Universität Bielefeld

(some selected) Recent Developments in Lattice Studies for Quarkonia

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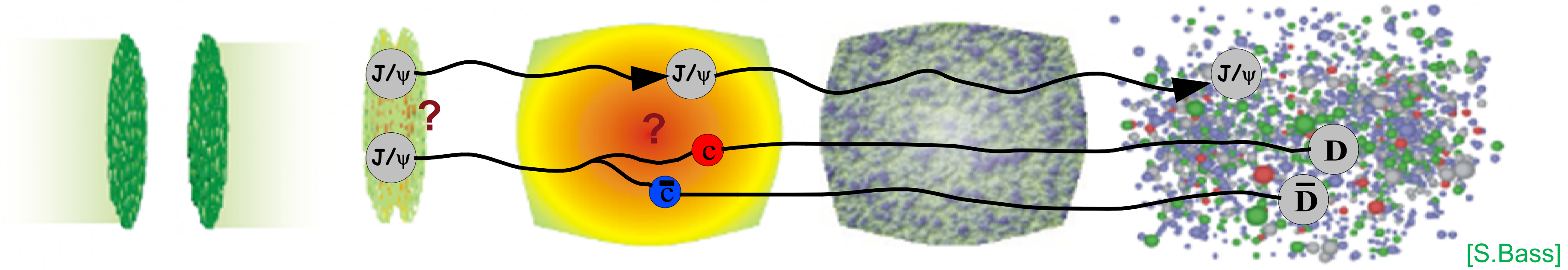
5th International Workshop on
Heavy Quark Production in Heavy-Ion Collisions
Utrecht
14.11.-17.11.2012

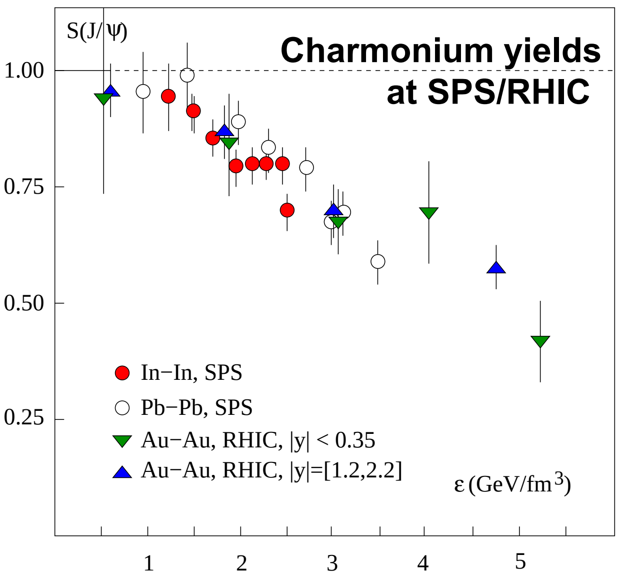
Motivation - Quarkonium in Heavy Ion Collisions



Charmonium+Bottmonium is produced (mainly) in the early stage of the collision

Depending on the Dissociation Temperature

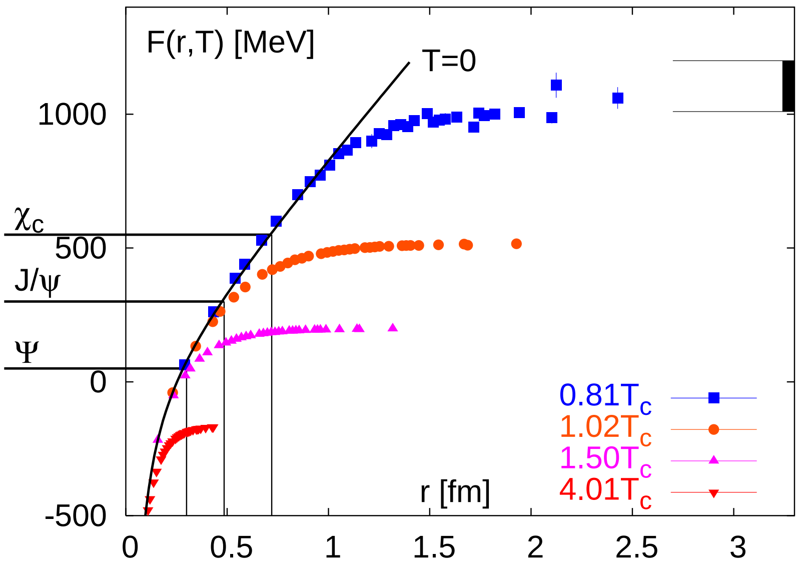
- **remain as bound states in the whole evolution**
- **release their constituents in the plasma**



First estimates on Dissociation Temperatures

from detailed knowledge of Heavy Quark Free Energies and Potential Models

[Kaczmarek, Zantow]



Motivation - Transport Coefficients

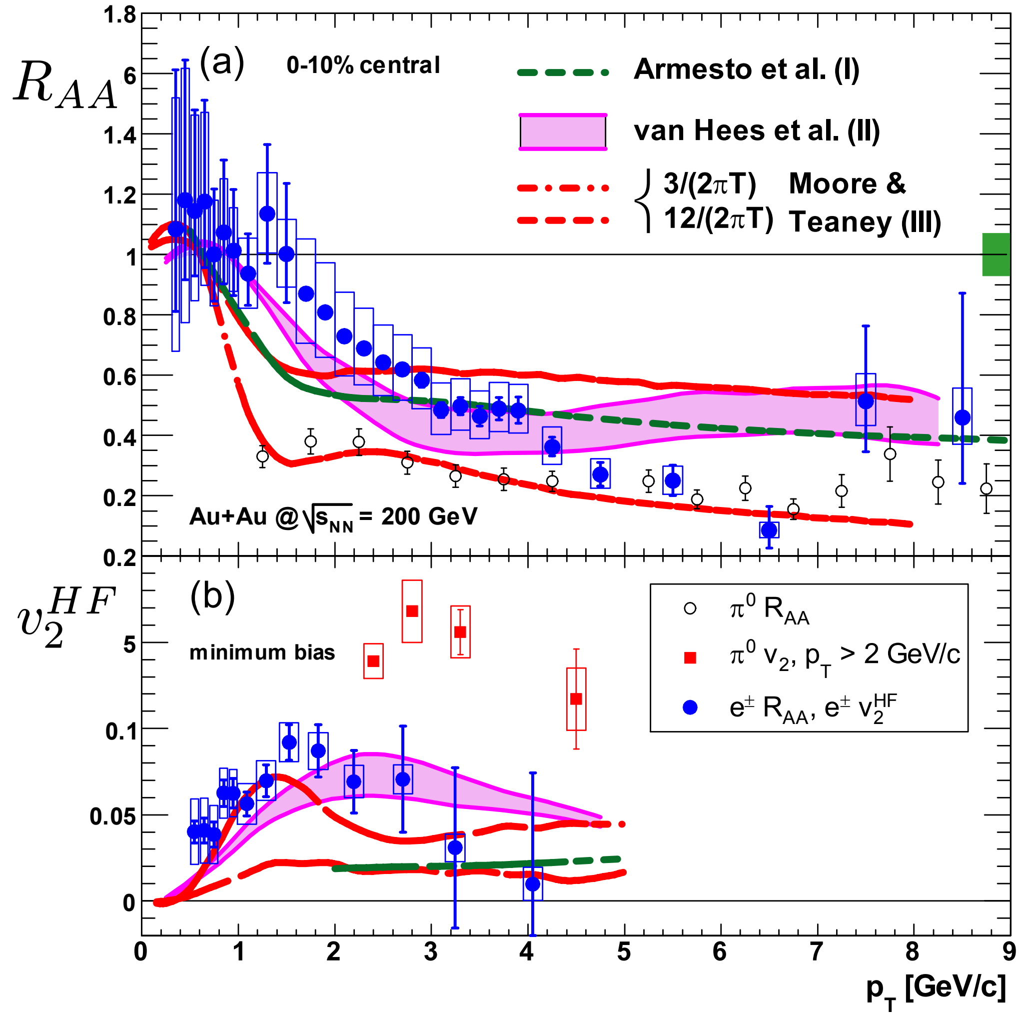
Transport Coefficients are important ingredients into **hydro models for the evolution of the system.**

Usually determined by matching to experiment (see right plot)

here: Heavy Quark Diffusion Constant D

also: Electrical conductivity σ (light quarks)

Need to be determined from QCD using first principle lattice calculations!



[PHENIX Collaboration, Adare et al.,arXiv:1005.1627 & PRL98(2007)172301]

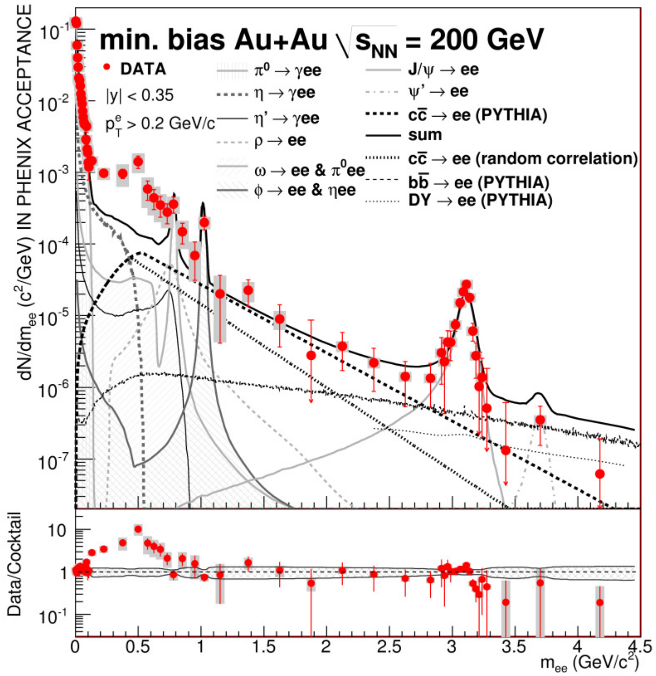
Motivation – PHENIX/STAR results for the low-mass dilepton rates

pp-data well understood by hadronic cocktail

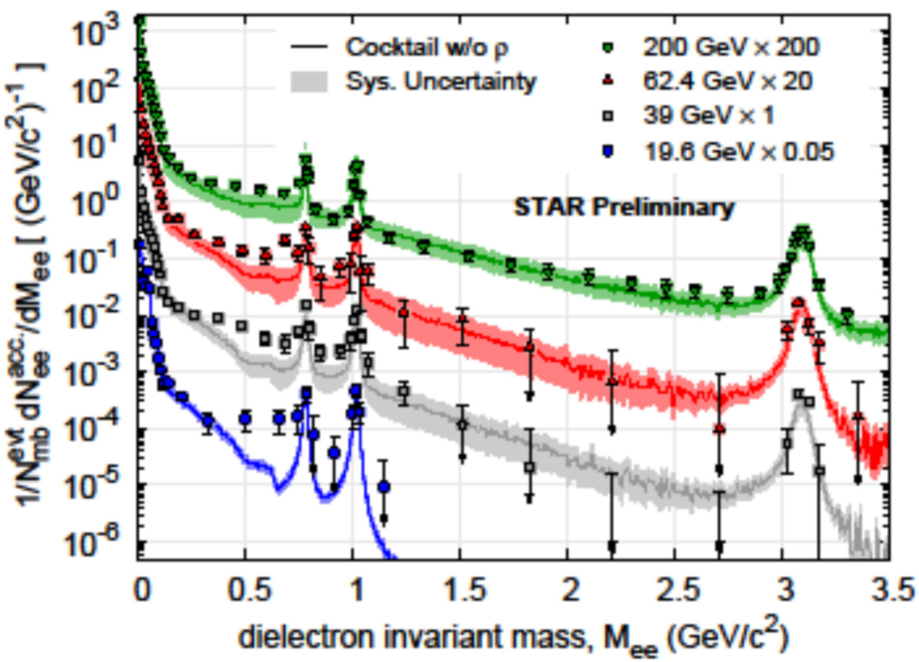
large enhancement in Au+Au between 150-750 MeV

indications for thermal effects!?

