Resolving the Neutrino Ambiguity

By: Kelvin Mei (Rutgers University)

Advisors: Konstantinos Kousouris
           Andrea Giammanco
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CMS – Big Picture

• **General Purpose Experiment**
  • SUSY – Does this explain the unification of the electromagnetic, weak, and strong forces?
  • Higgs – Does this particle exist? Is the Standard Model accurate?
  • Dark Energy/Dark Matter – Why is the universe accelerating?
  • Extra Dimensions – Explain the weakness of gravity?
**Introduction – Single Top Decays**

- Single top quarks are produced through electroweak processes.
- Normally the top quark decays hadronically into many jets.
- About 30% of the time, the top quark decays through a semileptonic channel, resulting in a lepton, its corresponding neutrino, a bottom quark.
Introduction – Neutrino Ambiguity

• Leptons can be detected with a very high efficiency in the CMS detector (with a relative isolation requirement >.95, only 12% of the signal is removed, but almost the entire background is cut out with this requirement).

• The bottom quark can with relatively high efficiency be reconstructed using b-tagging techniques (90.4% efficiency in distinguishing the correct b-jet in a signal with many jets).

• The neutrino cannot be detected, and its properties are deduced from the missing energy and the conservation of 4-momentum from $W^+ \rightarrow l^+ + \nu$, but...
Principal Equation:

\[ P_W = P_\nu + P_\mu \]

after lots of mathematics, and quite a few approximations:

\[ P_{A,B}^{z,\nu} = \frac{\mu \cdot P_{z,\mu}}{P_T^{2,\mu}} \pm \sqrt{\frac{\mu^2 \cdot P_{z,\mu}^2}{P_T^{4,\mu}}} - \frac{E_{\mu}^2 \cdot E_{T}^{\text{miss}^2}}{P_T^{2,\mu}} \]

\[ \mu = \frac{M_W^2}{2} + P_{T,\mu} \cdot E_T^{\text{miss}}. \]

For more detailed calculations, check the Appendix.
Purpose

Find a method by which to solve the neutrino ambiguity, such that the neutrino longitudinal momentum is as close to its true value as possible.
Motivation

• Provides an independent, unbiased estimate of one of the components of the CKM matrix ($V_{tb}$).
• If we can more accurately reconstruct the single top, then this channel will become more sensitive to new physics searches.
• If we can more accurately reconstruct the single top, then channels with single top as a background will also become more sensitive to new physics.
• Approach can be applied, perhaps, to other channels with neutrino ambiguities, such as:
  • T-tbar with one of them undergoing semileptonic decay
  • Observation of exotic WZ resonances (3 leptons and 1 neutrino)
  • Semi-leptonic kaon decay where there is a W boson decaying into a lepton and neutrino.
Procedure

• Research and come up with a new method (or use an already established one).

• Implement the method onto a test Monte Carlo tree using ROOT and create plots of the neutrino $P_z$, the W boson, and the top quark mass.

• Use a Gaussian fit for the neutrino $P_z$ and a Landau fit on the top quark mass in order to compare methods and to get rough estimates for the spread and mean values of the momentum and mass distributions.
Traditional Methods

- Of the positive methods, choosing the smaller root is more advantageous.
- Of the negative methods, scaling the $\text{MET}$ is better, but does not improve significantly to dropping the imaginary root.
- A new approach beyond traditional methods is necessary, especially for negative discriminants.

<table>
<thead>
<tr>
<th>Method</th>
<th>Gaussian $\sigma$</th>
<th>Landau MPV</th>
<th>Landau $\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parton</td>
<td>79.07</td>
<td>171.56</td>
<td>0.35</td>
</tr>
<tr>
<td>Positive: Smaller Root</td>
<td>71.15</td>
<td>148.30</td>
<td>17.17</td>
</tr>
<tr>
<td>Positive: Weighing the Roots</td>
<td>88.65</td>
<td>120.71</td>
<td>21.10</td>
</tr>
<tr>
<td>Negative: Drop Imaginary Part</td>
<td>72.02</td>
<td>204.48</td>
<td>37.69</td>
</tr>
<tr>
<td>Negative: Let $W$ Mass Fluctuate</td>
<td>183.37</td>
<td>193.86</td>
<td>32.33</td>
</tr>
<tr>
<td>Negative: Scaling the $\text{MET}$</td>
<td>79.53</td>
<td>196.68</td>
<td>33.18</td>
</tr>
</tbody>
</table>
Pure Traditional Method

**Implemented Method:**

Just a combination of the simple positive discriminant method (choose the smaller root) and the simple negative discriminant method (drop the imaginary part).

