A-dependence of $Z^0$ production at LHC

Jian-Wei Qiu
Brookhaven National Laboratory

Based on work done with Kang, ...

CERN Workshop on “pA@LHC”, June 4 - 8, 2012, Geneva, Switzerland
Outline of my talk

- Why $Z^0$ and its $p_T$ distribution?
- Two-scale cross sections and QCD resummation
- Collins-Soper-Sterman (CSS) formalism
- The $b$-space and role of nonperturbative large $b$-region
- $pA$ collision and nuclear $A$-dependence
- Rapidity dependence and shadowing
- Summary
**What do we know?**

- **Z⁰ in AA-collision:**

  - **Final-state:** No strong interaction
  - **Initial-state:** Not too small $x \sim 0.03$ (weak antishadowing in EPS09)

Very small centrality (or A) dependence
Transverse momentum distribution

**Z^0-PT distribution in pp collisions:**

- CMS
  
  \[ \text{L dt = 36 pb}^{-1} \text{ at } \sqrt{s} = 7 \text{ TeV} \]

- ATLAS
  
  \[ \text{ATLAS (b) } \int \text{L dt = 35-40 pb}^{-1} \]

- **POWHEG + CT10**

- **Data (e + \mu \text{ combined})**

- **ResBos**

- **FEWZ \( O(\alpha_s^2) \)**

- **PYTHIA**

- \(| \eta | < 2.4 \)

- \( p_T > 20 \text{ GeV} \)

- \( 66 \text{ GeV} < m_\perp < 116 \text{ GeV} \)

\( P_T \) as low as \([0, 2.5] \text{ GeV bin} \) (or about 1.25 GeV)
Cross section of two very different scales

- Interesting region: $p_T \ll M_Z \sim 91$ GeV

- Fixed order pQCD calculation is not stable!

- Large logarithmic contribution from gluon shower:

  
  \[ \propto \frac{1}{q_T^2} \rightarrow \infty \]

  
  \[ \mathcal{O}_s \ln^2 \left( \frac{M_Z^2}{q_T^2} \right)^n \]

  Resummation is necessary!
Early approach to resummation

- **LO Differential $Q_T$-distribution as $Q_T \to 0$**:
  \[
  \frac{d\sigma}{dy dQ_T^2} \approx \left( \frac{d\sigma}{dy} \right)_{\text{Born}} \times 2C_F \left( \frac{\alpha_s}{\pi} \right) \frac{1}{Q_T^2} \ln \left( \frac{Q^2}{Q_T^2} \right) \Rightarrow \infty
  \]

\[
\int_0^{Q^2} \frac{d\sigma}{dy dQ_T^2} \approx \left( \frac{d\sigma}{dy} \right)_{\text{Born}} \quad + \mathcal{O}(\alpha_s)
\]

- **Integrated $Q_T$-distribution**:
  \[
  \int_0^{Q_I^2} \frac{d\sigma}{dy dp_T^2} \approx \left[ \int_0^{Q_T^2} \frac{d\sigma}{dy dp_T^2} \right] \left( \frac{d\sigma}{dy} \right)_{\text{Born}} \times \left[ 1 - \int_0^{Q_T^2} \frac{2C_F \alpha_s}{\pi} \frac{1}{p_T^2} \ln \left( \frac{Q^2}{p_T^2} \right) dp_T^2 \right]
  \]

\[
\approx \left( \frac{d\sigma}{dy} \right)_{\text{Born}} \times \exp \left[ -C_F \frac{\alpha_s}{\pi} \frac{1}{n^2} \ln \left( \frac{Q^2}{Q_T^2} \right) \right]
\]

**Effect of gluon emission**

**Assume this exponentiates**
Resummed $Q_T$ distribution

- Differentiate the integrated $Q_T$-distribution:

$$\frac{d\sigma}{dydQ_T^2} \approx \left(\frac{d\sigma}{dy}\right)_{\text{Born}} \times 2C_F \left( \frac{\alpha_s}{\pi} \right)^{1n\left(\frac{Q^2}{Q_T^2}\right)} \times \exp\left[-C_F \left( \frac{\alpha_s}{\pi} \right) 1n^2 \left(\frac{Q^2}{Q_T^2}\right)\right] \Rightarrow 0$$

as $Q_T \to 0$

- compare to the explicit LO calculation:

$$\frac{d\sigma}{dydQ_T^2}_{\text{LO}} \approx \left(\frac{d\sigma}{dy}\right)_{\text{Born}} \times 2C_F \left( \frac{\alpha_s}{\pi} \right)^{1n\left(\frac{Q^2}{Q_T^2}\right)} \Rightarrow \infty \quad \text{Q}_T\text{-spectrum (as }Q_T\to 0\text{) is completely changed!}$$