Need to understand the contribution from QGP$\rightarrow$ spectral functions from lattice QCD



[PHENIX PRC81, 034911 (2010)]



[STAR preliminary, arXiv:1210.5549]

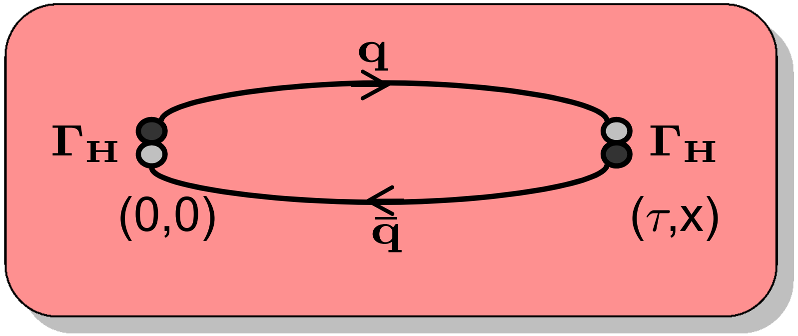
Dileptonrate directly related to vector spectral function:

$$\frac{\mathrm{d}W}{\mathrm{d}\omega \mathrm{d}^3p} = \frac{5\alpha^2}{54\pi^3}\frac{1}{\omega^2(e^{\omega/T}-1)}\,\rho_{\mathbf{V}}(\omega,\vec{\mathbf{p}},\mathbf{T})$$

Vector correlation functions at high temperature

$$G(\tau,\vec{p},T) = \int_0^\infty \frac{d\omega}{2\pi} \rho(\omega,\vec{p},T) K(\tau,\omega,T)$$

$$K(\tau,\omega,T) = \frac{\cosh\left(\omega(\tau - \frac{1}{2T})\right)}{\sinh\left(\frac{\omega}{2T}\right)}$$



Lattice observables:

$$G_{\mu\nu}(\tau,\vec{x}) = \langle J_\mu(\tau,\vec{x}) J_\nu^\dagger(0,\vec{0})\rangle$$

$$J_\mu(\tau,\vec{x}) = 2\kappa Z_V \bar{\psi}(\tau,\vec{x})\Gamma_\mu \psi(\tau,\vec{x})$$ ← local, non-conserved current, needs to be renormalized

$$G_{\mu\nu}(\tau,\vec{p}) = \sum_{\vec{x}} G_{\mu\nu}(\tau,\vec{x}) e^{i\vec{p}\vec{x}}$$ ← only $\vec{p}=0$ used here

How to extract spectral properties from correlation functions?

Spectral functions at high temperature

Free theory (massless case):

free non-interacting vector spectral function (infinite temperature):

$$
\begin{aligned}
\rho_{00}^{free}(\omega) &= 2\pi T^2 \omega \delta(\omega) \\
\rho_{ii}^{free}(\omega) &= 2\pi T^2 \omega \delta(\omega) + \frac{3}{2\pi}\, \omega^2 \tanh(\omega/4T)
\end{aligned}
$$

δ-functions exactly cancel in $\rho_V(\omega)$=-$\rho_{00}(\omega)$+$\rho_{ii}(\omega)$

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δ-functions exactly cancel in $\rho_V(\omega)$=-$\rho_{00}(\omega)$+$\rho_{ii}(\omega)$

With interactions (but without bound states):

while ρ_{00} is protected, the δ-funtion in ρ_{ii} gets smeared:

$\kappa = \frac{\alpha_s}{\pi}$ at leading order

Ansatz:

$$\rho_{00}(\omega) = 2\pi \chi_q \omega \delta(\omega)$$
$$\rho_{ii}(\omega) = 2\chi_q c_{BW} \frac{\omega \Gamma/2}{\omega^2 + (\Gamma/2)^2} + \frac{3}{2\pi}(1+\kappa)\,\omega^2 \tanh(\omega/4T)$$

Ansatz with 3-4 parameters: $(\chi_q), c_{BW}, \Gamma, \kappa$

works fine for light quarks at 1.5 Tc

["Thermal dilepton rate and electrical conductivity…", H.T.-Ding, OK et al., PRD83 (2011) 034504]

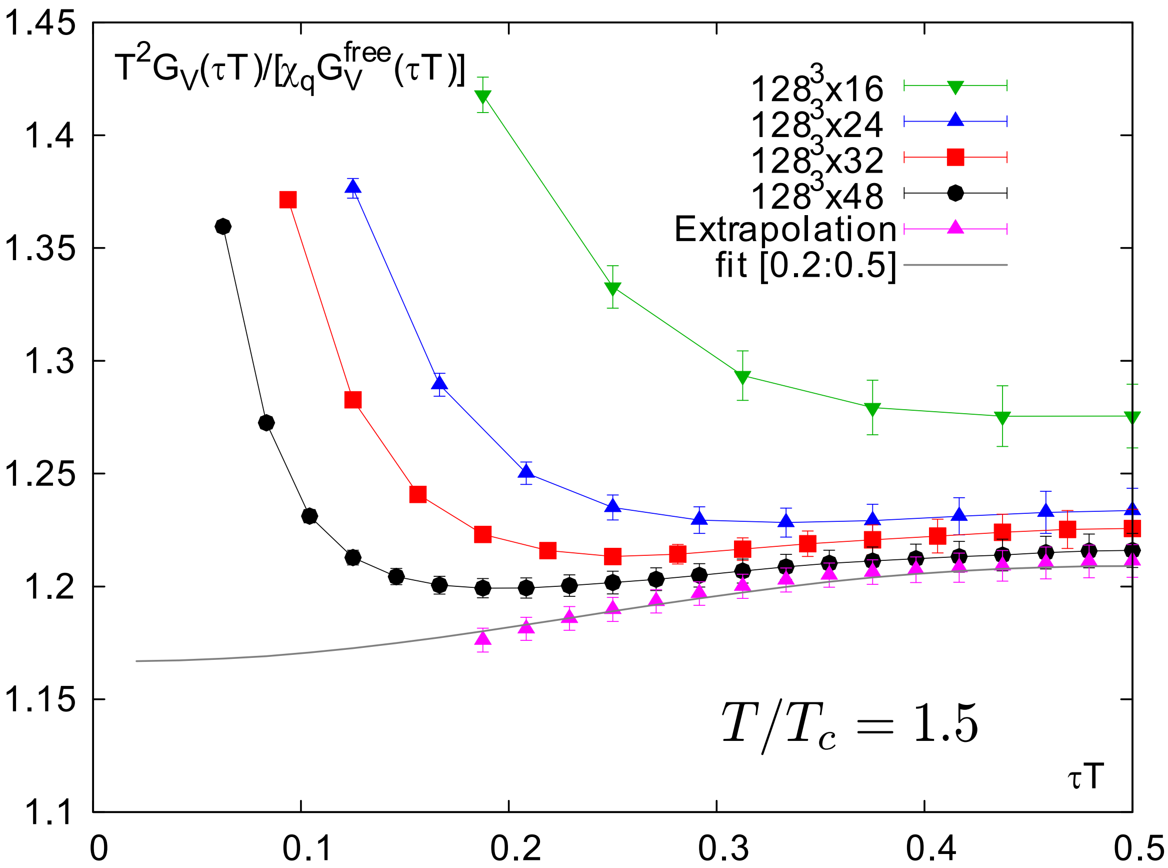
Light Quarks - Vector Correlation Function – continuum extrapolation

Use our Ansatz for the spectral function

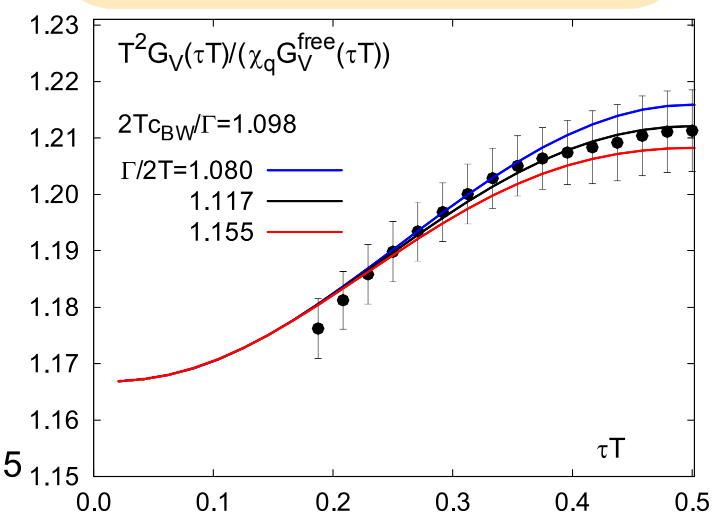
["Thermal dilepton rate and electrical conductivity…", H.T.-Ding, OK et al., PRD83 (2011) 034504]

$$\rho_{00}(\omega) = 2\pi\chi_q\omega\delta(\omega)$$
$$\rho_{ii}(\omega) = 2\chi_q c_{BW}\frac{\omega\Gamma/2}{\omega^2+(\Gamma/2)^2} + \frac{3}{2\pi}(1+\kappa)\,\omega^2\ \tanh(\omega/4T)$$

and fit to the continuum extrapolated values $\frac{\mathbf{G_V(\tau,T)}}{\mathbf{\bar{G}_{00}G_V^{free}(\tau,T)}}$ & $\mathbf{G_V^{(2)}}$



$$\frac{2C_{BW}\chi_q}{\Gamma} = 1.098(27)$$
$$\frac{\Gamma}{T} = 2.235(75)$$
$$\kappa = 0.0465(27)$$



