Multi-turn Extraction:
Splitting the PS Beam in Transverse Phase Space

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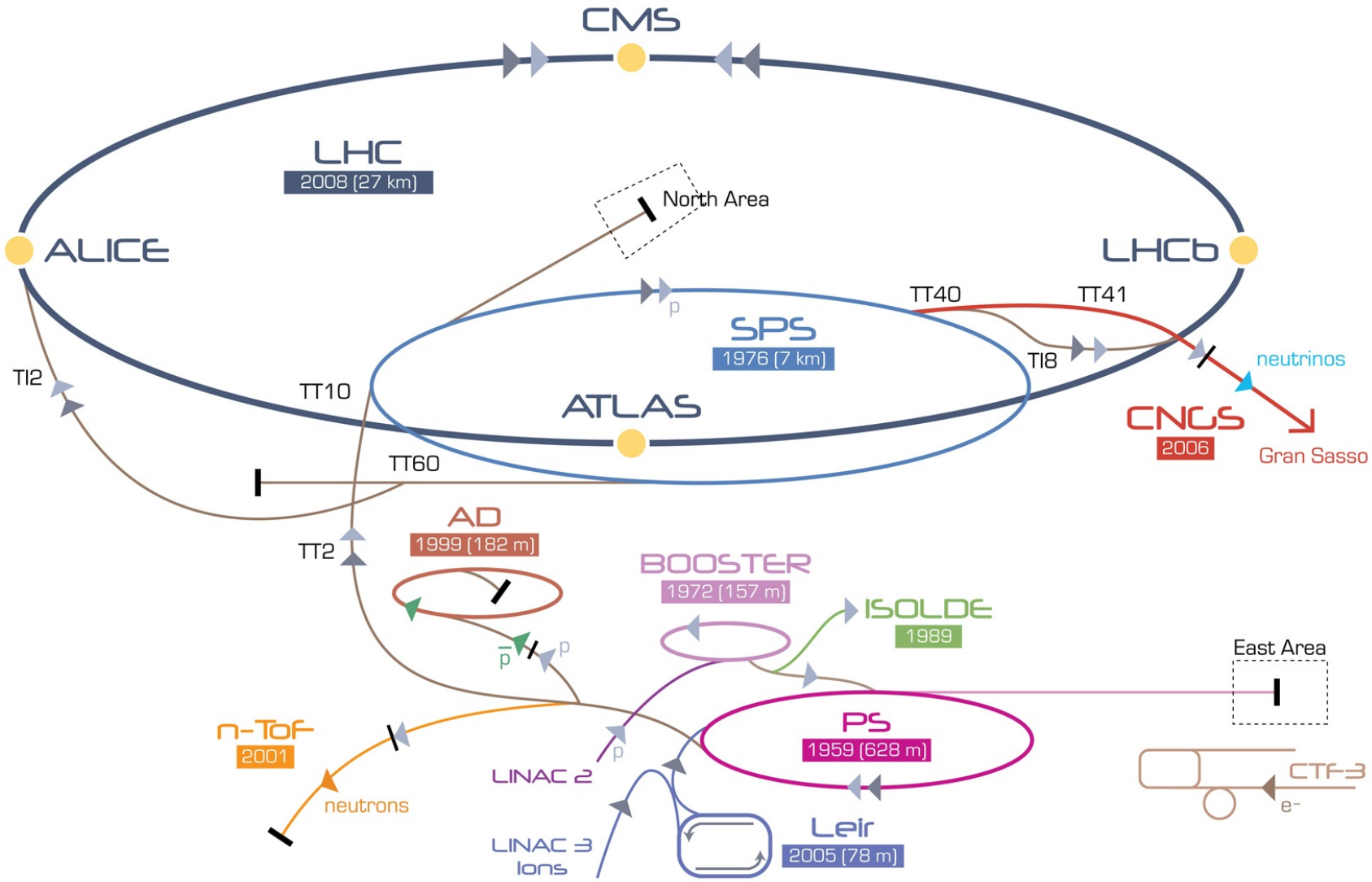
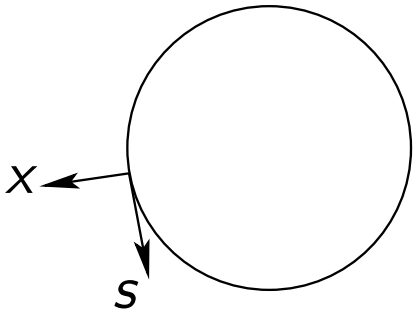
CERN Accelerators ... Focus: PS → SPS transfer for fixed-target experiments.



