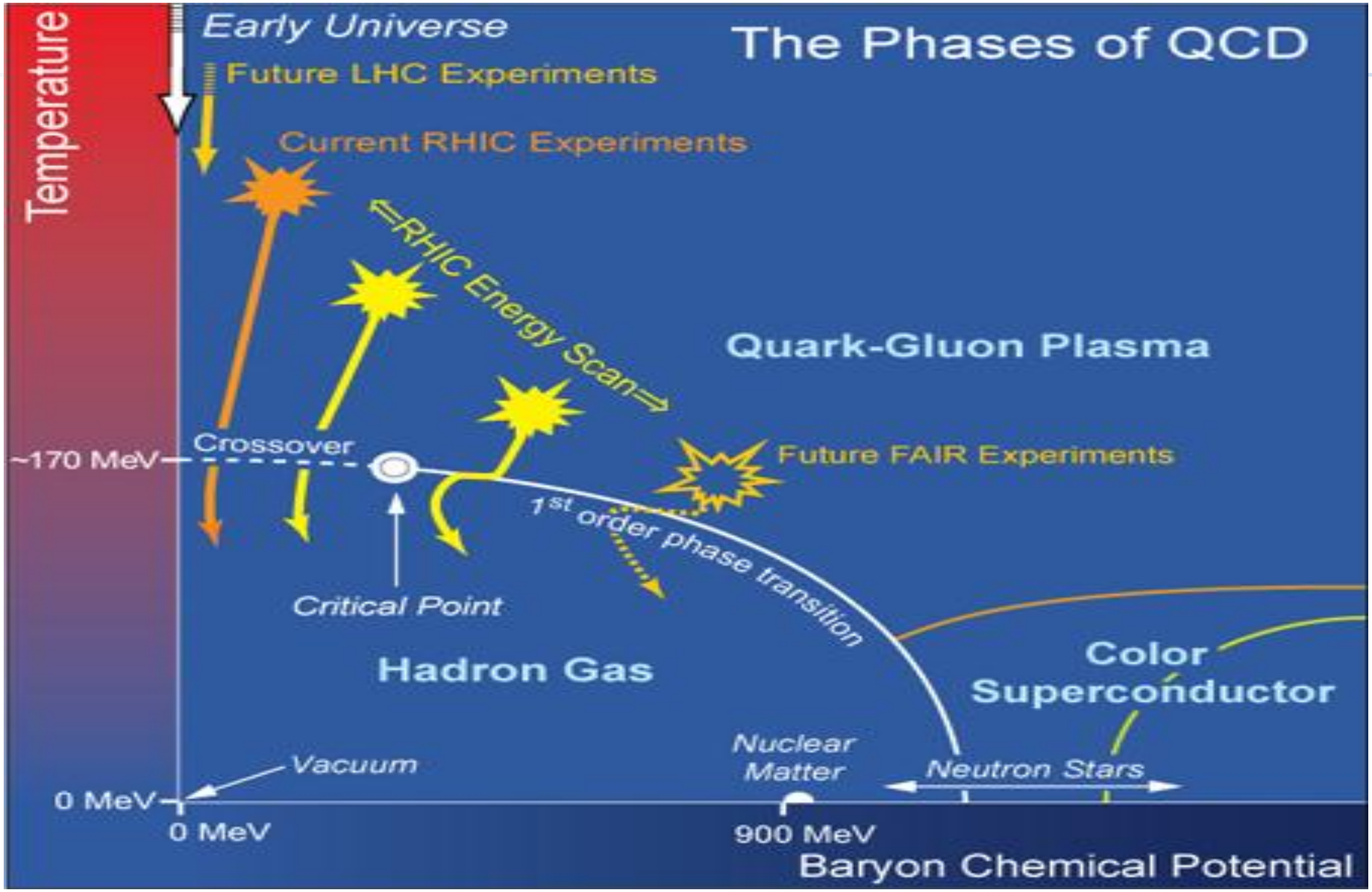
Zhang,Hou, Ren,Yin JHEP1107:035(2011)
Zhang,Hou, Ren, [arxiv1210.5187]

HPT 2012, Wuhan

Outline

- Motivations
- AdS/CFT correspondence
- Heavy quark potential and jet quenching parameter
- Summary and discussion

Motivations



Many interesting phenomena in QCD lie in the strongly coupled region

Lattice QCD: a first-principle computation.

