EFFECTS OF MOMENTUM DEPENDENT DRAG AND EQUATION OF STATE ON HEAVY FLAVOUR SUPPRESSION

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Questions to be addressed:

Part I:

- Why heavy quarks are used to extract the properties of QGP?
- How are they used as a probe in QGP?
- What are the different initial conditions considered here?
- What is the scope of the EOS to specify the initial conditions?
- What are the relevant transport coefficients of heavy quarks?
- What is nuclear modification factor (R_{AA}) and how is it determined in this formalism?
- What effect does the momentum dependence of transport coefficients have on R_{AA} of heavy flavours?

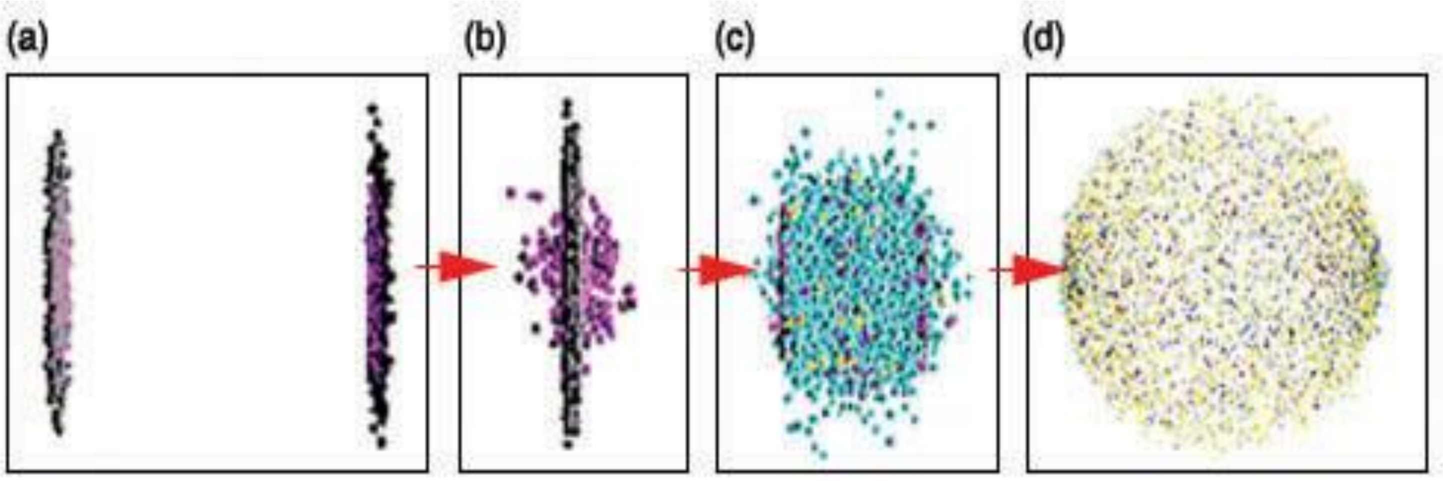
FORMATION OF QUARK GLUON PLASMA:

✓Two heavy ions colliding at ultra-relativistic energies produce a system where the degrees of freedom are deconfined quarks and gluons.

✓After some time it is expected to thermalise and to form Quark Gluon Plasma.

✓SPS, RHIC and LHC are such experimental facilities.

✓The physics of QGP is governed by QCD at finite temperature and density.



HEAVY QUARKS AS PROBE:

✓Mostly produced in early stage by hard collisions.
✓Witness the evolution of the system to be probed from the beginning.
✓Do not decide the bulk of the system. Criterion for a good probe.
✓Probability of finding a charm or anti charm is very little. Do not get 'killed' easily while propagating in QGP.
✓Masses ~ 1.2 GeV and 5 GeV >> Temperature of QGP. Production inside QGP is not copious.
✓Number conservation is there.
✓The probability of thermalisation within the life time of QGP is less compared to the light partons.

$$q+\bar{q}\rightarrow Q+\bar{Q} \qquad g+g\rightarrow Q+\bar{Q} \qquad q+\bar{q}\rightarrow Q++\bar{Q}+g$$

$$g+g\rightarrow Q+\bar{Q}+g \qquad q(\bar{q})+g\rightarrow Q+\bar{Q}+(\bar{q})q$$

FORMALISM:

- Heavy quarks like charm, beauty originated from early hard collisions and propagating in the medium of QGP act as Brownian particles.
- Distribution functions of both the expanding bath and the Brownian particle are evolving with time. QGP is in local thermal equilibrium whereas heavy quarks are out of equilibrium.
- Expansion of bath is controlled by Relativistic Hydrodynamic equations

$$\partial_\mu T^{\mu\nu} = 0$$

- Evolution of Brownian particle is governed by the Boltzmann Transport Equation

$$\left[\frac{\partial}{\partial t} + \frac{\mathbf{p}}{E}.\frac{\partial}{\partial \mathbf{x}} + \mathbf{F}.\frac{\partial}{\partial \mathbf{p}}\right] f(\mathbf{x},\mathbf{p},t) = \left[\frac{\partial f}{\partial t}\right]_{coll}$$

✓When external force, F=0 and QGP is uniform:

$$\frac{\partial f(\mathbf{p},t)}{\partial t} = \left[\frac{\partial f}{\partial t}\right]_{coll} \qquad f(\mathbf{p},t) = \frac{1}{V}\int d^3\mathbf{x}\, f(\mathbf{x},\mathbf{p},t)$$

⇩

Collision integral c(f). It comes under an integral sign. A non-linear integro-differential equation

w(**p**,**k**) → rate of collisions which change the momentum of probe from **p** to **p-k** .

$$C(f) = \int d^3k [w(\mathbf{p}+\mathbf{k},\mathbf{k}) f(\mathbf{p}+\mathbf{k}) - w(\mathbf{p},\mathbf{k}) f(\mathbf{p})]$$

➢Hard to solve. Therefore we adopt different approximations to make it simpler.

➢Landau approximation → Soft scattering → small momentum transfer → trajectory of the probe remains more or less straight.

➢Fokker-Planck Equation (FPE): One of the collision partners in equilibrium

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial p_i}\left[A_i(\mathbf{p}) f + \frac{\partial}{\partial p_j}\{B_{ij}(\mathbf{p}) f\}\right]$$

A_i and B_{ij} are related to the drag and diffusion coefficients respectively.

$$A_i(p) = \gamma(p) p_i \qquad B_{ij}(p) = D(p)\delta_{ij}$$

✓Drag coefficient is the measure of average momentum transfer of the probe weighted by the interaction.

✓ The dependence on strong coupling comes from the interaction part.

✓ The medium effect is considered through the relevant phase space.

