Resolving the Neutrino Ambiguity

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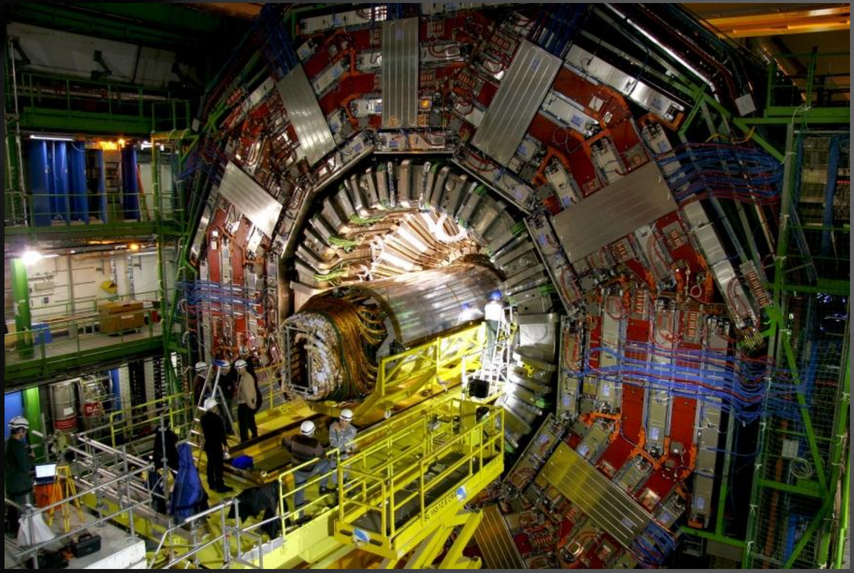
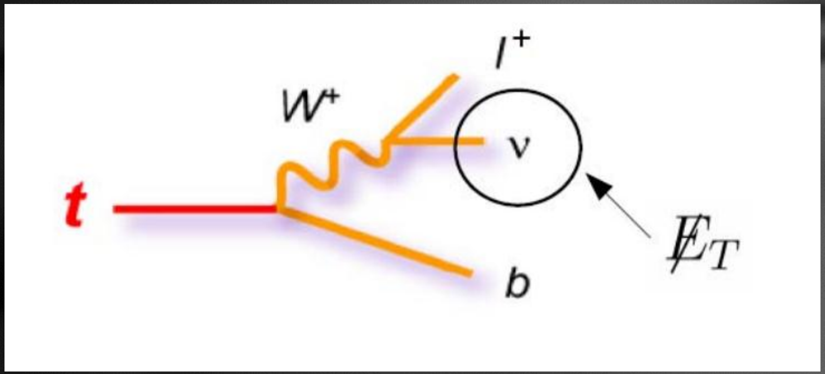
  - Extra Dimensions – Explain the weakness of gravity?



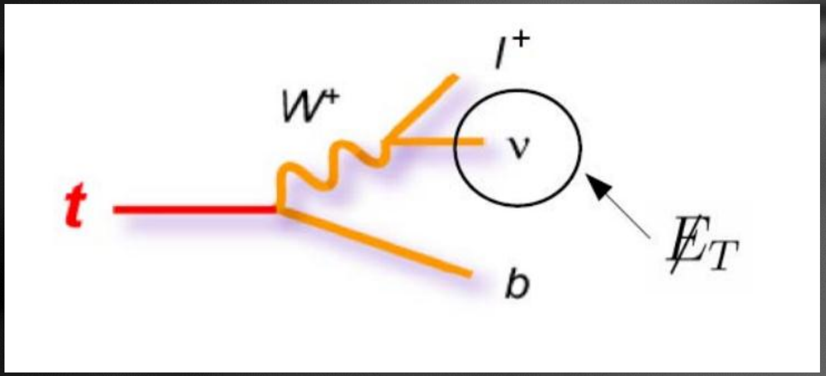
Introduction – Single Top Decays

- Single top quarks are produced through electroweak processes.
- Normally the top quark decays hadronically into many jets.
- About 30% of the time, the top quark decays through a semileptonic channel, resulting in a lepton, its corresponding neutrino, a bottom quark.



Introduction – Neutrino Ambiguity

- Leptons can be detected with a very high efficiency in the CMS detector (with a relative isolation requirement >.95, only 12% of the signal is removed, but almost the entire background is cut out with this requirement).
- The bottom quark can with relatively high efficiency be reconstructed using b-tagging techniques (90.4% efficiency in distinguishing the correct b-jet in a signal with many jets).
- The neutrino cannot be detected, and its properties are deduced from the missing energy and the conservation of 4-momentum from $W^+ \rightarrow l^+ + \nu$, but…



Principal Equation:

$$P_W = P_\nu + P_\mu$$

after lots of mathematics, and quite a few approximations:

$$P_{z,\nu}^{A,B} = \frac{\mu \cdot P_{z,\mu}}{P_{\mathrm{T},\mu}^2} \pm \sqrt{\frac{\mu^2 \cdot P_{z,\mu}^2}{P_{\mathrm{T},\mu}^4} - \frac{E_\mu^2 \cdot {\mathbf{E}_{\mathrm{T}}^{\mathrm{miss}}}^2 - \mu^2}{P_{\mathrm{T},\mu}^2}} \ ,$$

$$\mu = \frac{M_W^2}{2} + \mathbf{P}_{\mathrm{T},\mu} \cdot \mathbf{E}_{\mathrm{T}}^{\mathrm{miss}} \ .$$

For more detailed calculations, check the Appendix.

Purpose

Find a method by which to solve the neutrino ambiguity, such that the neutrino longitudinal momentum is as close to its true value as possible.

Motivation

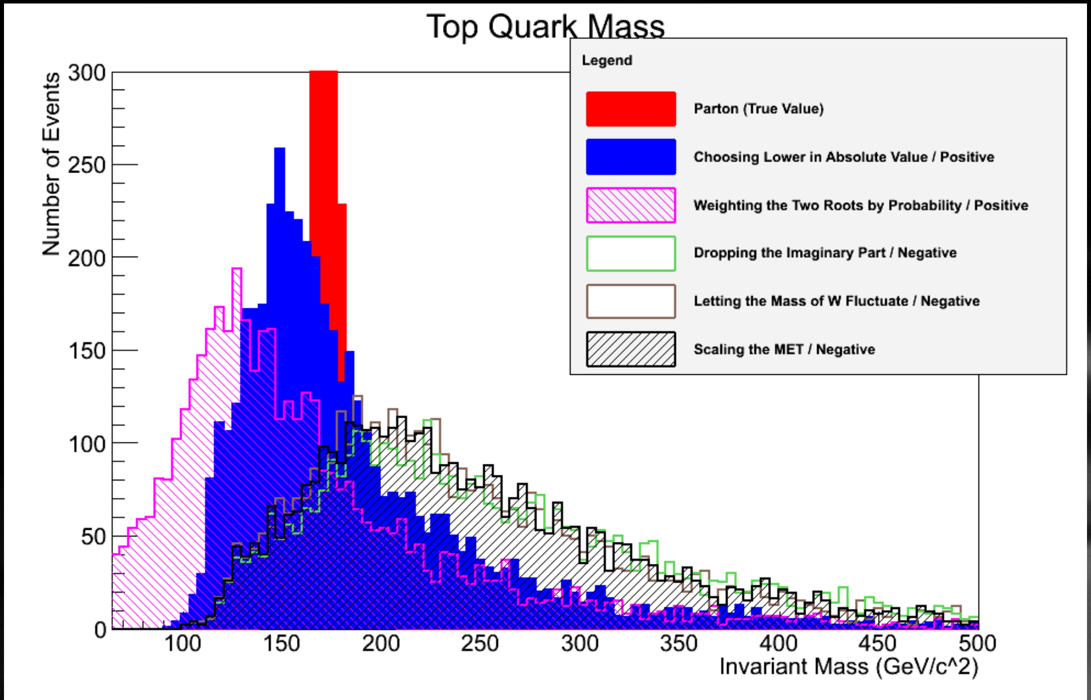
- Provides an independent, unbiased estimate of one of the components of the CKM matrix (V_{tb}).
- If we can more accurately reconstruct the single top, then this channel will become more sensitive to new physics searches.
- If we can more accurately reconstruct the single top, then channels with single top as a background will also become more sensitive to new physics.
- Approach can be applied, perhaps, to other channels with neutrino ambiguities, such as:
  - T-tbar with one of them undergoing semileptonic decay
  - Observation of exotic WZ resonances (3 leptons and 1 neutrino)
  - Semi-leptonic kaon decay where there is a W boson decaying into a lepton and neutrino.

Procedure

- Research and come up with a new method (or use an already established one).
- Implement the method onto a test Monte Carlo tree using ROOT and create plots of the neutrino Pz, the W boson, and the top quark mass.
- Use a Gaussian fit for the neutrino Pz and a Landau fit on the top quark mass in order to compare methods and to get rough estimates for the spread and mean values of the momentum and mass distributions.

Traditional Methods

- *Of the positive methods, choosing the smaller root is more advantageous.*
- *Of the negative methods, scaling the ME_T is better, but does not improve significantly to dropping the imaginary root.*
- *A new approach beyond traditional methods is necessary, especially for negative discriminants.*



Method	Gaussian σ	Landau MPV	Landau σ
Parton	79.07	171.56	0.35
Positive : Smaller Root	71.15	148.30	17.17
Positive: Weighing the Roots	88.65	120.71	21.10
Negative: Drop Imaginary Part	72.02	204.48	37.69
Negative: Let W Mass Fluctuate	183.37	193.86	32.33
Negative: Scaling the ME_T	79.53	196.68	33.18

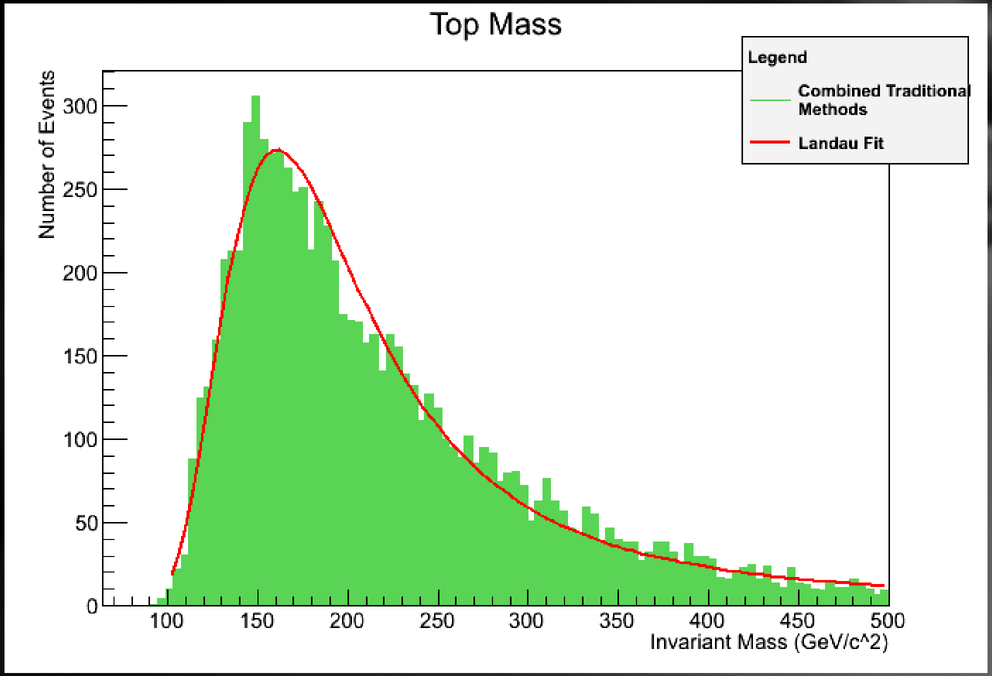
Pure Traditional Method

Implemented Method:

Just a combination of the simple positive discriminant method (choose the smaller root) and the simple negative discriminant method (drop the imaginary part).

Reasoning:

Just to be used for comparison.



VARIABLE NAME	VALUE	ERROR
1 Constant	1530.61	24.01
2 MPV	166.69	0.67
3 Sigma	26.21	0.33

TMVA (Multivariate Regression)

- Several different multivariate regressions were tried, but the most promising one was chosen.
- There are ten variables in this multivariate regression:
  - Missing Transverse Energy angle in the x-y plane (ϕ_{MET})
  - Missing Transverse Energy (ME_T)
  - Lepton Transverse Momentum ($P_{T,l}$)
  - Lepton Transverse Momentum angle in the x-y plane (ϕ_l)
  - Lepton Pseudorapidity (η_l)
  - B-jet Transverse Momentum ($P_{T,b}$)
  - B-jet Transverse Momentum angle in the x-y plane (ϕ_b)
  - B-jet Pseudorapidity (η_b)
  - B-tag value
  - Rho – a variable that takes into account pile-up.
- A Boosted Decision Tree Method was applied with the target being the momentum of the neutrino.
  - Boosted decision trees are less susceptible to overtraining than neural networks and regular decisions trees.
  - For correlation matrices and regression output deviation graphs, see backup slides.

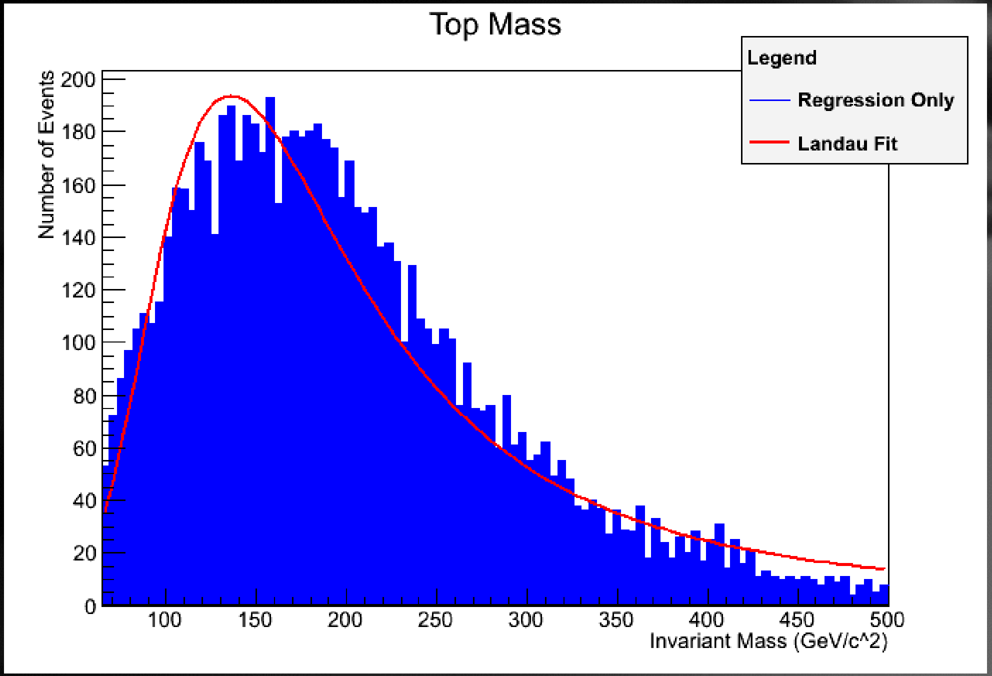
Pure Regression Method

<u>Implemented Method:</u>

Boosted Decision Tree with 10 variables.

<u>Reasoning:</u>

A multivariate analysis may be able to avoid the shortcomings of the traditional methods



VARIABLE NAME	VALUE	ERROR
1 Constant	1073.37	17.07
2 MPV	143.77	0.97
3 Sigma	36.63	0.51

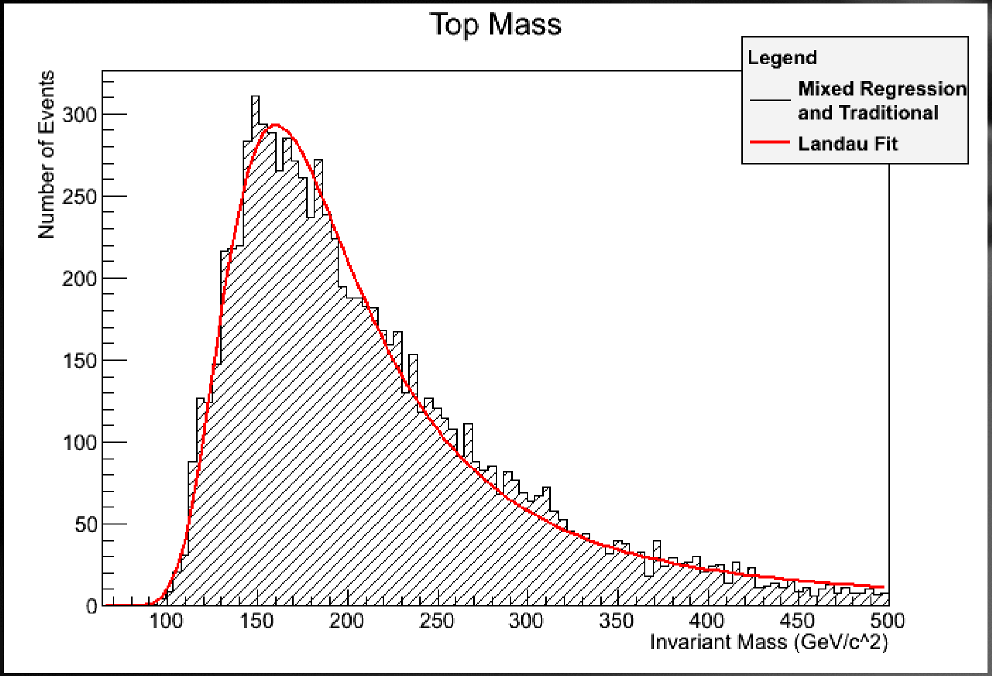
Mixed Traditional and Regression Method

Implemented Method:

The regression method as a whole is not that much better, so the regression method was used on just the negative discriminants.

Reasoning:

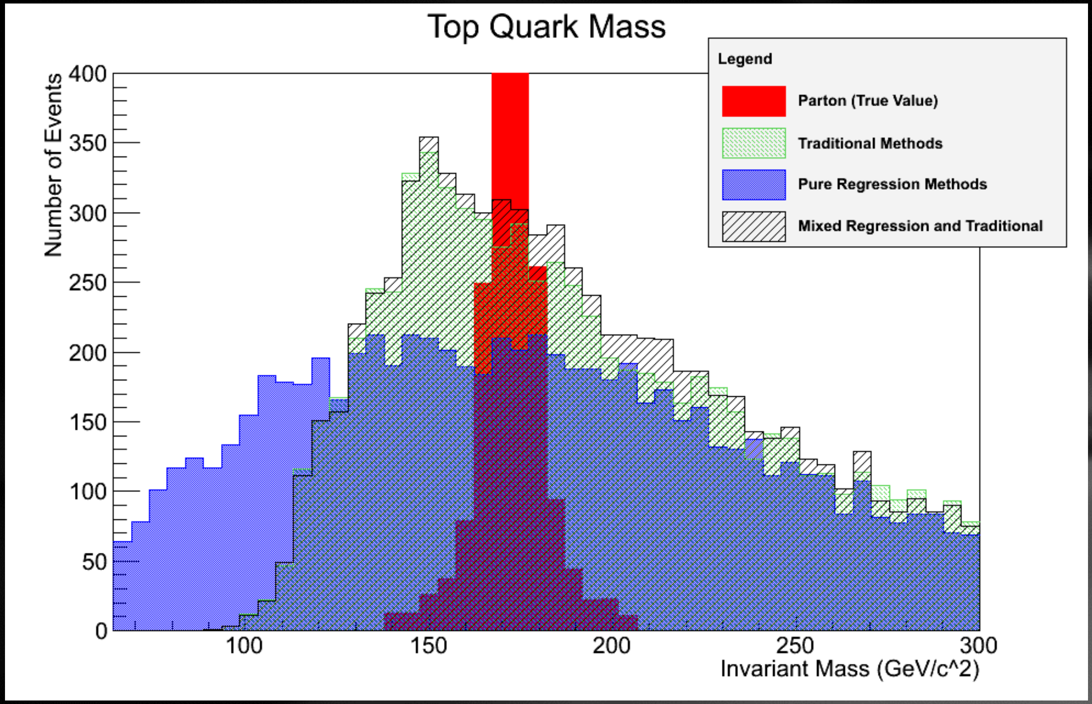
A multivariate analysis may not on the whole be better than traditional methods, but may be better for just the negative discriminants.