However, there are technical difficulties in the computations if the system has

1. Finite baryon chemical potential
2. Real time dynamics

…

Maldacena conjecture: *Maldacena, Witten*

$N=4$ SUSY YM on the boundary $\Leftrightarrow$ Type IIB string theory in the bulk

'‘t Hooft coupling $\lambda \equiv N_c g_{YM}^2 = \frac{1}{\alpha'^2}$ (string tension $= \frac{1}{2\pi\alpha'}$)

$$\frac{\lambda}{N_c} = 4\pi g_s$$

$$< e^{\int d^4x \phi_0(x) O(x)} > = Z_{\text{string}}[\phi(x,0)=\phi_0(\text{x})]$$

In the limit $N_c \to \infty$ and $\lambda \to \infty$

$$Z_{\text{string}}[\phi(x,0)=\phi_0(x)] = e^{-I_{\text{sugra}}[\phi]}\Big|_{\phi(x,0)=\phi_0(x)}$$

$$I_{\text{sugra}}[\phi] = \text{classical supergravity action}$$

QCD versus
N=4 Super Yang-Mills from gravity dual

	QCD	Super YM
N_c	3	>>1
t'Hooft coupling	5.5-18.8	>>1
Quarks	Fundamental	Adjoint
Conformal symmetry	No	Yes at zero T No at nonzero T
Supersymmetry	No	Yes at zero T No at nonzero T

AdS/CFT applied to RHIC physics

- **Viscosity ratio, η/s.** $\frac{\eta}{s} = \frac{1}{4\pi}$ — Policastro, Son and Starinet
- **Thermodynamics.** $s = \frac{3}{4} s^{(0)}$ — Gubser
- **Jet quenching** $\hat{q} = \pi^{\frac{3}{2}} \frac{\Gamma\left(\frac{3}{4}\right)}{\Gamma\left(\frac{5}{4}\right)} \sqrt{\lambda} T^3$ — Liu, Rajagopal and Wiederman
- **Photon production**
- **Friction**
- **Heavy quarkonium**
- **Hardron spectrum (ADS/QCD)**

Heavy quark potential

The gravity dual of a Wilson loop at large N_c and large λ

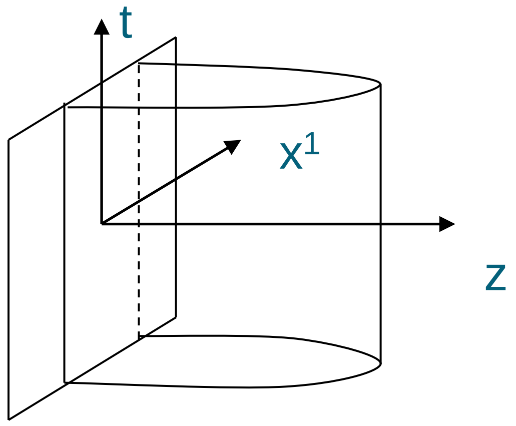
$$\mathrm{tr} < W(C) > = e^{-\sqrt{\lambda} S_{\min}[C]}$$

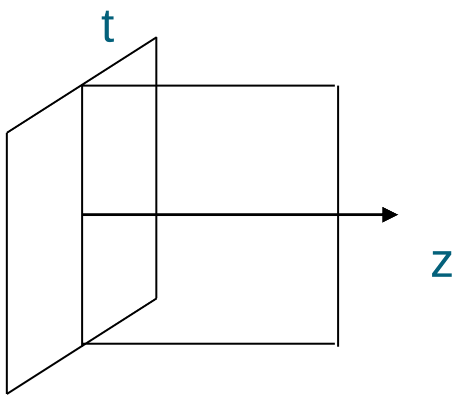
the min.area of string world sheet in the AdS_5

$$W(C) = P e^{-i \oint_C dx^\mu A_\mu(x)}$$

Heavy quark potential probes confinement hadronic phase and meson melting in plasma