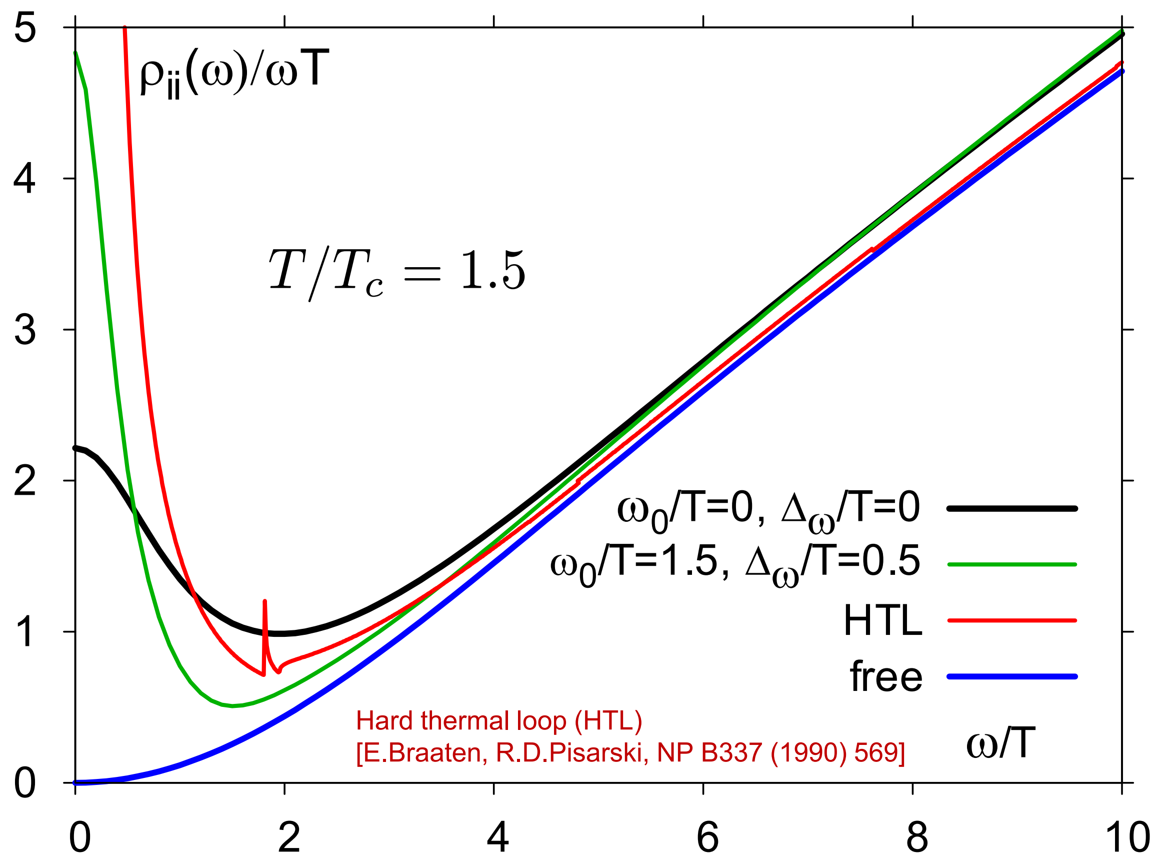
Light Quarks - Spectral function and electrical conductivity

Use our Ansatz for the spectral function

["Thermal dilepton rate and electrical conductivity…", H.T.-Ding, OK et al., PRD83 (2011) 034504]

$$\rho_{00}(\omega) = 2\pi\chi_q\omega\delta(\omega)$$

$$\rho_{ii}(\omega) = 2\chi_q c_{BW}\frac{\omega\Gamma/2}{\omega^2+(\Gamma/2)^2} + \frac{3}{2\pi}(1+\kappa)\,\omega^2\ \tanh(\omega/4T)\times\Theta(\omega_0,\Delta_\omega)$$



Analysis of the systematic errors

using truncation of the large ω contribution

$$\Theta(\omega_0,\Delta_\omega) = \left(1+e^{(\omega_0^2-\omega^2)/\omega\Delta_\omega}\right)^{-1}$$

$$\frac{\sigma}{T} = \frac{C_{em}}{6}\lim_{\omega\to 0}\frac{\rho_{ii}(\omega,\vec{p}=0,T)}{\omega T}$$

electrical conductivity

$$1/3 \le \frac{1}{C_{em}}\frac{\sigma}{T} \le 1$$

Light Quarks - Dilepton rates and electrical conductivity

["Thermal dilepton rate and electrical conductivity…", H.T.-Ding, OK et al., PRD83 (2011) 034504]

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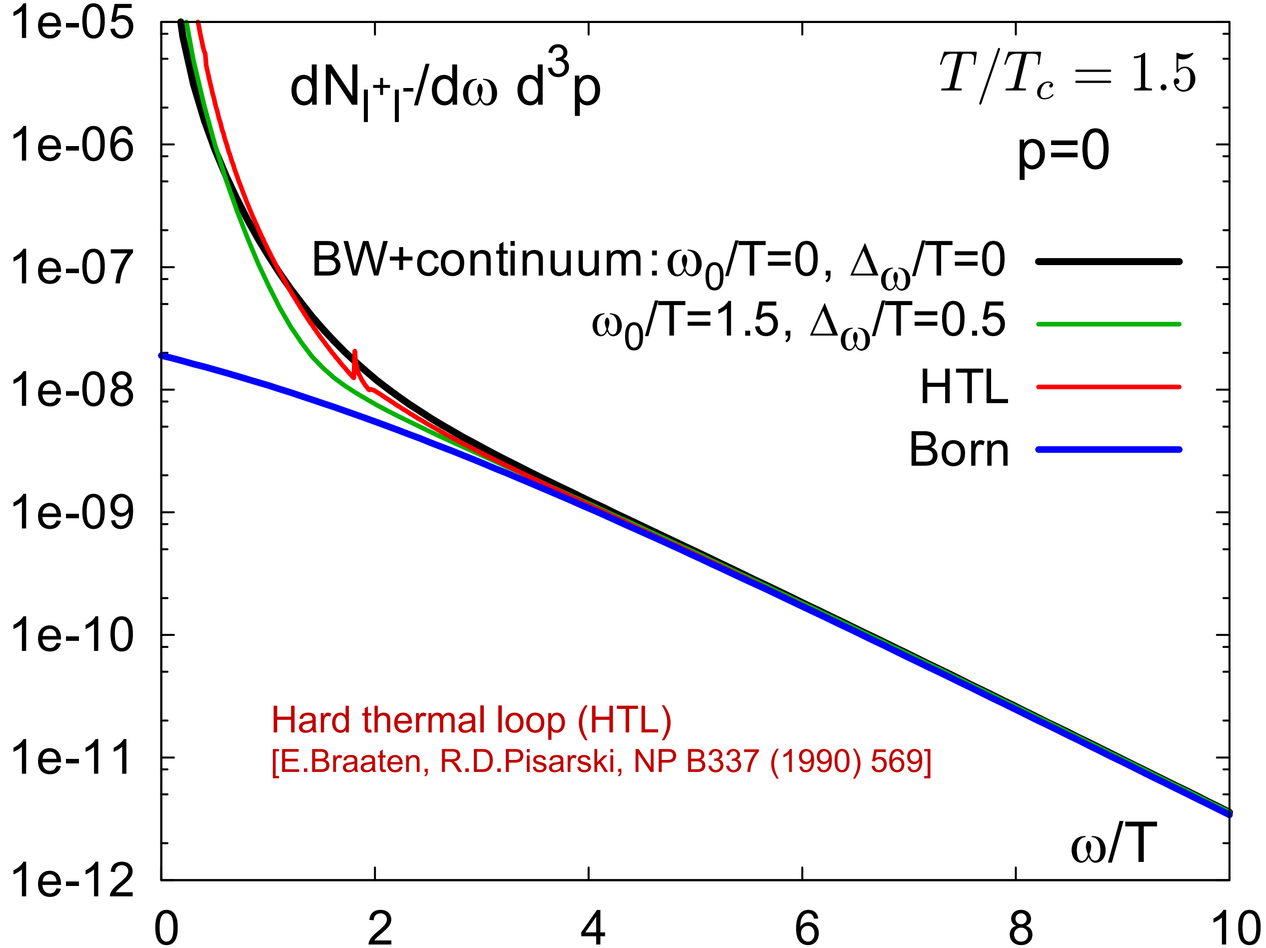

Preliminary results at 1.1 T_c

[M.Müller et al., preliminary 2012]

PRACE-Project:
Thermal Dilepton Rates and
Electrical Conductivity in the QGP
(JUGENE Bluegene/P in Jülich)

$1.09 T_c$ Lattices (all $N_\sigma / N_\tau = 3$)

$$1/T = a \cdot N_\tau$$

N_τ	N_σ	β	κ	$1/a$[GeV]	$\approx a$[fm]	#conf
32	96	7.192	0.13440	9.65	0.020	223
48	144	7.544	0.13383	14.21	0.015	226
64	192	7.793	0.13345	19.30	0.010	165

study of T-dependence of dilepton rates and electrical conductivity

fixed aspect ratio to allow continuum limit at finite momentum:

$$\frac{\vec{p}}{T} = 2\pi \vec{k} \frac{N_\tau}{N_\sigma}$$

Preliminary results at 1.1 T_c

[M.Müller et al., preliminary 2012]

Use our Ansatz for the spectral function

$$\rho_{00}(\omega) = 2\pi\chi_q\omega\delta(\omega)$$

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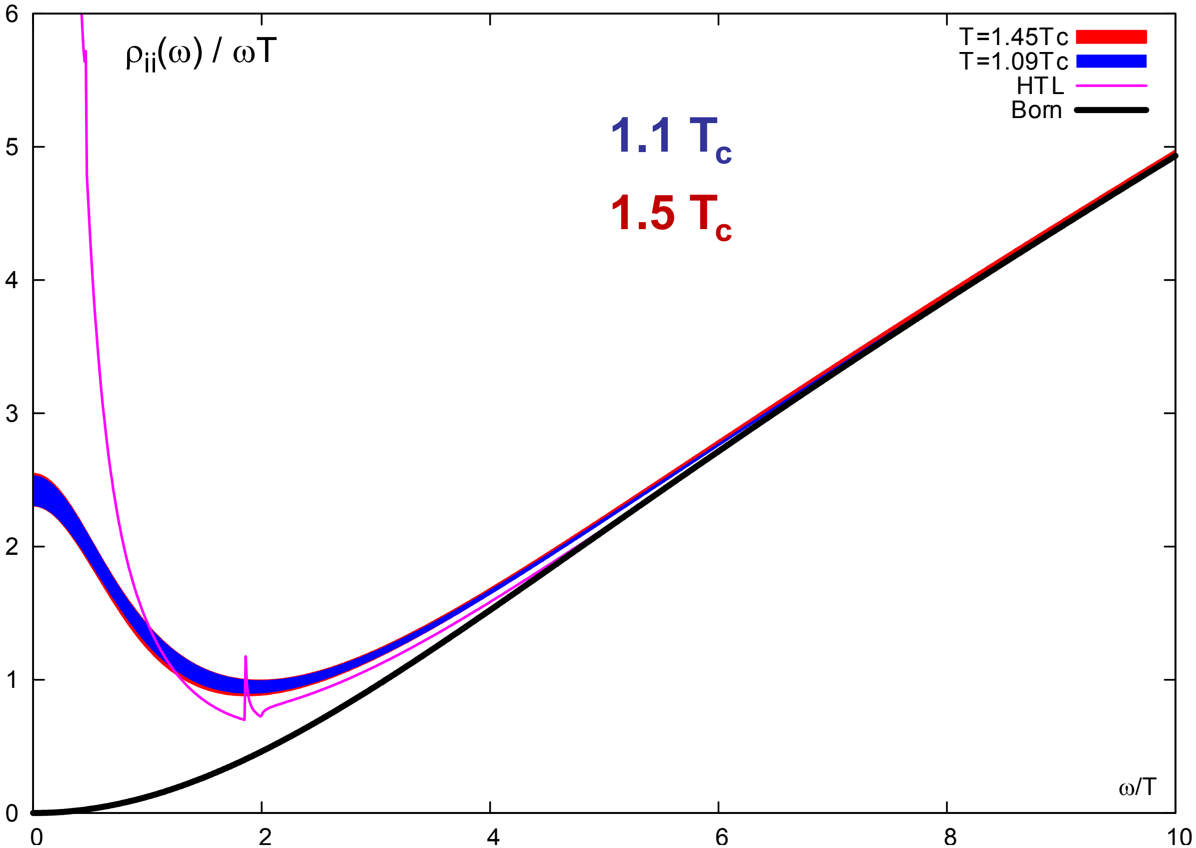

