Multi-turn Extraction:
Splitting the PS Beam in Transverse Phase Space

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CERN Accelerators … Focus: PS → SPS transfer for fixed-target experiments.
Multi-turn Extraction

- Particle’s trajectory in the beam is oscillatory...

First-order approximation:

\[ x(s) = \sqrt{\varepsilon \beta(s)} \sin \psi(s). \]

↑ emittance

- Tune: \( \nu = \frac{\psi(L)}{2\pi}. \)

\[ \nu \in \mathbb{Q} \implies \text{fixed pts.} \]
My Goals

- Understand particle trapping theoretically.
- Control trapping fractions precisely.
- Minimize the emittance blow-up.

My strategy:

1. Examine a simplified Hamiltonian.
   \[ \mathcal{H} = \frac{1}{2} [p - \delta(t)]^2 - [1 + \beta(t)] \cos q. \]

2. Move on to a perturbative approximation of a realistic Hamiltonian.
   \[ \mathcal{H} = \alpha(t) \ p + \beta(t) \ p^2 + \gamma(t) \ p^2 \cos(4q). \]
Simple Adiabatic Trapping Fraction

- We examined $H(t)$ in which $\delta(t)$ governs travel and $\beta(t)$ governs growth:

$$H = \frac{1}{2} [p - \delta(t)]^2 - [1 + \beta(t)] \cos q.$$  

- We fixed a rectangle of IC’s and set the island in the center with $\Sigma_{\text{island}} = 0.\ldots$

  **Travel**: increasing $\delta(t)$ linearly.

  **Growth**: increasing $\beta(t)$ quadratically to $\beta(T)$. 

![Graph showing Island Trapping Fraction over $\beta(T)$ range from -1.0 to 1.5, with values ranging from 0.0 to 1.0.](image-url)
Simple $\Delta I$ & $\sigma(\Delta I)$ upon Separatrix Crossing

- $I$ not conserved upon separatrix crossing. If particle passes from above to below island...

\[
\Delta I \propto \frac{\dot{\delta}}{\omega} \ln(2 \sin \pi \xi) + \text{offset}
\]

\[
\sigma(\Delta I) \propto \sigma \left[ \frac{\dot{\delta}}{\omega} \ln(2 \sin \pi \xi) \right] \propto \left| \frac{\dot{\delta}}{\omega} \right|
\]
Moving to

\[ \mathcal{H} = \alpha(t) p + \beta(t) p^2 + \gamma(t) p^2 \cos(4q) \]

We set IC's in a rectangle and let islands begin with zero size at bottom of rectangle.
Moving Forward

- Perturbative Hamiltonian:
  - Fully understand trapping fraction curves and examine $\sigma(\Delta I)$.

- Way Ahead . . . Real PS Map:
  - Optimize time-dependent parameters in real $\mathcal{H}(t)$ numerically.
  - Test predictions by making measurements in the PS.
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Questions?