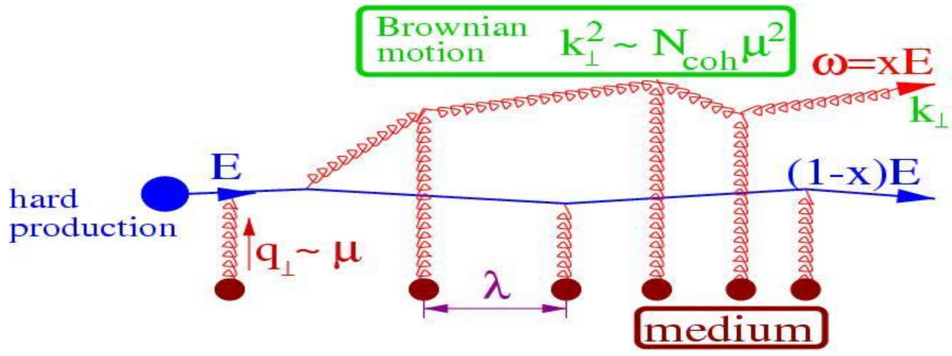
$$F(r,T) = T\left(S_{\min}[\text{parallel lines}] - 2S_{\min}[\text{single line}]\right)$$





Jet quenching parameter

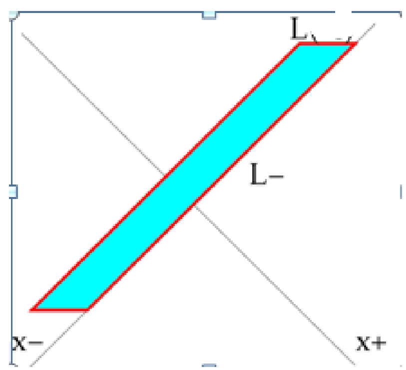
Nestor Armesto ,et, al, JHEP 0609 (2006) 039



Jet quenching parameter describes the energy-loss of a jet of fast moving heavy quarks in QGP for TL<<1

$$\int_{\mathbf{y}=0=\mathbf{r}(y_l)}^{\mathbf{u}=\mathbf{r}(\bar{y}_l)} \mathcal{D}\mathbf{r}\,\exp\left[i\int_{y_l}^{\bar{y}_l} d\xi\,\frac{\omega}{2}\left(\dot{\mathbf{r}}^2 - \frac{\hat{q}(\xi)\mathrm{r}^2}{2i\,\omega}\right)\right] \longrightarrow \exp\left[-\frac{1}{4}\hat{q}(y_l-\bar{y}_l)\mathrm{r}^2\right] = \langle W^A(\mathcal{C})\rangle_T$$

$$W^A[C] = \exp\left(-\frac{\hat{q}\,L_- L^2}{4\sqrt{2}}\right)$$



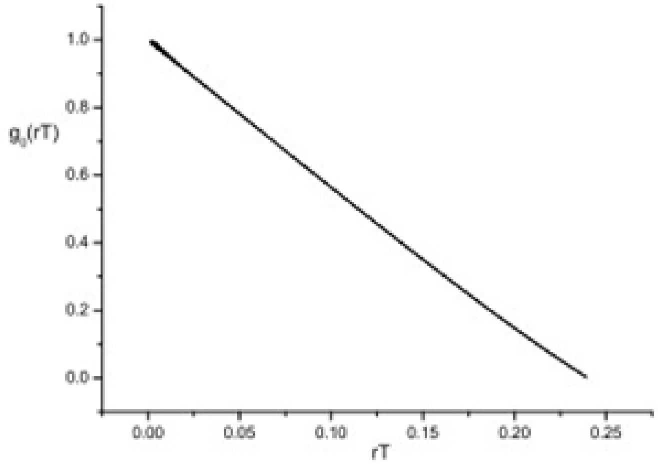
Semi-classical expansion	Strong coupling expansion
The type Ⅱ B superstring	N=4 SYM
The path integral of the string-sigma action	Wilson loop expectation
Classical limit	Leading order
One loop correction	Sub-leading order