Analysis of the systematic errors

in progress

electrical conductivity

$$\frac{\sigma}{T} = \frac{C_{em}}{6}\lim_{\omega\to 0}\frac{\rho_{ii}(\omega,\vec{p}=0,T)}{\omega T}$$

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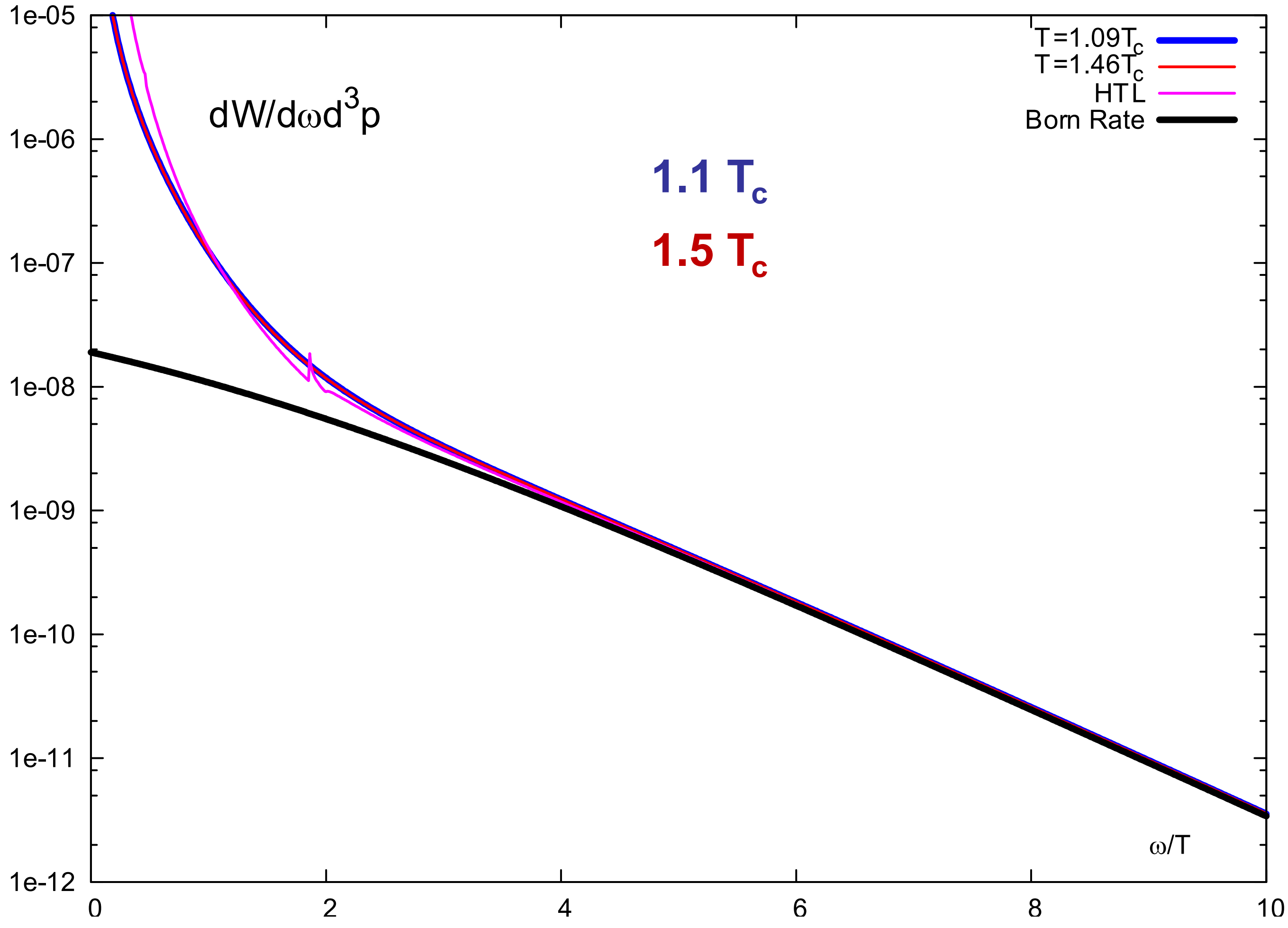
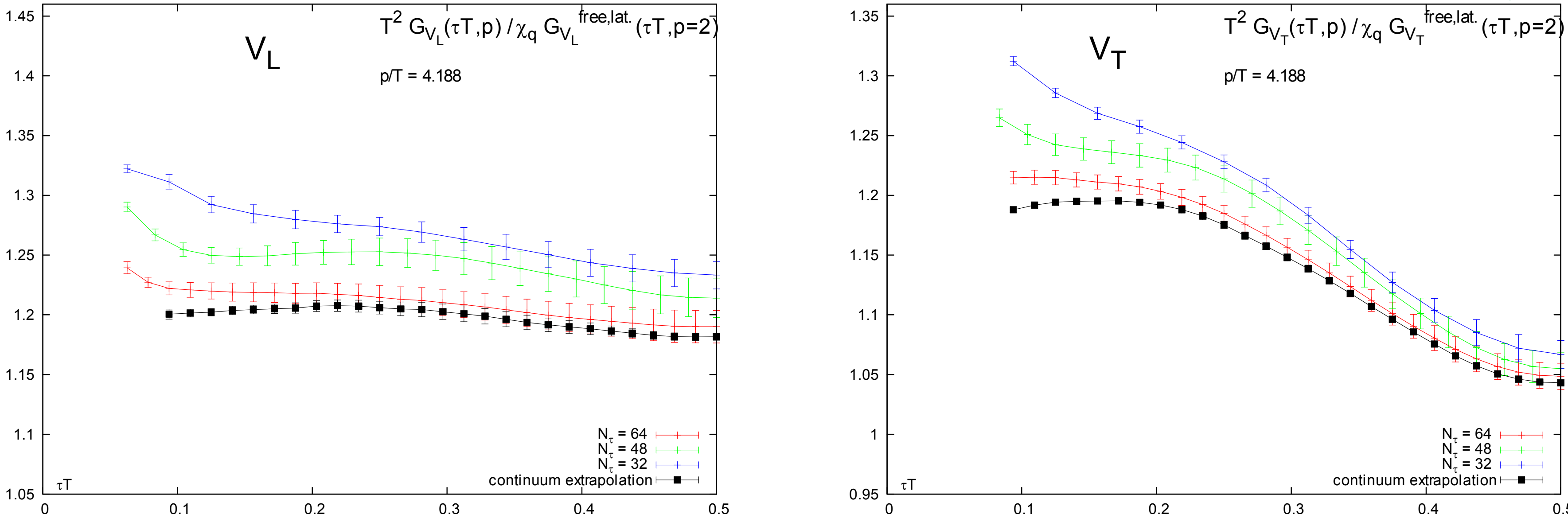
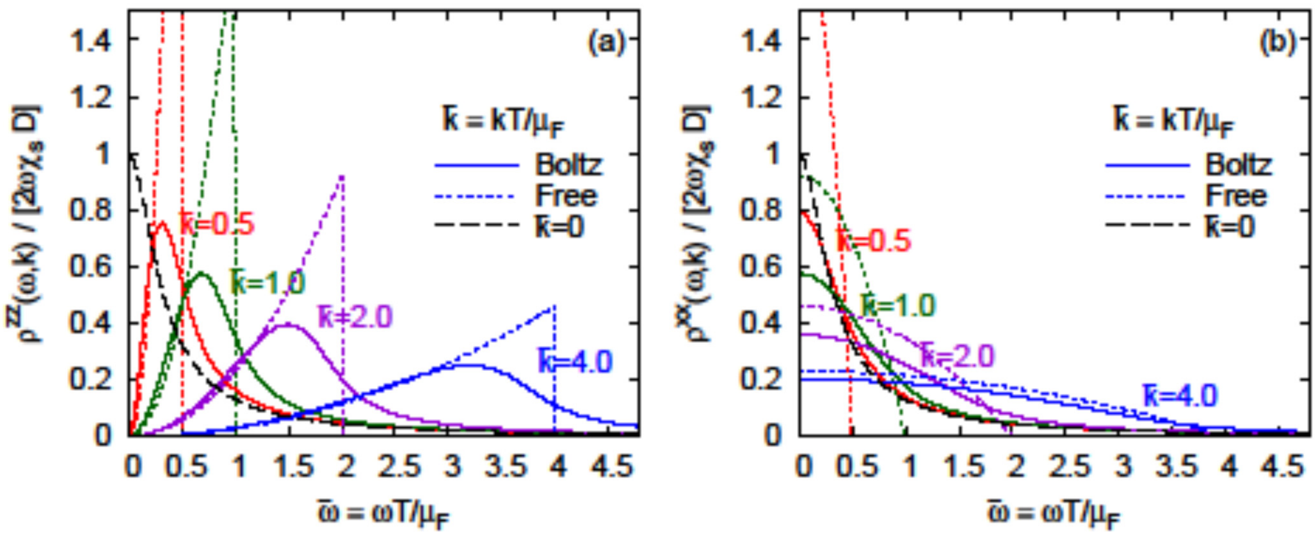


Non-zero momentum

[M.Müller et al., preliminary 2012]