DETERMINATION OF DRAG AND DIFFUSION COEFFICIENTS:

$$\langle F(p)\rangle = \frac{1}{2E_p}\int\frac{d^3\mathbf{q}}{(2\pi)^3 2E_q}\int\frac{d^3\mathbf{q}'}{(2\pi)^3 2E_{q'}}\int\frac{d^3\mathbf{p}'}{(2\pi)^3 2E_{p'}}\frac{1}{\gamma_C}\sum|M|^2(2\pi)^4\delta^4(p+q-p'-q')\hat{f}(\mathbf{q})F(p')$$

✓ Includes elastic scattering of charm or beauty with quark, anti-quark and gluon present in the plasma.

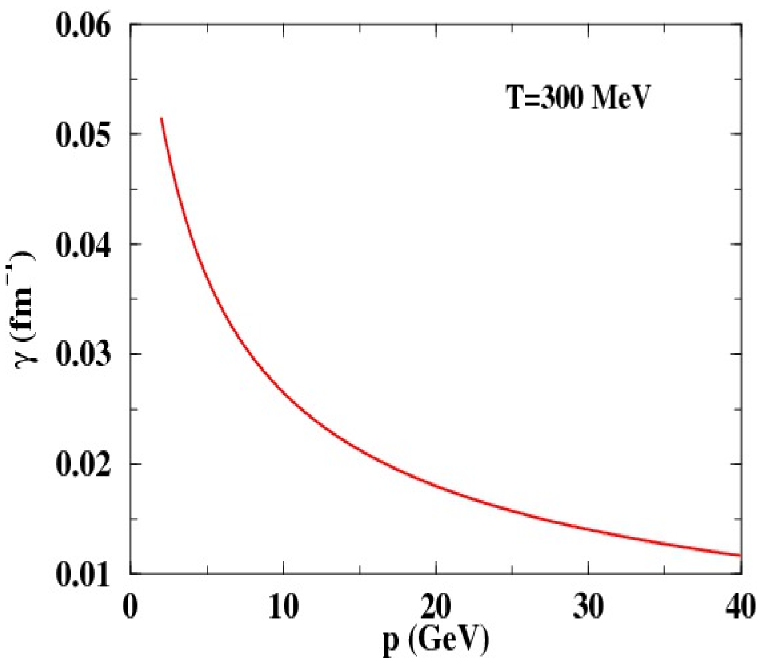
✓ Appropriate choice of the function $F(p')$ gives drag and diffusion coefficient.

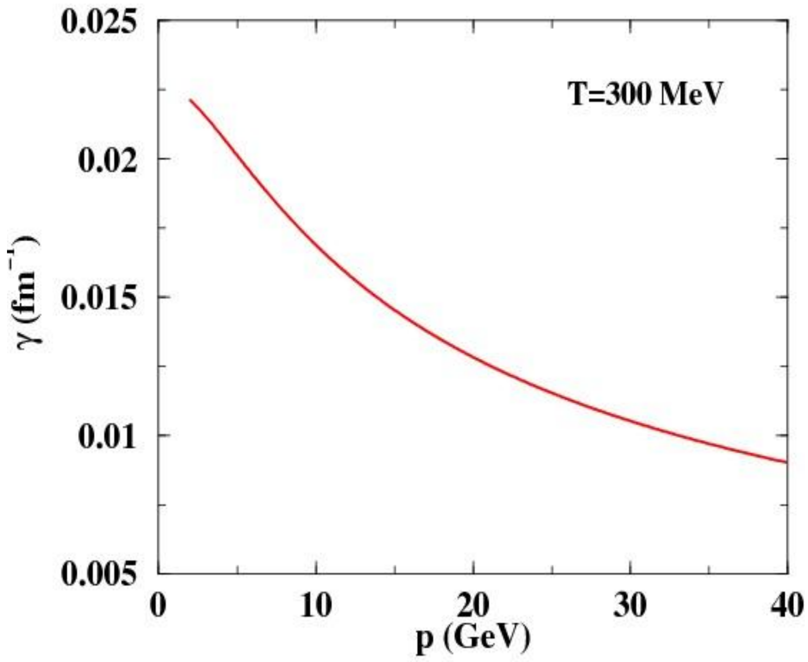
Ref: **B.Svetitsky, PRD 37, 2484(1988)**

$$F(p') = 1 - \frac{\mathbf{p}.\mathbf{p}'}{p^2}$$

$$F(p') = \frac{1}{4}[p'^2 - \frac{(\mathbf{p}.\mathbf{p}')^2}{p^2}]$$

✓ Collisional drag obtained for charm and beauty.



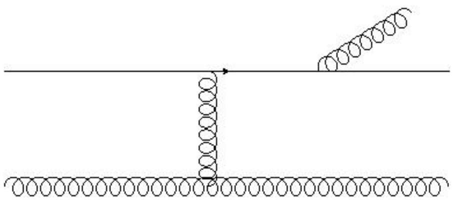


Charm **Bottom**

- In case of charm, at higher p, γ drops off to about 1/5th of its value at lower p and in case of bottom this factor is half. Had we considered the value of drag at lower p only, we would have overestimated it.
- Therefore, we have kept full momentum dependence of drag and diffusion coefficients.

RADIATIVE ENERGY LOSS OF HEAVY QUARK IN QGP:

The radiative processes under consideration are the gluon bremsstrahlung. A generic diagram of this process →



$$-[\frac{dE}{dx}]_{rad} = \gamma_{rad} p \quad \rightarrow \boxed{7}$$

➢ Evaluation of energy loss:

$$-[\frac{dE}{dx}]_{rad} = \Lambda \varepsilon$$

$\Lambda \longrightarrow$ elastic scattering rate $\longrightarrow$ Expression for Λ :

$$\Lambda = \frac{1}{2E_p}\int\frac{d^3\mathbf{q}}{(2\pi)^3 2E_q}\int\frac{d^3\mathbf{q}'}{(2\pi)^3 2E_{q'}}\int\frac{d^3\mathbf{p}'}{(2\pi)^3 2E_{p'}}\frac{1}{\gamma_Q}\sum|M|^2(2\pi)^4\delta^4(p+q-p'-q')\,\widehat{f}(\mathbf{q})$$

ε → average energy per collision and is given by:

$$\varepsilon = \int d\eta d^2 k_\perp \frac{dn_g}{d\eta d^2 k_\perp} k_0 \times \Theta(\tau_{sc} - \tau_F) \times \Theta(E - k_0) \times F^2$$

LPM effect

Bound on energy

Dead Cone effect

$$\frac{C_A \alpha_s}{\pi^2} \frac{q_\perp{}^2}{k_\perp{}^2[(\mathbf{q}_\perp - \mathbf{k}_\perp)^2 + m_D{}^2]}$$

→ emitted gluon multiplicity distribution in Gunion-Bertsch approximation

$k_5 = (k_0, k_\perp, k_3)$ → four momentum and η → rapidity of emitted gluon.