Heavy quark potential leading order from AdS/CFT

$$V(r) \simeq -\frac{4\pi^2}{\Gamma^4\left(\frac{1}{4}\right)}\frac{\sqrt{\lambda}}{r}\; g_0(rT)$$

Maldacena PRL.80, (1998) 4859

where $g_0(rT)$ is a monotonic decreasing function corresponds to the **leading order**



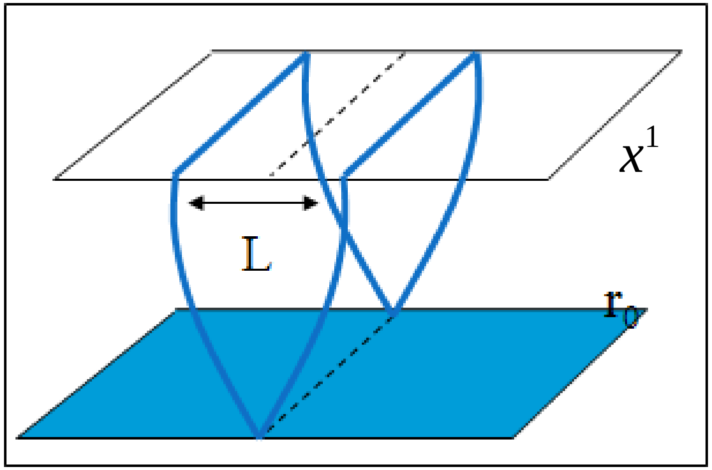
S.J.Rey , Nucl.Phys.B 527,171(1998)

Jet quenching leading order from AdS/CFT

$$\hat{q} = \frac{\pi^{3/2}\Gamma(\frac{3}{4})}{\Gamma(\frac{5}{4})}\sqrt{\lambda}T^3$$

Liu, Rajagopal & Wiedemann, PRL,97,182301(2006)

Dipole amplitude: two parallel Wilson lines in the light cone:



Leading orders are strictly valid when $\lambda \to \infty$, $N_c \to \infty$

Real QCD, $5.5 < \lambda < 6\pi$ $N_C = 3$

Finite λ corrections may be essential
for more precise theoretical predictions.

That is why we compute their next order corrections!

Next order corrections

Formulation:

$$W[C] \equiv \left\langle \exp\left(i\oint_C dx^\mu A_\mu \right) \right\rangle = \int [dX][d\theta] \exp\left[\frac{i}{2\pi\alpha'} S(X,\theta) \right]$$

Strong coupling expansion ⇄ Semi-classical expansion

$$\ln W[C] = i\sqrt{\lambda}\left[S\left(\bar{X},\ 0\right) + \frac{b[C]}{\sqrt{\lambda}} + \dots \right] = S_{eff}[C]$$

$\bar{X}$ = the solution of the classical equation of motion;

$b[C]$ comes from the fluctuation of the string world sheet around $\bar{X}$

Formulation:

$$\frac{1}{2\pi\alpha'} S(X,\theta) = \text{the superstring action in } AdS_5 \times S^5$$ *Metsaev and Tseytlin*

$Schwarzschid - AdS_5 \times S^5$ M. Cvetic,et,al. Nucl. Phys. B 573,149 (2000)

$$S(X,0) = \frac{1}{2}\int d^2\sigma \sqrt{g}\, g^{\alpha\beta} G_{\mu\nu} \frac{\partial X_\mu}{\partial \sigma^\alpha} \frac{\partial X_\nu}{\partial \sigma^\beta}$$