VARIABLE NAME	VALUE	ERROR
1 Constant	1623.99	24.74
2 MPV	165.72	0.61
3 Sigma	25.18	0.31

Summary

- *The regression did not significantly improve on the traditional methods, and even the mixed method did not help with the reconstruction that much better.*
- *A different method or a more comprehensive multivariate analysis is necessary to improve further from the traditional methods*



Method	Gaussian σ	Landau MPV	Landau σ
Parton	79.07	171.56	0.35
Traditional Methods	70.31	166.69	26.21
Regression Methods	91.47	143.77	36.63
Mixed Regression and Traditional	78.65	165.72	25.18

Thanks

- Special thanks to my advisors, Dr. Konstantinos Kousouris and Dr. Andrea Giammanco for their guidance and dealing with my rudimentary coding experience and lack of particle physics knowledge.
- Thanks to the University of Michigan advisors, Dr. Homer Neal, Dr. Steven Goldfarb, Dr. Jean Krisch, and Dr. Junjie Zhu, as well as the National Science Foundation, for their assistance in all matters big and small at CERN and for giving me this opportunity.
- Thanks to the CMS Collaboration and CERN for a wonderful time here on its premises and for hosting the summer student program. Thanks also to all the wonderful lecturers who took time out of their schedule to teach us particle physics.
- Finally, thanks to everyone this past summer who has helped me or supported me.

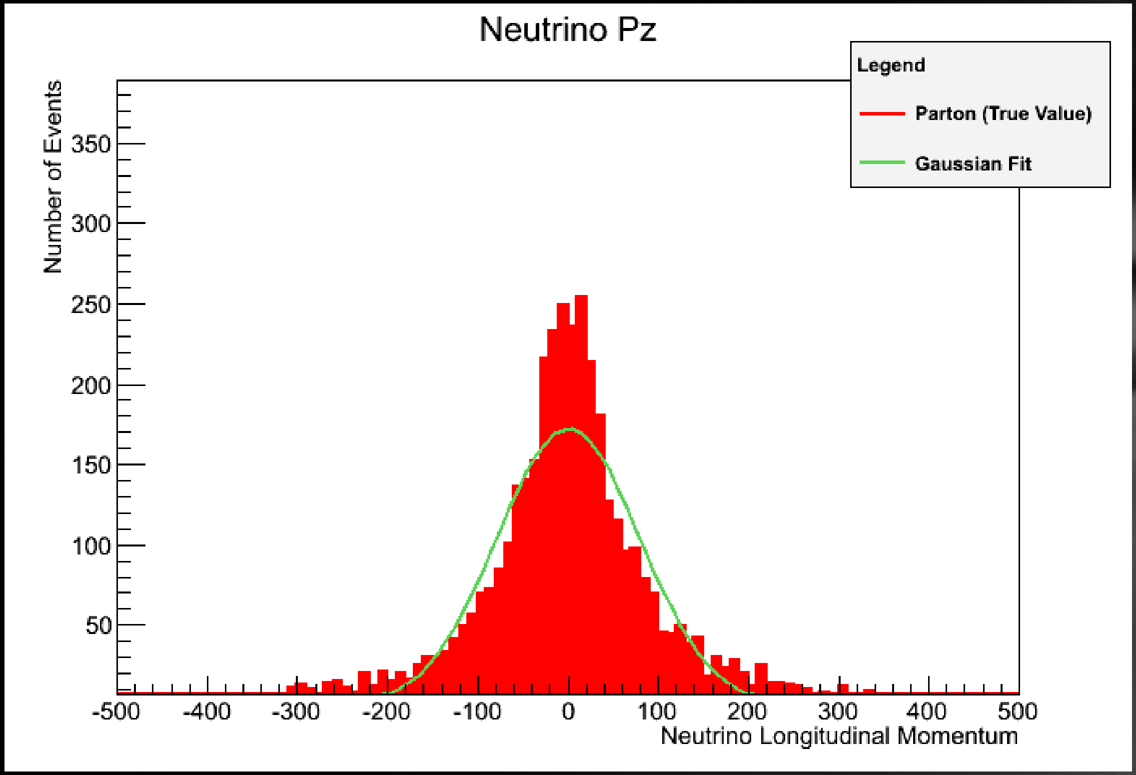
Works Cited

- CMS collaboration. "Measurement of the t-channel single top quark production cross section in pp collisions at $\sqrt{s}=7$ TeV", arXiv:1106.3052 [hep-ex], Phys. Rev. Lett. 107, 091802 (2011), doi:10.1103/PhysRevLett.107.091802.
  - Traditional methods implemented were derived from the above thesis.
  - Equation and Feynmann diagrams were also taken from the above thesis.
- CMS logo is the official logo for the CMS Group at CERN.

Appendix – Extra Plots

Neutrino Pz Graphs

Goal: Neutrino Pz



VARIABLE NAME	VALUE	ERROR
1 Constant	172.20	4.99
2 Mean	0.00	fixed
3 Sigma	79.07	1.85

Choosing the Smallest Root

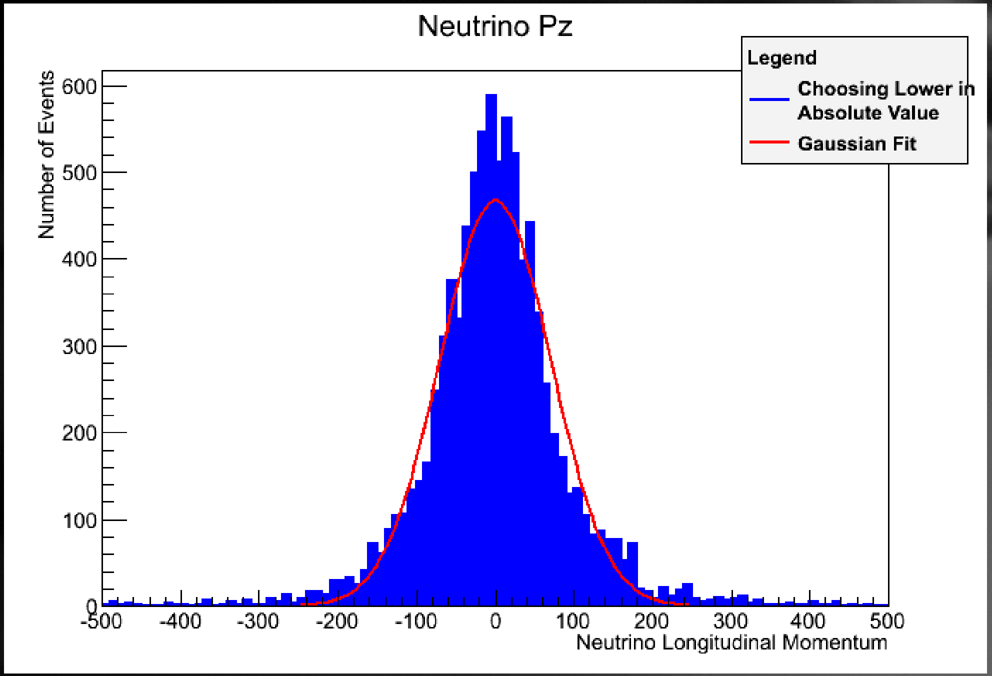
__Implemented Method:__

Choosing the smaller root by absolute value.

__Reasoning:__

Current implemented method

Gives a more accurate value of the neutrino longitudinal momentum about 60% of the time.



VARIABLE NAME	VALUE	ERROR
1 Constant	464.51	7.54
2 Mean	0.00	fixed
3 Sigma	71.15	0.85

Weighing the Two Roots

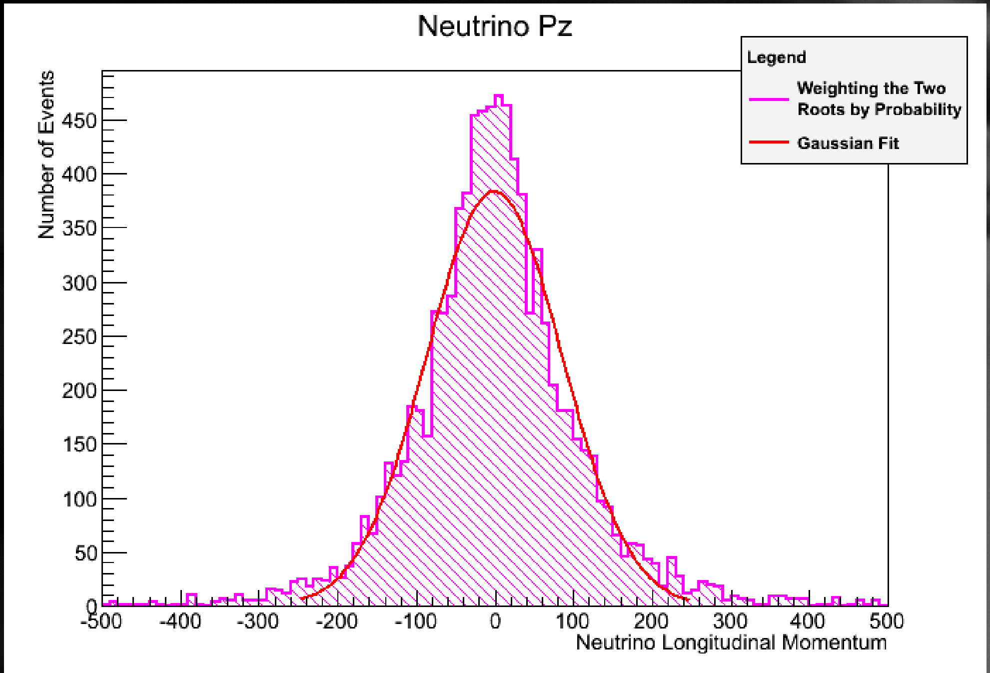
Implemented Method:

First calculate the probability that the smaller root is closer to the actual neutrino longitudinal momentum.

Then weigh the two roots by their corresponding probabilities and use that value as the Pz.

Reasoning:

Should theoretically average out the two roots in a way that they should recreate the top mass accurately on average.



VARIABLE NAME	VALUE	ERROR
1 Constant	373.48	5.85
2 Mean	0.00	fixed
3 Sigma	88.65	0.99

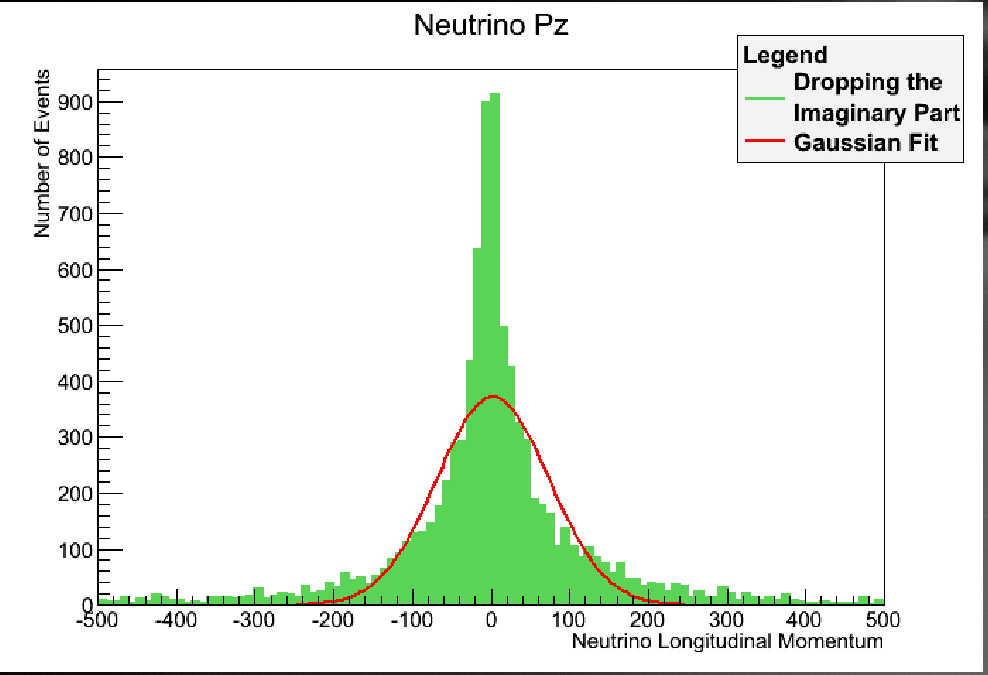
Dropping the Imaginary Part

Implemented Method:

Just drop the imaginary part of the root, leaving you with a constant.

Reasoning:

Current implemented method



VARIABLE NAME	VALUE	ERROR
1 Constant	368.96	9.45
2 Mean	0.00	fixed
3 Sigma	72.02	1.62

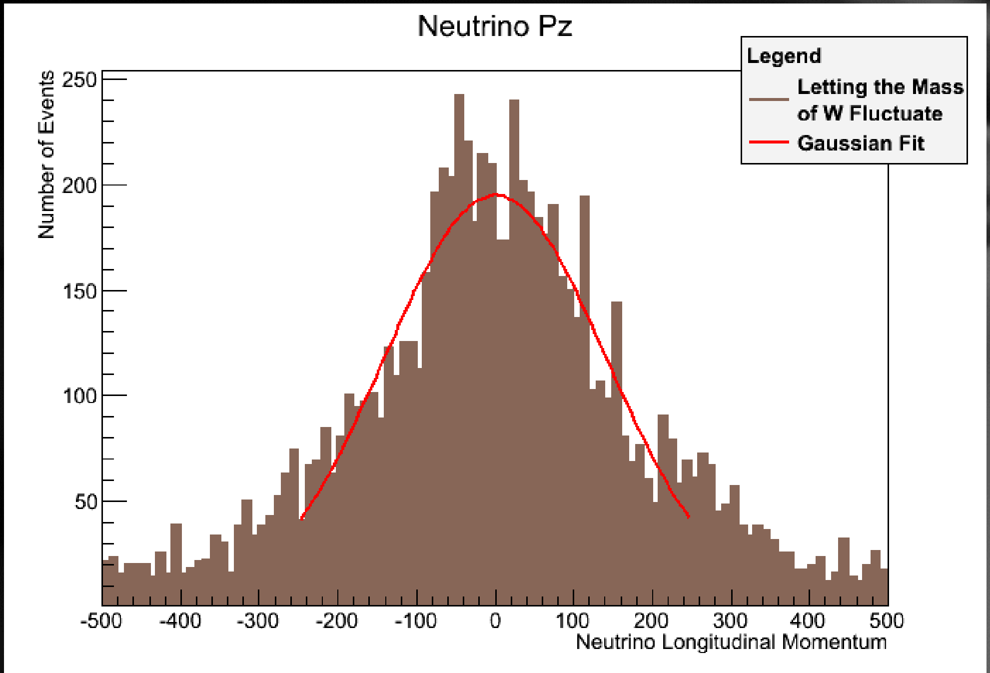
Letting the Mass of the W Boson Change

Implemented Method:

Set the determinant equal to zero and let the m_W change, resulting in a different constant

Reasoning:

The invariant mass of the W boson histogram has finite width, so the W boson is not always 80.4 $\frac{GeV}{c^2}$



VARIABLE NAME	VALUE	ERROR
1 Constant	165.87	2.70
2 Mean	0.00	fixed
3 Sigma	183.37	2.23

Scaling the ME_T

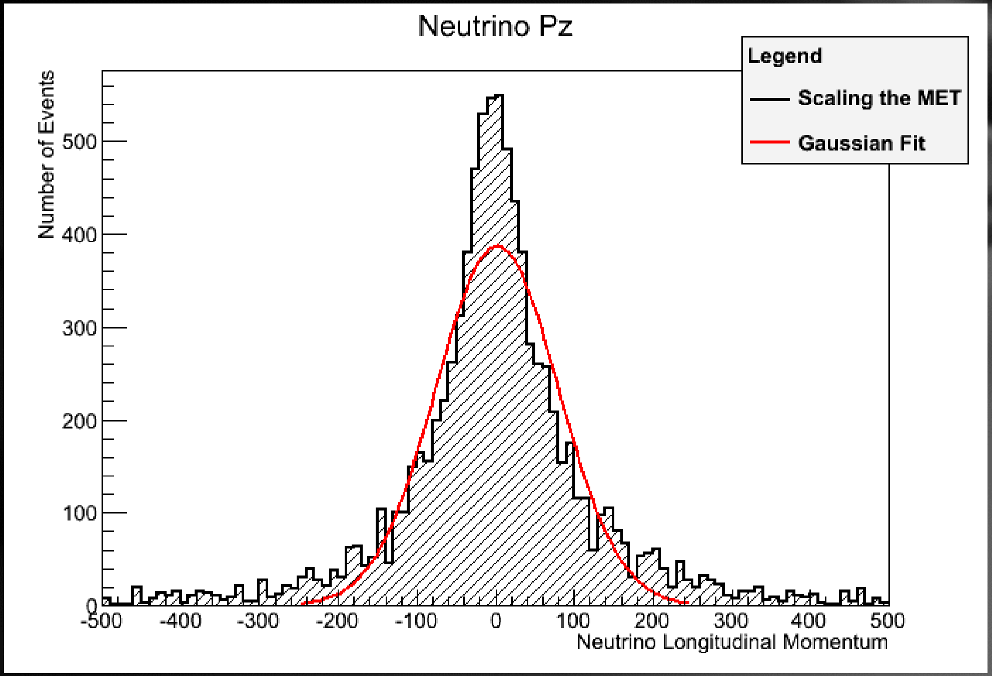
Implemented Method:

Let the Missing Transverse Energy fluctuate such that the discriminant is zero. This will then change the value of the constant, giving yet another estimate.

Reasoning:

More often than not, the neutrino is not the sole carrier of the missing transverse energy. Other culprits include light recoil jets and other bottom jets that may have be produced with the top quark.

Therefore, in theory, changing the missing transverse energy to set the discriminant is allowed due to the presence of these other particles.