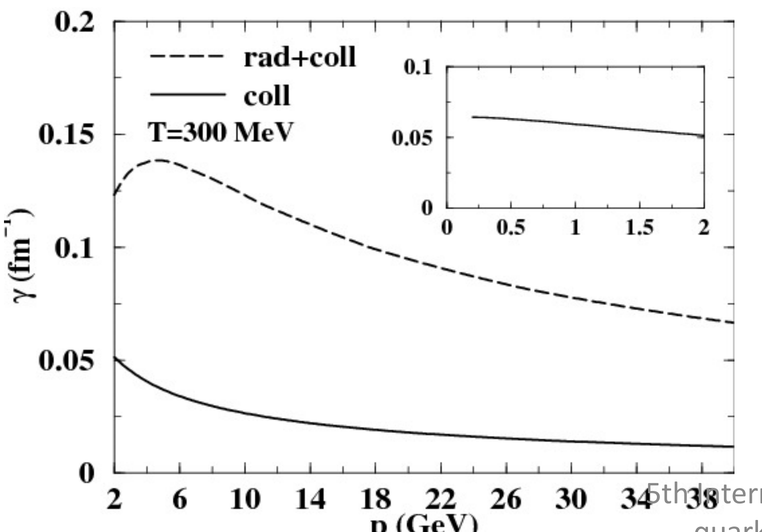
$q = (q_0, q_\perp, q_3)$ → four momentum of the propagator gluon.

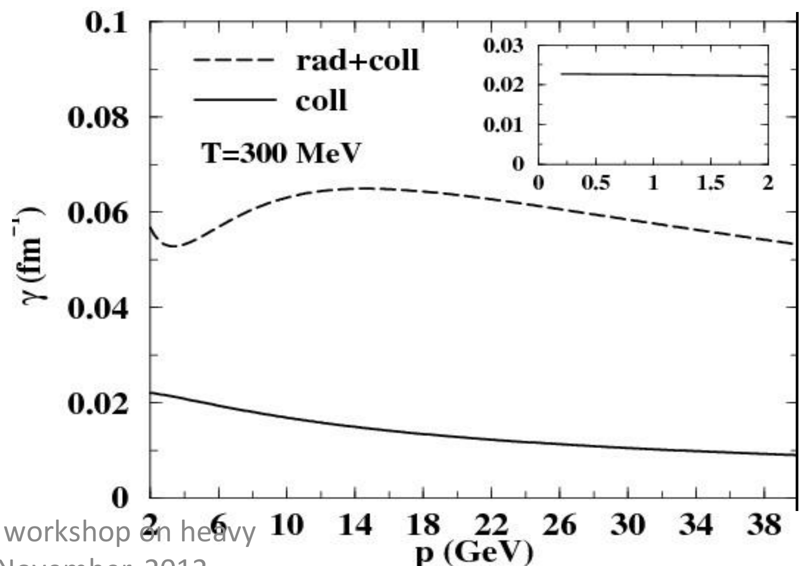
✓We have also included drag due to radiative processes and we add the collisional and radiative drag to obtain the effective drag
[S. Mazumder , T.Bhattacharyya, J. Alam and S. Das PRC 84, 044901(2011)]

$$\gamma = \gamma_{coll} + \gamma_{rad}$$

✓Full momentum and temperature dependence of drag coefficients are taken.

✓ Radiative drag is dominant over the collisional one, especially at large momentum domain.





INITIAL CONDITIONS:

- ✓ Evolutions of bath (Hydro) and the probe (Fokker Planck) are controlled by the differential equations.

- ✓ Mathematically, we need to provide initial conditions for both.

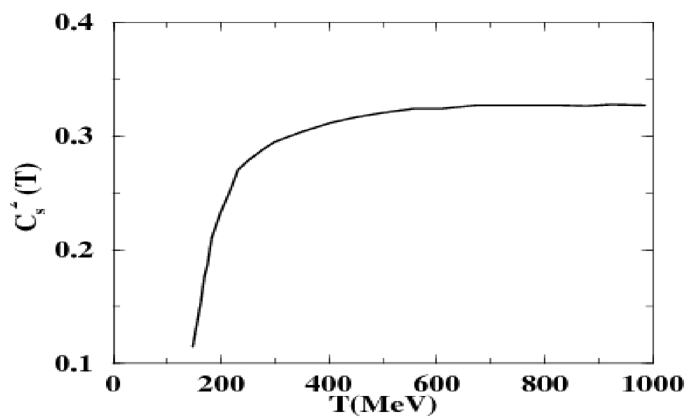
- ✓ Heavy quarks are generated from early hard collisions→ coupling is weak→ initial distribution can be calculated from 1st principle pQCD techniques→ provided from MNR code.

- ✓ In case of plasma it is not possible, as of now, from any 1st principle calculations to provide initial thermalisation time and initial temperature.

Initial Conditions and Equation of State:

$$T_i^3 \tau_i \sim \frac{dN}{dy} \qquad (4)$$

- Only one equation, but two unknowns to be determined.
- In order to minimize the uncertainties, we determine a bound on initial temperature keeping multiplicity fixed from experiment.
- Lattice calculation shows significant variation of c_s^2 with T.



- space time evolution of QGP is sensitive to the Equation of State ($P = c_s^2 \varepsilon$). Lower value of c_s^2 makes the rate of expansion of QGP slower.

BOUND ON INITIAL TEMPERATURE:

✓At $c_s^2 = 1/3$, the nuclear modification factor for electrons originated from the semileptonic decays of D and B mesons is calculated with a particular value of initial temperature and initial thermalisation time and contrasted with RHIC data.

$$R_{AA}^{D(B)\to e} = \frac{f_{final}^{D(B)\to e}(p_T, T_c)}{f_{in}^{D(B)\to e}(p_T, T_i)}$$

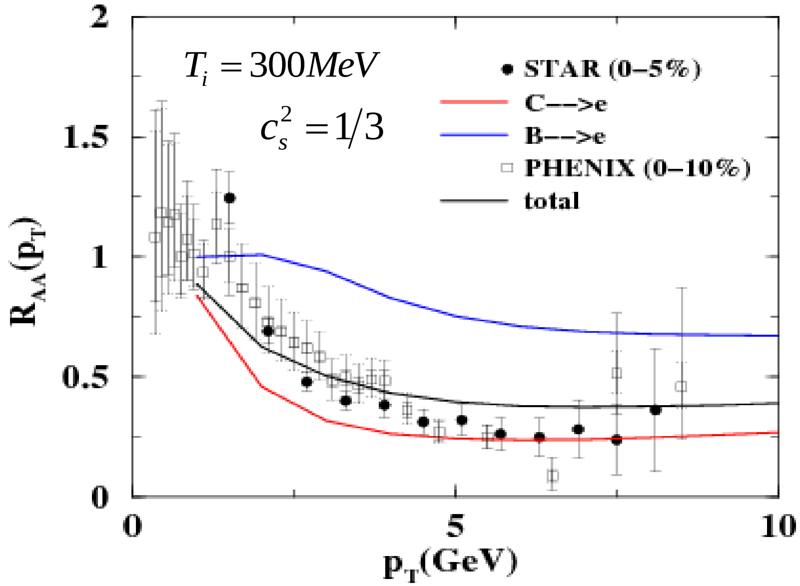
✓ Same procedure is repeated for $c_s^2 = 1/4$ and 1/5.