**Reasoning:**

Just to be used for comparison.

<table>
<thead>
<tr>
<th>VARIABLE NAME</th>
<th>VALUE</th>
<th>ERROR</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Constant</td>
<td>1530.61</td>
<td>24.01</td>
</tr>
<tr>
<td>2 MPV</td>
<td>166.69</td>
<td>0.67</td>
</tr>
<tr>
<td>3 Sigma</td>
<td>26.21</td>
<td>0.33</td>
</tr>
</tbody>
</table>
Several different multivariate regressions were tried, but the most promising one was chosen.

There are ten variables in this multivariate regression:
- Missing Transverse Energy angle in the x-y plane ($\phi_{\text{MET}}$)
- Missing Transverse Energy ($\text{MET}$)
- Lepton Transverse Momentum ($P_{T,l}$)
- Lepton Transverse Momentum angle in the x-y plane ($\phi_l$)
- Lepton Pseudorapidity ($\eta_l$)
- B-jet Transverse Momentum ($P_{T,b}$)
- B-jet Transverse Momentum angle in the x-y plane ($\phi_b$)
- B-jet Pseudorapidity ($\eta_b$)
- B-tag value
- Rho – a variable that takes into account pile-up.

A Boosted Decision Tree Method was applied with the target being the momentum of the neutrino.
- Boosted decision trees are less susceptible to overtraining than neural networks and regular decisions trees.
- For correlation matrices and regression output deviation graphs, see backup slides.

TMVA (Multivariate Regression)
**Pure Regression Method**

*Implemented Method:*

*Boosted Decision Tree with 10 variables.*

*Reasoning:*

*A multivariate analysis may be able to avoid the shortcomings of the traditional methods*
Mixed Traditional and Regression Method

**Implemented Method:**

The regression method as a whole is not that much better, so the regression method was used on just the negative discriminants.

**Reasoning:**

A multivariate analysis may not on the whole be better than traditional methods, but may be better for just the negative discriminants.

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<tr>
<th>VARIABLE NAME</th>
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</tr>
</thead>
<tbody>
<tr>
<td>1 Constant</td>
<td>1623.99</td>
<td>24.74</td>
</tr>
<tr>
<td>2 MPV</td>
<td>165.72</td>
<td>0.61</td>
</tr>
<tr>
<td>3 Sigma</td>
<td>25.18</td>
<td>0.31</td>
</tr>
</tbody>
</table>
Summary

- The regression did not significantly improve on the traditional methods, and even the mixed method did not help with the reconstruction that much better.

- A different method or a more comprehensive multivariate analysis is necessary to improve further from the traditional methods.

<table>
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<tr>
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<tr>
<td>Parton</td>
<td>79.07</td>
<td>171.56</td>
<td>0.35</td>
</tr>
<tr>
<td>Traditional Methods</td>
<td>70.31</td>
<td>166.69</td>
<td>26.21</td>
</tr>
<tr>
<td>Regression Methods</td>
<td>91.47</td>
<td>143.77</td>
<td>36.63</td>
</tr>
<tr>
<td>Mixed Regression and Traditional</td>
<td>78.65</td>
<td>165.72</td>
<td>25.18</td>
</tr>
</tbody>
</table>
Thanks

• Special thanks to my advisors, Dr. Konstantinos Kousouris and Dr. Andrea Giammanco for their guidance and dealing with my rudimentary coding experience and lack of particle physics knowledge.

• Thanks to the University of Michigan advisors, Dr. Homer Neal, Dr. Steven Goldfarb, Dr. Jean Krisch, and Dr. Junjie Zhu, as well as the National Science Foundation, for their assistance in all matters big and small at CERN and for giving me this opportunity.

• Thanks to the CMS Collaboration and CERN for a wonderful time here on its premises and for hosting the summer student program. Thanks also to all the wonderful lecturers who took time out of their schedule to teach us particle physics.

• Finally, thanks to everyone this past summer who has helped me or supported me.
Works Cited

  - Traditional methods implemented were derived from the above thesis.
  - Equation and Feynmann diagrams were also taken from the above thesis.
- CMS logo is the official logo for the CMS Group at CERN.
Appendix – Extra Plots
Neutrino Pz Graphs
Goal: Neutrino $P_z$
Choosing the Smallest Root

*Implemented Method:*

Choosing the smaller root by absolute value.