VARIABLE NAME	VALUE	ERROR
1 Constant	379.17	7.39
2 Mean	0.00	fixed
3 Sigma	79.53	1.25

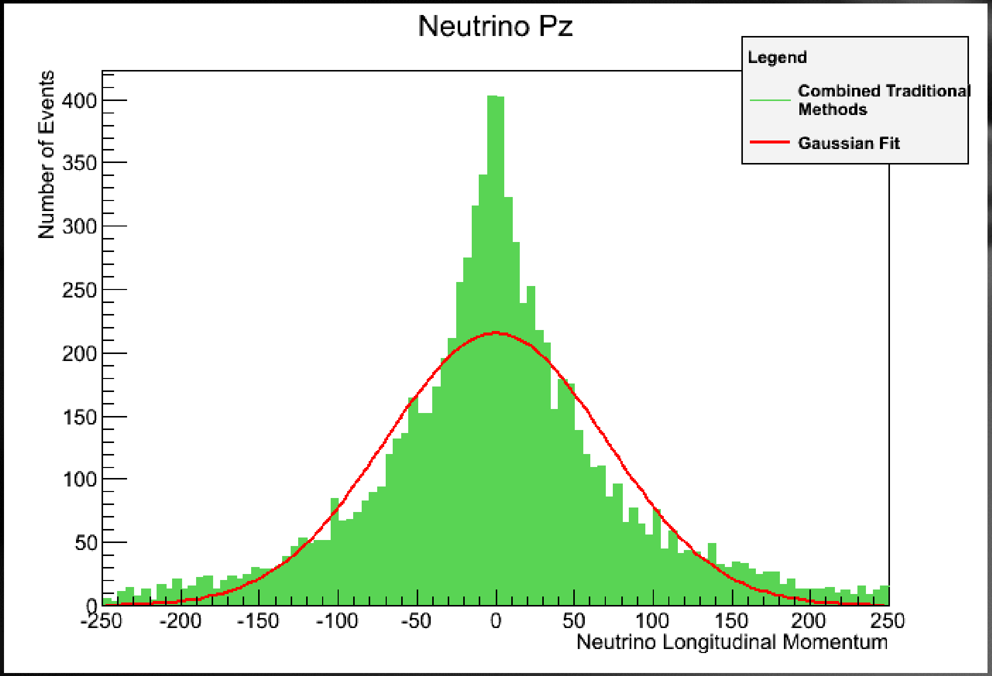
Pure Traditional Method

Implemented Method:

Just a combination of the simple positive discriminant method (choose the smaller root) and the simple negative discriminant method (drop the imaginary part).

Reasoning:

Just to be used for comparison.



VARIABLE NAME	VALUE	ERROR
1 Constant	215.62	4.07
2 Mean	0.00	fixed
3 Sigma	70.31	1.06

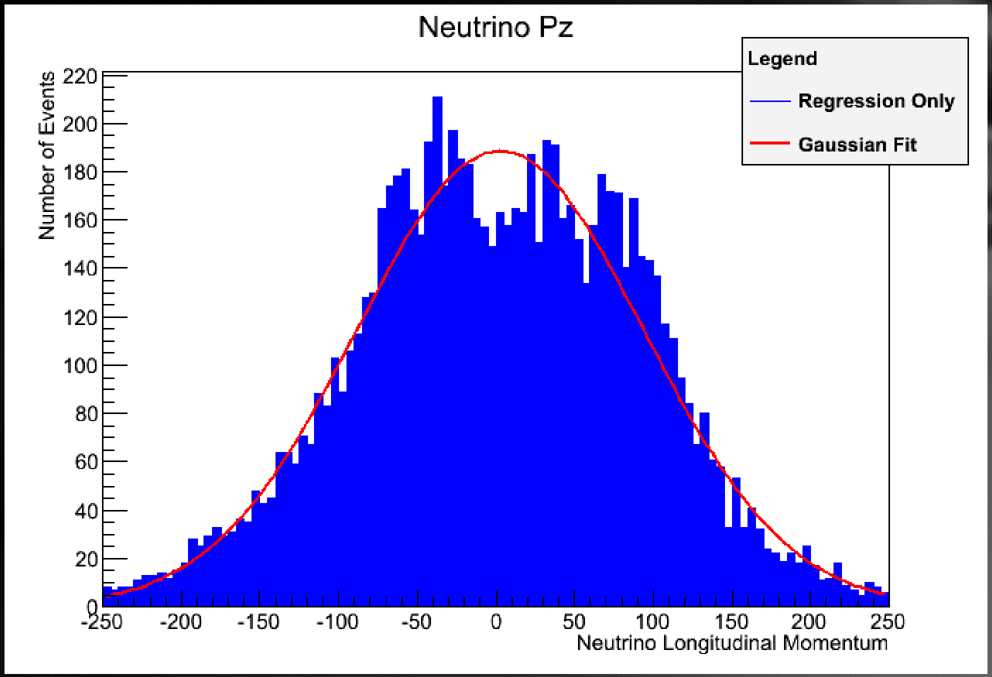
Pure Regression Method

Implemented Method:

Boosted Decision Tree with 10 variables.

Reasoning:

A multivariate analysis may be able to avoid the shortcomings of the traditional methods



VARIABLE NAME	VALUE	ERROR
1 Constant	188.25	2.54
2 Mean	0.00	fixed
3 Sigma	91.47	0.78

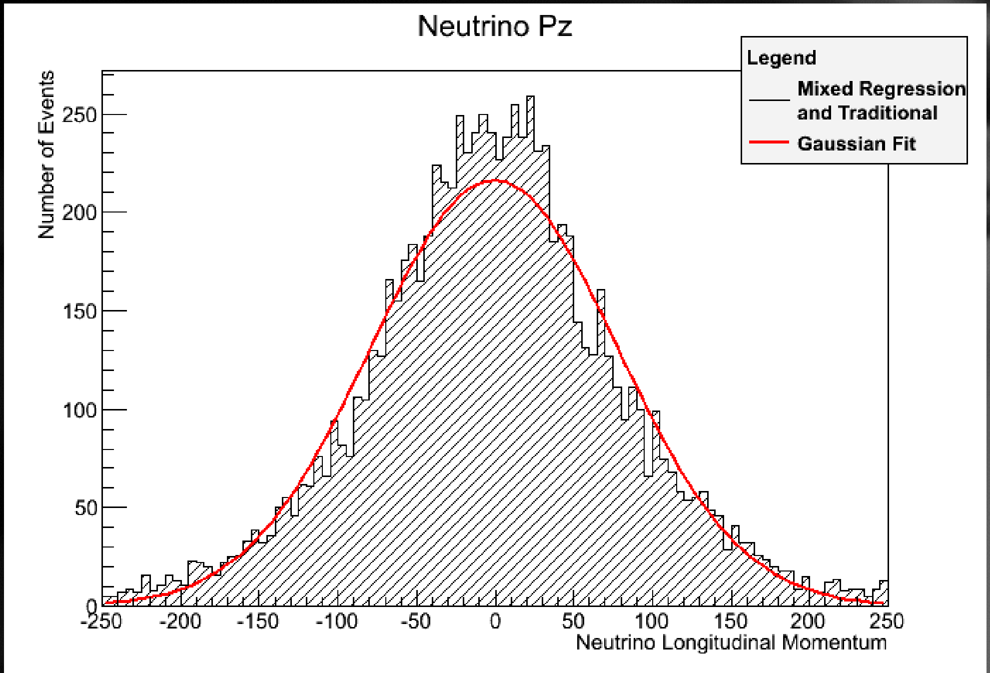
Mixed Traditional and Regression Method

Implemented Method:

The regression method as a whole is not that much better, so the regression method was used on just the negative discriminants.

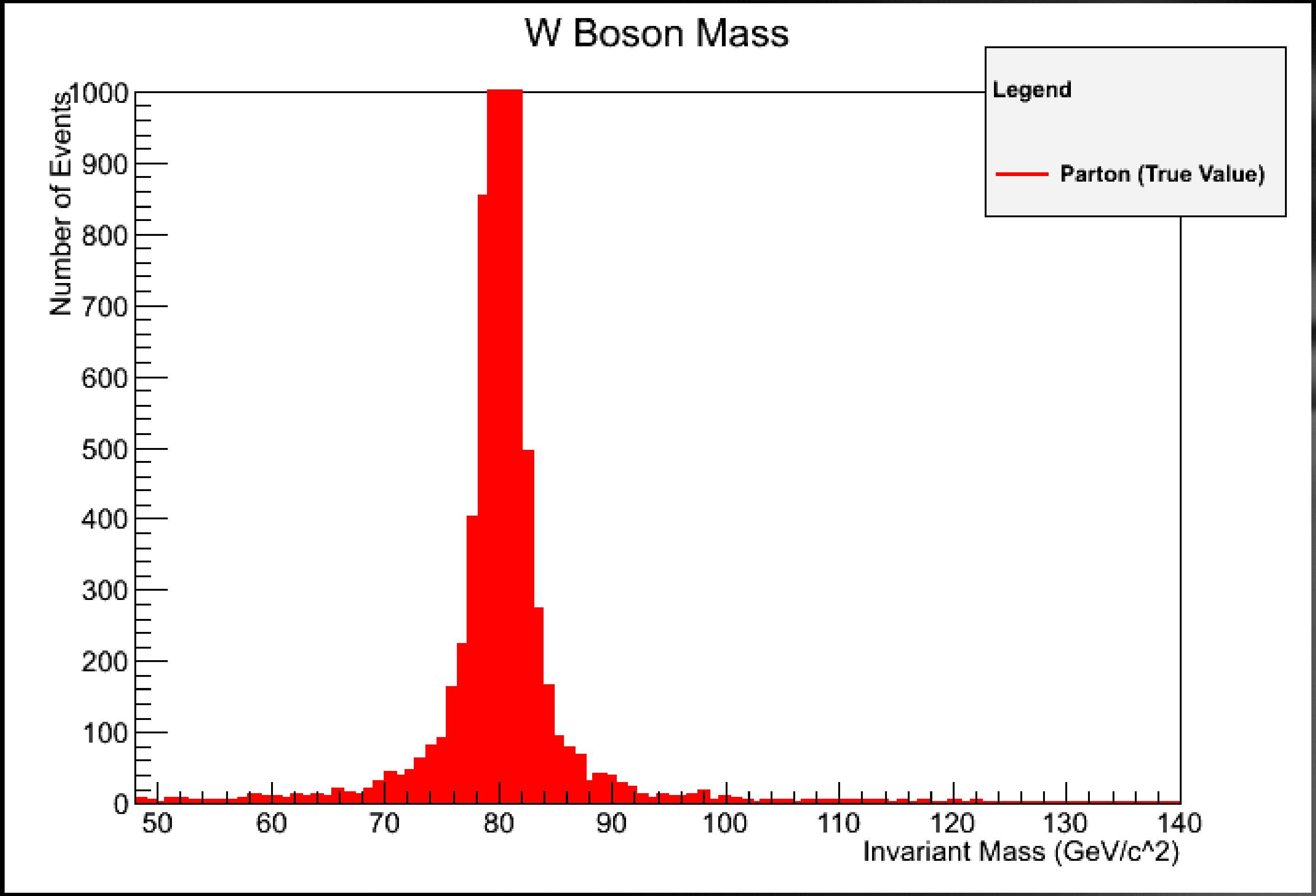
Reasoning:

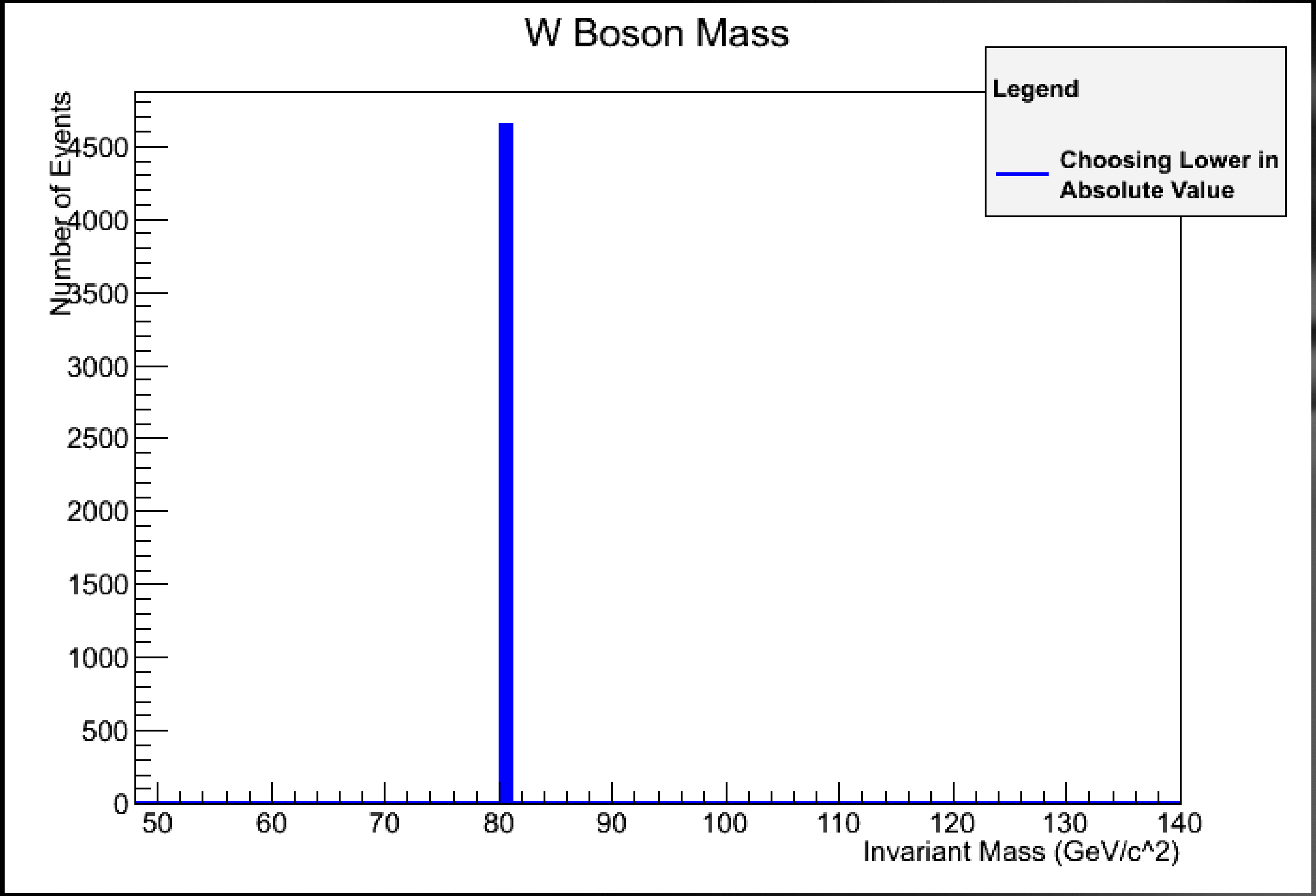
A multivariate analysis may not on the whole be better than traditional methods, but may be better for just the negative discriminants.



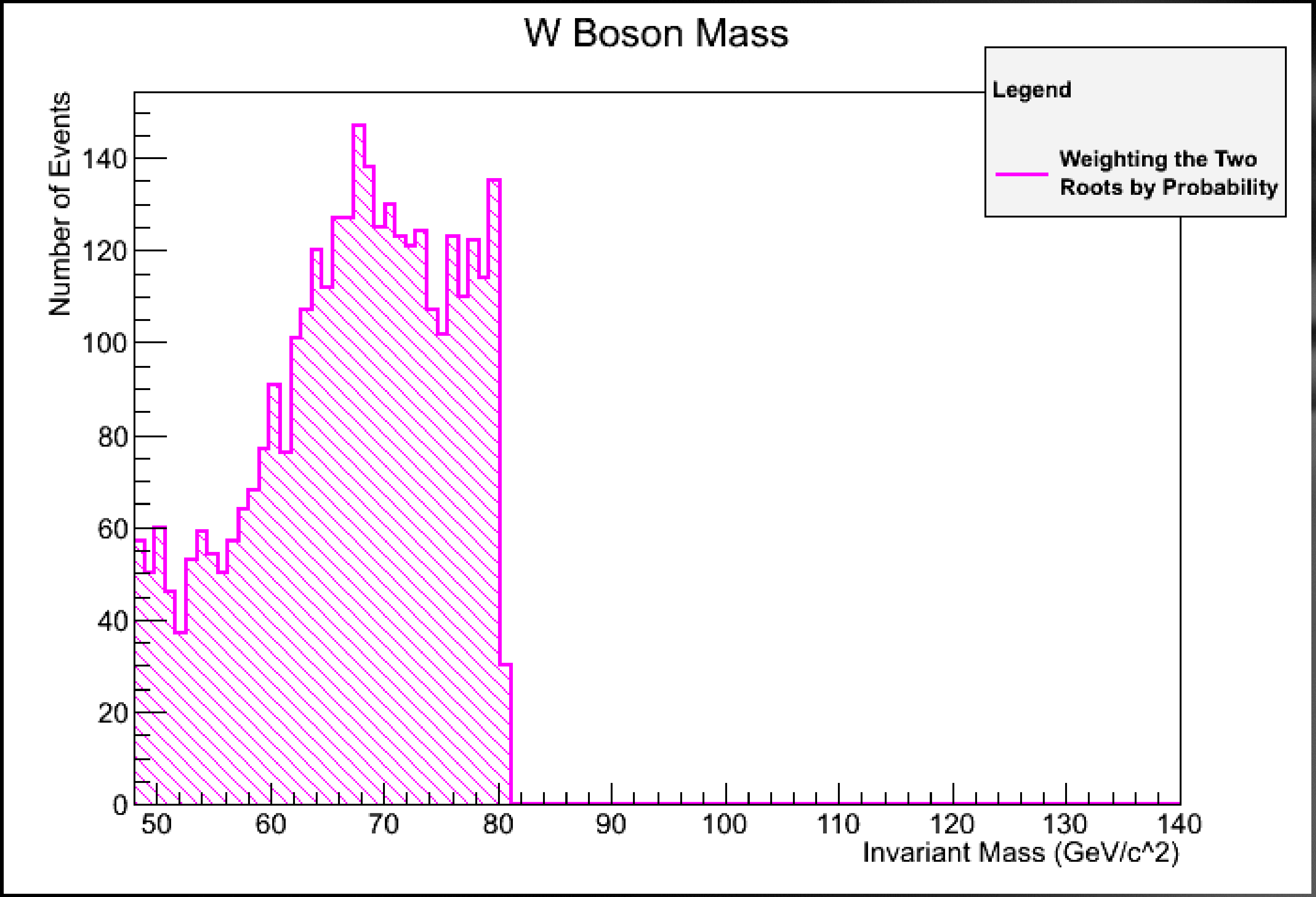
VARIABLE NAME	VALUE	ERROR
1 Constant	216.25	3.19
2 Mean	0.00	fixed
3 Sigma	78.65	0.80