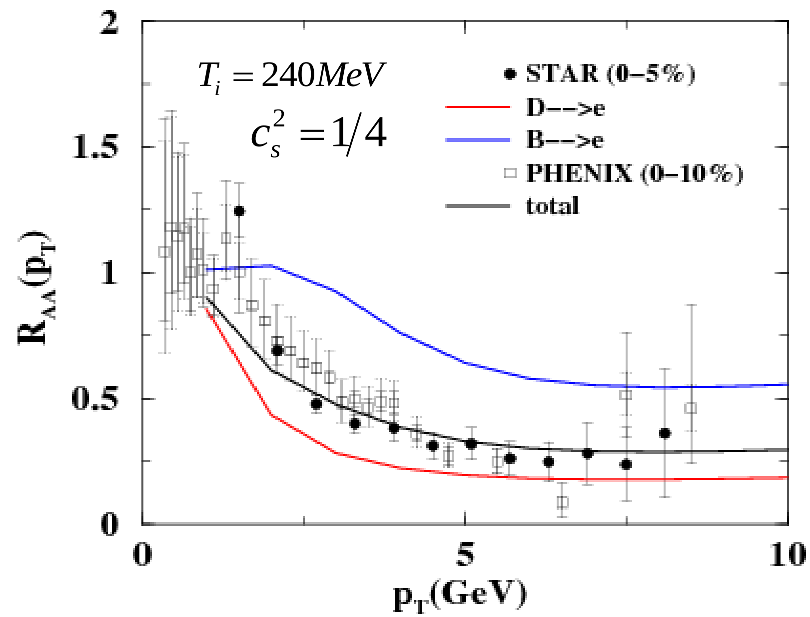
✓Throughout the process multiplicity is kept constant.

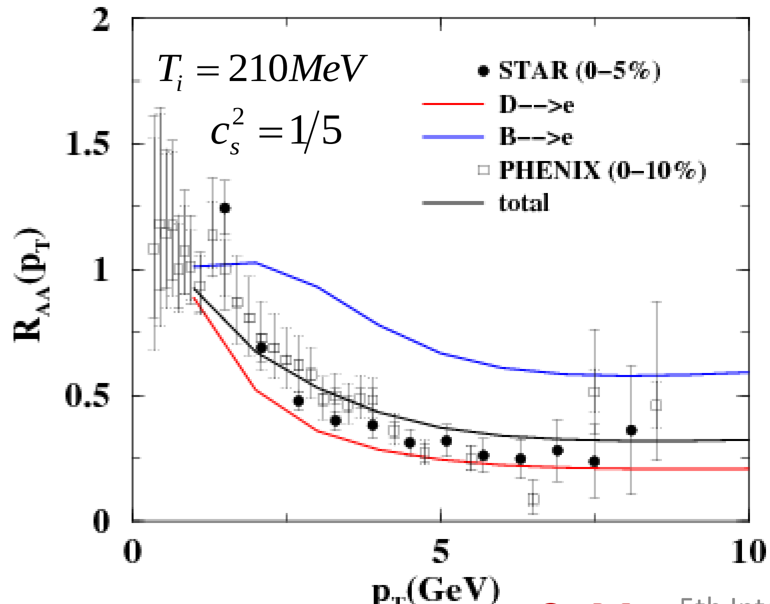
✓ A range of T_i and τ_i is found out:

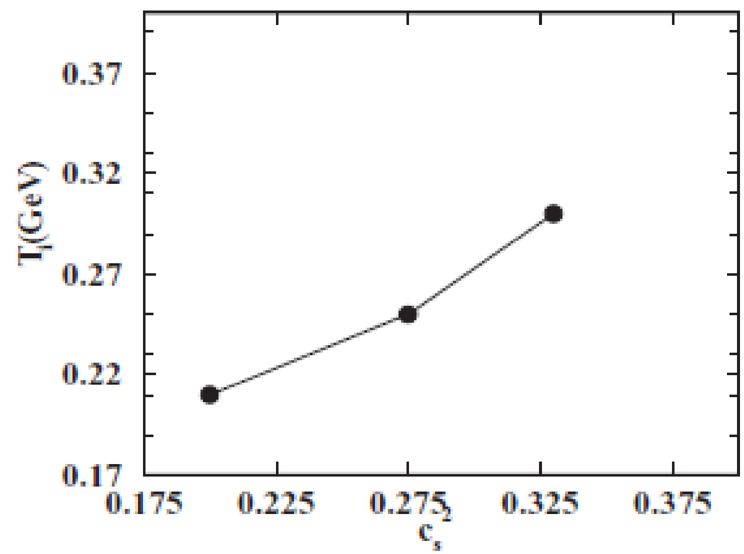
c_s^2	T_i (MeV)	τ_i (fm/c)
1/3	300	0.5
1/4	240	0.9
1/5	210	1.3

BOUND ON INITIAL TEMPERATURE FROM PRESENT ANALYSIS:









S. Mazumder and J. Alam PRC 85(2012) 044918

Conclusion from the first part:

❖ Two constraints are used to fix the initial temperature→ multiplicity and R_{AA} at RHIC.

❖ Value of T_i depends upon the nature of EOS, hard or soft.

❖ When the EOS is soft, i.e. $c_s^2 = 1/5$, HQ spends more time in QGP. Therefore, the value of the initial temperature is forced by the value of sound velocity squared.

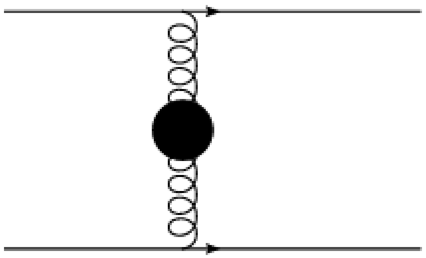
❖The T_i vs c_s^2 graph is a constant multiplicity graph.

❖ The lowest value of initial temperature (210 MeV) is well above the transition temperature (175 MeV) indicating that the system produced in RHIC might be in the deconfined phase.

Part II

❖Transport coefficients in Hard Thermal Loop (HTL) approximations and its bearing on the nuclear modification factor.

Hard Thermal Loop (HTL) approximations:



Problem of normal perturbation theory in non-zero T QCD:

✓ Inside a thermal medium, there are a few momentum scales.
✓ The scale ~ T is the characteristic of individual particle. Average distance between them ~ 1/T→ 'Hard' scale
✓ The scale ~ gT is due to the collective motion which takes place over distances 1/gT→ 'Soft' scale

✓ Due to the soft gluon exchange between the probe and bath particle, higher order diagrams in g can contribute to the lower order calculations:

✓ Consider a diagram ~ g^{α}

✓ While calculating any physical quantity:

$$\sim g^{\alpha} \times \frac{1}{p^2} = g^{\alpha-2} \qquad \text{(p~gT)}$$

✓ If we want to do a calculation up to order g^{α}, we have to find out all the terms contributing to this power due to the soft momentum .

✓ Another difficulty is that the fractional power of g may appear.