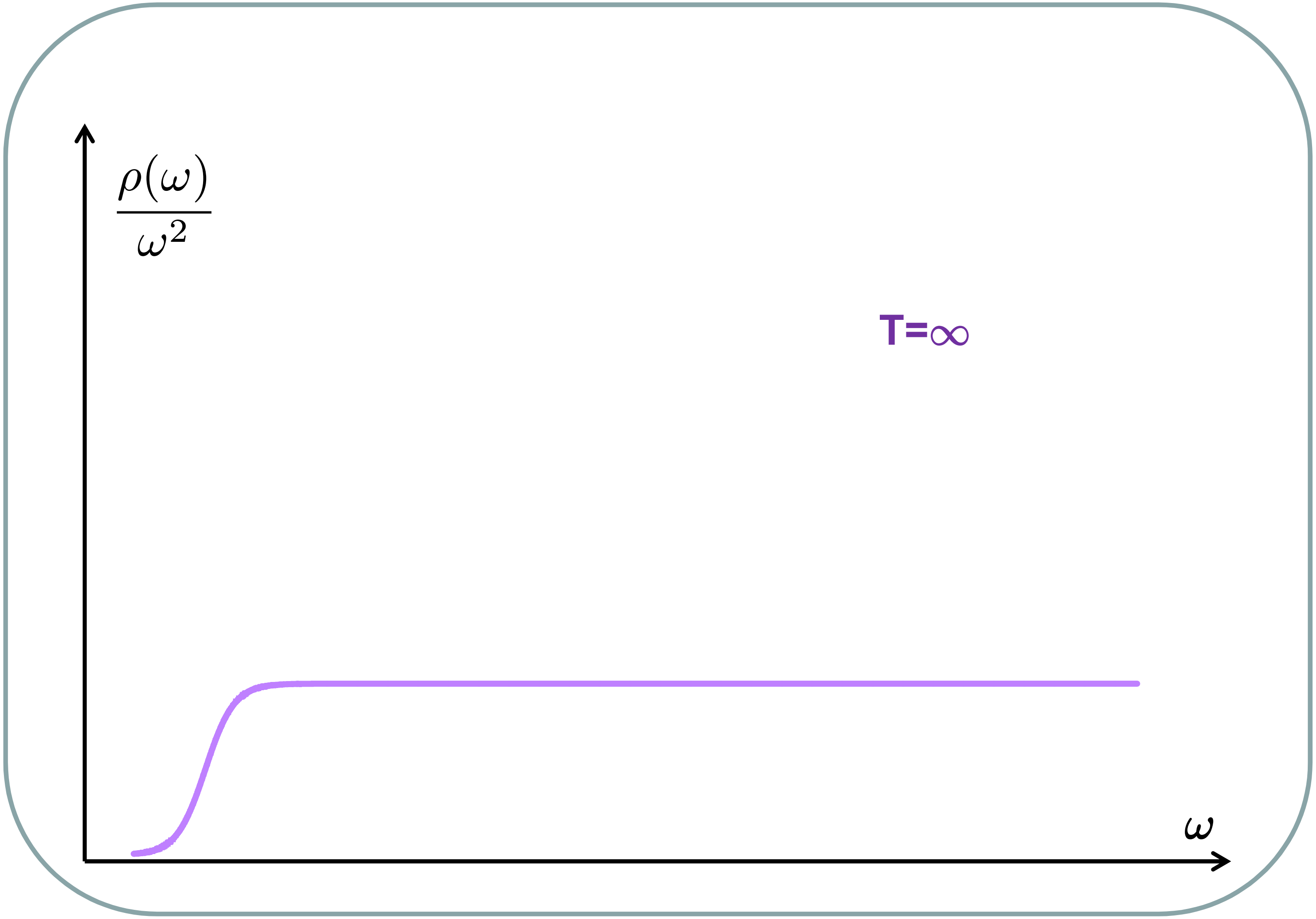


indications for non-trivial behaviour of spectral functions at small frequencies:



[Hong+Teaney, PRC82 (2010)044908]

Quarkonium spectral function – What do we expect!?



+ zero-mode contribution at ω=0: $\rho(\omega) = 2\pi\chi_{00}\,\omega\delta(\omega)$

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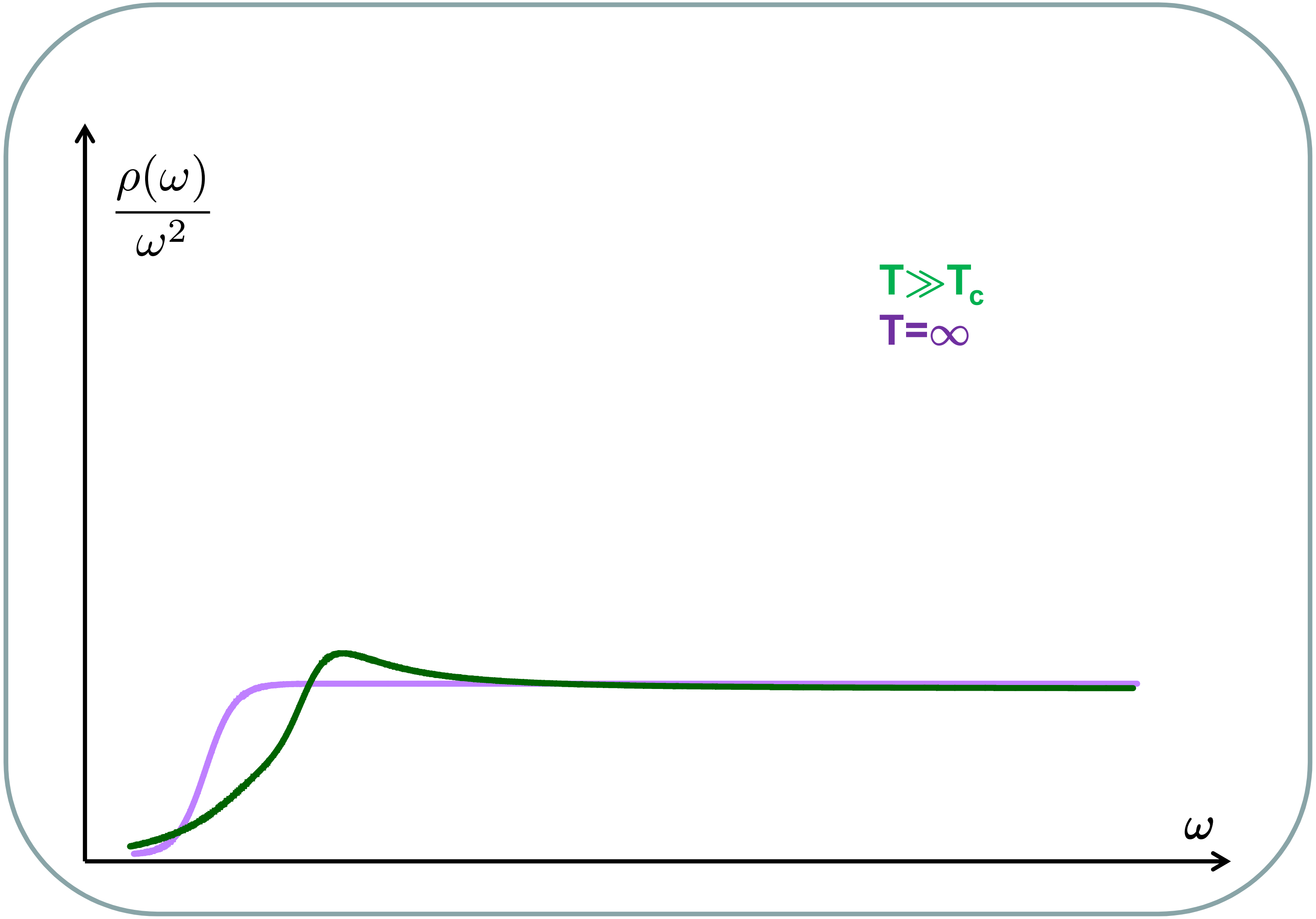

+ zero-mode contribution at ω=0: $\rho(\omega) = 2\pi\chi_{00}\ \omega\delta(\omega)$

+ transport peak at small ω: $\rho(\omega \ll T) \simeq 2\chi_{00}\frac{T}{M}\frac{\omega\eta}{\omega^2+\eta^2}\,, \qquad \eta = \frac{T}{MD}$

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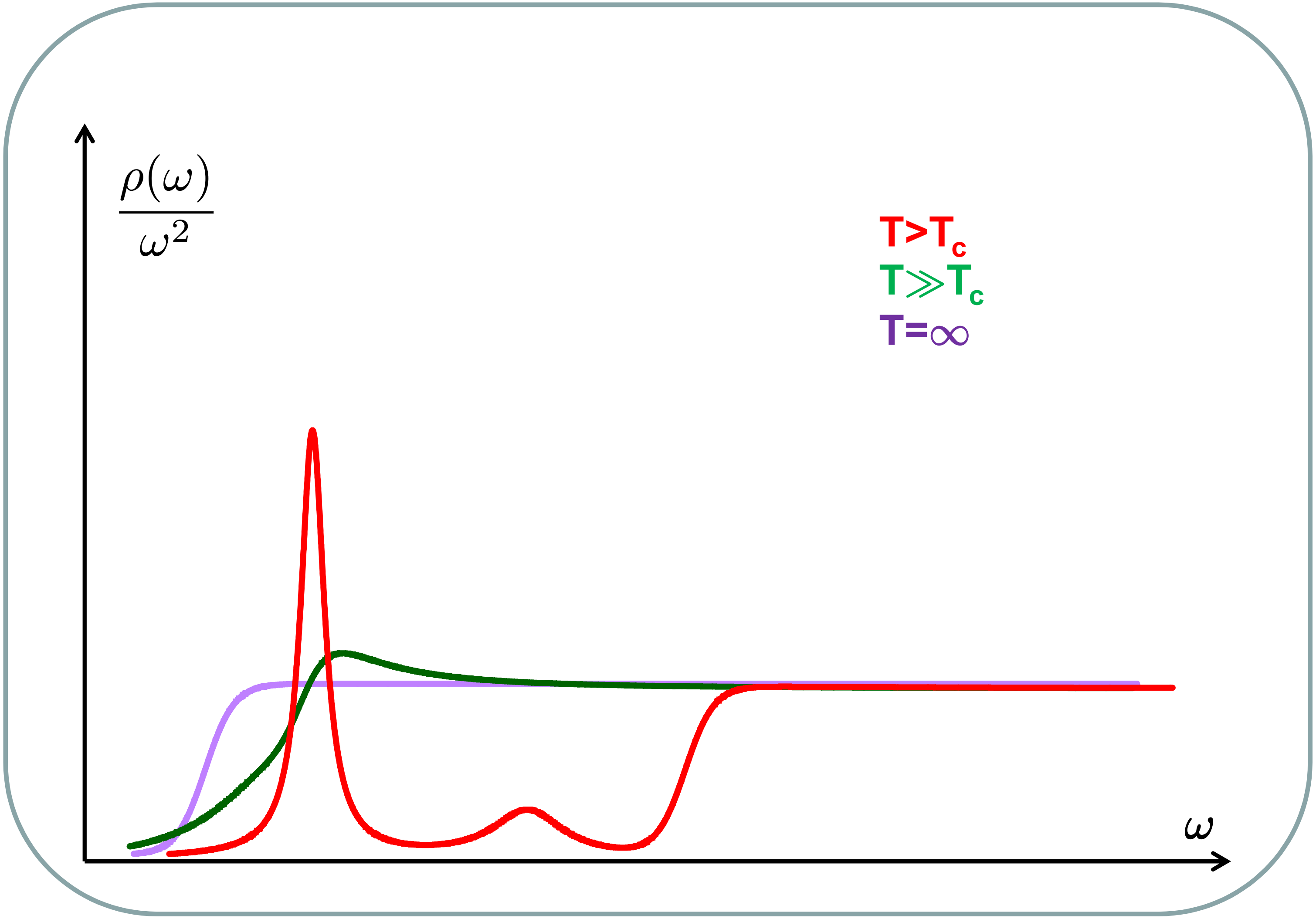