- We just resummed (exponentiated) an infinite series of soft gluon emissions – double logarithms

$$e^{-\alpha_s L^2} \approx 1 - \alpha_s L^2 + \frac{(\alpha_s L^2)^2}{2!} - \frac{(\alpha_s L^2)^3}{3!} + \ldots$$

$L \propto 1n\left(\frac{Q^2}{Q_T^2}\right)$

Soft gluon emission treated as uncorrelated
Still a wrong $Q_T$-distribution

- **Experimental fact:**
  \[ \frac{d\sigma}{dydQ_T^2} \Rightarrow \text{finite [neither } \infty \text{ nor } 0!] \text{ as } Q_T \to 0 \]

- **Double Leading Logarithmic Approximation (DLLA):**
  - Radiated gluons are both soft and collinear with strong ordering in their transverse momenta
  - Ignores the overall vector momentum conservation
  - Double logs ~ random work ~ zero probability to be $Q_T = 0$

DLLA over suppress small $Q_T$ region

Resummation of uncorrelated soft gluon emission leads to too strong suppression at $Q_T=0$
Importance of momentum conservation

- Vector momentum conservation:
  
  Particle can receive many finite $k_T$ kicks via soft gluon radiation, but, yet still has $Q_T=0$

  Need vector sum!

- Subleading logarithms are equally important at $Q_T=0$

- Solution:
  
  Impose 4-momentum conservation at each step of soft gluon resummation
Leading order $K_T$-factorized cross section:

\[ \frac{d\sigma_{AB}}{dQ^2 dQ_T^2} = \sum_f \int d\xi_a \int d\xi_b \int \frac{d^2k_A}{(2\pi)^6} \frac{d^2k_B}{(2\pi)^6} \frac{d^2k_{s,T}}{(2\pi)^6} \]

\[ \times P_{f/A}(\xi_a, k_A) P_{\bar{f}/B}(\xi_b, k_B) H_{\bar{f}f}(Q^2) S(k_{s,T}) \]

\[ \times \delta^2(Q_T - k_{A_T} - k_{B_T} - k_{s,T}) \]

\[ \delta^2(Q_T - \prod_i k_{i,T}) = \frac{1}{(2\pi)^2} \int d^2b \ e^{\frac{r}{ib \cdot Q_T}} \prod_i e^{\frac{r}{ib \cdot k_{i,T}}} \]

The $Q_T$-distribution is determined by the $b$-space function:

\[ b W_{AB}(b, Q) \]
Role of each term in CSS formalism

\[ \sigma_{\text{TOT}} = \sigma_{\text{RESUM}} + \sigma_{\text{PERT}} - \sigma_{\text{ASYM}} \]

- \( \sigma_{\text{RESUM}} \) for \( q_T \sim 0 \)
- \( \sigma_{\text{PERT}} \) for \( q_T \sim M_W \)

Graph showing the cross section \( d\sigma/dq_T^2 \) vs. transverse momentum \( q_T \).
b-space resummation

- b-space distribution:
  \[ W_{AB}(b, Q) = \sum_{ij} W_{ij}(b, Q) \hat{\sigma}_{ij}(Q) \]

- Collins-Soper equation:
  \[ \frac{\partial}{\partial \ln Q^2} \tilde{W}_{ij}(b, Q) = [K(b\mu, \alpha_s) + G(Q/\mu, \alpha_s)] \tilde{W}_{ij}(b, Q) \]

- Evolution kernels satisfy RG equation:
  \[ \frac{\partial}{\partial \ln \mu^2} K(b\mu, \alpha_s) = -\frac{1}{2} \gamma_K(\alpha_s(\mu)) \]
  \[ \frac{\partial}{\partial \ln \mu^2} G(Q/\mu, \alpha_s) = \frac{1}{2} \gamma_K(\alpha_s(\mu)) \]

- Solution - resummation:
  \[ W_{ij}(b, Q) = W_{ij}(b, 1/b) e^{-S_{ij}(b, Q)} \]

  Boundary condition – perturbative if b is small!

  Sudakov form factor
  All large logs
Boundary condition – collinear factorization:

\[ W_{ij}(b, Q) = \sum_{a,b} \sigma_{ij \rightarrow Z} \left[ \phi_{a/A} \otimes C_{a\rightarrow i} \right] \otimes \left[ \phi_{b/B} \otimes C_{b\rightarrow j} \right] \]

Perturbative solution:

\[ W_{AB}^{\text{pert}}(b, Q) = \sum_{a,b,i,j} \sigma_{ij \rightarrow Z} \left[ \phi_{a/A} \otimes C_{a\rightarrow i} \right] \otimes \left[ \phi_{b/B} \otimes C_{b\rightarrow j} \right] \times e^{-S_{ij}(b,Q)} \]

Extrapolation to large-b?