*Reasoning:*

Current implemented method

Gives a more accurate value of the neutrino longitudinal momentum about 60% of the time.

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<tr>
<th>VARIABLE NAME</th>
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</tr>
</thead>
<tbody>
<tr>
<td>1 Constant</td>
<td>464.51</td>
<td>7.54</td>
</tr>
<tr>
<td>2 Mean</td>
<td>0.00</td>
<td>fixed</td>
</tr>
<tr>
<td>3 Sigma</td>
<td>71.15</td>
<td>0.85</td>
</tr>
</tbody>
</table>
Weighing the Two Roots

**Implemented Method:**

First calculate the probability that the smaller root is closer to the actual neutrino longitudinal momentum.

Then weigh the two roots by their corresponding probabilities and use that value as the $P_z$.

**Reasoning:**

Should theoretically average out the two roots in a way that they should recreated the top mass accurately on average.

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<tr>
<th>VARIABLE NAME</th>
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<tbody>
<tr>
<td>1 Constant</td>
<td>373.48</td>
<td>5.85</td>
</tr>
<tr>
<td>2 Mean</td>
<td>0.00</td>
<td>fixed</td>
</tr>
<tr>
<td>3 Sigma</td>
<td>88.65</td>
<td>0.99</td>
</tr>
</tbody>
</table>
Dropping the Imaginary Part

**Implemented Method:**

Just drop the imaginary part of the root, leaving you with a constant.

**Reasoning:**

Current implemented method

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<th>VARIABLE NAME</th>
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<tbody>
<tr>
<td>1 Constant</td>
<td>368.96</td>
<td>9.45</td>
</tr>
<tr>
<td>2 Mean</td>
<td>0.00</td>
<td>fixed</td>
</tr>
<tr>
<td>3 Sigma</td>
<td>72.02</td>
<td>1.62</td>
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Letting the Mass of the W Boson Change

**Implemented Method:**

Set the determinant equal to zero and let the $m_W$ change, resulting in a different constant

**Reasoning:**

The invariant mass of the W boson histogram has finite width, so the W boson is not always $80.4 \, \frac{GeV}{c^2}$

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<tr>
<td>1 Constant</td>
<td>165.87</td>
<td>2.70</td>
</tr>
<tr>
<td>2 Mean</td>
<td>0.00 fixed</td>
<td></td>
</tr>
<tr>
<td>3 Sigma</td>
<td>183.37</td>
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Scaling the ME_T

**Implemented Method:**

Let the Missing Transverse Energy fluctuate such that the discriminant is zero. This will then change the value of the constant, giving yet another estimate.

**Reasoning:**

More often than not, the neutrino is not the sole carrier of the missing transverse energy. Other culprits include light recoil jets and other bottom jets that may have be produced with the top quark.

Therefore, in theory, changing the missing transverse energy to set the discriminant is allowed due to the presence of these other particles.

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<td>1  Constant</td>
<td>379.17</td>
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<td>1 Constant</td>
<td>215.62</td>
<td>4.07</td>
</tr>
<tr>
<td>2 Mean</td>
<td>0.00</td>
<td>fixed</td>
</tr>
<tr>
<td>3 Sigma</td>
<td>70.31</td>
<td>1.06</td>
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<td>0.00 fixed</td>
<td></td>
</tr>
<tr>
<td>3 Sigma</td>
<td>78.65</td>
<td>0.80</td>
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</table>
W Boson Plots
Single Top Plots
Goal: Top Mass

<table>
<thead>
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<th>VALUE</th>
<th>ERROR</th>
</tr>
</thead>
<tbody>
<tr>
<td>1  Constant</td>
<td>164402.00</td>
<td>25796.00</td>
</tr>
<tr>
<td>2  MPV</td>
<td>171.56</td>
<td>0.03</td>
</tr>
<tr>
<td>3  Sigma</td>
<td>0.35</td>
<td>0.03</td>
</tr>
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Traditional Methods

Positive Discriminants / Two Real Roots
Choosing the Smallest Root

Implemented Method:

Choosing the smaller root by absolute value.

Reasoning:

Current implemented method

Gives a more accurate value of the neutrino longitudinal momentum about 60% of the time.