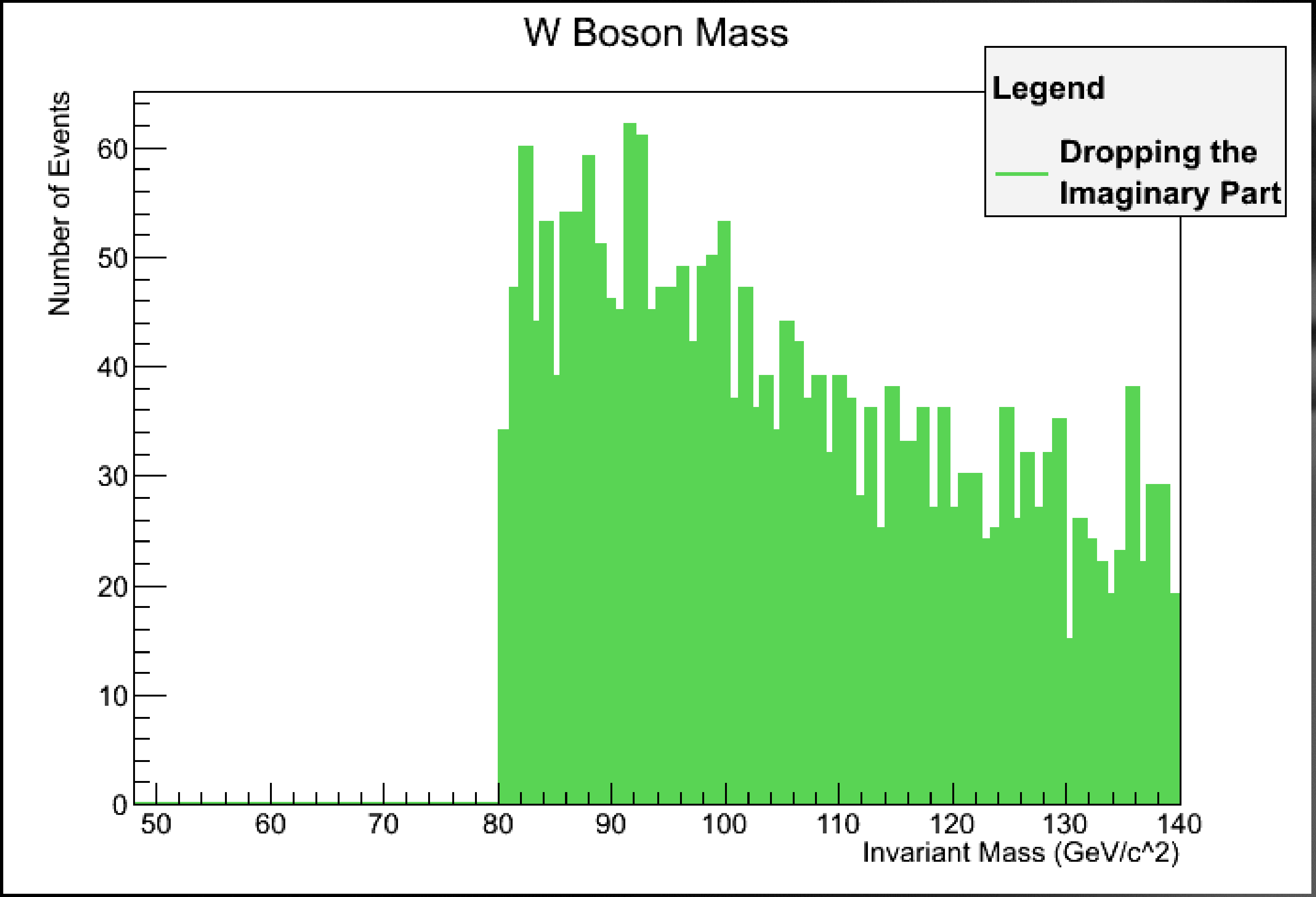
W Boson Plots

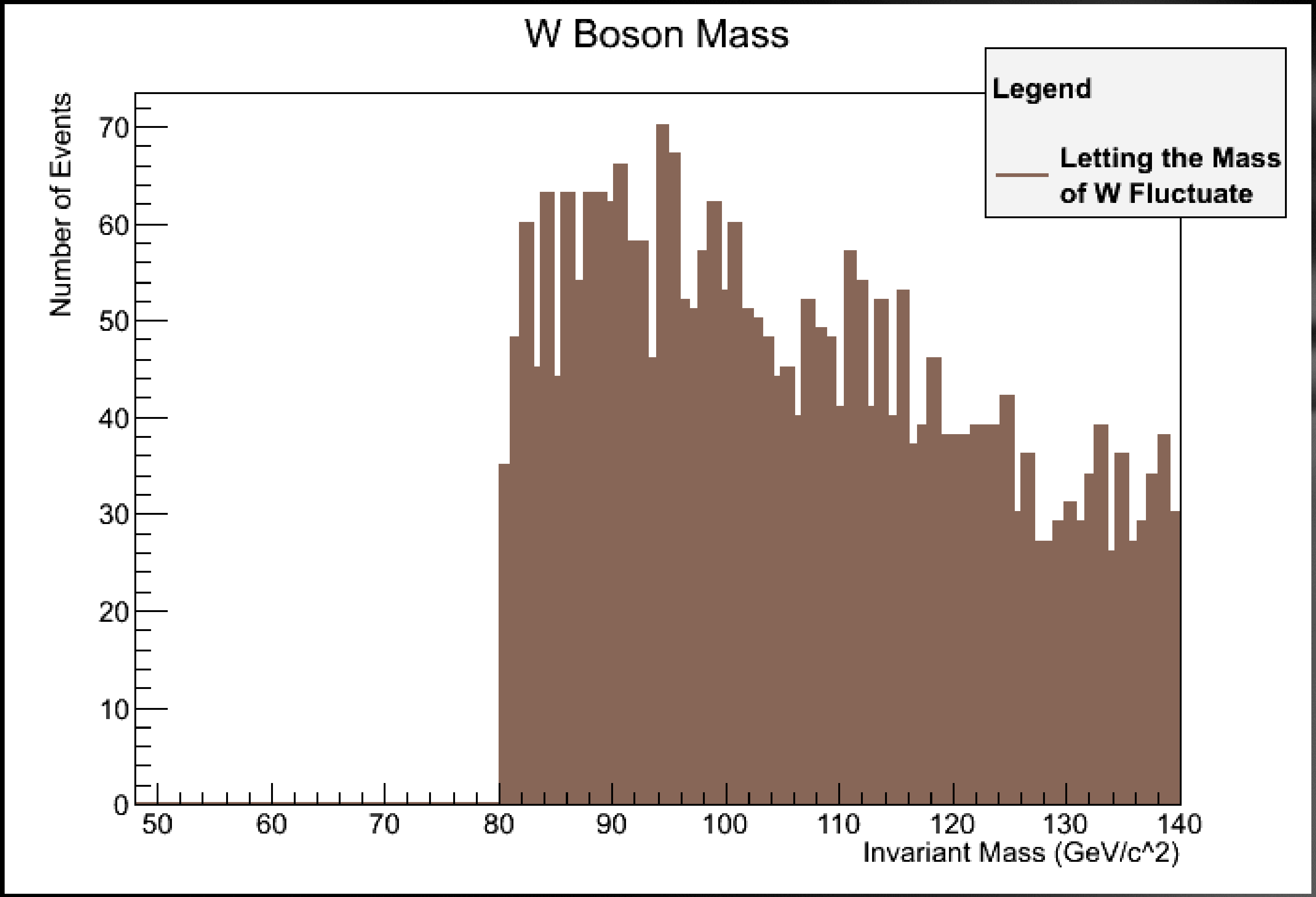

W Boson Mass
Legend
Parton (True Value)
Number of Events
1000
900
800
700
600
500
400
300
200
100
0
50
60
70
80
90
100
110
120
130
140
Invariant Mass (GeV/c^2)

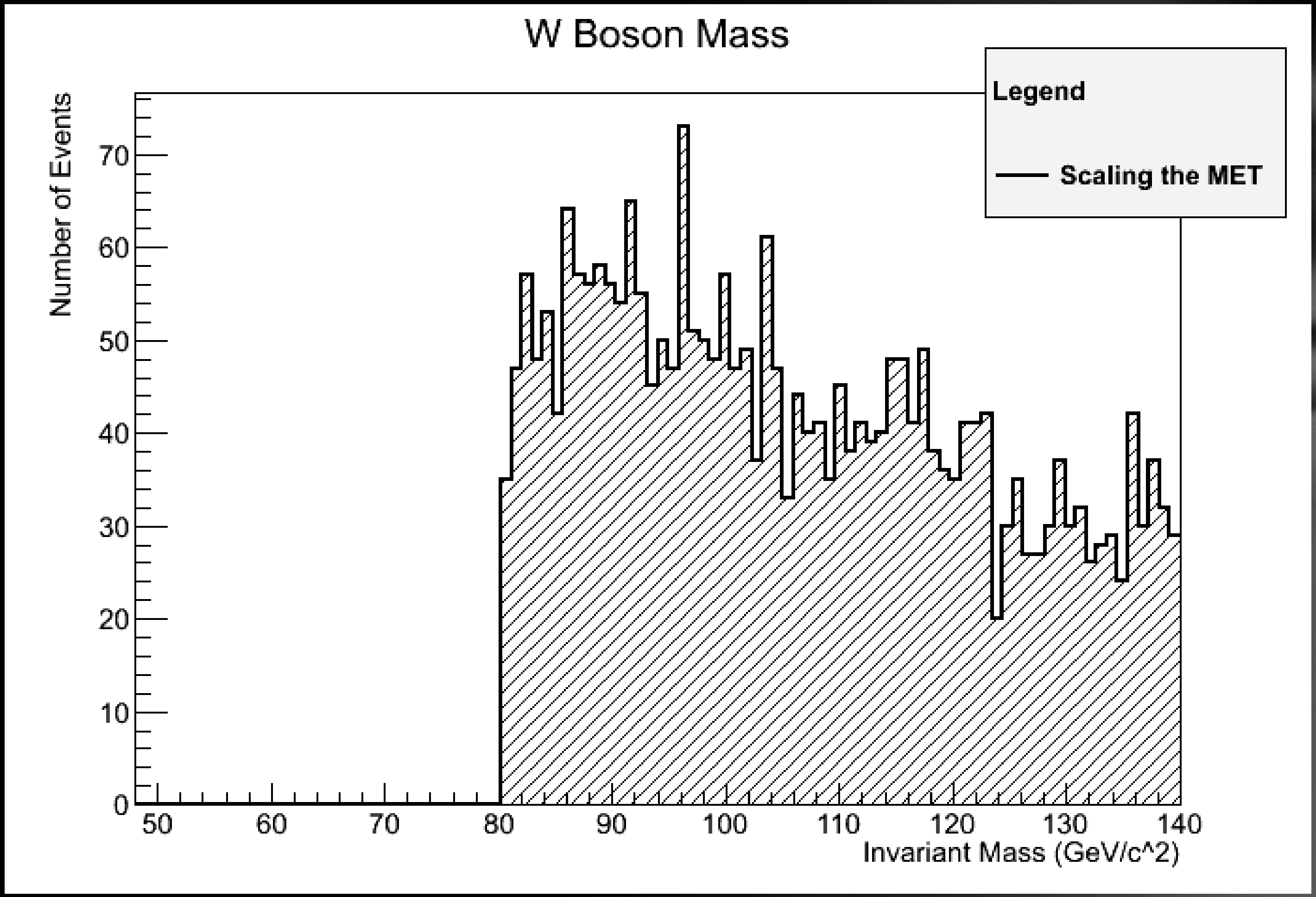


W Boson Mass
Legend
Choosing Lower in Absolute Value
Number of Events
0
500
1000
1500
2000
2500
3000
3500
4000
4500
50
60
70
80
90
100
110
120
130
140
Invariant Mass (GeV/c^2)

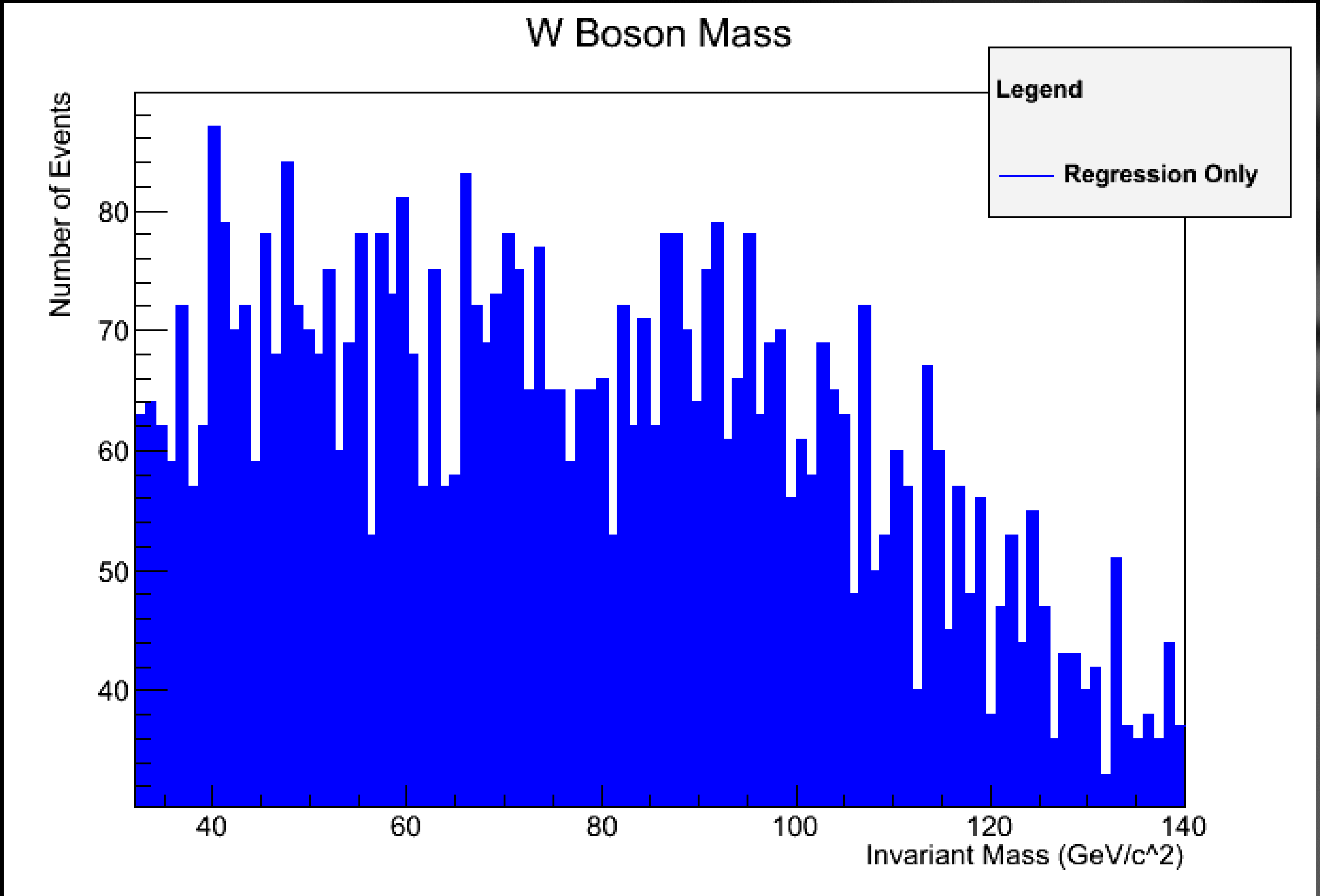

W Boson Mass
Legend
Weighting the Two Roots by Probability
Number of Events
0
20
40
60
80
100
120
140
50
60
70
80
90
100
110
120
130
140
Invariant Mass (GeV/c^2)


W Boson Mass
Legend
Dropping the Imaginary Part
Number of Events
0
10
20
30
40
50
60
50
60
70
80
90
100
110
120
130
140
Invariant Mass (GeV/c^2)

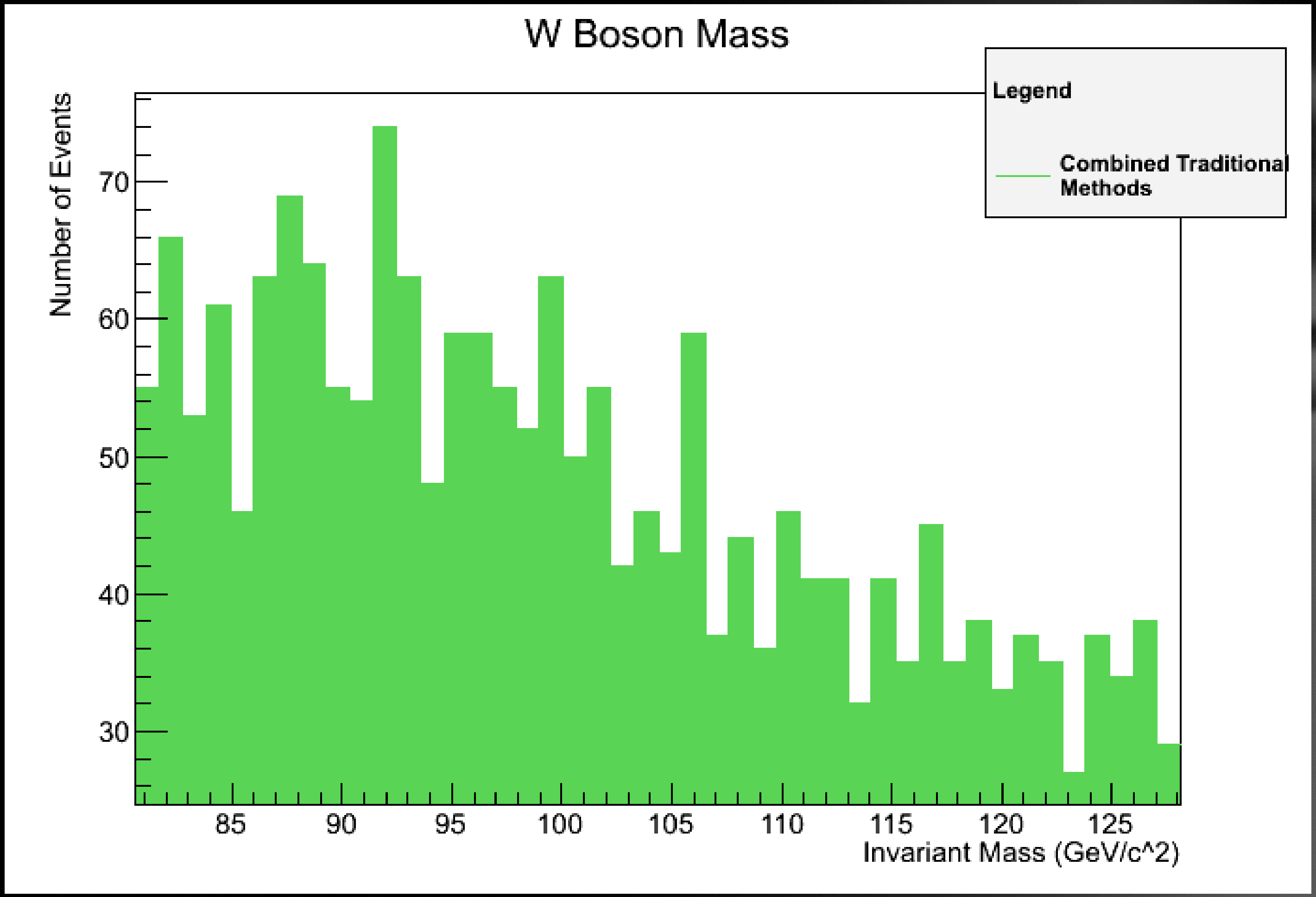

W Boson Mass
Legend
Letting the Mass of W Fluctuate
Number of Events
0
10
20
30
40
50
60
70
50
60
70
80
90
100
110
120
130
140
Invariant Mass (GeV/c^2)

