CGC predictions for $R_{pA}$, multiplicities and KNO scaling in pA@LHC

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Energy and centrality dependence of multiplicities

i) KLN model

(updated predictions from arXiv:1111.3031)

\[
\frac{dN}{dy} = K \frac{4\pi N_c}{N_c^2 - 1} \int d^2r_t \int_0^\infty \frac{dp_t^2}{p_t^4} \alpha_s x_2 G_{A_2}(x_2, p_t^2) x_1 G_{A_1}(x_1, p_t^2)
\]

with

\[
x G(x, p_t^2) = \begin{cases} 
\frac{1}{\alpha_s(Q_s)} p_t^2 (1 - x)^4, & p_t < Q_s(x) \\
\frac{1}{\alpha_s(Q_s)} Q_s^2 (1 - x)^4, & p_t > Q_s(x)
\end{cases}
\]

\[
Q_s^2(y) = Q_0^2 N_{\text{part}} \left( x_0 \frac{\sqrt{s}}{Q_0} e^{\mp y} \right)^{\bar{\lambda}}
\]
probe $A^{1/3}$ and $\sqrt{s}$ dependence of $Q_s$
ii) $k_\perp$ factorization with rcBK UGDs

BK equation (incl. non-linear terms $\rightarrow$ saturation of scattering amplitude!)

$$\frac{\partial \mathcal{N}(r, Y)}{\partial Y} = \int d^2r_1 \, K(r, r_1, r_2) \left[ \mathcal{N}(r_1, Y) + \mathcal{N}(r_2, Y) - \mathcal{N}(r, Y) - \mathcal{N}(r_1, Y) \mathcal{N}(r_2, Y) \right]$$

running-coupling kernel (Balitsky prescription)

$$K(r, r_1, r_2) = \frac{N_c \alpha_s(r^2)}{2\pi^2} \left[ \frac{1}{r_1^2} \left( \frac{\alpha_s(r_1^2)}{\alpha_s(r_2^2)} - 1 \right) + \frac{r^2}{r_1^2 r_2^2} + \frac{1}{r_2^2} \left( \frac{\alpha_s(r_2^2)}{\alpha_s(r_1^2)} - 1 \right) \right]$$

dipole scattering amplitude in adj. rep.

$$\mathcal{N}_A = 2 \mathcal{N}_F - \mathcal{N}_F^2$$

(generalized) unintegrated gluon distribution:

$$\varphi(k, Y; b, A) = \frac{C_F \, k^2}{\alpha_s(k)} \int \frac{d^2r}{(2\pi)^3} \, e^{-i k \cdot r} \mathcal{N}_A(r, Y; b, A)$$
\( k_\perp \)-factorization, multiplicity in A+B \( \rightarrow \) g+X

(generalized) unintegrated gluon distribution:

\[
\varphi(k, Y; b, A) = \frac{C_F k^2}{\alpha_s(k)} \int \frac{d^2 r}{(2\pi)^3} e^{-ik \cdot r} N_A(r, Y; b, A)
\]

Multiplicity: (Kharzeev, Levin, Nardi ansatz)

\[
\frac{dN^{A+B \rightarrow g}}{dy \, d^2 b} = K \frac{1}{2C_F} \int \frac{d^2 p_t}{p_t^2} \int^{p_t} d^2 k_t \alpha_s(Q) \varphi \left( \frac{|p_t + k_t|}{2}, x_1 \right) \varphi \left( \frac{|p_t - k_t|}{2}, x_2 \right)
\]

- finite at \( p_t \rightarrow 0 \) if UGD does not blow up
- \( x_{1,2} = (p_t/\sqrt{s}) \exp (\pm y) \); \( Y_{1,2} = \log(x_0/x_{1,2}) \)
  where \( x_0 = 0.01 \) is assumed onset of rcBK evol.
AA: centrality and energy dependence of multiplicities

\( N_{\text{had}} \sim N_{\text{glue}} \)

\[ 2 \frac{dN_{\text{ch}}}{d\eta} \sim \frac{1}{\alpha_s(k)} \]

in UGD

\[ \sim 1/\alpha_s(k) \]

Albacete & Dumitru: arXiv:1011.5161
if $dN/d\eta$ works out, we have a very economical description of multiplicities in terms of single scale $Q_s(A,\sqrt{s})$
IP-Sat model and an independent rcBK study:

Tribedy & Venugopalan: arXiv:1112.2445
CGC based approaches (constrained by RHIC, LHC-pp & reasonable model for A) predict similar $dN/d\eta$ around $\eta \sim 0$ for upcoming p+Pb at LHC!