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<tbody>
<tr>
<td>1 Constant</td>
<td>1183.98</td>
<td>26.29</td>
</tr>
<tr>
<td>2 MPV</td>
<td>148.30</td>
<td>0.56</td>
</tr>
<tr>
<td>3 Sigma</td>
<td>17.17</td>
<td>0.30</td>
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<tr>
<td>1 Constant</td>
<td>890.34</td>
<td>21.62</td>
</tr>
<tr>
<td>2 MPV</td>
<td>120.71</td>
<td>0.77</td>
</tr>
<tr>
<td>3 Sigma</td>
<td>21.10</td>
<td>0.43</td>
</tr>
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Traditional Methods

Negative Discriminants / Two Complex Roots
Dropping the Imaginary Part

**Implemented Method:**

Just drop the imaginary part of the root, leaving you with a constant.

**Reasoning:**

Current implemented method

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<tbody>
<tr>
<td>1 Constant</td>
<td>521.17</td>
<td>11.50</td>
</tr>
<tr>
<td>2 MPV</td>
<td>204.48</td>
<td>1.33</td>
</tr>
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<td>37.69</td>
<td>0.71</td>
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Another look at the Equation:

\[ P_{z,\nu}^{A,B} = \frac{\mu \cdot P_{z,\mu}}{P_{T,\mu}^2} \pm \sqrt{\frac{\mu^2 \cdot P_{z,\mu}^2}{P_{T,\mu}^4} - \frac{E_{\mu}^2 \cdot E_{T}^{\text{miss}^2}}{P_{T,\mu}^2} - \mu^2} \]

\[ \mu = \frac{M_W^2}{2} + P_{T,\mu} \cdot E_T^{\text{miss}}. \]

For more detailed calculations, check the Appendix.
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<tbody>
<tr>
<td>1 Constant</td>
<td>596.55</td>
<td>13.23</td>
</tr>
<tr>
<td>2 MPV</td>
<td>193.86</td>
<td>1.11</td>
</tr>
<tr>
<td>3 Sigma</td>
<td>32.33</td>
<td>0.59</td>
</tr>
</tbody>
</table>
Yet another look at the Equation:

\[
\begin{align*}
P_{z,\nu}^{A,B} &= \frac{\mu \cdot P_{z,\mu}}{P_{T,\mu}^2} \pm \sqrt{\frac{\mu^2 \cdot P_{z,\mu}^2}{P_{T,\mu}^4} - \frac{E_{\mu}^2 \cdot E_{T,\mu}^{\text{miss}}^2}{P_{T,\mu}^2} - \mu^2}, \\
\mu &= \frac{M_W^2}{2} + P_{T,\mu} \cdot E_T^{\text{miss}}.
\end{align*}
\]

For more detailed calculations, check the Appendix.
Scaling the $\text{ME}_T$

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Let the Missing Transverse Energy fluctuate such that the discriminant is zero. This will then change the value of the constant, giving yet another estimate.

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More often than not, the neutrino is not the sole carrier of the missing transverse energy. Other culprits include light recoil jets and other bottom jets that may have been produced with the top quark.

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<td>1.13</td>
</tr>
<tr>
<td>3 Sigma</td>
<td>33.18</td>
<td>0.60</td>
</tr>
</tbody>
</table>
Combined Plots
Regression Slides
As can be seen, the variables are not highly correlated, except for the pseudorapidities of the lepton and the bottom jet, but that should not impact the regression significantly because they are independent variables.
This analysis used regression methods, so there is no clear test for overtraining, unlike for classification, where the Kolmogorov-Smirnov test is used.

Training graph shows a relatively wide distribution in deviations, even with small Pz.
The deviations here are much larger for the test distribution, which is to be expected. These massive deviations show that even at small $Pz$, there can be massive deviations in the regression model used.
Extra Calculations
The Principal Equation:

\[ P_{z,\nu}^{A,B} = \frac{\mu \cdot P_{z,\mu}}{P_{T,\mu}^2} \pm \sqrt{\frac{\mu^2 \cdot P_{z,\mu}^2}{P_{T,\mu}^4} - \frac{E_{T}^2 \cdot E_{miss}^2}{P_{T,\mu}^2} - \mu^2}, \]

\[ \mu = \frac{M_W^2}{2} + P_{T,\mu} \cdot E_{T}^{miss}. \]
Calculations for Principal Equation