Multi-turn Extraction



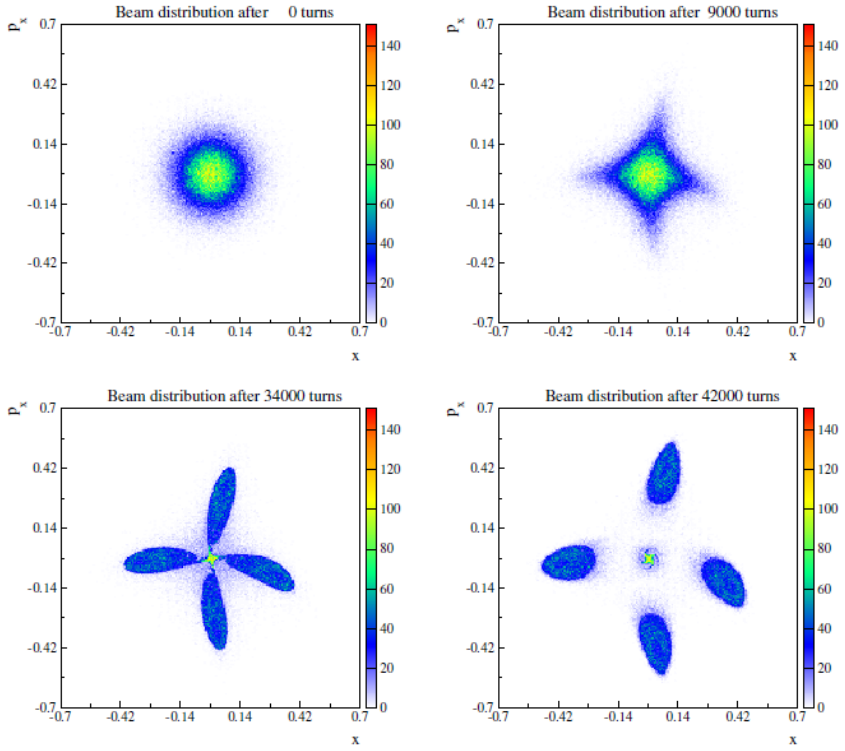
- Particle's trajectory in the beam is oscillatory...

  First-order approximation:

$$x(s) = \sqrt{\underset{\substack{\uparrow \\ \text{emittance}}}{\varepsilon}\,\beta(s)}\,\sin\psi(s)\,.$$

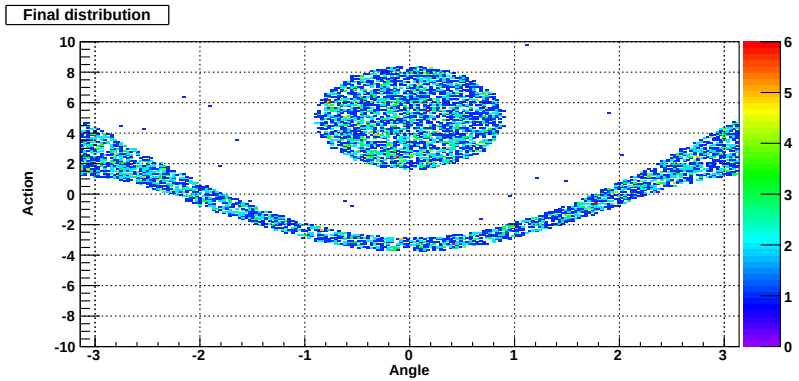
- Tune: $\nu = \psi(L)/2\pi$.

$$\nu \in \mathbb{Q} \quad \Rightarrow \quad \text{fixed pts.}$$



My Goals

- Understand particle trapping theoretically.
- Control trapping fractions precisely.
- Minimize the emittance blow-up.