An example with $\lambda\phi^4$-theory:

The T-dependent part of the self-energy:

$$\Pi_{\beta} = 24\lambda \int \frac{d^3k}{(2\pi)^3} \frac{1}{\omega\left(e^{\beta\omega} - 1\right)}$$

After calculating we get :

$$\Pi_{\beta} = \lambda T^2 \left(1 - 3\left(\frac{\lambda}{\pi^2}\right)^{1/2} + \dots \right)$$

⇨ **Non-analytic function of coupling constant. Therefore, normal perturbation theory breaks down.**

✓ We have to device a new resummation technique to sum infinite set of diagrams.

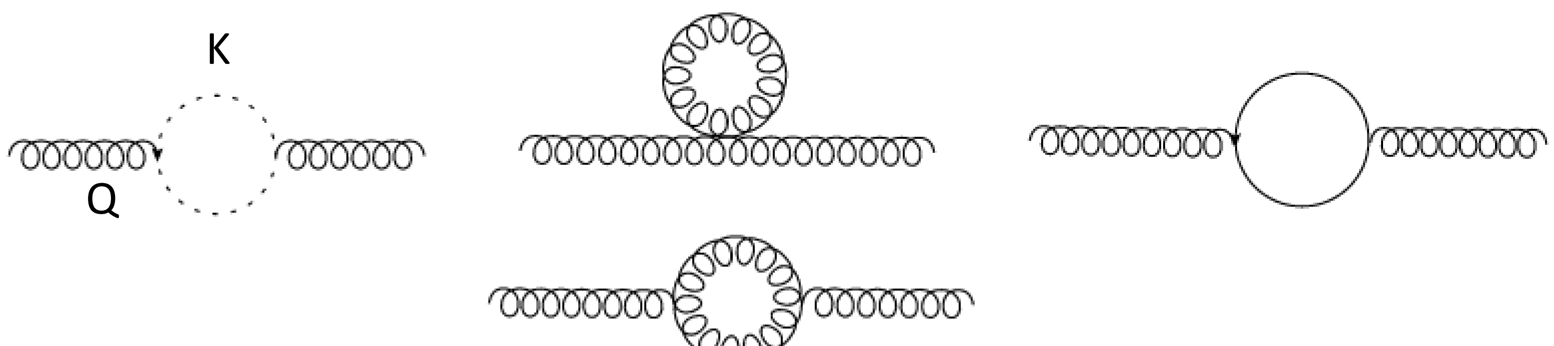
✓ Each of the diagrams is infra-red divergent, but the sum is finite.

✓ We have to have an effective propagator for the quasi-particles in the plasma.

✓ After resumming the Dyson-Schwinger series, we get the effective gluon propagator.

✓ To find out the resummed gluon propagator, the HTL approximated gluon self-energy goes as input.

✓ HTL approximated gluon self-energy is calculated up to one loop. Four diagrams are contributing:



✓ The HTL approximation means: K>>Q. With this the gluon self-energy is calculated taking one loop contributions only. We have neglected two or higher loop diagrams which can also contribute.

✓ Thus obtained effective propagator:

$$\Delta^{\mu\nu} = \frac{P_T^{\mu\nu}}{-Q^2+\Pi_T} + \frac{P_L^{\mu\nu}}{-Q^2+\Pi_L} + (\alpha-1)\frac{Q^\mu Q^\nu}{Q^2}$$

✓ α is the gauge-fixing parameter taken to be 1 → Feynman gauge

✓ Longitudinal tensor:

$$P_L^{\mu\nu} = -\frac{1}{Q^2 q^2}\left(\omega Q^\mu - Q^2 u^\mu\right)\left(\omega Q^\nu - Q^2 u^\nu\right)$$

✓ Transverse tensor:

$$P_T^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu + \frac{\left(Q^\mu - u^\mu (Q.u)\right)\left(Q^\nu - u^\nu (Q.u)\right)}{q^2}$$

$$Q_\mu P_L^{\mu\nu} = Q_\mu P_T^{\mu\nu} = P_{L\nu}^{\mu} P_T^{\nu\rho} = 0 \qquad P_i^{\mu\rho} P_{i\nu\rho} = P_{i\nu}^{\mu}, i = L/T$$

$$\Pi_L(Q) = \left(1-x^2\right)\pi_L(x), \Pi_T(Q) = \pi_T(x)$$

Transport coefficients in HTL approximations:

✓ We have calculated drag and diffusion coefficients with the effective gluon propagator.

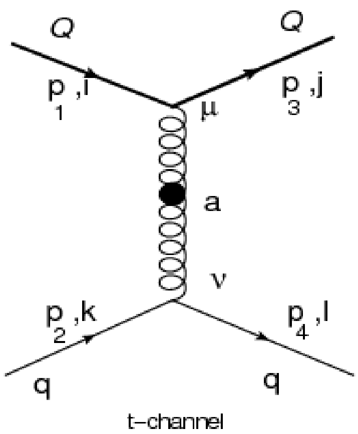
✓ All the invariant amplitude squared and the cross terms are calculated.

✓ No. of terms in each of them is from 100 to 200 in case of $Qg \rightarrow Qg$.

✓ In case of s-channel, u-channel and their interference terms in $Qg \rightarrow Qg$, bare propagator is used because the heavy quark used as propagator is not thermalised.

Invariant amplitude squared for Qq→Qq:

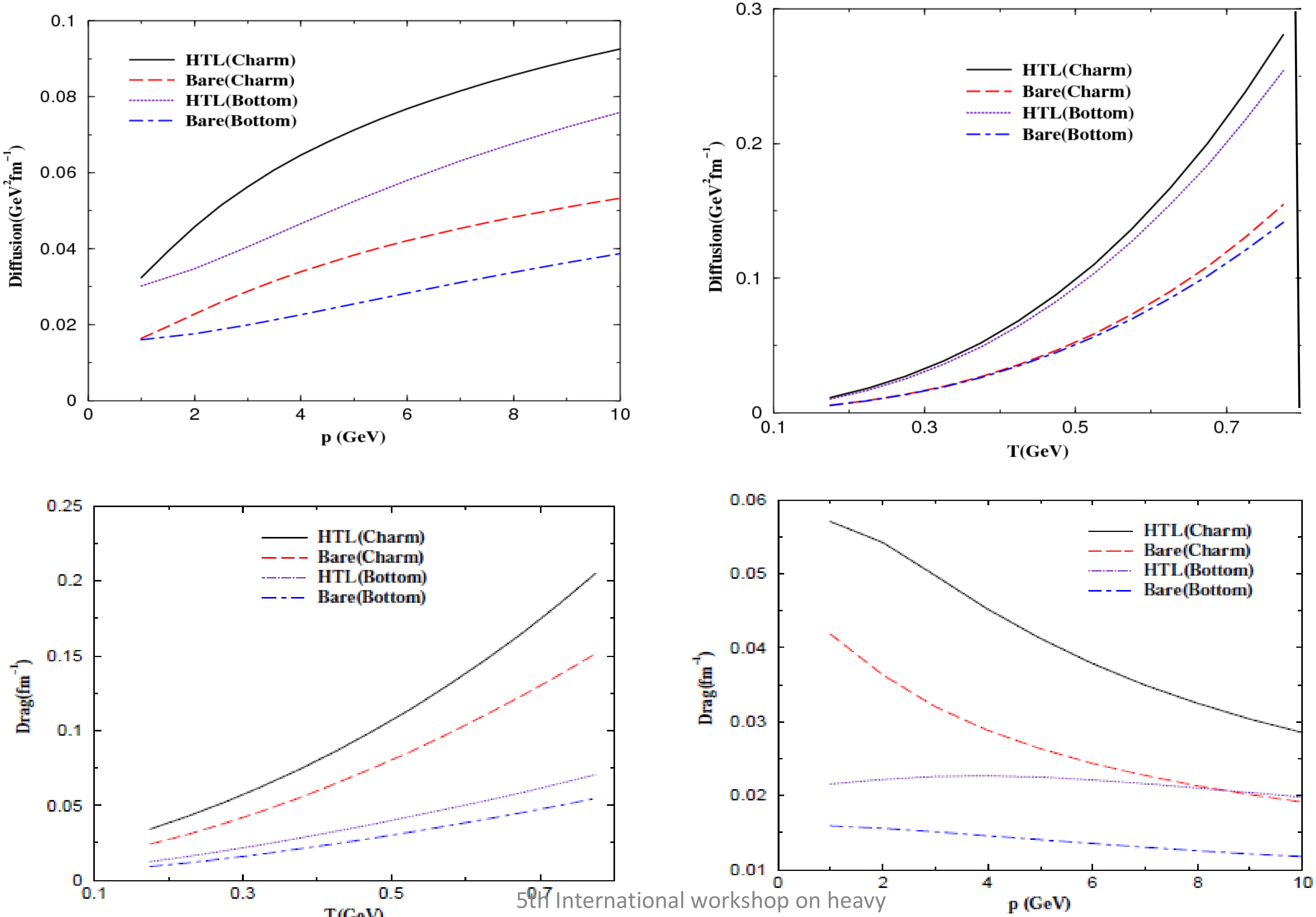
- ✓ **For $Qq \to Qq$, only t-channel diagram appears**



$$M_t = \bar{u}(p_3)(-ig\gamma^{\mu} t^{a}_{ji})u(p_1)[\Delta_{\mu\nu}]\bar{u}(p_4)(-ig\gamma^{\nu} t^{a}_{lk})u(p_2)$$

- ✓ **t-channel of Qg→Qg is more complicated due to the 3-gluon vertex.**

Plots of drag and diffusion from present analysis:



Conclusion from the second part:

- ✓ Effective transport coefficients are larger than bare ones.

- ✓ At T=400 MeV, HTL drag of charm is 33% more than the bare drag. For bottom it is 25%.

- ✓ This difference increases with the increase in T. It will have more observable effect in LHC regime.

- ✓ p-dependence is also affected: For a 5 GeV charm, this difference is 50%.

- ✓ The inclusion of imaginary part of self-energy introduces new processes like Landau damping etc, which may be the reason behind larger drag.

- ✓ Larger drag has significant importance in the equilibration of heavy quarks in QGP.

Back up slides

w(**p**,**k**) → rate of collisions which change the momentum of probe from **p** to **p-k** .

$$C(f) = \int d^3k[w(\mathbf{p}+\mathbf{k},\mathbf{k})f(\mathbf{p}+\mathbf{k}) - w(\mathbf{p},\mathbf{k})f(\mathbf{p})] \quad \rightarrow \boxed{2}$$

- Landau approximation → Soft scattering → small momentum transfer → trajectory of the probe remains more or less straight

- w(**p**,**k**) falls off rapidly with k. Expanding Eq.2 in powers of **k** :

$$w(\mathbf{p}+\mathbf{k},\mathbf{k})f(\mathbf{p}+\mathbf{k}) \approx w(\mathbf{p},\mathbf{k})f(\mathbf{p}) + k_i\frac{\partial}{\partial p_i}(wf) + \frac{1}{2}k_ik_j\frac{\partial^2}{\partial p_i \partial p_j}(wf)$$

The 2nd and 3rd term of the above expansion are of the same order of magnitude; the reason is that the quantity k_i in the 2nd term, whose sign is variable, involves a greater degree of cancellation than the quadratic 3rd term. The later terms in the expansion are all small compared to the first three.

With this replacement, the collision integral becomes →

$$C(f) \approx \int d^3\mathbf{k}[\mathbf{k}.\frac{\partial}{\partial \mathbf{p}} + \frac{1}{2}k_ik_j\frac{\partial^2}{\partial p_i \partial p_j}](wf)$$

$$\frac{\partial f}{\partial t} = C_1 \frac{\partial^2 f}{\partial p_x^{\ 2}} + C_2 \frac{\partial^2 f}{\partial p_y^{\ 2}} + C_3 \frac{\partial f}{\partial p_x} + C_4 \frac{\partial f}{\partial p_y} + C_5 f + C_6. \qquad \rightarrow \boxed{3}$$

Second order partial differential equation → numerically complicated to solve

$$C_1 = C_2 = D, \qquad C_3 = \gamma p_x + 2\frac{\partial D}{\partial p_T}\frac{p_x}{p_T}, \qquad C_4 = \gamma p_y + 2\frac{\partial D}{\partial p_T}\frac{p_y}{p_T},$$

$$C_5 = 2\gamma + \frac{\partial \gamma}{\partial p_T}\frac{p_x^{\ 2}}{p_T} + \frac{\partial \gamma}{\partial p_T}\frac{p_y^{\ 2}}{p_T}, \qquad C_6 = 0$$

Coefficients dependent on drag and diffusion coefficients and their derivatives with respect to momentum.

A simplified form can be achieved if we ignore momentum dependence of and γD

$$\frac{\partial f}{\partial t} = \gamma \frac{\partial}{\partial p}(pf) + D\frac{\partial^2 f}{\partial p^2}$$

Simple to solve → closed form analytical solution

To understand the basic physical interpretation of drag and diffusion coefficients, let us take the following 1-D Eq.

$$\frac{\partial f}{\partial t} = \gamma \frac{\partial}{\partial p}(pf) + D\frac{\partial^2 f}{\partial p^2} \quad \rightarrow 4$$

Solving Eq.4 with delta function initial condition, $f(p,t=0)=\delta(p-p_0)$ we get:

$$f(p,t) = [\frac{\gamma}{2\pi D}(1-e^{-2\gamma t})]^{-1/2} \times \exp[-\frac{\gamma}{2D}\frac{(p-p_0e^{-\gamma t})^2}{1-e^{-2\gamma t}}] \quad \rightarrow 5$$

Resembles to a Gaussian distribution: $g(x) = \frac{1}{(2\pi\sigma^2)^{1/2}}\exp[-\frac{(x-\mu)^2}{2\sigma^2}]$ $\rightarrow 6$