\[ P_W = P_\nu + P_l \]
\[ P_W^2 = M_W^2 = (P_\nu + P_l)^2 \approx 0 + P_l^2 \approx 0 + 2P_\nu P_l \]

\[
= 2 \begin{pmatrix}
E_l \\
p_{l, x} \\
p_{l, y} \\
p_{l, z}
\end{pmatrix}
\begin{pmatrix}
E_\nu \\
p_{\nu, T} \cos \phi \\
p_{\nu, T} \sin \phi \\
p_{\nu, z}
\end{pmatrix}
\]

\[
= 2 \left[ E_l E_\nu - p_{l, x} p_{\nu, T} \cos \phi - p_{l, y} p_{\nu, T} \sin \phi - p_{l, z} p_{\nu, z} \right]
\]

\[
= 2 \left[ E_l E_\nu - p_{\nu, T} \left( p_{l, x} \cos \phi - p_{l, y} \sin \phi \right) - p_{l, z} p_{\nu, z} \right]
\]

\[
E_l E_\nu = E_l \sqrt{E_{\nu, T}^2 + E_{\nu, z}^2} = E_l \sqrt{p_{\nu, T}^2 + p_{\nu, z}^2}
\]

\[
p_{l, x} = p_{l, T} \cos \phi, \quad p_{l, y} = p_{l, T} \sin \phi, \quad \phi \equiv \phi_\nu
\]

\[
= 2 \left[ E_l \sqrt{p_{\nu, T}^2 + p_{\nu, z}^2} - p_{\nu, T} p_{l, T} \left( \cos \phi_\nu \cos \phi_\nu + \sin \phi_\nu \sin \phi_\nu \right) - p_{l, z} p_{\nu, z} \right]
\]

\[
= 2 \left[ E_l \sqrt{p_{\nu, T}^2 + p_{\nu, z}^2} - p_{\nu, T} p_{l, T} \cos \left( \phi_\nu - \phi_\nu \right) - p_{l, z} p_{\nu, z} \right]
\]
Calculations for Principal Equation

\[
E_l \sqrt{p_{v,T}^2 + p_{v,z}^2} = \frac{M_W^2}{2} + p_{v,T} p_{l,T} \cos (\phi_l - \phi_v) + p_{l,z} p_{v,z}
\]

\[a = \frac{p_{l,z} p_{v,z}}{E_l - p_{l,z}} + \frac{2a p_{l,z} p_{v,z}}{E_l - p_{l,z}} + \frac{a^2 - E_l^2 p_{v,T}^2}{E_l - p_{l,z}} + \frac{p_{v,z}^2}{E_l - p_{l,z}}
\]

\[0 = -a^2 - 2a p_{l,z} p_{v,z} + E_l^2 p_{v,T}^2 + p_{v,z}^2 (E_l^2 - p_{l,z}^2)
\]

\[0 = p_{v,z}^2 - \frac{2a p_{l,z}}{E_l^2 - p_{l,z}^2} p_{v,z} - \frac{a^2 - E_l^2 p_{v,T}^2}{E_l^2 - p_{l,z}^2}
\]

\[p_{v,z}^{1,2} = \pm \frac{a p_{l,z}}{E_l^2 - p_{l,z}^2} \pm \sqrt{\left( \frac{a p_{l,z}^2}{E_l^2 - p_{l,z}^2} \right)^2 + \frac{a^2 - E_l^2 p_{v,T}^2}{E_l^2 - p_{l,z}^2}}
\]

\[= \frac{a p_{l,z}}{E_l^2 - p_{l,z}^2} \pm \frac{1}{E_l^2 - p_{l,z}^2} \sqrt{(a p_{l,z})^2 + (a^2 - E_l^2 p_{v,T}^2) (E_l^2 - p_{l,z}^2)}
\]

\[= \frac{1}{E_l^2 - p_{l,z}^2} \left( a p_{l,z} \pm E_l \sqrt{a^2 - E_l^2 p_{v,T}^2 + p_{l,z}^2 p_{v,T}^2} \right)
\]
Calculations for Complex Roots Method 2

\[
\frac{\mu^2 \cdot P_{Z,\mu}^2}{P_{T,\mu}^4} - \frac{E^2_\mu \cdot E_T^{miss^2} - \mu^2}{P_{T,\mu}^2} = 0
\]