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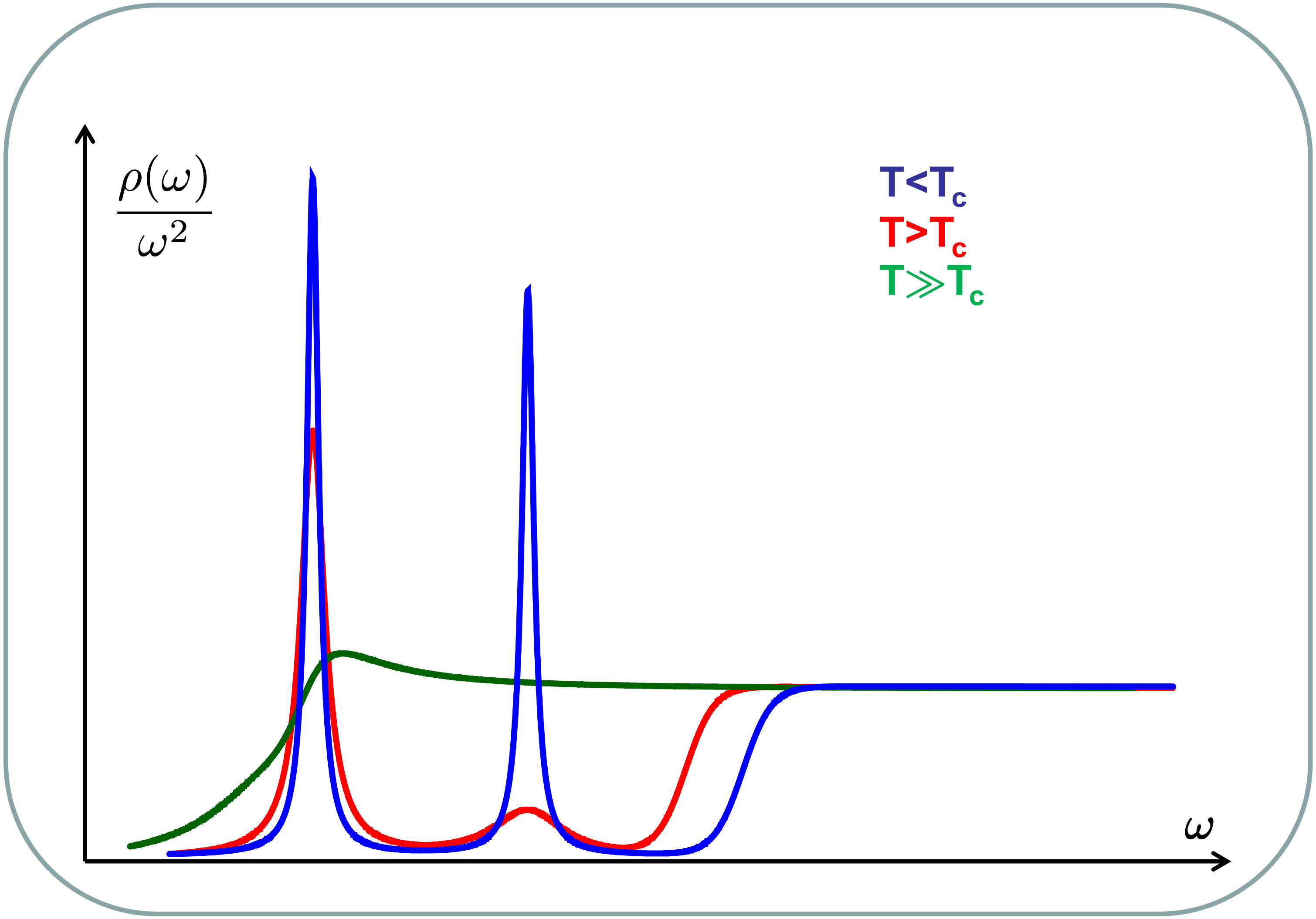

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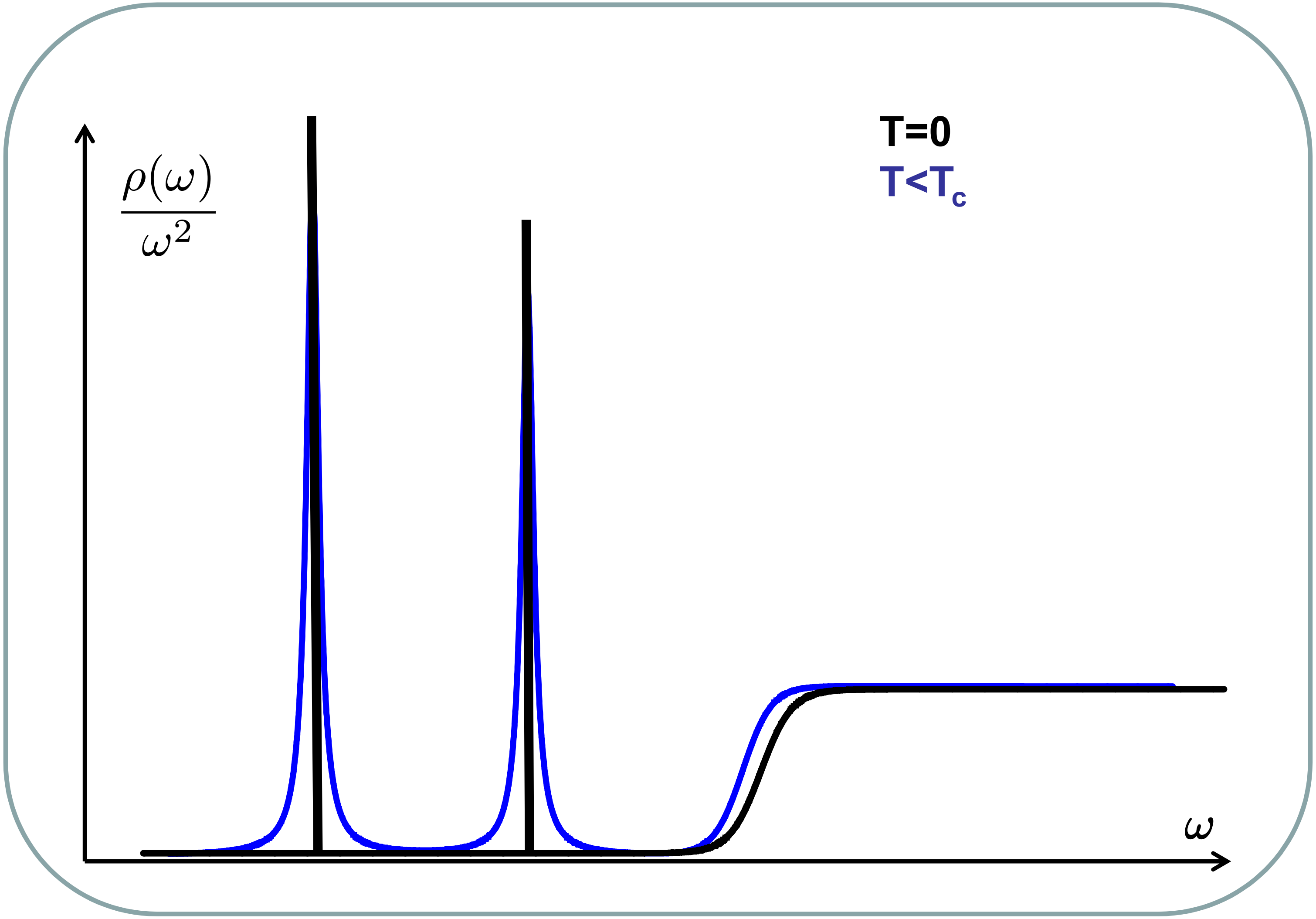

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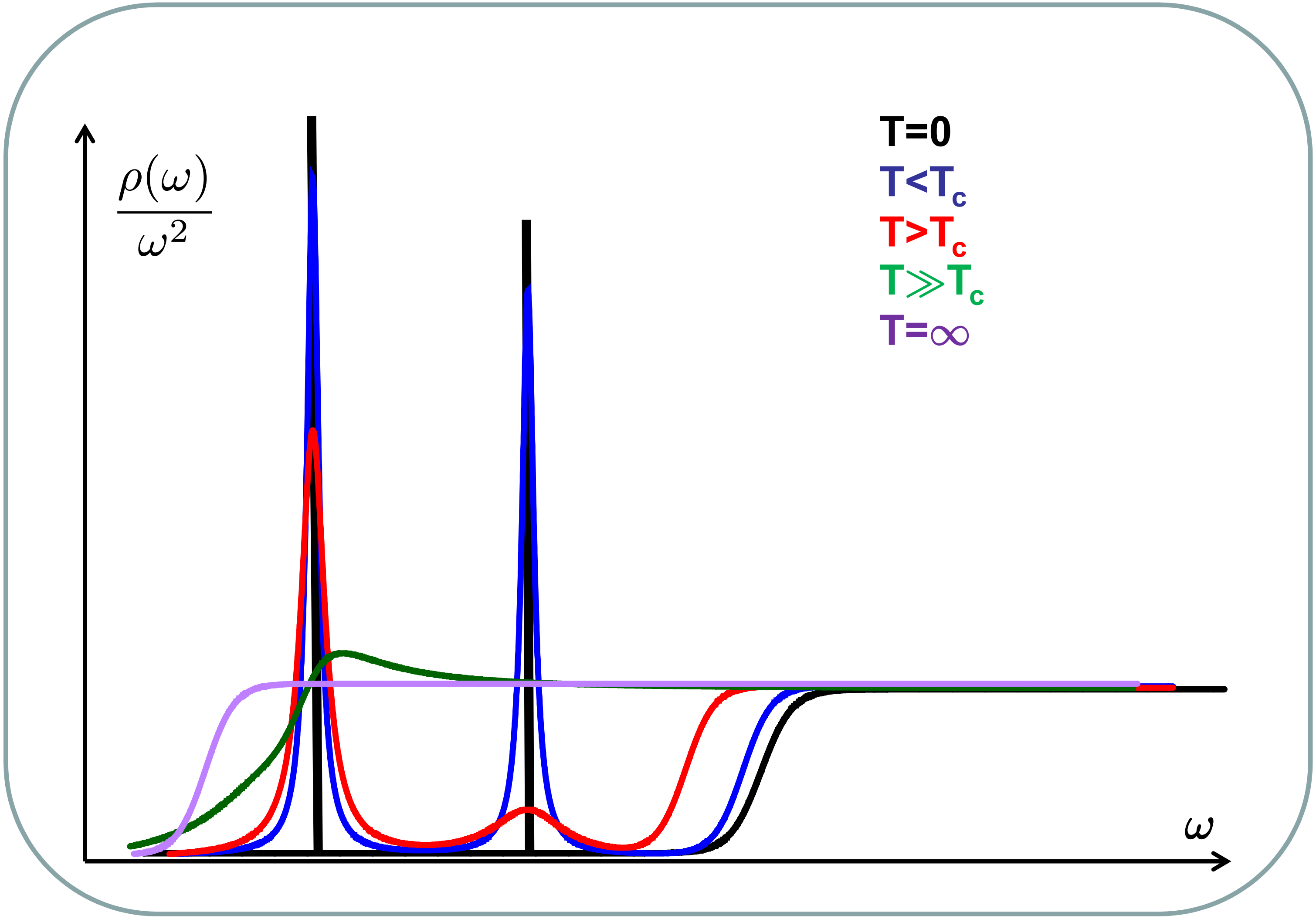

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Spatial Correlation Function and Screening Masses