= Polyakov action of a bosonic string

and is equivalent to the Nambu-Goto action

$$X^\mu = \bar{X}^\mu + \delta X^\mu, \qquad \theta \neq 0 \qquad g_{\alpha\beta} = \bar{g}_{\alpha\beta} + \delta g_{\alpha\beta}$$

Quadratic expansion in δX^μ, θ and $\delta g^{\alpha\beta}$

$$S(X,\theta) = S(\bar{X},0) + S_B^{(2)}(\delta X) + S_F^{(2)}(\theta) + \ldots$$

Bosonic and fermionic fluctuations decouple.

$$W[C] = e^{iS(\bar{X},0)} Z \qquad Z = Z_B Z_F$$

Partition function at finite T with fluctuation underlying the heavy quark potential

Hou, Liu, Ren, PRD80,2009

Straight line：

$$Z = Z_B Z_F = \frac{\det^2\left(-\nabla_+^2 + 1 + \frac{1}{4}R^{(2)}\right)\det^2\left(-\nabla_-^2 + 1 + \frac{1}{4}R^{(2)}\right)}{\det^{\frac{3}{2}}\left(-\nabla^2 + \frac{8}{3} + \frac{1}{2}R^{(2)}\right)\det^{\frac{5}{2}}(-\nabla^2)}$$

Parallel lines：

$$Z = \frac{\det^2\left(-\nabla_+^2 + 1 + \frac{1}{4}R^{(2)}\right)\det^2\left(-\nabla_-^2 + 1 + \frac{1}{4}R^{(2)}\right)}{\det^{\frac{1}{2}}\left(-\nabla^2 + 4 + R^{(2)} - 2\delta\right)\det(-\nabla^2 + 2 + \delta)\det^{\frac{5}{2}}(-\nabla^2)}$$

Partition function at finite T with fluctuation underlying the jet quenching parameter.

Zhang,Hou, Ren, arxiv1210.5187

$$Z = \frac{\det^2 \mathcal{A}_+ \det^2 \mathcal{A}_-}{\det^{\frac{1}{2}} \mathcal{A}_{\xi\eta} \det^{\frac{1}{2}} \mathcal{A}_{\zeta} \det^{\frac{5}{2}} (-\nabla^2)}$$

where $\mathcal{A}_\zeta = -e^{-2\phi} \left(\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial \rho^2} \right) + M_1^2$

The other A refer to the correspondence operators too

The one loop effective action

$$S_{eff} = S[\bar{X}, 0] - i \ln Z$$

Computing of the determinant ratio

$$\frac{\det H_2}{\det H_1} = \frac{\Lambda[u_2, v_2]}{\Lambda[u_1, v_1]}$$

I.M.Gelfand, et, al, J.Math.Phys.,1,48(1960)

(u_i, v_i) *are 2 independent solutions* 。

$$\Lambda[u_i, v_i] = \frac{u_i(a)v_i(b) - u_i(b)v_i(a)}{W[u_i, v_i]}$$

Wronskian determinant

Reduce evaluating functional determinants to a set of 2nd order ordinary differential equations, which are solved numerically

Heavy quark potential including the next order correction

$$V(r) \simeq -\frac{4\pi^2}{\Gamma^4\left(\frac{1}{4}\right)}\frac{\sqrt{\lambda}}{r}\left[g_0(rT) - \frac{1.33460 g_1(rT)}{\sqrt{\lambda}} + O\left(\frac{1}{\lambda}\right)\right]$$