$\longrightarrow$ My strategy:

(1) Examine a simplified Hamiltonian.

$$\mathcal{H} = \frac{1}{2}\left[p - \delta(t)\right]^2 - \left[1 + \beta(t)\right] \cos q \,.$$

(2) Move on to a perturbative approximation of a realistic Hamiltonian.

$$\mathcal{H} = \alpha(t)\, p + \beta(t)\, p^2 + \gamma(t)\, p^2 \cos(4q) \,.$$

Simple Adiabatic Trapping Fraction

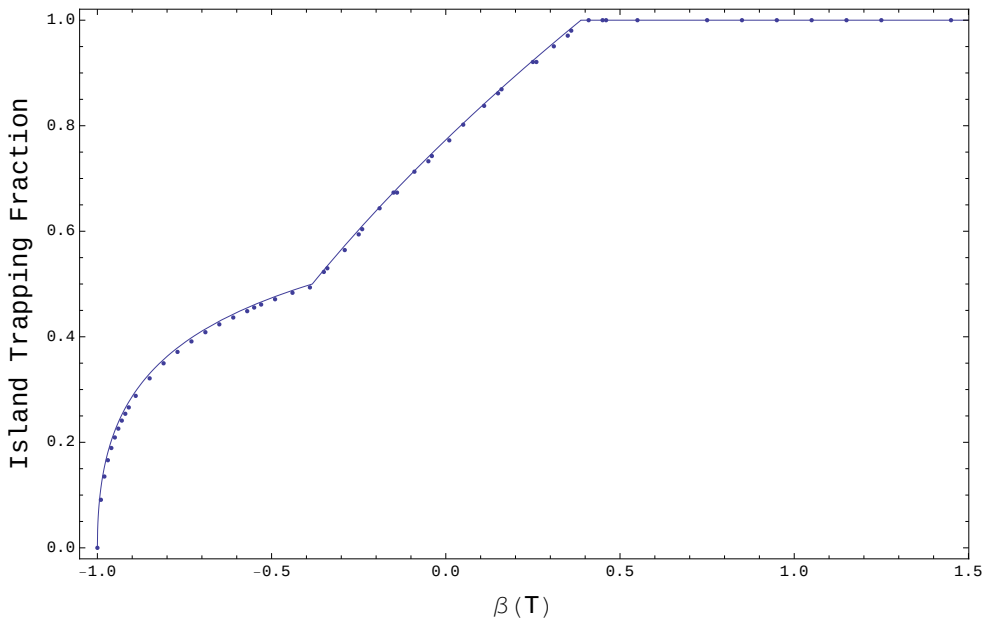
- We examined $\mathcal{H}(t)$ in which $\delta(t)$ governs travel and $\beta(t)$ governs growth:

$$\mathcal{H} = \frac{1}{2}\left[p - \delta(t)\right]^2 - \left[1 + \beta(t)\right] \cos q \,.$$

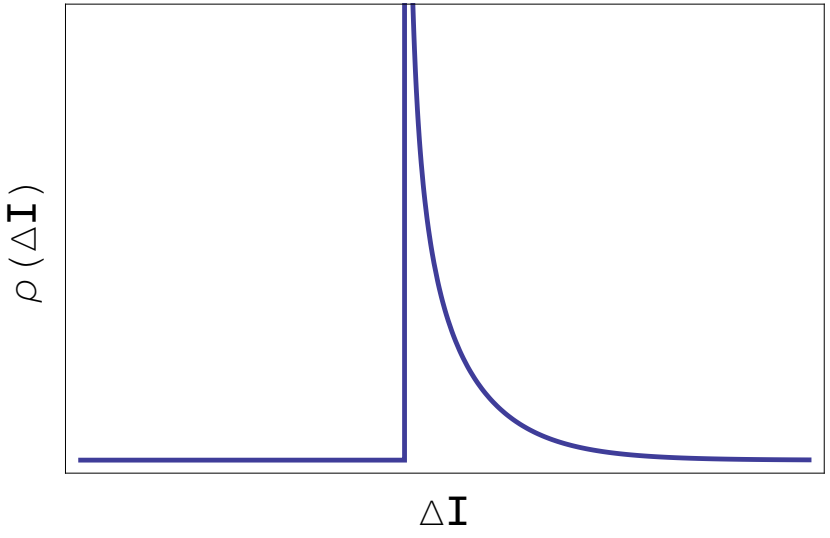
- We fixed a rectangle of IC's and set the island in the center with $\Sigma_{\text{island}} = 0\ldots$