$$\frac{dN_{ch}}{d\eta} \sim 17 \pm 2 \quad (\eta \sim 0, \text{min bias})$$

if ok, we have a very economical description of multiplicities in terms of single scale $Q_s(A, \sqrt{s})$!
1. Global fits to e+p data at small-x

2. Extract NP fit parameters

\[ \phi(x, k_t) \sim \frac{1}{k_t^{2\gamma}} \quad \gamma > 1 \]

<table>
<thead>
<tr>
<th>Set</th>
<th>( Q^2_{s0, proton} ) (GeV^2)</th>
<th>( \gamma )</th>
</tr>
</thead>
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<td>MV</td>
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<td>1</td>
</tr>
<tr>
<td>MV'</td>
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</tr>
<tr>
<td>MV'</td>
<td>0.157</td>
<td>1.101</td>
</tr>
</tbody>
</table>

4. Apply gained knowledge in the study of other systems (theory driven extrapolation)

LO kt-factorization:

\[ \frac{dN^g}{d\eta d^2 p_T} \sim K \alpha_s(Q^2) \phi(x_1, k_t) \otimes \phi(x_2, k_t - pt) \otimes FF(Q^2_f) \]
Minimum Bias $p+Pb$ @ 4400 GeV
● “Shadowing” depends on $k_T$

● Target nucleon fluct. can either cause suppr. or enhancement, depending on $k_T$
LO only; Altinoluk-Kovner term in the works
Multiplicity distributions in \textit{pp} collisions

$P(n)$: negative binomial distribution

\[ P(n) = \frac{\Gamma(k + n)}{\Gamma(k)\Gamma(n + 1)} \frac{\langle n \rangle^n k^k}{(\langle n \rangle + k)^{n+k}} \]
KNO scaling in high-energy pp data

\[ \bar{n} P(n) \equiv \psi(z) \text{ is universal} \]

(independent of energy); \( z \equiv n/\bar{n} \)
KNO scaling (even $p+Pb$ approx.; \textit{prediction})

for $A+B$:

\[ k_{AB} \sim k_{pp} \min(T_A, T_B) \]

\[
\frac{N_{\text{ch}}}{\langle N_{\text{ch}} \rangle} = 1 \quad \text{for} \quad z = 0
\]

\[
\frac{N_{\text{ch}}}{\langle N_{\text{ch}} \rangle} = 0.1 \quad \text{for} \quad z = 3
\]

$\psi(z)$ = $\langle N_{\text{ch}} \rangle P(N_{\text{ch}})$

\[
z = \frac{N_{\text{ch}}(|\eta|<0.5)}{\langle N_{\text{ch}} \rangle}
\]

Dumitru + Nara, arXiv:1201.6382

MC-rcBK, $p+Pb$ \(\sqrt{s}=4.4\) TeV
MC-rcBK, $d+Au$ \(\sqrt{s}=0.2\) TeV
MC-rcBK, $p+p$ \(\sqrt{s}=0.9\) TeV
MC-rcBK, $p+p$ \(\sqrt{s}=2.36\) TeV
MC-rcBK, $p+p$ \(\sqrt{s}=7\) TeV

$pA \@ \text{LHC}$
$d+Au \@ \text{RHIC}$
$N_{\text{part}}$ fluctuations in p+Pb:

$p+Pb$, $\sqrt{s} = 4.4$ TeV
KNO scaling in p+Pb @ 4.4TeV on linear scale

Glauber fluct. ~ 20% correction
small because *particle production is coherent*!

\[
\frac{\psi_{pPb}(Z)}{\psi_{pp}(Z)}
\]
Eccentricities $\epsilon_n$ in Au+Au

Dumitru + Nara, arXiv:1201.6382

Glauber fluc only

Glauber + NBD $k \sim \min(T_A, T_B)$
KNO scaling: \[ \bar{n} P(n) \equiv \psi(z) \] is universal (independent of energy); \( z \equiv n/\bar{n} \)

Note that if \( k \ll \bar{n} \), NBD can be written as

\[ \bar{n} P(n) \, dz \sim z^{k-1} e^{-kz} \, dz \quad , \quad z \equiv n/\bar{n} \]

So, if \( k \approx \text{const} \), this leads to KNO scaling!

fit to pp @ LHC: \( k / \bar{n} \sim 0.16 \) at 2360 GeV

\[ \text{but why is} \]

i) \( P(n) \) a NBD?