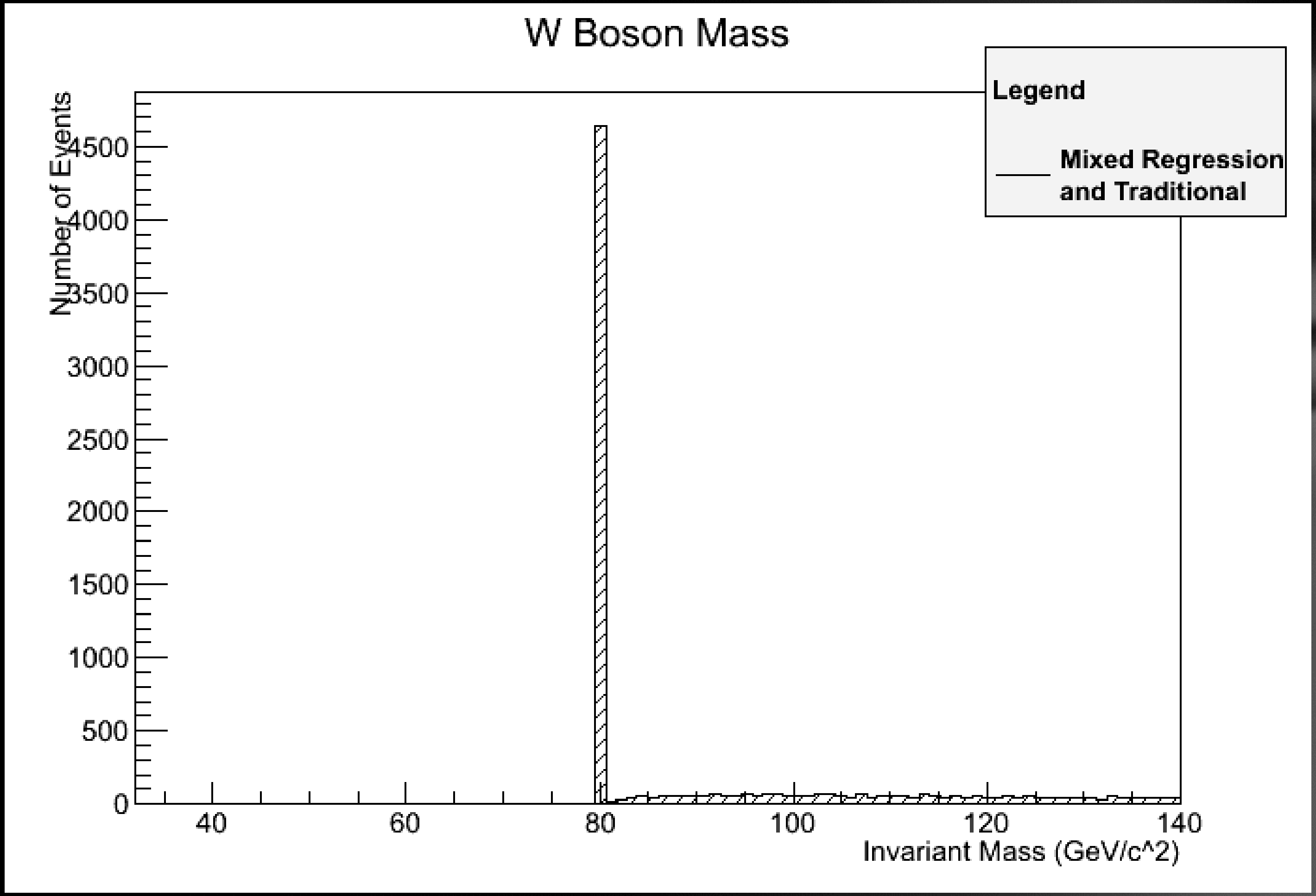


W Boson Mass
Legend
Scaling the MET
Number of Events
0
10
20
30
40
50
60
70
50
60
70
80
90
100
110
120
130
140
Invariant Mass (GeV/c^2)

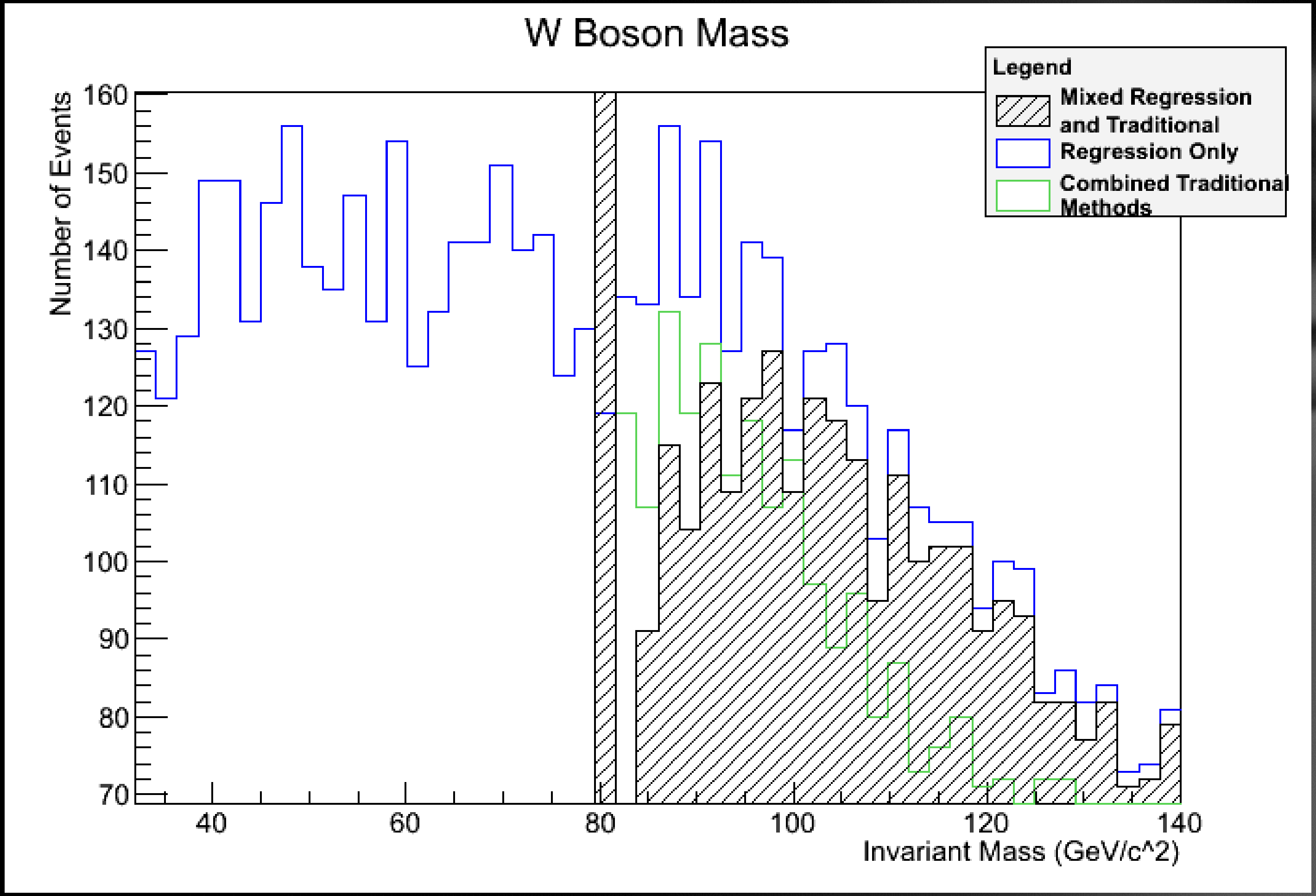


W Boson Mass
Legend
Regression Only
Number of Events
40
50
60
70
80
40
60
80
100
120
140
Invariant Mass (GeV/c^2)


W Boson Mass
Legend
Combined Traditional Methods
Number of Events
30
40
50
60
70
85
90
95
100
105
110
115
120
125
Invariant Mass (GeV/c^2)

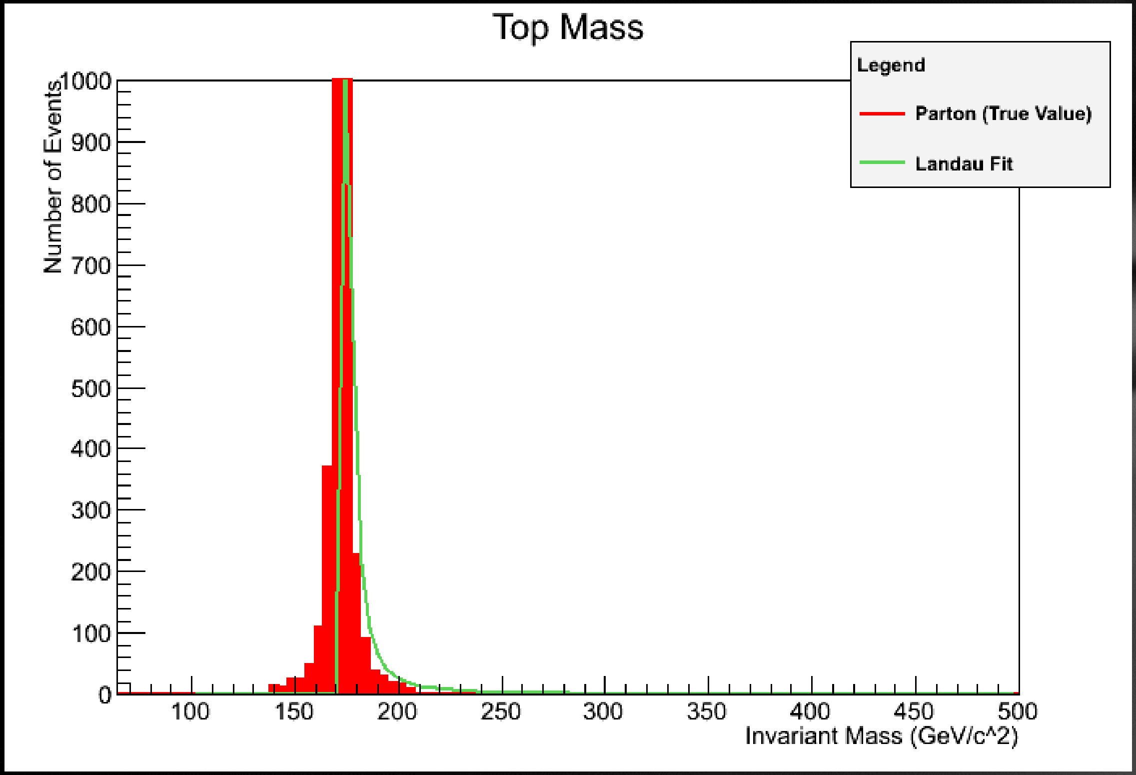


W Boson Mass
Legend
Mixed Regression
and Traditional
Number of Events
4500
4000
3500
3000
2500
2000
1500
1000
500
0
40
60
80
100
120
140
Invariant Mass (GeV/c^2)


W Boson Mass
Legend
Mixed Regression and Traditional
Regression Only
Combined Traditional Methods
Number of Events
160
150
140
130
120
110
100
90
80
70
40
60
80
100
120
140
Invariant Mass (GeV/c^2)

Single Top Plots

Goal: Top Mass



VARIABLE NAME	VALUE	ERROR
1 Constant	164402.00	25796.00
2 MPV	171.56	0.03
3 Sigma	0.35	0.03

Traditional Methods

Positive Discriminants / Two Real Roots

Choosing the Smallest Root

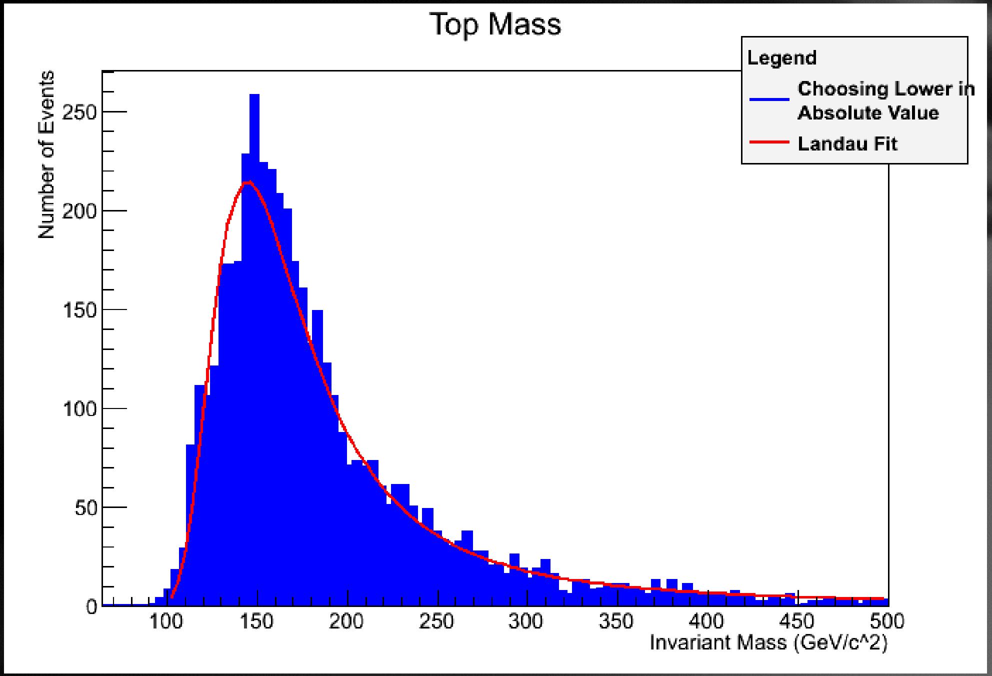
Implemented Method:

Choosing the smaller root by absolute value.

Reasoning:

Current implemented method

Gives a more accurate value of the neutrino longitudinal momentum about 60% of the time.



VARIABLE NAME	VALUE	ERROR
1 Constant	1183.98	26.29
2 MPV	148.30	0.56
3 Sigma	17.17	0.30

Weighing the Two Roots

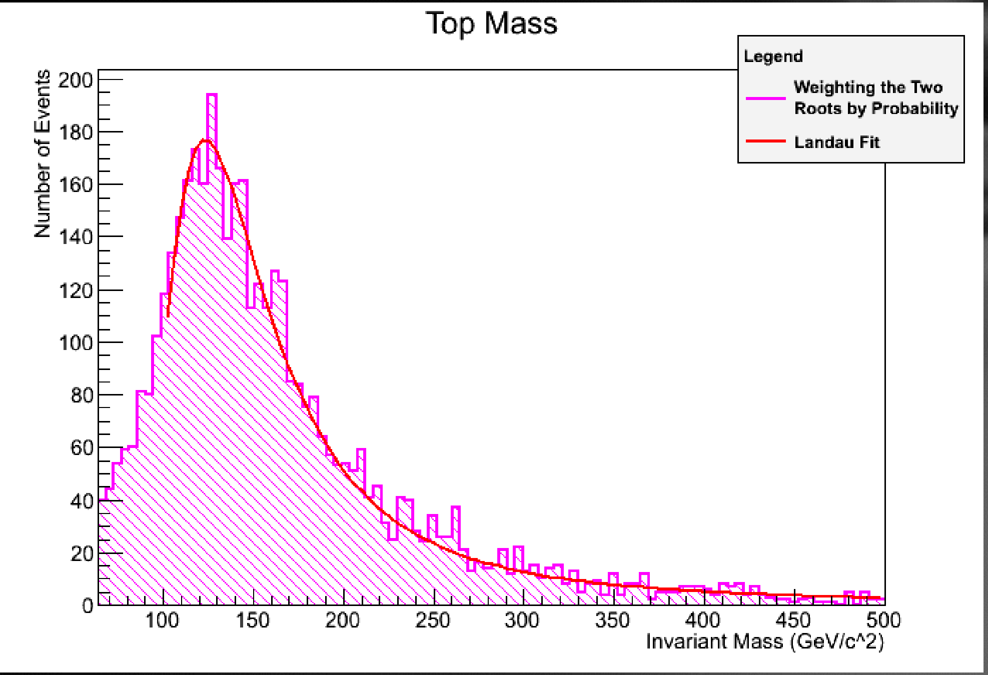
Implemented Method:

First calculate the probability that the smaller root is closer to the actual neutrino longitudinal momentum.

Then weigh the two roots by their corresponding probabilities and use that value as the Pz.

Reasoning:

Should theoretically average out the two roots in a way that they should recreate the top mass accurately on average.



VARIABLE NAME	VALUE	ERROR
1 Constant	890.34	21.62
2 MPV	120.71	0.77
3 Sigma	21.10	0.43

Traditional Methods

Negative Discriminants / Two Complex Roots

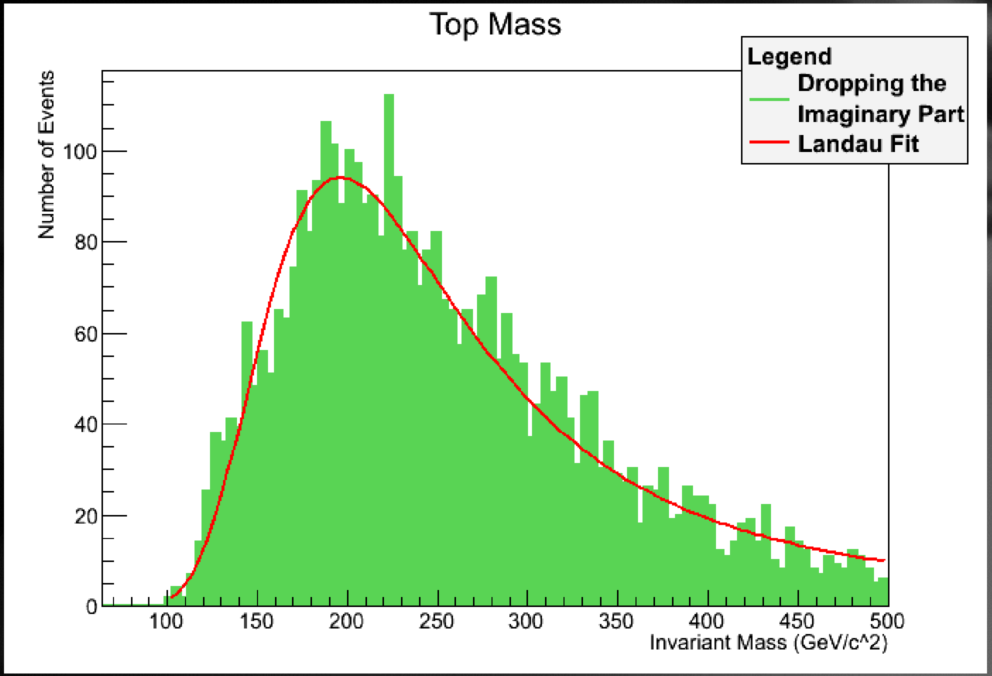
Dropping the Imaginary Part

Implemented Method:

Just drop the imaginary part of the root, leaving you with a constant.

Reasoning:

Current implemented method



VARIABLE NAME	VALUE	ERROR
1 Constant	521.17	11.50
2 MPV	204.48	1.33
3 Sigma	37.69	0.71

Another look at the Equation:

$$P_{z,\nu}^{A,B} = \frac{\mu \cdot P_{z,\mu}}{P_{\mathrm{T},\mu}^2} \pm \sqrt{\frac{\mu^2 \cdot P_{z,\mu}^2}{P_{\mathrm{T},\mu}^4} - \frac{E_\mu^2 \cdot \mathbf{E}_{\mathrm{T}}^{\mathrm{miss}^2} - \mu^2}{P_{\mathrm{T},\mu}^2}} \,,$$

$$\mu = \frac{M_W^2}{2} + \mathbf{P}_{\mathrm{T},\mu} \cdot \mathbf{E}_{\mathrm{T}}^{\mathrm{miss}} \,.$$

For more detailed calculations, check the Appendix.