Comparing Eq.5 and 6, mean momentum: $< p >= p_0 e^{-\gamma t}$

Relaxation time ~ γ^{-1}

Variance, $\sigma^2 = \frac{D(1-e^{-2\gamma t})}{\gamma}$

➢Effect of diffusion: Putting $\gamma = 0$, Eq.4 becomes Einstein's diffusion equation having <p>= and p_0 .$\sigma^2 = 2Dt$

➢As t increases <p> remains same but variance increases resulting in the following figure(FIG-1):

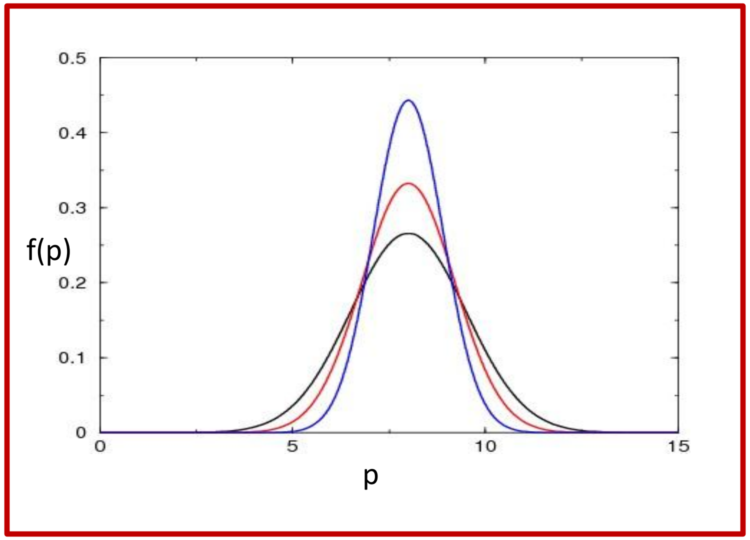


FIG-1

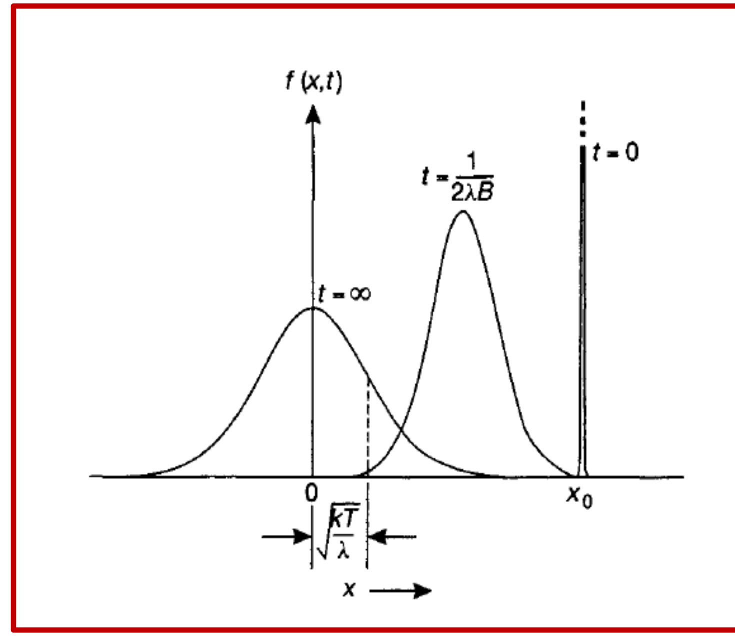


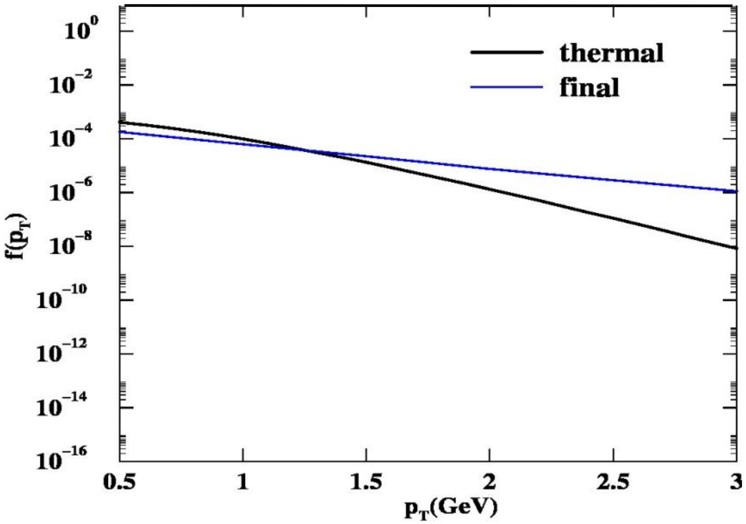
FIG-2

➢Effect of drag: If drag $\gamma \neq 0$, as t increases <p> will decrease and as well as variance will increase. The effective behaviour looks like FIG-2.

Energy loss is calculated:

$$-[\frac{dE}{dx}]_{rad} = \Lambda\varepsilon$$

- ✓ Λ is the scattering rate and ε is the energy per collision.
- ✓ Dead cone effect taken into account→ while propagating through the plasma, heavy quarks deviate lesser than the light quarks→ Consequently, gluon radiation from heavy quark is suppressed.
- ✓ LPM effect taken into consideration→ competition between two time scales of the system, mean scattering time, τ_{sc} and formation time of the radiated gluon, τ_F .
- ✓ Formation time is the minimal time needed to resolve the emitted gluon from the high energy parent quark.
- ✓ In order that the bremsstrahlung gluon is emitted we have to have $\tau_{sc} > \tau_F$



Final momentum distribution has not yet reached thermal distribution. To thermalise, the heavy quark need to lose more energy. As the final distribution will approach the thermal one, no. of heavy quarks with higher p_T will decrease and that with lower p_T will increase.

EFFECT OF DRAG AND DIFFUSION ON NUCLEAR SUPPRESSION

We will observe the effect of the transport coefficients of the medium on the nuclear modification factor → Physically, R_{AA} is the modification of the initial momentum distribution of the probe due to its propagation through the medium. We can measure it from the following relation :