ii) \( k \ll \bar{n} \)?
NBD from MV model

for large nucleus, $A^{1/3} \to \infty$

$$S_{MV} = \int d^2 x_\perp \frac{1}{\mu^2} \rho^a \rho^a + \text{soft YM fields} + \text{coupling of soft} \leftrightarrow \text{hard}$$

$$\left\langle \frac{dN}{dy_1 \cdots dy_n} \right\rangle_{\text{conn.}} = \beta_n \left\langle \frac{dN}{dy_1} \right\rangle \cdots \left\langle \frac{dN}{dy_n} \right\rangle$$

$$\beta_n = (n - 1)! k^{1-n}$$

$$\bar{n} = \# \frac{N_c (N_c^2 - 1)}{\alpha_s} Q_s^2 \pi R^2$$

$$k = \# \frac{N_c^2 - 1}{2\pi} Q_s^2 \pi R^2 \sim T_A$$

Gelis, Lappi, McLerran:
arXiv:0905.3234
So, why KNO then?

\[ \bar{n} = \# \frac{N_c (N_c^2 - 1)}{\alpha_s} Q_s^2 \pi R^2 \]
\[ k = \# \frac{N_c^2 - 1}{2\pi} Q_s^2 \pi R^2 \]

Why is \( k = O(\alpha_s^0) \)?

\[ \frac{\bar{n}}{k} \sim \frac{N_c}{\alpha_s} \gg 1 \]
New interpretation for KNO at high energy:

KNO scaling emerges if
  i) Gaussian action
  ii) high occupation number

How about
  i) quantum evolution
  ii) corrections to Gaussian action?
Beyond MV action...

\[ S = \int d^2 x_\perp \left\{ \frac{1}{2\mu^2} \rho^a \rho^a + \frac{1}{\kappa_4} \left( \delta^{ab} \delta^{cd} + \delta^{ac} \delta^{bd} + \delta^{ad} \delta^{bc} \right) \rho^a \rho^b \rho^c \rho^d \right\} + \text{soft YM fields} + \text{coupling of soft} \leftrightarrow \text{hard} \]

- \( \mu^2 \sim g^2 A^{1/3} \); \( \kappa_4 \sim g^4 A \)
Recalculate width $k^{-1}$ of mult. distribution

\[
\left\langle \frac{dN_2}{dy_1 dy_2} \right\rangle_{\text{conn.}} = \frac{1}{k} \left\langle \frac{dN}{dy_1} \right\rangle \left\langle \frac{dN}{dy_2} \right\rangle
\]
compute $k^{-1}$ from 2-particle connected diagrams:

$$\frac{Q_s^2 S_\perp}{2\pi} \frac{1}{k} \approx \frac{1}{N_c^2 - 1} - 3\frac{N_c^2 + 1}{N_c^2 - 1} \beta + \cdots$$

MV $\rho^4$

$\beta > 0$ makes $k$ bigger, should be sufficiently small so as not to ruin KNO!

$$\beta \equiv \frac{C_F^2}{6\pi^3} \frac{g^8}{Q_s^2 \kappa_4} \left[ \int_{-\infty}^{\infty} dz^- \mu^4(z^-) \right]^2$$

$$\approx 0.01 \text{ A}^{-2/3}$$
can provide fundamental information about couplings in small-x action!
JIMWLK evolution appears to preserve Gaussianity of initial MV action!
(more to come)

Quadrupole evolution

$$Q \equiv \frac{1}{N_c} \langle \text{tr} \ V_x V_y^\dagger V_u V_w^\dagger \rangle$$

A.D., Jalilian-Marian, Lappi, Schenke, Venugopalan:
arXiv:1108.4764
JIMWLNK Hamiltonian:

\[ H = -\frac{1}{16\pi^3} \int_{uvw} \mathcal{M}_{uvw} \left( 1 + V_u^\dagger V_v - V_u^\dagger V_z - V_z^\dagger V_v \right)^{ab} \frac{\delta}{\delta \alpha_u^a} \frac{\delta}{\delta \alpha_v^b} \]

\[ \mathcal{M}_{uvw} = \frac{(u-v)^2}{(u-z)^2 (v-z)^2} \]

Gaussian evolution described by eff. Hamiltonian with only virt. parts:

\[ H = -\frac{1}{8\pi^2} \int_{uv} \log (u-v)^2 Q_s^2(Y) \left( 1 + V_u^\dagger V_v \right)^{ab} \frac{\delta}{\delta \alpha_u^a} \frac{\delta}{\delta \alpha_v^b} \]
Kinematics at \(y \approx 0\) @ LHC and \(y \approx 3-4\) @ RHIC is NOT the same

( elementary kinematics)

much higher gluon-\(p_T\) are probed at LHC, even if \(x\) is roughly similar, due to very different \(<z>\) in fragmentation!