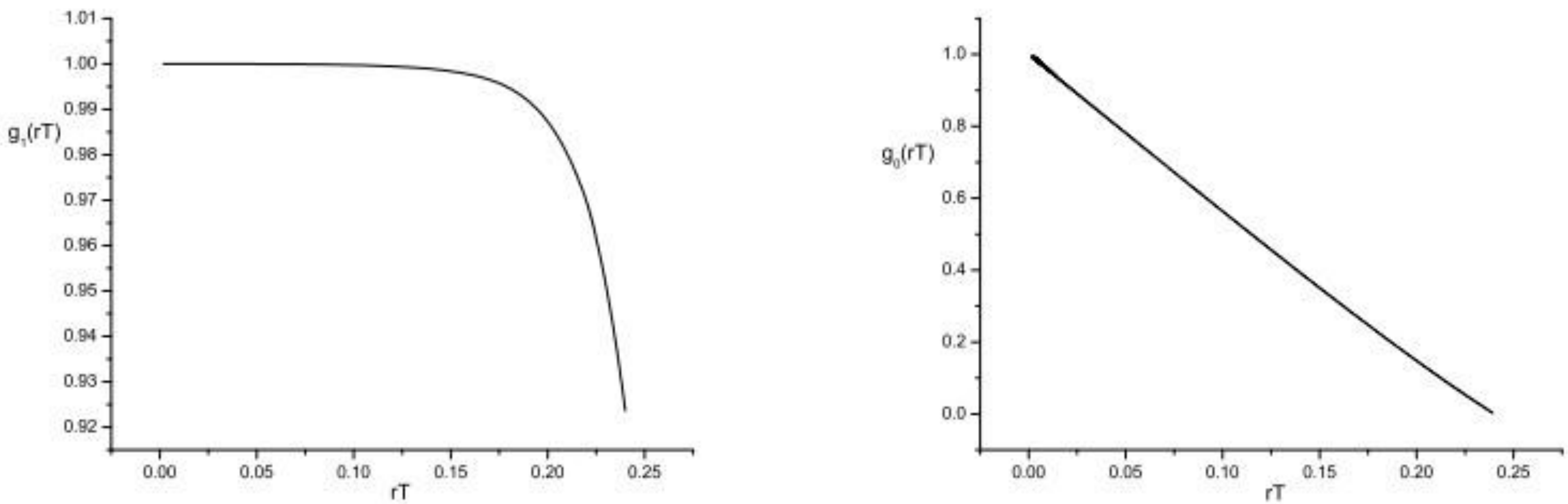


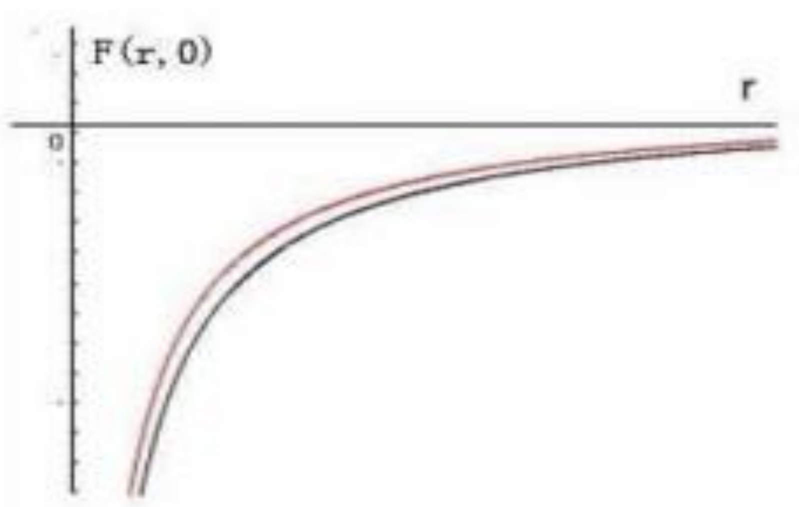
Figure 3. The left curve represents $g_1(rT)$, while the right represents $g_0(rT)$.