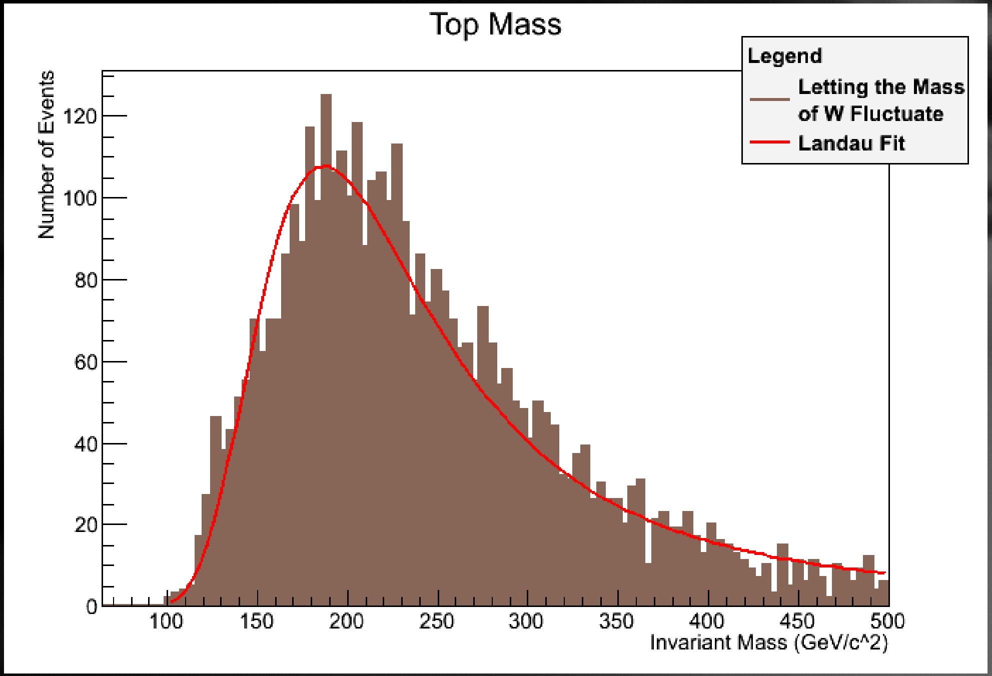
Letting the Mass of the W Boson Change

Implemented Method:

Set the determinant equal to zero and let the m_W change, resulting in a different constant

Reasoning:

The invariant mass of the W boson histogram has finite width, so the W boson is not always $80.4\,\frac{GeV}{c^2}$



VARIABLE NAME	VALUE	ERROR
1 Constant	596.55	13.23
2 MPV	193.86	1.11
3 Sigma	32.33	0.59

Yet another look at the Equation:

$$P_{z,\nu}^{A,B} = \frac{\mu \cdot P_{z,\mu}}{P_{\mathrm{T},\mu}^2} \pm \sqrt{\frac{\mu^2 \cdot P_{z,\mu}^2}{P_{\mathrm{T},\mu}^4} - \frac{E_\mu^2 \cdot \mathbf{E}_\mathrm{T}^{\mathrm{miss}^2} - \mu^2}{P_{\mathrm{T},\mu}^2}}\,,$$

$$\mu = \frac{M_W^2}{2} + \mathbf{P}_{\mathrm{T},\mu} \cdot \mathbf{E}_\mathrm{T}^{\mathrm{miss}}.$$

For more detailed calculations, check the Appendix.

Scaling the ME_T

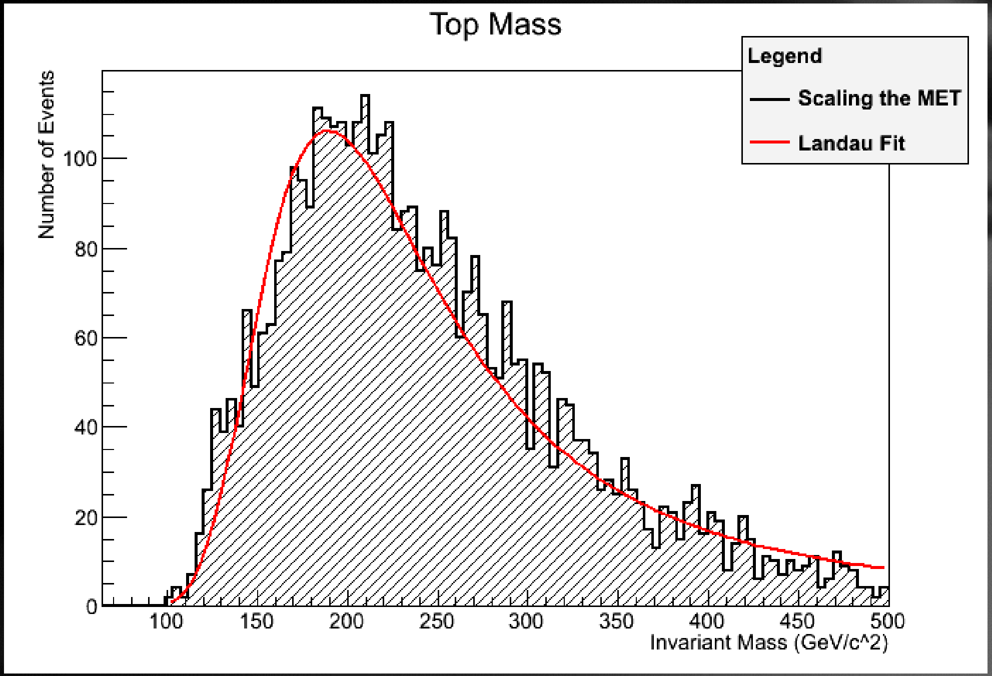
<u>*Implemented Method:*</u>

Let the Missing Transverse Energy fluctuate such that the discriminant is zero. This will then change the value of the constant, giving yet another estimate.

<u>*Reasoning:*</u>

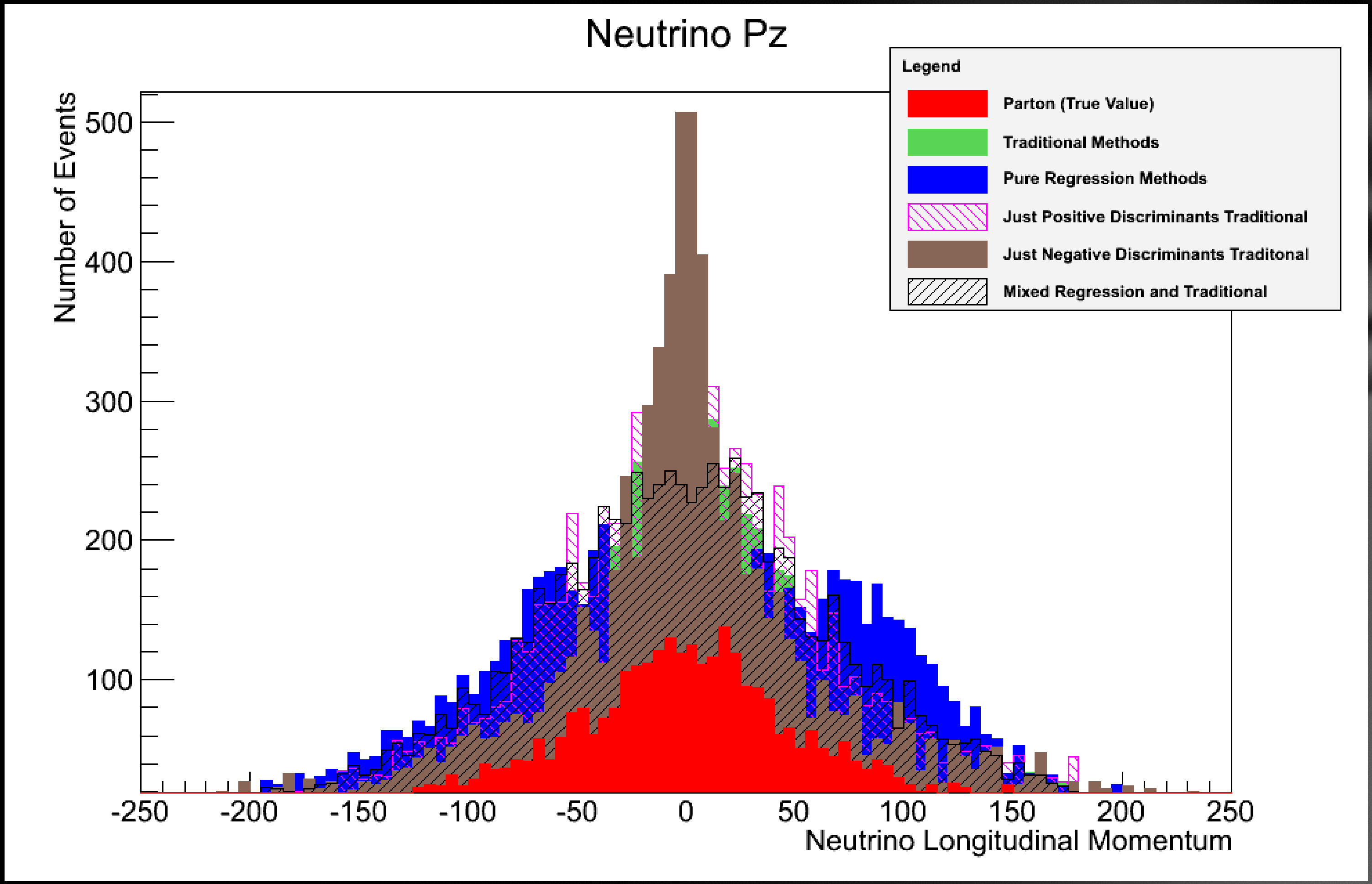
More often than not, the neutrino is not the sole carrier of the missing transverse energy. Other culprits include light recoil jets and other bottom jets that may have be produced with the top quark.

Therefore, in theory, changing the missing transverse energy to set the discriminant is allowed due to the presence of these other particles.

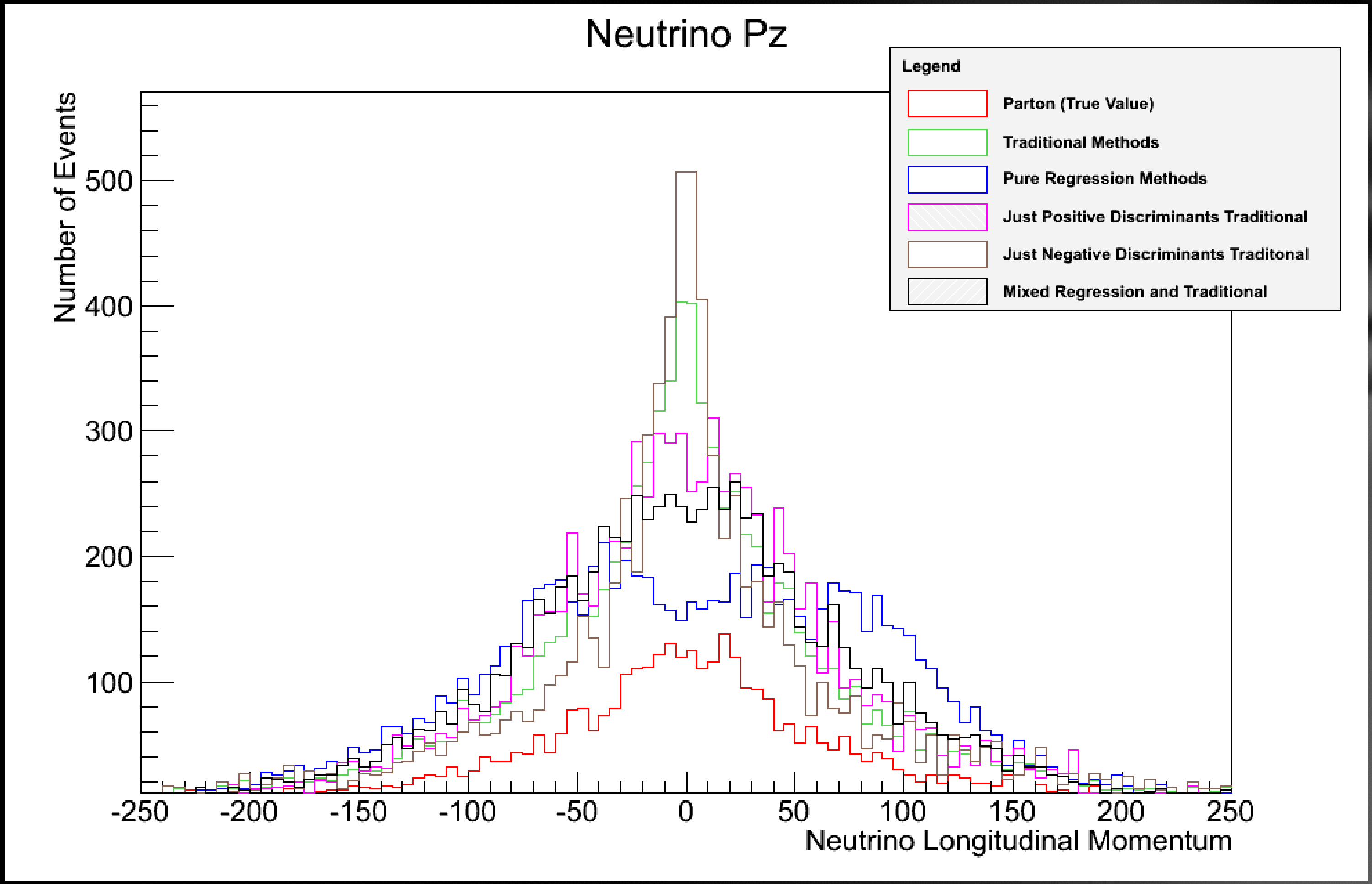


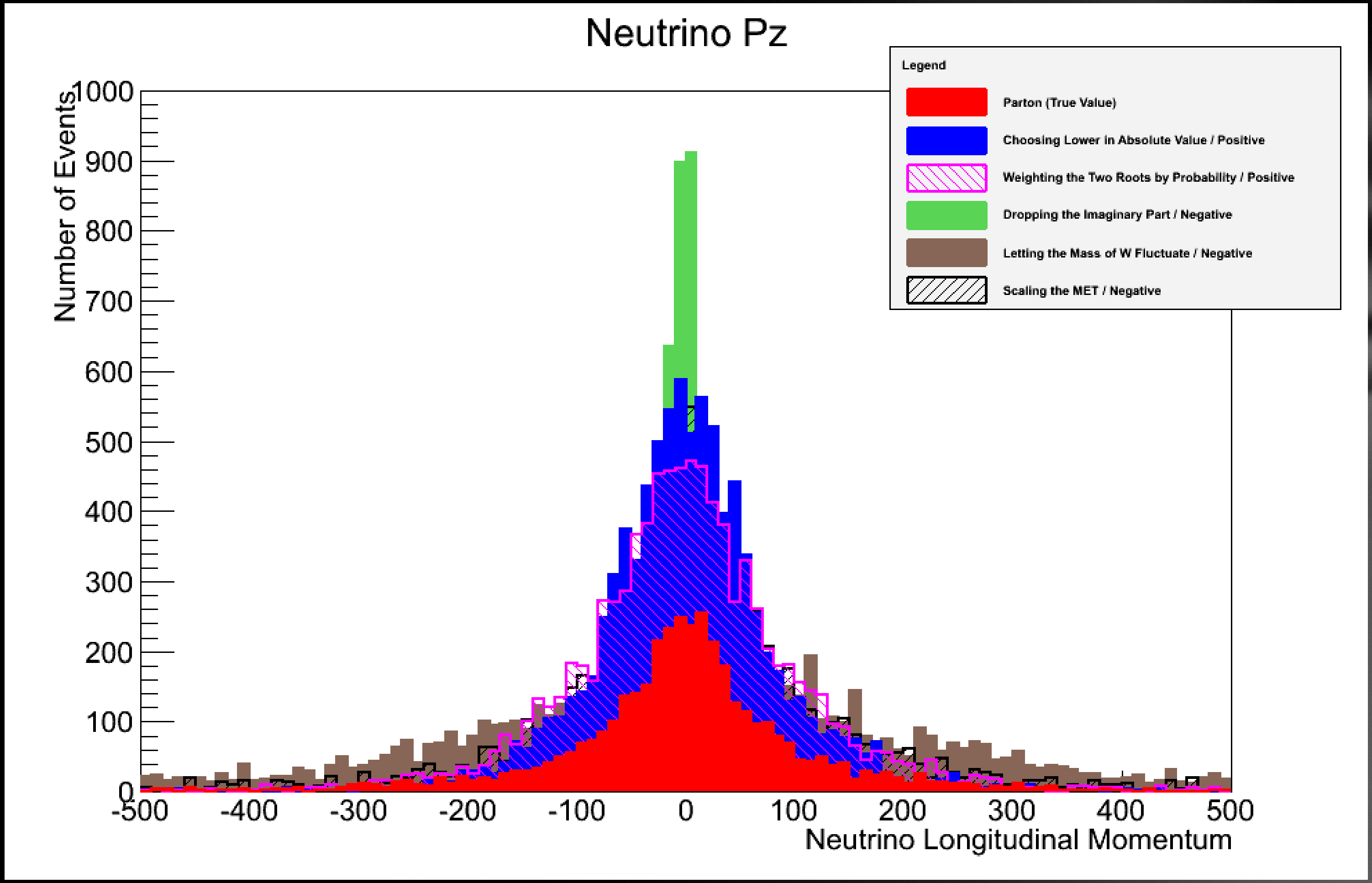
VARIABLE NAME	VALUE	ERROR
1 Constant	588.12	12.95
2 MPV	196.68	1.13
3 Sigma	33.18	0.60