Correlation functions along the **spatial direction**

$$G(z,T) = \int dx dy \int_0^{1/T} d\tau \langle J(x,y,z,\tau) J(0,0,0,0) \rangle$$

are related to the meson spectral function at **non-zero spatial momentum**

$$G(z,T) = \int_{-\infty}^{\infty} dp_z e^{ip_z z} \int_0^{\infty} d\omega \frac{\sigma(\omega, p_z, T)}{\omega}$$

exponential decay defines **screening mass M_{scr} :**

$$\xrightarrow[z \gg 1/T]{} \quad \mathbf{e^{-M_{scr} z}}$$

bound state contribution

$$\sigma(\omega, p_z, T) \sim \delta(\omega^2 - p_z^2 - M^2)$$

high-T limit (non-interacting free limit)

$$\sigma(\omega, p_z, T) \sim \sigma_{free}(\omega, p_z, T)$$

$$M_{scr} = M$$ $\longrightarrow$ indications for medium modifications/dissociation $\longleftarrow$ $$M_{scr} = 2\sqrt{(\pi T)^2 + {m_c}^2}$$

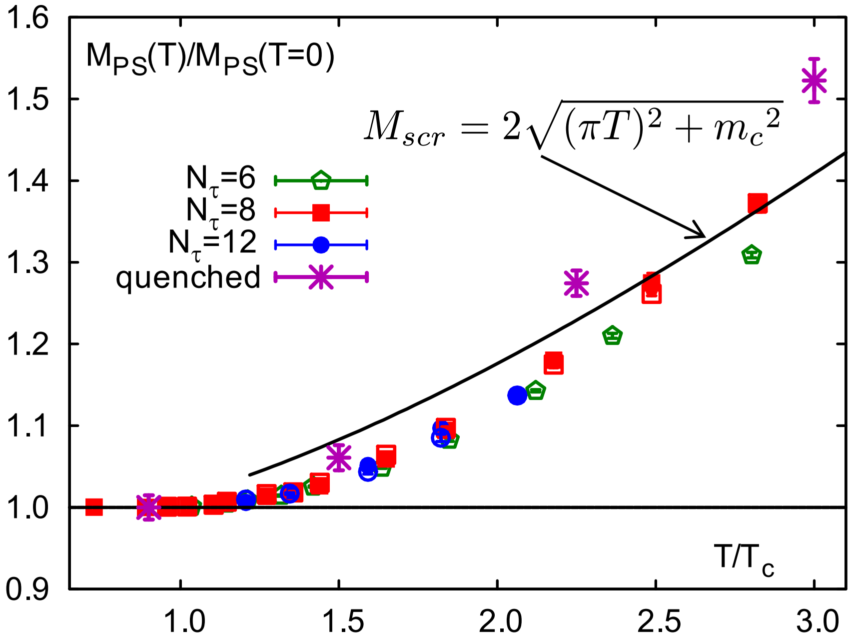
Spatial Correlation Function and Screening Masses

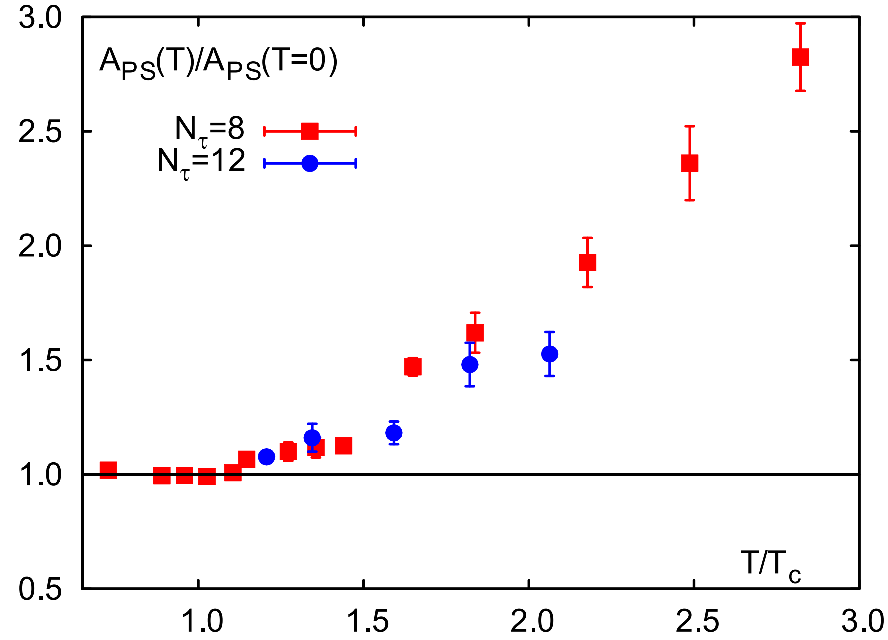
[“Signatures of charmonium modification in spatial correlation functions“, F.Karsch, E.Laermann, S.Mukherjee, P.Petreczky, (2012) arXiv:1203.3770]

2+1 flavor QCD using p4-improved staggered action

$32^3 \times N_t$ with N_t=6,8,12 and 32

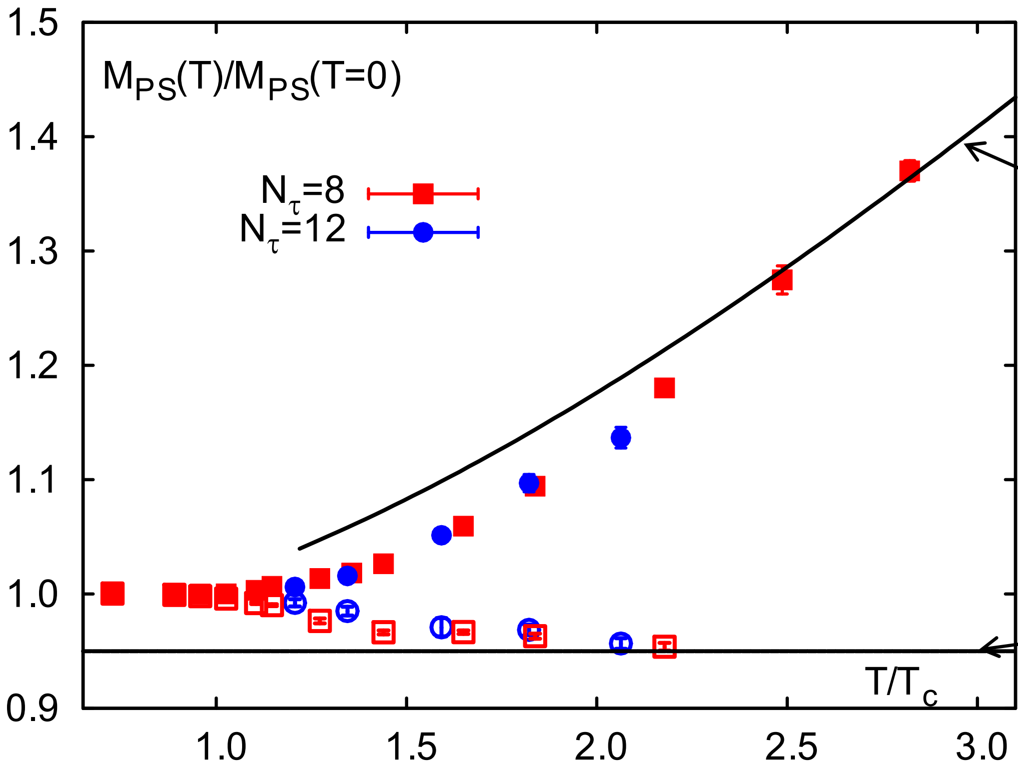
physical m_s and m_l=ms/10 ($m_\pi \simeq$ 220 MeV)





$M_{scr} = M$ ⟶ indications for medium modifications/dissociation ⟵ $M_{scr} = 2\sqrt{(\pi T)^2 + m_c{}^2}$

Spatial Correlation Function and Screening Masses



anti-periodic boundary conditions (closed symbols)

$$M_{scr} = 2\sqrt{(\pi T)^2 + {m_c}^2}$$

periodic boundary conditions (open symbols)

$$M_{scr} = 2m_c$$

$$M_{scr} = M$$

screening masses for bound states insensitive to boundary conditions due to bosonic nature of the basic degrees of freedom

"… the change in the behavior of the charmonium screening masses around T=1.5T_c is likely due to the melting of the meson states."

["Signatures of charmonium modification in spatial correlation functions", F.Karsch, E.Laermann, S.Mukherjee, P.Petreczky (2012) arXiv:1203.3770]

Vector correlation and spectral function at finite temperature

$$G(\tau,\vec{p},T) = \int_0^\infty \frac{d\omega}{2\pi} \rho(\omega,\vec{p},T) K(\tau,\omega,T)$$

$$K(\tau,\omega,T) = \frac{\cosh\left(\omega(\tau - \frac{1}{2T})\right)}{\sinh\left(\frac{\omega}{2T}\right)}$$

Lattice observables:

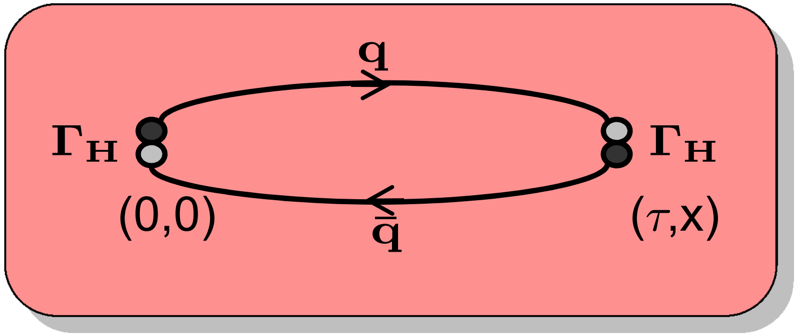
$$G_{\mu\nu}(\tau,\vec{x}) = \langle J_\mu(\tau,\vec{x}) J_\nu^\dagger(0,\vec{0})\rangle$$

$$J_\mu(\tau,\vec{x}) = 2\kappa Z_V \bar{\psi}(\tau,\vec{x})\Gamma_\mu\psi(\tau,\vec{x})$$

← local, non-conserved current, needs to be renormalized

$$G_{\mu\nu}(\tau,\vec{p}) = \sum_{\vec{x}} G_{\mu\nu}(\tau,\vec{x}) e^{i\vec{p}\vec{x}}$$

← only $\vec{p} = 0$ used here



Charmonium spectral function in quenched QCD

[H.T.Ding, OK et al., arXiv:1204.4945]

Quenched SU(3) gauge configurations (separated by 500 updates) at 4 temperatures

Lattice size $N_\sigma^3 N_\tau$ with $N_\sigma = 128$
$N_\tau = 16, 24, 32, 48, 96$

Non-perturbatively O(a) clover improved Wilson fermions

Non-perturbative renormalization constants

Quark masses close to charm quark mass

β	Mass in GeV J/ψ	η_c	χ_{c1}	χ_{c0}
6.872	3.1127(6)	3.048(2)	3.624(38)	3.540(25)
7.457	3.147(1)(25)	3.082(2)(21)	3.574(8)	3.486(4)
7.793	3.472(2)(114)	3.341(2)(104)	4.02(2)(23)	4.52(2)(37)