Ex.: \(p_{T,had.} \approx 4\) GeV at \(y = 3.2\) RHIC probes regime \(\sim Q_{s,adj}\) or just above same \(p_{T,had.}\) at \(y = 0\) LHC is also sensitive to \(p_{T,\text{glue}} > Q_{s,adj}\)
Summary II

min bias pA is sensitive to large $b$, and Glauber fluctuations (push up $R_{pA}$)

cut on more central collisions?
Summary

- predicted $\langle dN_{ch}/d\eta \rangle = 17 \pm 2$
- multiplicity distributions in pp @ LHC exhibit KNO scaling ($\eta=0$, $n/\bar{n} \leq 3$)
- can be described by NBD with $k \ll \bar{n}$
- approx. KNO scaling predicted for p+Pb @ LHC (slight distortion of KNO due to Glauber flucs)
- higher-order eccentricities $\varepsilon_3$ etc. in HIC increase

- theoretical studies of fluctuations:
  - constrain magnitude of higher $\rho^n$ operators
  - evolution with energy to test validity of Gaussian approximation from JIMWLK
Backup Slides
fluctuations of valence partons in plane

1. Initial conditions for the evolution (x=0.01)

\[ N(R) = \sum_{i=1}^{A} \Theta \left( \sqrt{\frac{\sigma_0}{\pi}} - |R - r_i| \right) \rightarrow Q^2_{s0}(R) = N(R) \cdot Q^2_{s0,\text{nucl}} \]

\[ \varphi(x_0 = 0.01, k_t, R) \]

2. Solve local running coupling BK evolution at each transverse point

rcBK equation or KLN model

\[ \varphi(x, k_t, R) \]

3. Calculate gluon production at each transverse point according to kt-factorization

\[ N_{\text{part}, A}(\vec{b}) = \sum_{i=1 \cdots A} \Theta \left( P(\vec{b} - \vec{r}_i) - \nu_i \right). \]

\[ P(b) = 1 - \exp[-\sigma_g T_{pp}(b)], \quad T_{pp}(b) = \int d^2s \, T_p(s) \, T_p(s - b) \]

\[ T_p(r) = \frac{1}{2\pi B} \exp[-r^2/(2B)] \quad \sigma_{NN}(\sqrt{s}) = \int d^2b \left( 1 - \exp[-\sigma_g T_{pp}(b)] \right) \]

INPUT: \( \varphi(x = 0.01, k_t) \) FOR A SINGLE NUCLEON:
need KNO flucs also for d+Au@RHIC
MC-KLN initial condition ($k_T$ factorization with Glauber fluct. only) leads to underestimate of $v_3$!
result for constant \[ k = \frac{1}{\pi} \Delta x_\perp^2 \Delta \eta \Lambda_{QCD}^2 \]
energy dependent $k \sim E_{CM}^{0.2}$

MC-rcBK, KNO scaling with $k \propto (\sqrt{s} / 900\,\text{GeV})^{0.2}$

$p+p \sqrt{s}=0.9\,\text{TeV}$
$p+p \sqrt{s}=2.36\,\text{TeV}$
$p+p \sqrt{s}=7\,\text{TeV}$
Non-Gaussian initial conditions for high-energy evolution

- Odderon operator \(-d^{abc} \rho^a \rho^b \rho^c / \kappa_3\) 

S. Jeon and R. Venugopalan, 

- Effective action for a system of \(k \sim N_c A^{1/3} \gg 1\) valence quarks in SU(3);
- Random walk of SU(3) color charges in the space of representations (m,n);

- Probability \(P(m, n) = e^{-S(m,n)}\)

\[ S(m, n; k) \sim \frac{N_c}{k} C_2(m, n) - \frac{1}{3} \left( \frac{N_c}{k} \right)^2 C_3(m, n) + \frac{1}{6} \left( \frac{N_c}{k} \right)^3 C_4(m, n) \]

\(C_2, C_3, C_4\) - Casimir operators for the representation (m,n)

- Define color charge per unit area \(\rho^a = g Q^a / \Delta^2 x\)

where \( |Q| = \sqrt{Q^a Q^a} \equiv \sqrt{C_2} \)