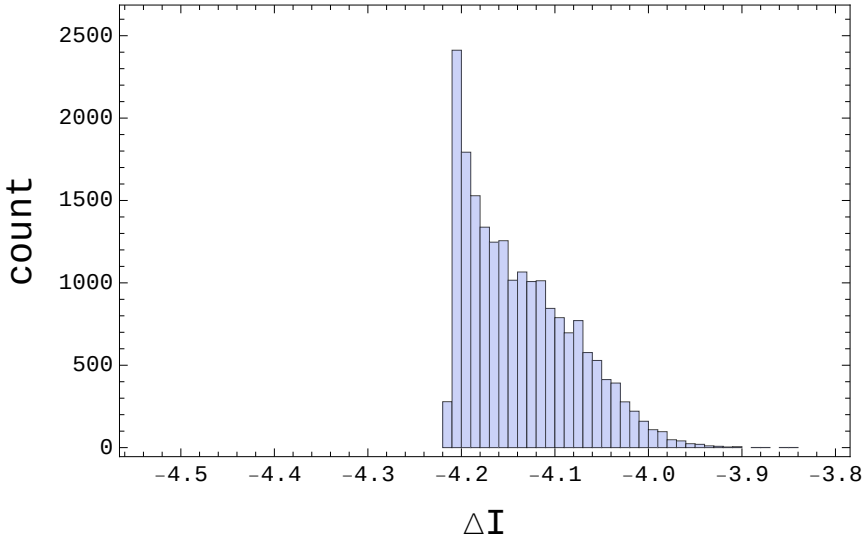
Travel : increasing $\delta(t)$ linearly.

Growth : increasing $\beta(t)$ quadratically to $\beta(T)$.



Simple $\Delta\mathcal{I}$ & $\sigma(\Delta\mathcal{I})$ upon Separatrix Crossing

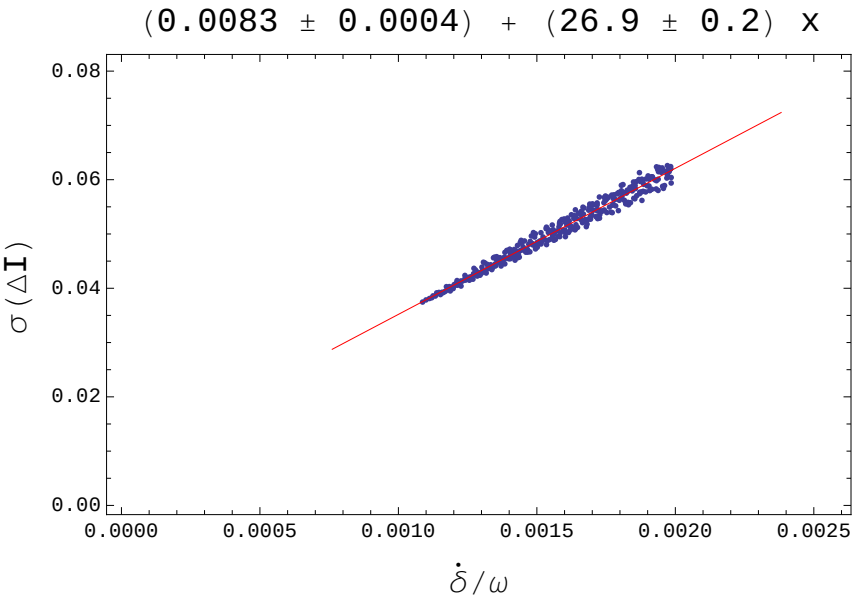




- $\mathcal{I}$ not conserved upon separatrix crossing. If particle passes from above to below island...

$$\Delta\mathcal{I} \propto \frac{\dot{\delta}}{\omega} \ln(2\sin\pi\xi) + \text{offset}$$

$$\sigma(\Delta\mathcal{I}) \propto \sigma\left[\frac{\dot{\delta}}{\omega} \ln(2\sin\pi\xi)\right] \propto \left|\frac{\dot{\delta}}{\omega}\right| .$$