\[
\mu^2 \cdot P_{Z,\mu}^2 - P_{T,\mu}^2 \cdot (E^2_\mu \cdot E_T^{miss^2} - \mu^2) = 0
\]

\[
\mu^2 \cdot (P_{Z,\mu}^2 + P_{T,\mu}^2) = P_{T,\mu}^2 \cdot (E^2_\mu \cdot E_T^{miss^2})
\]

\[
\mu^2 = P_{T,\mu}^2 \cdot \frac{E^2_\mu \cdot E_T^{miss^2}}{(P_{Z,\mu}^2 + P_{T,\mu}^2)}
\]

\[
\left( \frac{M_W^2}{2} + P_{T,\mu} \cdot E_T^{miss} \right)^2 = P_{T,\mu}^2 \cdot \frac{E^2_\mu \cdot E_T^{miss^2}}{(P_{Z,\mu}^2 + P_{T,\mu}^2)}
\]

\[
M_W^2 = 2 \left( \sqrt{\frac{P_{T,\mu}^2 \cdot E^2_\mu \cdot E_T^{miss^2}}{(P_{Z,\mu}^2 + P_{T,\mu}^2)} - P_{T,\mu} \cdot E_T^{miss}} \right)
\]

\[
M_W = \sqrt{2 \left( \sqrt{\frac{P_{T,\mu}^2 \cdot E^2_\mu \cdot E_T^{miss^2}}{(P_{Z,\mu}^2 + P_{T,\mu}^2)} - P_{T,\mu} \cdot E_T^{miss}} \right)}
\]
Calculations for Complex Roots Method 2

\[ \mu_{\text{new}} = \left( \sqrt{\frac{P_T^2 \cdot E_\mu^2 \cdot E_{\text{miss}}^2}{(P_{z,\mu} + P_{T,\mu})}} \right) \]

\[ P_{z,\mu} = \frac{\left( \sqrt{\frac{P_T^2 \cdot E_\mu^2 \cdot E_{\text{miss}}^2}{(P_{z,\mu} + P_{T,\mu})}} \right) \cdot P_{z,\mu}}{P_T^2 \cdot \mu} \]

\[ P_{z,\mu} = \left( \sqrt{\frac{E_\mu^2 \cdot E_{\text{miss}}^2}{(P_{z,\mu} + P_{T,\mu})}} \right) \cdot \frac{P_{z,\mu}}{P_T^2 \cdot \mu} \]
Calculations for Complex Roots Method

\[
\frac{\mu^2 \cdot P_{z,\mu}^2}{P_{T,\mu}^4} - \frac{E_{\mu}^2 \cdot E_T^{\text{miss}^2} - \mu^2}{P_{T,\mu}^2} = 0
\]

If \( E_T^{\text{miss}^2} = \alpha ME_T \):

\[
\mu^2 \cdot P_{z,\mu}^2 + P_{T,\mu}^2 \mu^2 - P_{T,\mu}^2 E_{\mu}^2 \cdot (\alpha ME_T)^2 = 0
\]

\[
(P_{z,\mu}^2 + P_{T,\mu}^2) \mu^2 = E_{\mu}^2 \cdot (\alpha ME_T \cdot P_{T,\mu})^2
\]

\[
E_{\mu}^2 \mu^2 = E_{\mu}^2 \cdot (\alpha ME_T \cdot P_{T,\mu})^2
\]

\[
\mu^2 = (\alpha ME_T \cdot P_{T,\mu})^2
\]

\[
\frac{M_W^2}{2} + \alpha ME_T \cdot P_{T,\mu} \cdot \cos(\Delta \phi_{\nu,\ell}) = \alpha ME_T \cdot P_{T,\mu}
\]
Calculations for Complex Roots Method 3

\[ \frac{M_W^2}{2} = \alpha M E_T \cdot P_{T,\mu} \cdot (1 - \cos(\Delta \phi_{v,l})) \]

\[ \alpha = \frac{M_W^2}{2M E_T \cdot P_{T,\mu} \cdot (1 - \cos(\Delta \phi_{v,l}))} \]

\[ P_{T,v} = \frac{M_W^2}{2P_{T,\mu}(1 - \cos(\Delta \phi))} \text{, where } \Delta \phi = (M E T \phi - \phi_{\mu}) \]

\[ P_{z,v} = \frac{P_{z,\mu} \cdot P_{T,v}}{P_{T,\mu}} \]