Combined Plots

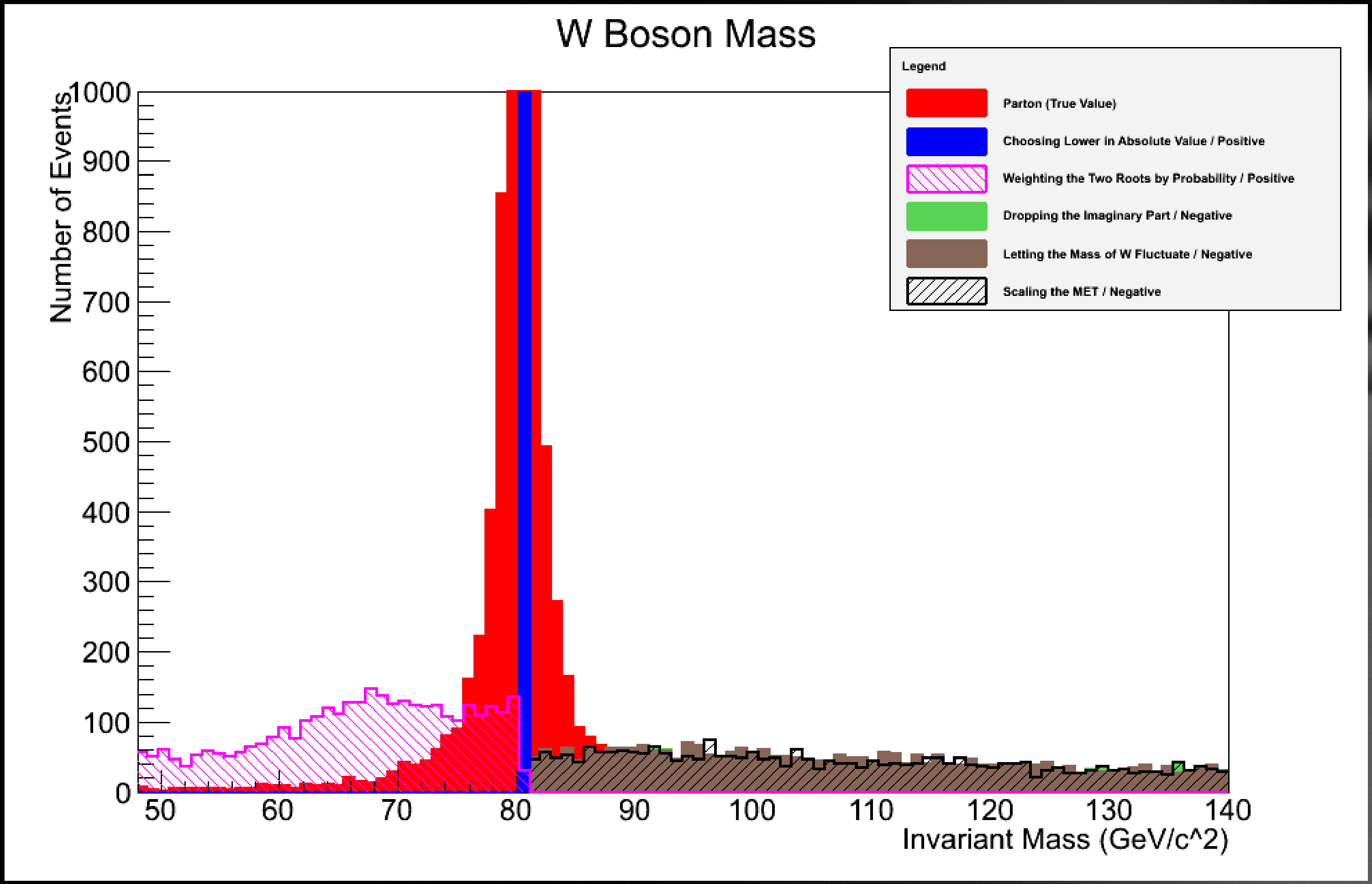


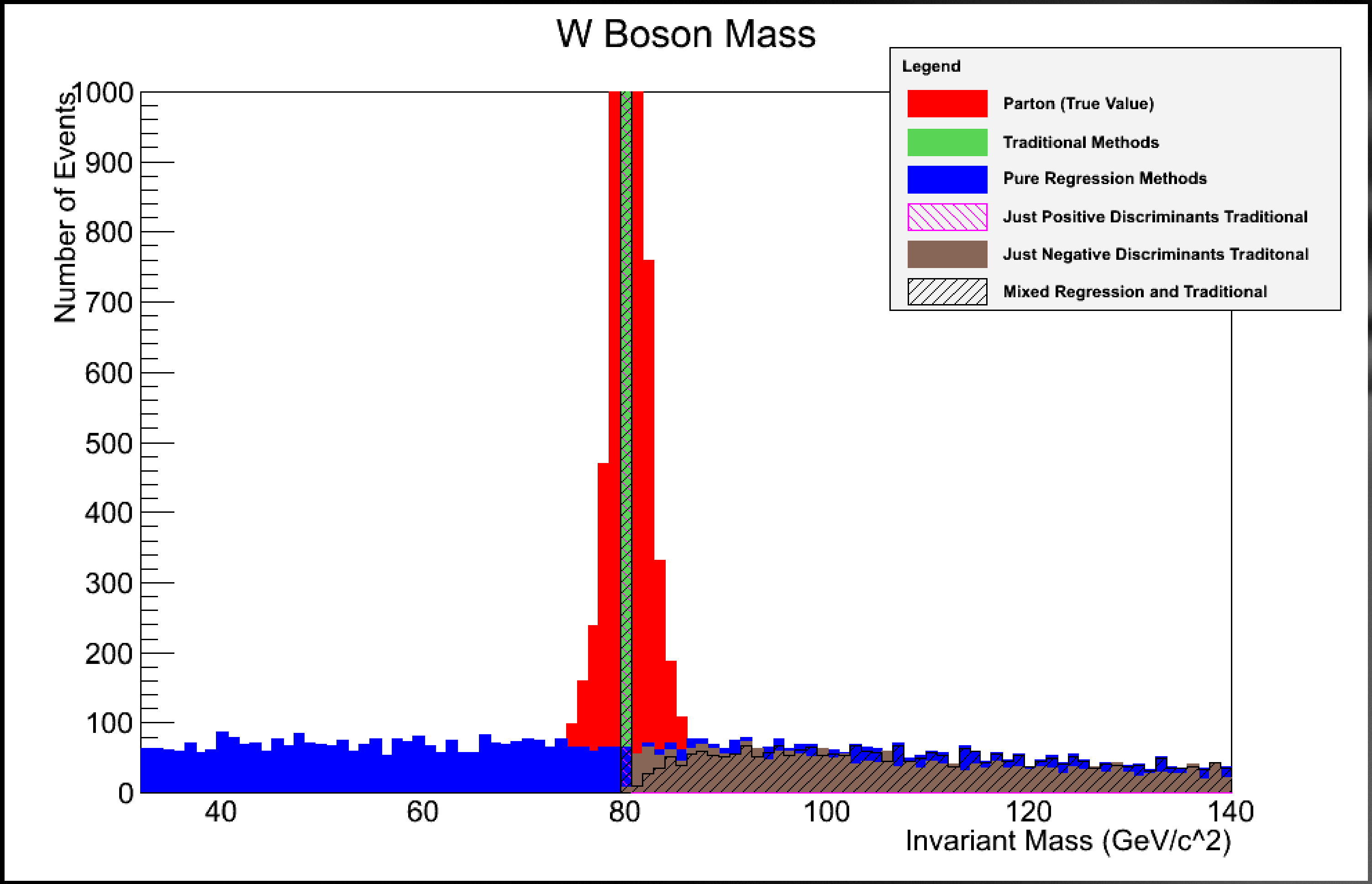
Neutrino Pz
Legend
Parton (True Value)
Traditional Methods
Pure Regression Methods
Just Positive Discriminants Traditional
Just Negative Discriminants Traditonal
Mixed Regression and Traditional
Number of Events
500
400
300
200
100
-250
-200
-150
-100
-50
0
50
100
150
200
250
Neutrino Longitudinal Momentum

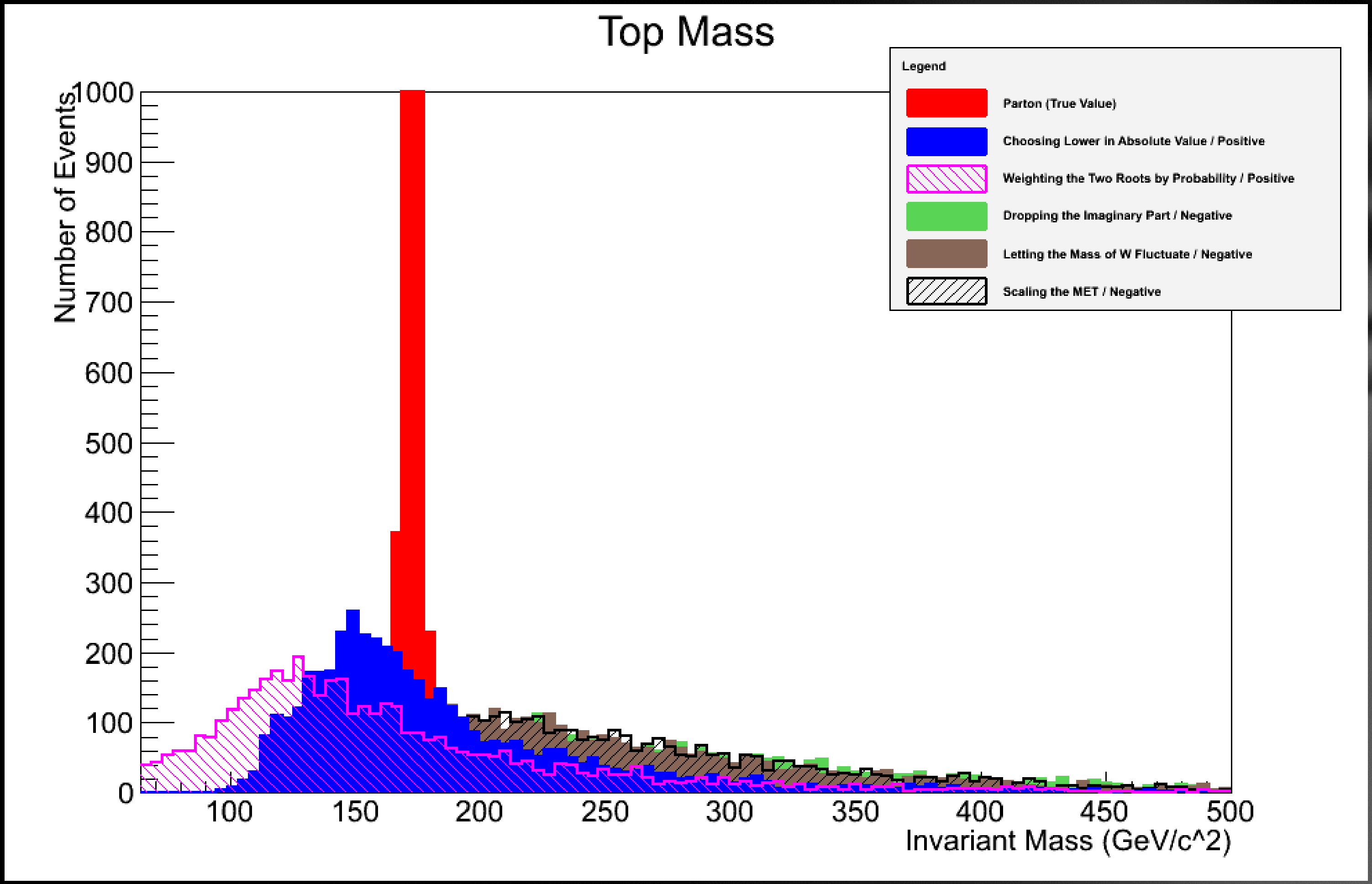

Neutrino Pz
Legend
Parton (True Value)
Traditional Methods
Pure Regression Methods
Just Positive Discriminants Traditional
Just Negative Discriminants Traditonal
Mixed Regression and Traditional
Number of Events
100
200
300
400
500
-250
-200
-150
-100
-50
0
50
100
150
200
250
Neutrino Longitudinal Momentum



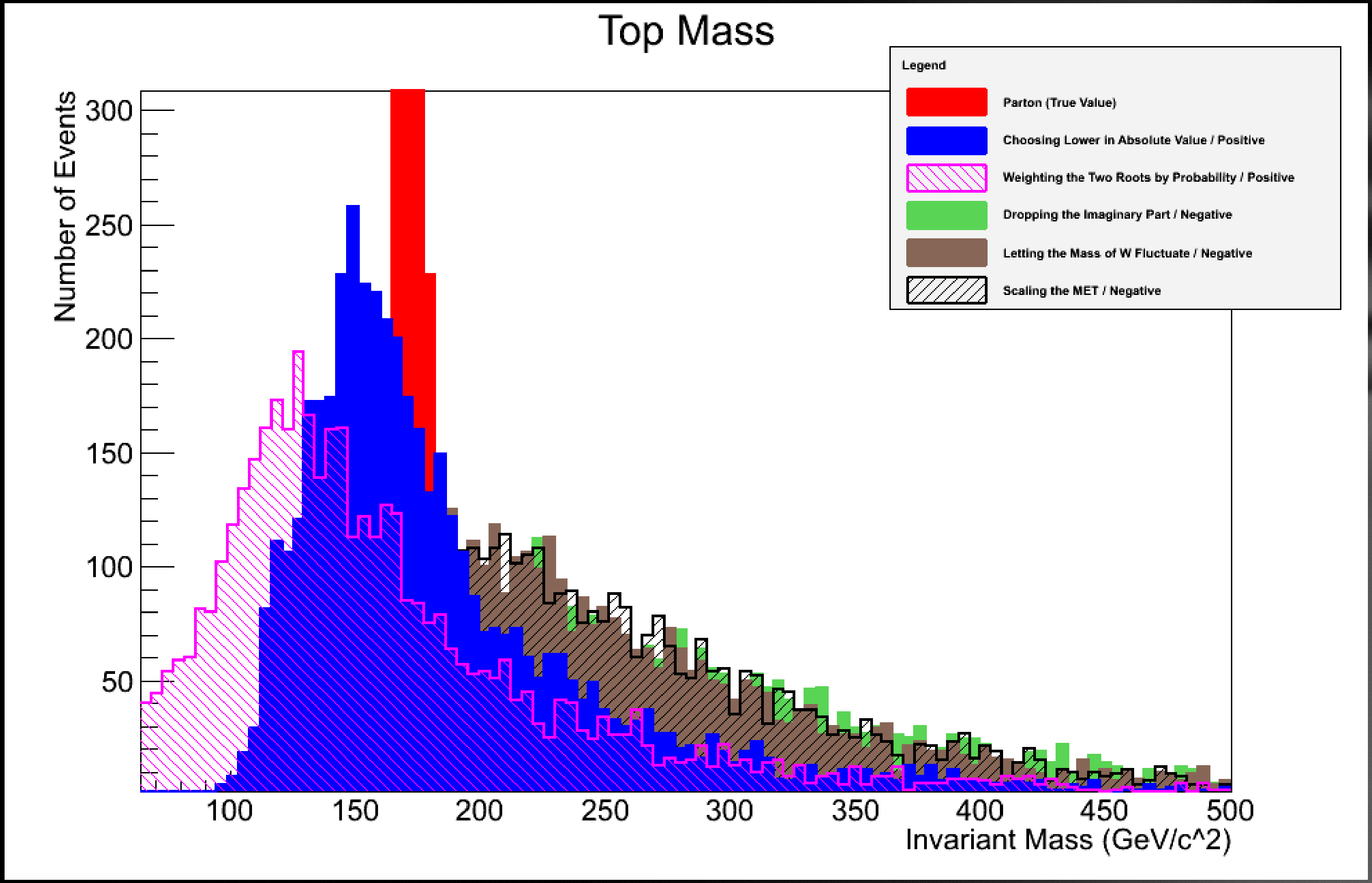
Neutrino Pz
Legend
Parton (True Value)
Choosing Lower in Absolute Value / Positive
Weighting the Two Roots by Probability / Positive
Dropping the Imaginary Part / Negative
Letting the Mass of W Fluctuate / Negative
Scaling the MET / Negative
Number of Events
1000
900
800
700
600
500
400
300
200
100
0
-500
-400
-300
-200
-100
0
100
200
300
400
500
Neutrino Longitudinal Momentum


W Boson Mass
Legend
Parton (True Value)
Choosing Lower in Absolute Value / Positive
Weighting the Two Roots by Probability / Positive
Dropping the Imaginary Part / Negative
Letting the Mass of W Fluctuate / Negative
Scaling the MET / Negative
Number of Events
0
100
200
300
400
500
600
700
800
900
1000
50
60
70
80
90
100
110
120
130
140
Invariant Mass (GeV/c^2)

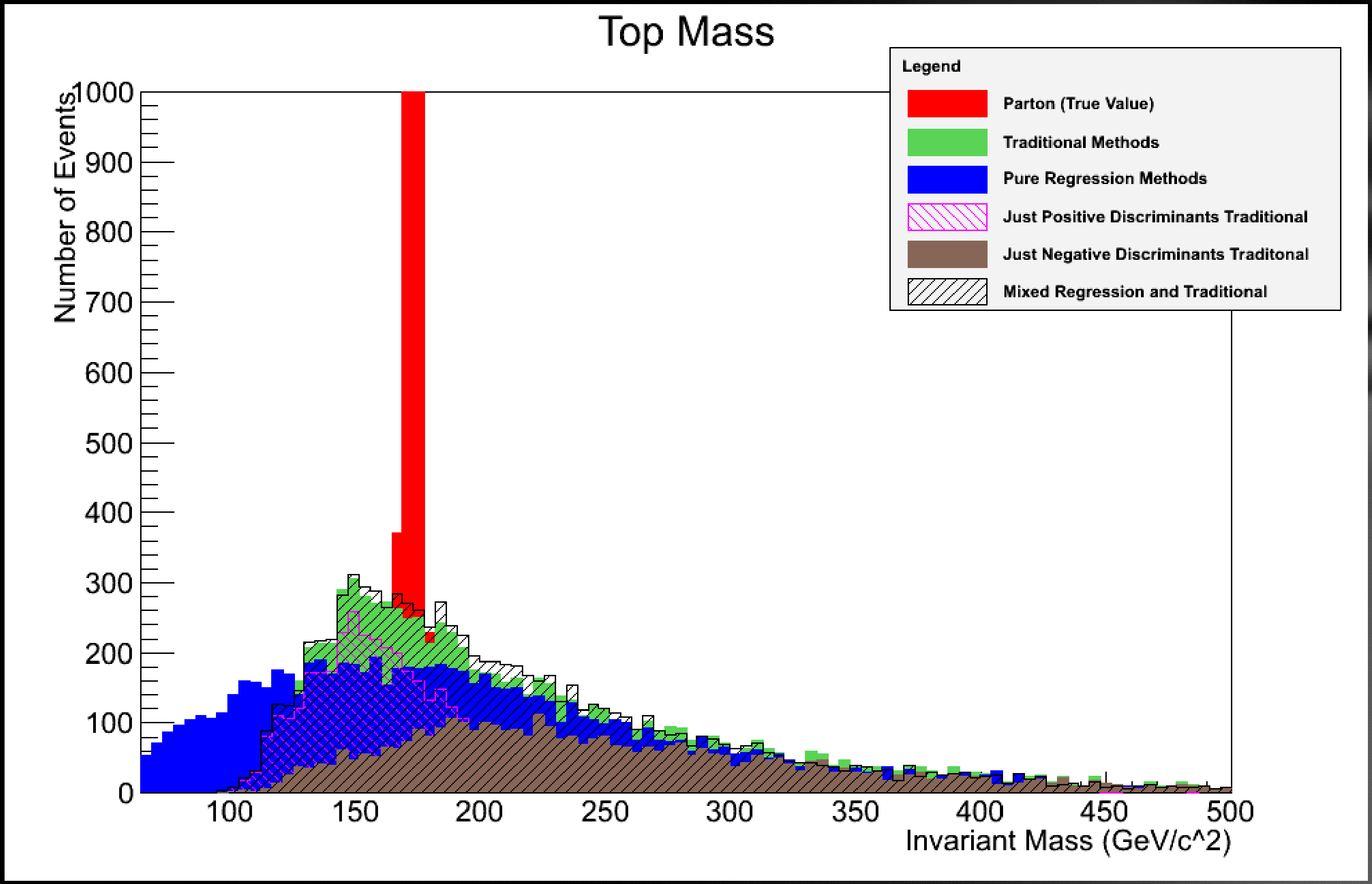

W Boson Mass
Legend
Parton (True Value)
Traditional Methods
Pure Regression Methods
Just Positive Discriminants Traditional
Just Negative Discriminants Traditonal
Mixed Regression and Traditional
Number of Events
0
100
200
300
400
500
600
700
800
900
1000
40
60
80
100
120
140
Invariant Mass (GeV/c^2)



Top Mass
Legend
Parton (True Value)
Choosing Lower in Absolute Value / Positive
Weighting the Two Roots by Probability / Positive
Dropping the Imaginary Part / Negative
Letting the Mass of W Fluctuate / Negative
Scaling the MET / Negative
Number of Events
0
100
200
300
400
500
600
700
800
900
1000
100
150
200
250
300
350
400
450
500
Invariant Mass (GeV/c^2)



Top Mass
Legend
Parton (True Value)
Choosing Lower in Absolute Value / Positive
Weighting the Two Roots by Probability / Positive
Dropping the Imaginary Part / Negative
Letting the Mass of W Fluctuate / Negative
Scaling the MET / Negative
Number of Events
50
100
150
200
250
300
100
150
200
250
300
350
400
450
500
Invariant Mass (GeV/c^2)