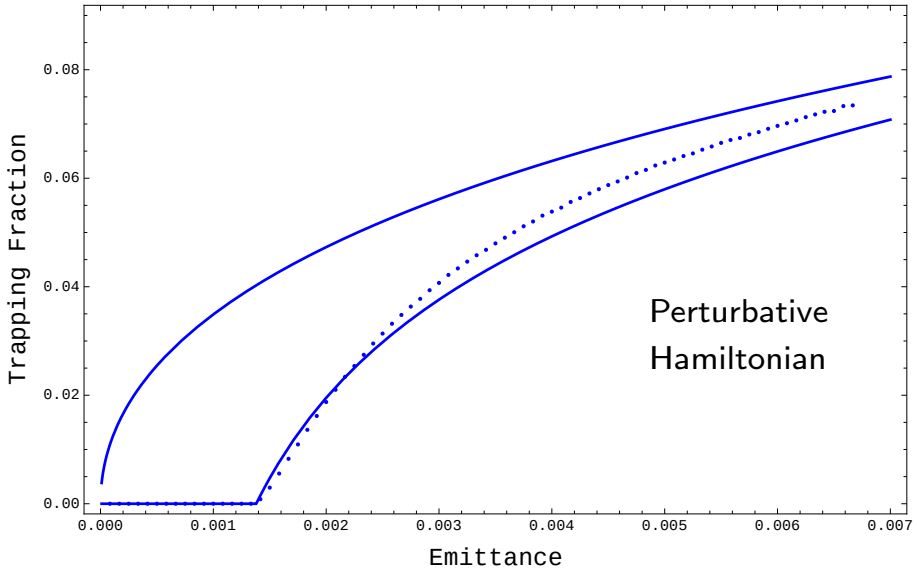


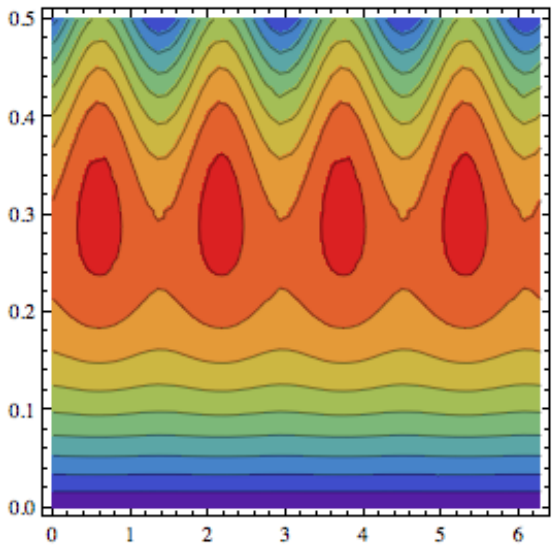
Perturbative Trapping Fraction

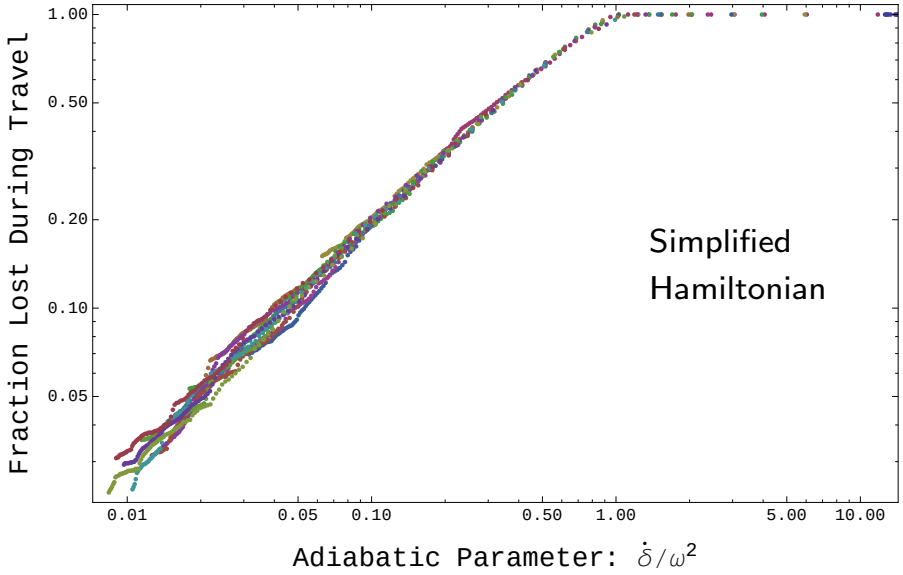
- Moving to

$$\mathcal{H} = \alpha(t)\, p + \beta(t)\, p^2 + \gamma(t)\, p^2 \cos(4q)$$

- We set IC's in a rectangle and let islands begin with zero size at bottom of rectangle.







Moving Forward

- Perturbative Hamiltonian:
  - Fully understand trapping fraction curves and examine $\sigma(\Delta\mathcal{I})$.
- Way Ahead . . . Real PS Map:
  - Optimize time-dependent parameters in real $\mathcal{H}(t)$ numerically.
  - Test predictions by making measurements in the PS.

ICPS — Utrecht, Netherlands







Questions?