- Non-perturbative
- Predictive power?

\[ \sigma^{\text{Resum}} \propto \int_0^\infty db J_0(q_T b) b W(b, Q) \]
Phenomenology

- **Resummed cross section:**

\[
\frac{d\sigma_{AB \rightarrow Z}^{\text{resum}}}{dq_T^2} \propto \int_0^\infty db \, J_0(q_T b) \, b \, W(b, Q)
\]

\[
W(b, Q) = \begin{cases} 
W^{\text{pert}}(b, Q) & b \leq b_{\text{max}} \\
W^{\text{pert}}(b_{\text{max}}, Q) F_{QZ}^{NP}(b, Q, b_{\text{max}}) & b > b_{\text{max}} 
\end{cases}
\]

- **Predictive power:**

- Larger Q
- Larger S \quad \rightarrow \quad Smaller \ b_{sp} \quad \rightarrow \quad Better prediction
Phenomenology – Tevatron

CDF Run-I
CTEQ-5
Qiu, Zhang 2001

CDF Run-II
CTEQ-6
Kang, Qiu 2012

No free fitting parameter!
Phenomenology – Higgs

Effectively no non-perturbative uncertainty!

$m_h = 125 \text{ GeV}$

$\sqrt{s} = 14 \text{ TeV}$

$y = 0$

Berger, Qiu, 2003
Effectively no non-perturbative uncertainty!
Nuclear dependence in pA

- Resummed partonic hard parts are insensitive to A:
  \[ W_{AB}^{\text{Pert}}(b, Q) = \sum_{a,b,i,j} \sigma_{i,j \rightarrow Z} \left[ \phi_{a/A} \otimes C_{a \rightarrow i} \right] \otimes \left[ \phi_{b/B} \otimes C_{b \rightarrow j} \right] \times e^{-S_{ij}(b,Q)} \]
  Only source of A-dependence is from nPDFs

- Large-b non-perturbative parts are sensitive to A:
  \[ W^{\text{Nonpert}}(b, Q) = W^{\text{Pert}}(b_{\text{max}}, Q) \ F_{QZ}^{NP}(b, Q, b_{\text{max}}) \]
  \[ F_{QZ}^{NP}(b, Q; b_{\text{max}}) = \exp \left\{ -\ln\left( \frac{Q^2 b_{\text{max}}^2}{c^2} \right) \left[ g_1 \left( (b^2)^\alpha - (b_{\text{max}}^2)^\alpha \right) \right] + g_2 \left( b^2 - b_{\text{max}}^2 \right) \right\} \]
  Leading twist
  Intrinsic power corrections
  Dynamical power corrections

- \( g_2 = \) Power correction to evolution of K and G:
  \[ g_2 \propto Q_s^2 \quad \Rightarrow \quad g_2 \Rightarrow g_2 A^{1/3} \]
Expectations – pA @ LHC

- Strong gluon shower $\langle p_T^2 \rangle >> Q_s^2$

- Dependence on nPDFs:
  \[
  \mu = \frac{C_2}{b} \approx \frac{1}{b} > b_{\text{max}}^{-1} > 1 - 2 \text{ GeV}
  \]

  Cover nPDFs for a much wider range of $Q$ (or $\mu$)!

  Strong shadowing effect for $Z^0$ – production!

  Small contribution from large-b
  Small A-dependent power corrections
“Cronin” effect for $Z^0$ production?

A$^{1/3}$-type power correction has “no” effect!
Transverse momentum broadening for $Z^0$

- **Broadening:**

\[
\Delta \langle q_T^2 \rangle_{pA} = \langle q_T^2 \rangle_{pA} - \langle q_T^2 \rangle_{pN}
\]

- **Much weak broadening:**

Kang, Qiu, 2008
$R_{pA}$ of $Z^0$ is determined by nPDFs:

Suppression in forward $y$ region even for $Z^0$
Summary

- Rapidity distribution, $d\sigma/dy$, clean probe of nPDFs at $M_Z$
- Gluon shower dominates the $p_T$ distribution of $Z^0$
- $R_{pA}$ vs $p_T$ could probe both shadowing and antishadowing at a scale $<< M_Z$
- $Z^0$ production in pA is an excellent benchmark:
  
  Theoretical calculation is insensitive to non-perturbative physics other than nPDFs!

Thank you!