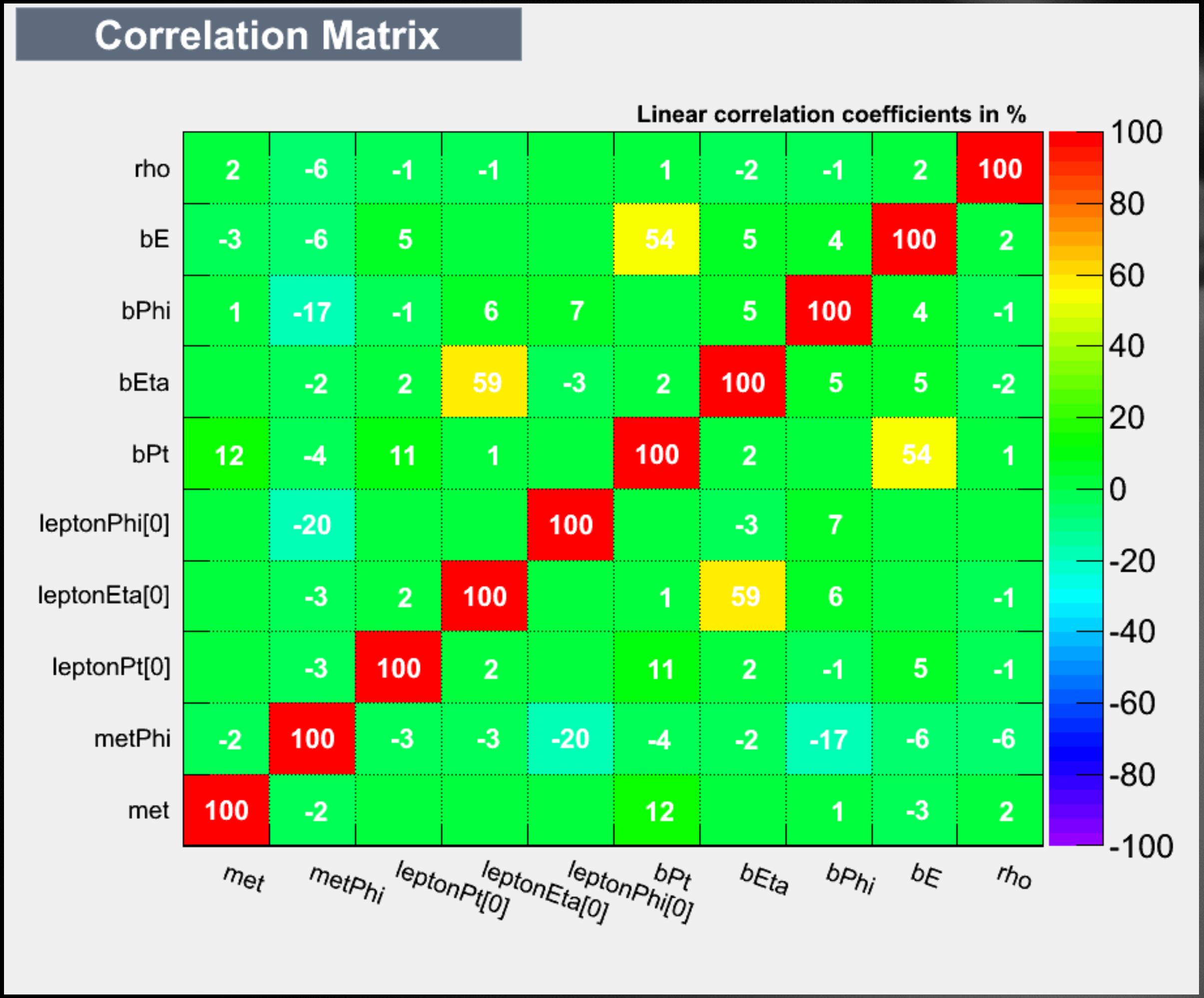


Top Mass
Legend
Parton (True Value)
Traditional Methods
Pure Regression Methods
Just Positive Discriminants Traditional
Just Negative Discriminants Traditonal
Mixed Regression and Traditional
Number of Events
0
100
200
300
400
500
600
700
800
900
1000
100
150
200
250
300
350
400
450
500
Invariant Mass (GeV/c^2)

Regression Slides

Correlation Matrix

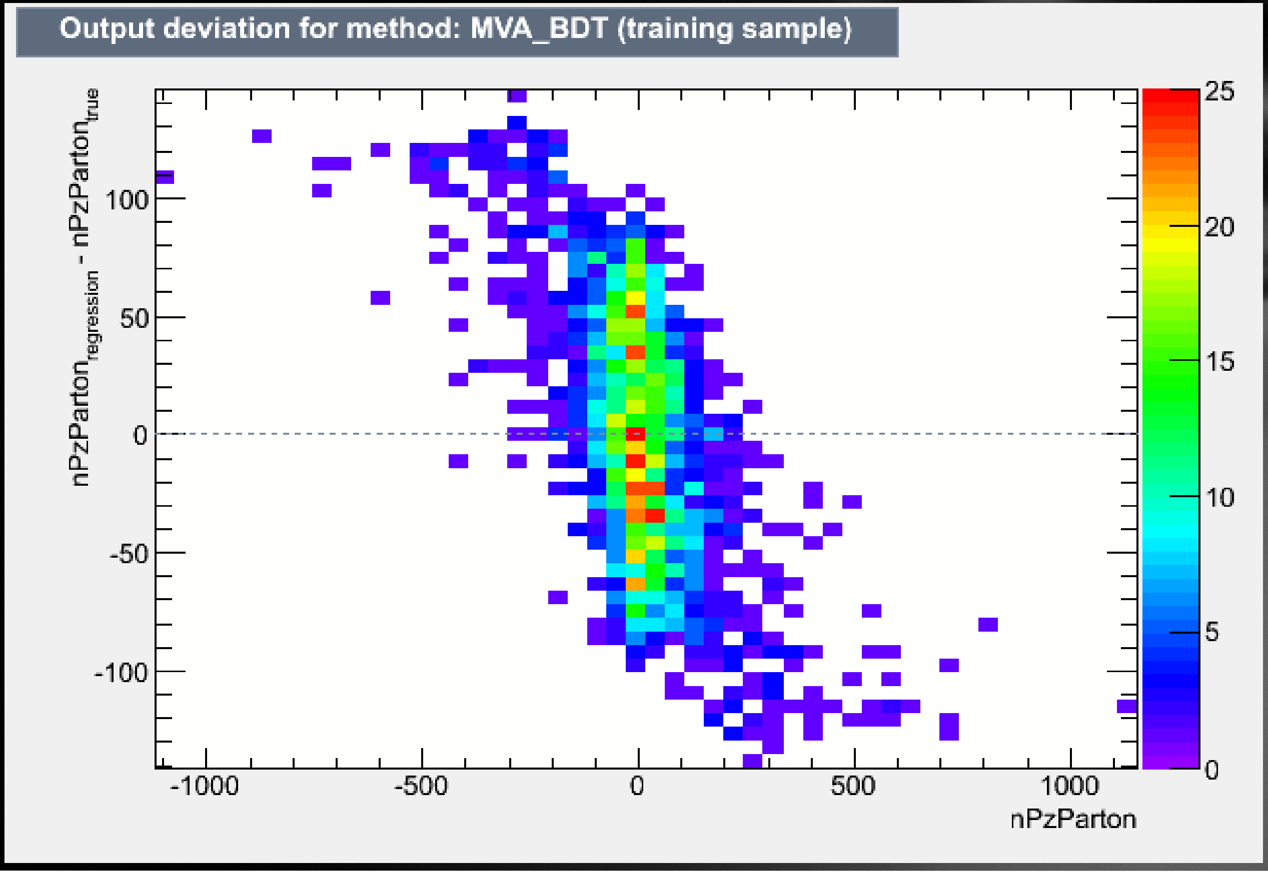
As can be seen, the variables are not highly correlated, except for the pseudorapidities of the lepton and the bottom jet, but that should not impact the regression significantly because they are independent variables.



Training Distribution

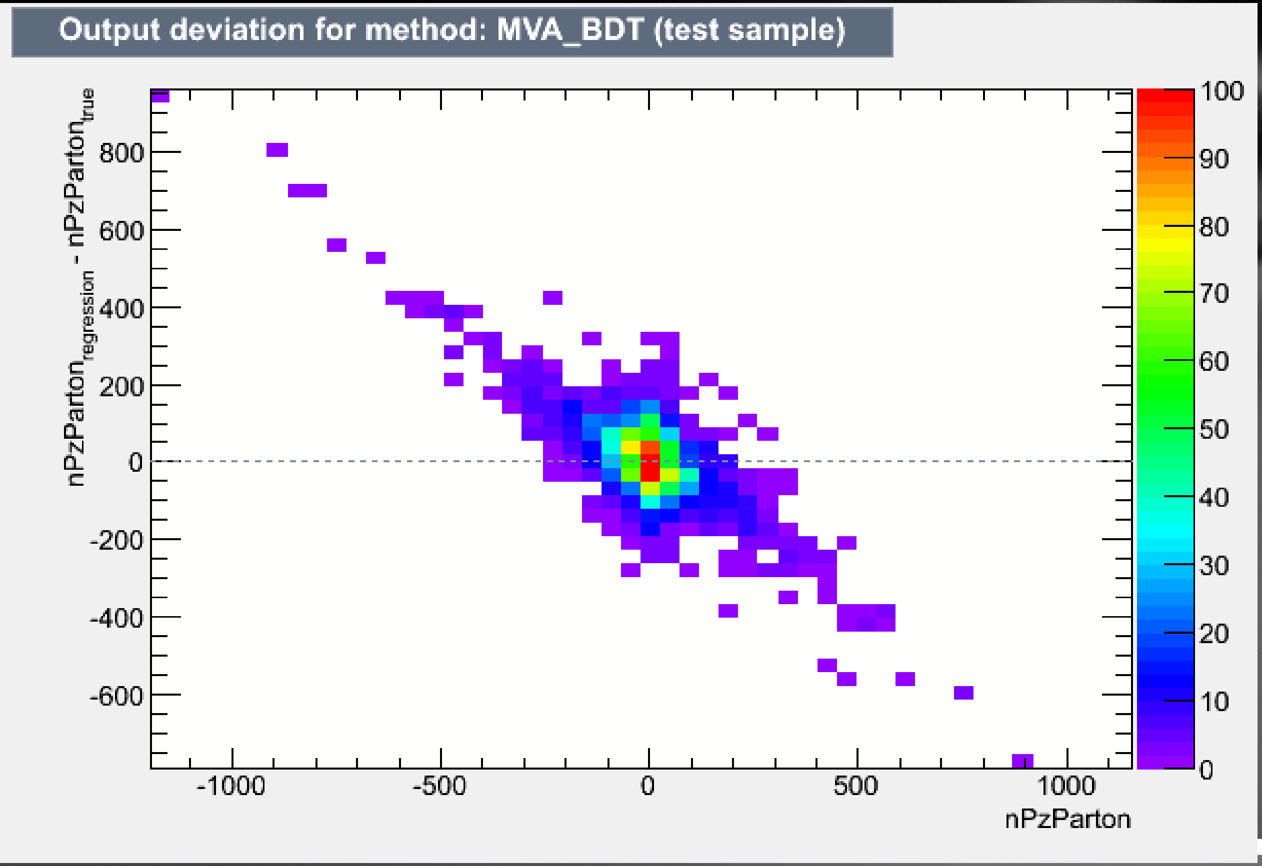
This analysis used regression methods, so there is no clear test for overtraining, unlike for classification, where the Kolmogorov-Smironov test is used.

Training graph shows a relatively wide distribution in deviations, even with small Pz.



Test Distribution

The deviations here are much larger for the test distribution, which is to be expected. These massive deviations show that even at small Pz, there can be massive deviations in the regression model used.



Extra Calculations

The Principal Equation:

$$P_{z,\nu}^{A,B} = \frac{\mu \cdot P_{z,\mu}}{P_{\mathrm{T},\mu}^2} \pm \sqrt{\frac{\mu^2 \cdot P_{z,\mu}^2}{P_{\mathrm{T},\mu}^4} - \frac{E_\mu^2 \cdot \mathbf{E}_\mathrm{T}^{\mathrm{miss}\,2} - \mu^2}{P_{\mathrm{T},\mu}^2}} \;,$$

$$\mu = \frac{M_W^2}{2} + \mathbf{P}_{\mathrm{T},\mu} \cdot \mathbf{E}_\mathrm{T}^{\mathrm{miss}} \;.$$

Calculations for Principal Equation

$$
\begin{aligned}
P_W &= P_\nu + P_l \\
P_W^2 &= M_W^2 = (P_\nu + P_l)^2 = \underbrace{P_\nu^2}_{\approx 0} + \underbrace{P_l^2}_{\approx 0} + 2P_\nu P_l \\
&= 2\begin{pmatrix} E_l \\ p_{l,x} \\ p_{l,y} \\ p_{l,z} \end{pmatrix}\begin{pmatrix} E_\nu \\ p_{\nu,T}\cos\phi \\ p_{\nu,T}\sin\phi \\ p_{\nu,z} \end{pmatrix} \\
&= 2\left[E_l E_\nu - p_{l,x}p_{\nu,T}\cos\phi - p_{l,y}p_{\nu,T}\sin\phi - p_{l,z}p_{\nu,z}\right] \\
&= 2\left[E_l E_\nu - p_{\nu,T}\left(p_{l,x}\cos\phi - p_{l,y}\sin\phi\right) - p_{l,z}p_{\nu,z}\right]
\end{aligned}
$$

$$E_l E_\nu = E_l\sqrt{E_{\nu,T}^2 + E_{\nu,z}^2} = E_l\sqrt{p_{\nu,T}^2 + p_{\nu,z}^2}$$

$$p_{l,x} = p_{l,T}\cos\phi, \quad p_{l,y} = p_{l,T}\sin\phi, \quad \phi \equiv \phi_\nu$$

$$= 2\left[E_l\sqrt{p_{\nu,T}^2 + p_{\nu,z}^2} - p_{\nu,T}p_{l,T}\underbrace{\left(\cos\phi_l\cos\phi_\nu + \sin\phi_l\sin\phi_\nu\right)}_{=\cos(\phi_l - \phi_\nu)} - p_{l,z}p_{\nu,z}\right]$$

Calculations for Principal Equation

$$
\begin{aligned}
E_l\sqrt{p_{\nu,T}^2 + p_{\nu,z}^2} &= \underbrace{\frac{M_W^2}{2} + p_{\nu,T}p_{l,T}\cos\left(\phi_l - \phi_\nu\right)}_{=:a} + p_{l,z}p_{\nu,z} \\
E_l\left(p_{\nu,T}^2 + p_{\nu,z}^2\right) &= a^2 + p_{l,z}^2 p_{\nu,z}^2 + 2ap_{l,z}p_{\nu,z} \\
0 &= -a^2 - 2ap_{l,z}p_{\nu,z} + E_l^2 p_{\nu,T}^2 + p_{\nu,z}^2\left(E_l^2 - p_{l,z}^2\right) \\
0 &= p_{\nu,z}^2 - \frac{2ap_{l,z}}{E_l^2 - p_{l,z}^2}p_{\nu,z} - \frac{a^2 - E_l^2 p_{\nu,T}^2}{E_l^2 - p_{l,z}^2} \\
p_{\nu,z}^{1,2} &= \frac{ap_{l,z}}{E_l^2 - p_{l,z}^2} \pm \sqrt{\left(\frac{ap_{l,z}}{E_l^2 - p_{l,z}^2}\right)^2 + \frac{a^2 - E_l^2 p_{\nu,T}^2}{E_l^2 - p_{l,z}^2}} \\
&= \frac{ap_{l,z}}{E_l^2 - p_{l,z}^2} \pm \frac{1}{E_l^2 - p_{l,z}^2}\sqrt{\left(ap_{l,z}\right)^2 + \left(a^2 - E_l^2 p_{\nu,T}^2\right)\left(E_l^2 - p_{l,z}^2\right)} \\
&= \frac{1}{E_l^2 - p_{l,z}^2}\left(ap_{l,z} \pm E_l\sqrt{a^2 - E_l^2 p_{\nu,T}^2 + p_{l,z}^2 p_{\nu,T}^2}\right)
\end{aligned}
$$

Calculations for Complex Roots Method 2

$$\frac{\mu^2 \cdot P_{z,\mu}^2}{P_{T,\mu}^4} - \frac{E_\mu^2 \cdot \boldsymbol{E}_T^{miss^2} - \mu^2}{P_{T,\mu}^2} = 0$$

$$\mu^2 \cdot P_{z,\mu}^2 - P_{T,\mu}^2 * (E_\mu^2 \cdot \boldsymbol{E}_T^{miss^2} - \mu^2) = 0$$

$$\mu^2 \cdot (P_{z,\mu}^2 + P_{T,\mu}^2) = P_{T,\mu}^2 * (E_\mu^2 \cdot \boldsymbol{E}_T^{miss^2})$$

$$\mu^2 = P_{T,\mu}^2 * \frac{E_\mu^2 \cdot \boldsymbol{E}_T^{miss^2}}{(P_{z,\mu}^2 + P_{T,\mu}^2)}$$

$$\left(\frac{M_W^2}{2} + \boldsymbol{P}_{T,\mu} \cdot \boldsymbol{E}_T^{miss}\right)^2 = P_{T,\mu}^2 * \frac{E_\mu^2 \cdot \boldsymbol{E}_T^{miss^2}}{(P_{z,\mu}^2 + P_{T,\mu}^2)}$$

$$M_W^2 = 2 * (\sqrt{\frac{P_{T,\mu}^2 * E_\mu^2 \cdot \boldsymbol{E}_T^{miss^2}}{(P_{z,\mu}^2 + P_{T,\mu}^2)}} - \boldsymbol{P}_{T,\mu} \cdot \boldsymbol{E}_T^{miss})$$

$$M_W = \sqrt{2 * (\sqrt{\frac{P_{T,\mu}^2 * E_\mu^2 \cdot \boldsymbol{E}_T^{miss^2}}{(P_{z,\mu}^2 + P_{T,\mu}^2)}} - \boldsymbol{P}_{T,\mu} \cdot \boldsymbol{E}_T^{miss})}$$

Calculations for Complex Roots Method 2

$$\mu_{new} = \left(\sqrt{\frac{P_{T,\mu}^2 * E_\mu^2 \cdot \boldsymbol{E}_T^{miss^2}}{(P_{z,\mu}^2 + P_{T,\mu}^2)}}\right)$$

$$P_{z,\mu} = \frac{\left(\sqrt{\frac{P_{T,\mu}^2 * E_\mu^2 \cdot \boldsymbol{E}_T^{miss^2}}{(P_{z,\mu}^2 + P_{T,\mu}^2)}}\right) \cdot P_{z,\mu}}{P_{T,\mu}^2}$$

$$P_{z,\mu} = \left(\sqrt{\frac{E_\mu^2 \cdot \boldsymbol{E}_T^{miss^2}}{(P_{z,\mu}^2 + P_{T,\mu}^2)}}\right) \cdot \frac{P_{z,\mu}}{P_{T,\mu}}$$

Calculations for Complex Roots Method 3

$$\frac{\mu^2 \cdot P_{z,\mu}^2}{P_{T,\mu}^4} - \frac{E_\mu^2 \cdot \boldsymbol{E}_T^{miss^2} - \mu^2}{P_{T,\mu}^2} = 0$$

$$If\ \boldsymbol{E}_T^{miss^2} = \alpha ME_T:$$

$$\mu^2 \cdot P_{z,\mu}^2 + P_{T,\mu}^2\mu^2 - P_{T,\mu}^2 E_\mu^2 \cdot (\alpha ME_T)^2 = 0$$

$$(P_{z,\mu}^2 + P_{T,\mu}^2)\mu^2 = E_\mu^2 \cdot \left(\alpha ME_T * P_{T,\mu}\right)^2$$

$$E_\mu^2 \mu^2 = E_\mu^2 \cdot \left(\alpha ME_T * P_{T,\mu}\right)^2$$

$$\mu^2 = \left(\alpha ME_T * P_{T,\mu}\right)^2$$

$$\frac{M_W^2}{2} + \alpha ME_T * P_{T,\mu} * \cos(\Delta\phi_{\nu,l}) = \alpha ME_T * P_{T,\mu}$$

Calculations for Complex Roots Method 3

$$\frac{M_W^2}{2} = \alpha ME_T * P_{T,\mu} * \left(1 - \cos\left(\Delta\phi_{\nu,l}\right)\right)$$

$$\alpha = \frac{M_W^2}{2ME_T * P_{T,\mu} * \left(1 - \cos\left(\Delta\phi_{\nu,l}\right)\right)}$$

$$P_{T,\nu} = \frac{M_W^2}{2P_{T,\mu}(1 - \cos(\Delta\phi))}\ , where\ \Delta\phi = \left(MET\phi - \phi_\mu\right)$$

$$P_{z,\nu} = \frac{P_{z,\mu} \cdot P_{T,\nu}}{P_{T,\mu}}$$