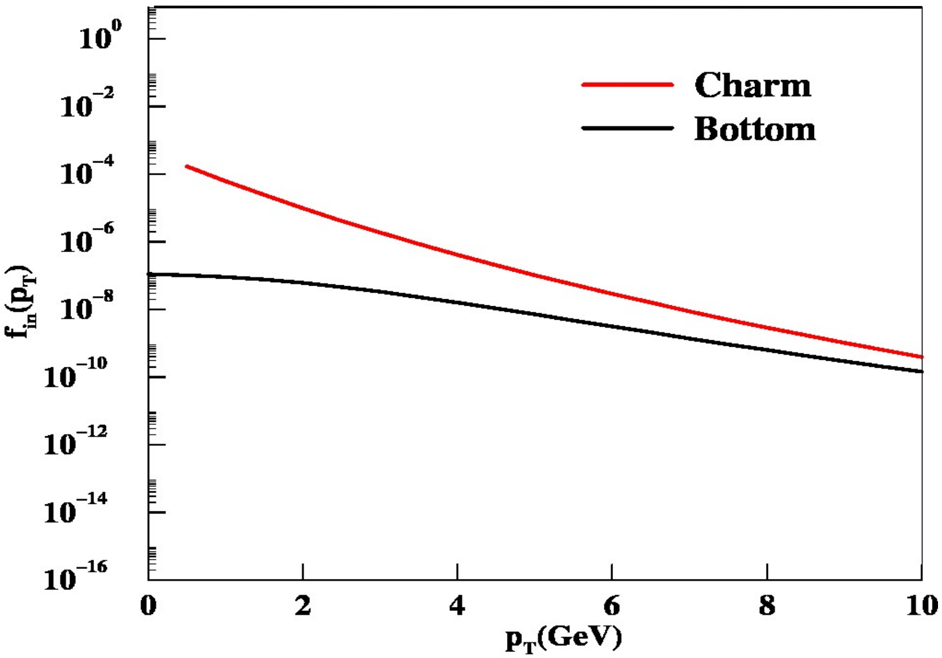
$$R_{AA} = \frac{f_{final}(p_T, T_C)}{f_{in}(p_T, T_i)}$$

Nuclear modification factor as a function of transverse momentum

f_{final} → Medium effect is encoded in it → measure of suppression caused by the medium

➢Less suppression of Bottom :

Smaller values of drag coefficients and harder initial transverse momentum distribution



The flatter initial distribution of bottom leading to higher average momentum than that of charm results in lesser suppression.

➢Results for RHIC and LHC :

As drag and diffusion coefficients have direct consequences on the observable , R_{AA}
We contrast our results with the experimental data available for RHIC and LHC.

FP Eq. solved for charm and bottom → we convolute this solution and the initial distribution with the Peterson fragmentation function of the heavy quarks → p_T distribution for B and D-meson obtained.

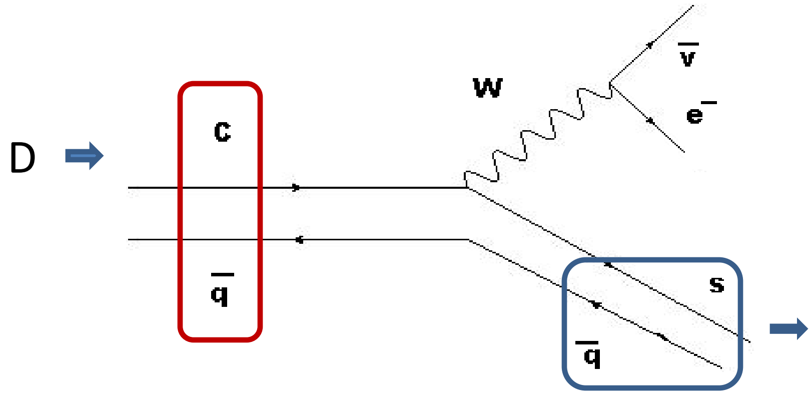
$$f(z) \propto \frac{1}{z[1 - \frac{1}{z} - \{\varepsilon_Q/(1-z)\}]^2}$$

Peterson fragmentation function → probability for a charm(bottom) to hadronise into D(B)-meson

Z is the fraction of momentum carried by the hadrons. for ε_Q charm is 0.05 and for bottom → $(M_c/M_b)^2 \varepsilon_c$ where $M_c (M_b)$ is the mass of charm (bottom) .

➢There are other fragmentation functions. But the final result is not that sensitive to the kind of fragmentation function used. The sensitivity of the result on f(z) is studied in the reference: S. K. Das et. al. PRC 80, 054916 (2009).

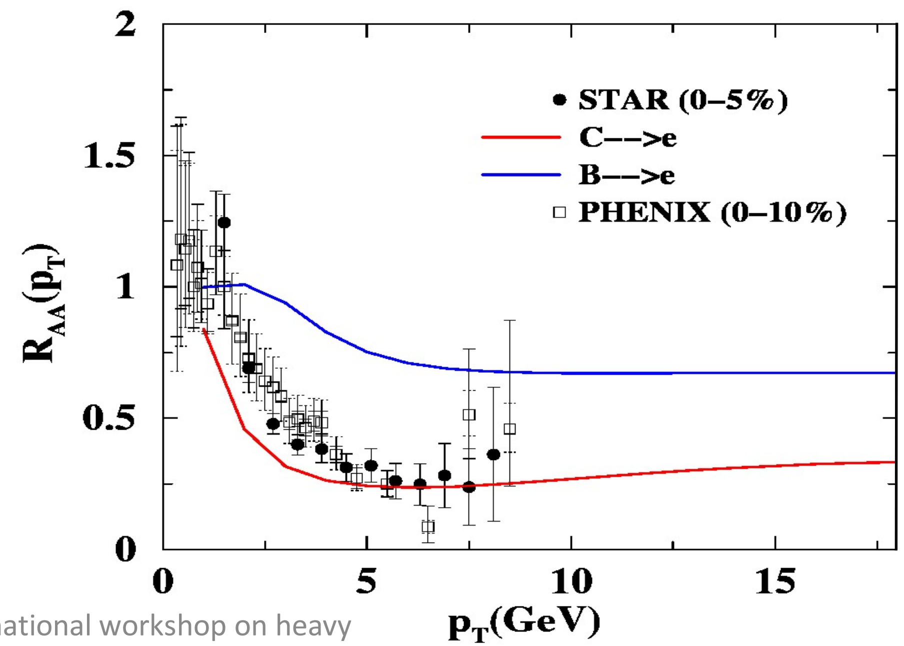
➢Single-electron spectra originated from the decays of heavy-flavoured mesons :



$$D \rightarrow K e^- \bar{\nu}$$

Single-electron spectra for initial distribution and the final solution has been obtained for D-meson and as well as B-meson.

$$R_{AA}^{D(B)\rightarrow e} = \frac{f^{D(B)\rightarrow e}(p_T, T_c)}{f^{D(B)\rightarrow e}(p_T, T_i)}$$



Longitudinal and transverse part of self-energy:

✓ In vacuum, photon and gluon are massless. Only, transverse degrees of freedom. Longitudinal d.o.f are unphysical.
✓ Inside a medium, they acquire effective mass. Therefore, they posses both d.o.f.
✓ Naively, medium at finite T breaks the Lorentz invariance due to the fact that **rest frame** of fluid already selects a frame.
✓ But, the formalism can be **manifestly Lorentz covariant** if we make use of **tensor decomposition**.
✓ In the rest frame of heat bath, the proper four velocity:

$$u^{\mu} = (1,0,0,0)$$

T is defined in the rest frame of fluid.
✓ Given u^{μ}, any four vector like Q^{μ} can be decomposed into parallel and perpendicular components to u^{μ} and vise versa.
✓ Tensors can also be decomposed in a similar fashion.