Zhang,Hou, Ren,Yin JHEP1107:03（2011）

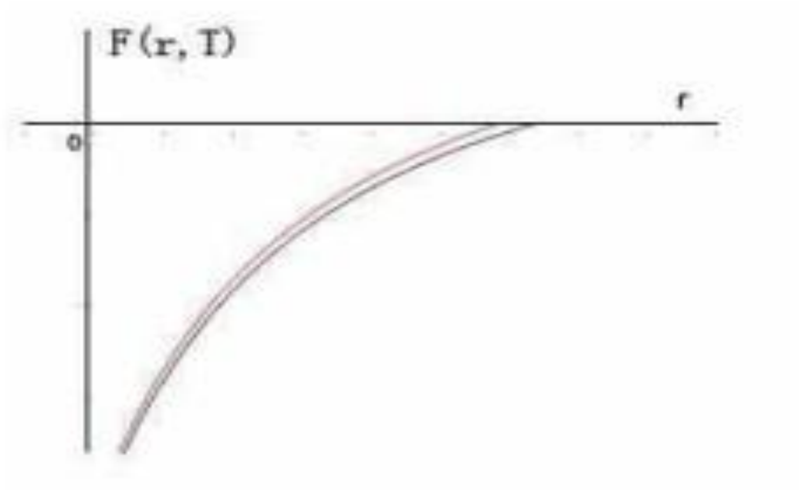
Heavy quark potential discussion

1.In the weak coupling, the heavy quark potential at T>0 is of Yukawa type, that is non-vanishing for arbitrarily large r.
2.To the leading order of strong coupling, the magnitude of the potential drops to zero at a finite r.

3. The $O\left(\frac{1}{\sqrt{\lambda}}\right)$ term decreases the screening radius.



Chu,Hou, Ren,JHEP0908,(2009)



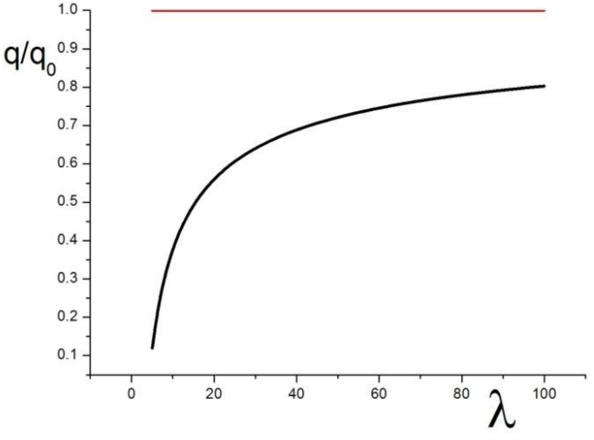
Zhang,Hou, Ren,Yin JHEP1107:03（2011）

Jet quenching parameter including the next order correction

$$\hat{q} = \frac{\pi^{3/2}\Gamma(\frac{3}{4})}{\Gamma(\frac{5}{4})}\sqrt{\lambda}T^3[1-1.97\lambda^{-1/2}+O(\lambda^{-1})]$$

Zhang,Hou, Ren, arxiv1210.5187

$$\hat{q}_0 = \frac{\pi^{3/2}\Gamma(\frac{3}{4})}{\Gamma(\frac{5}{4})}\sqrt{\lambda}T^3$$



Jet quenching parameter discussion

$$\hat{q}_{\exp} = 1 \rightarrow 15 GeV^2 / fm$$ Nestor Armesto ,et, al, JHEP 0609 (2006) 039

Take $N_c = 3, \alpha_{SYM} = \frac{1}{2} \Longleftrightarrow \lambda = 6\pi$

Choose T=300MeV

$$\hat{q}_0 = \frac{\pi^{3/2}\Gamma(\frac{3}{4})}{\Gamma(\frac{5}{4})}\sqrt{\lambda}T^3 = 4.48 GeV^2 / fm$$

1.Sub-leading order gives rise to 32% reduction of $\hat{q}$ from the leading order amount in this case.

2.The negative sign of sub-leading order is consistent with a monotonic behavior from strong coupling to weak coupling.

Summary and discussion

AdS/CFT provides a useful way to address the physics at strong coupling .

The partition function of Wilson loop with fluctuations in strongly coupling N=4 SYM plasma are derived.

We computed the jet quenching parameter and heavy quark potential up to sub-leading orders.

The applicability of these AdS/QCD results demands phenomenological work to explain them in a way which can be translated to real QCD.

Thanks