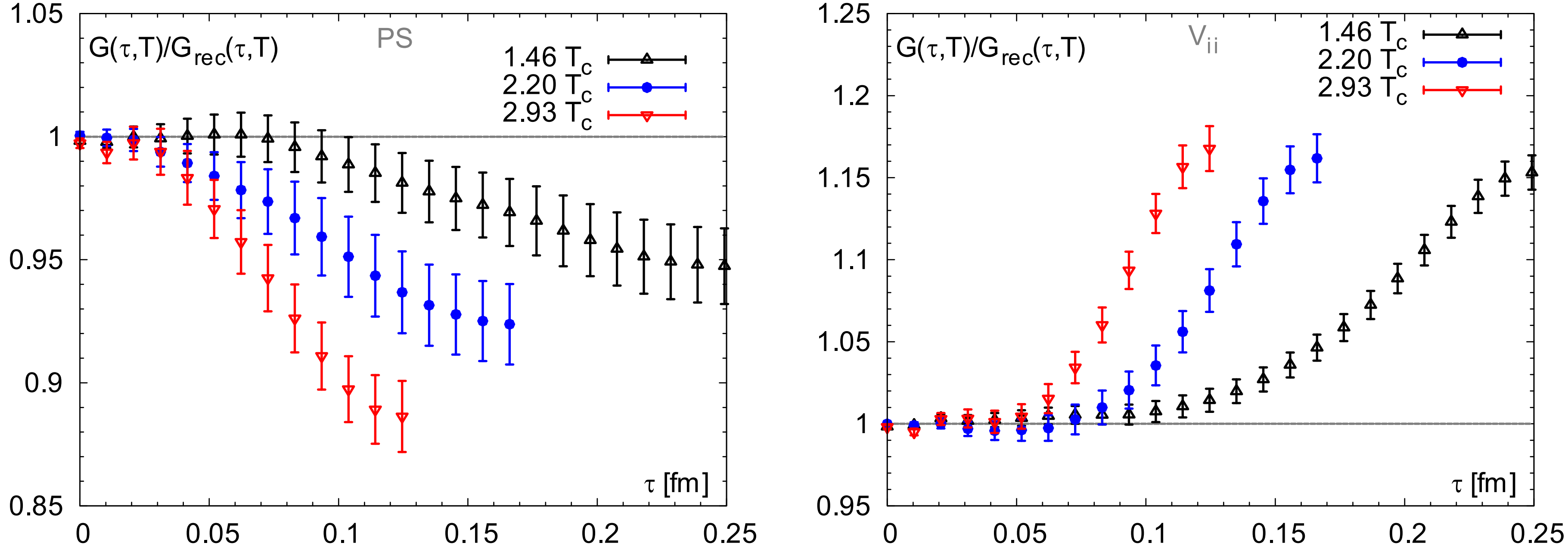
cut-off dependence | volume dependence

β	a[fm]	a^{-1}[GeV]	L_σ[fm]	csw	κ	$N_\sigma^3 \times N_\tau$	T/T_c	N_{conf}
6.872	0.031	6.432	3.93	1.412488	0.13035	$128^3 \times 32$	0.74	126
						$128^3 \times 16$	1.49	198
7.457	0.015	12.864	1.96	1.338927	0.13179	$128^3 \times 64$	0.74	179
						$128^3 \times 32$	1.49	250
7.793	0.010	18.974	1.33	1.310381	0.13200	$128^3 \times 96$	0.73	234
						$128^3 \times 48$	1.46	461
						$128^3 \times 32$	2.20	105
						$128^3 \times 24$	2.93	81

close to continuum ($m_c a \ll 1$)

Temperature dependence

Charmonium Correlators vs Reconstructed Correlators



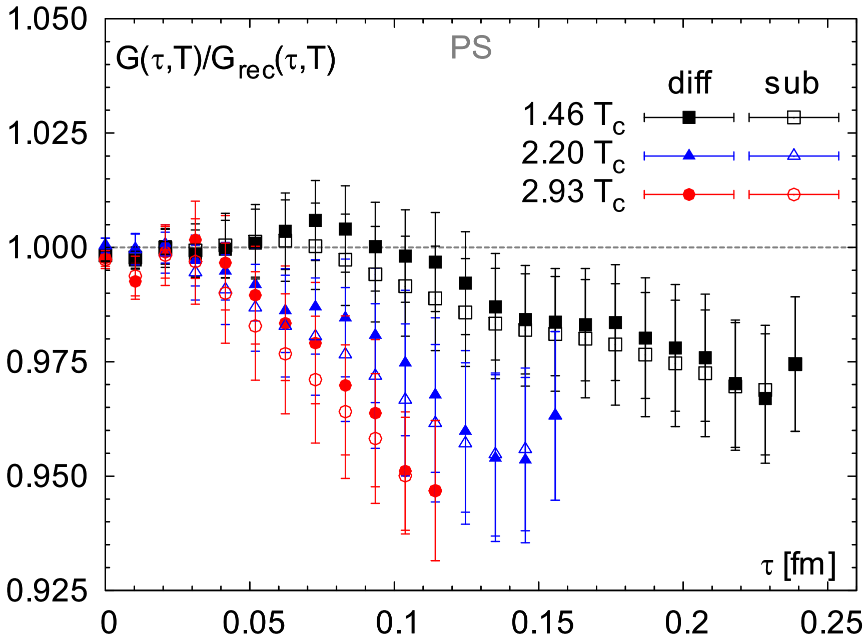
$$G_{rec}(\tau,T) = \int \sigma_0(\omega, 0.75T_c)K(\omega,\tau,T)$$

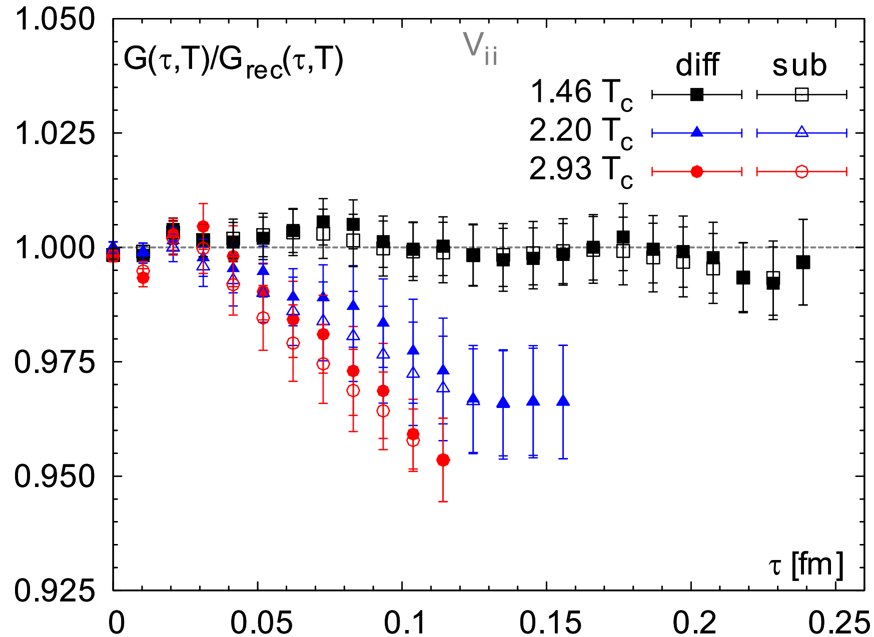
$$G_{\mu\mu}(\tau,T) = G_{ii}(\tau,T) + G_{00}(\tau,T)$$

- main T-effect due to zero-mode contribution
- well described by small ω-part of $\sigma_T(\omega,\text{T})$
- explains the rise in the vector channel
- no zero-mode contribution in PS-channel

(similar to discussions by Umeda, Petreczky)

Charmonium Correlators vs Reconstructed Correlators





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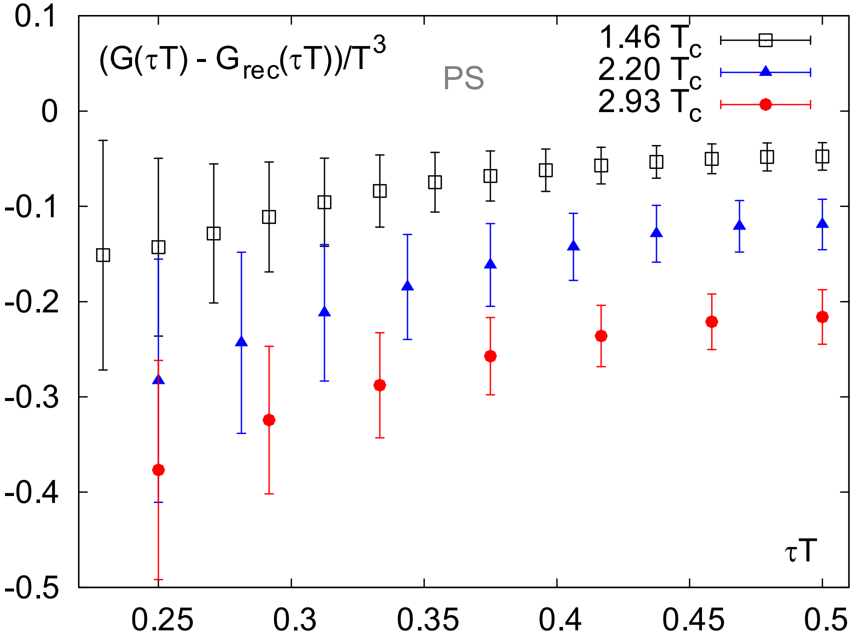
$$G_{\mu\mu}(\tau,T) = G_{ii}(\tau,T) + G_{00}(\tau,T)$$

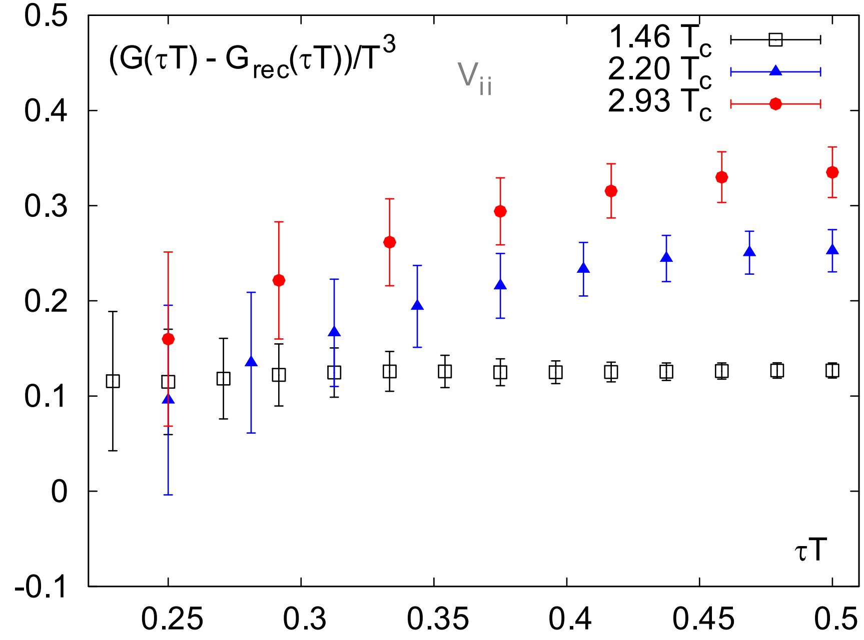
- main T-effect due to zero-mode contribution
- effectively removed by diff/sub correlators
- almost constant small-ω contribution
- similar T-dependence in PS and V channel in the large frequency part of spectral fct.

$$G_{diff}(\tau,T) = G(\tau,T) - G(\tau+1,T)$$

$$G_{sub}(\tau,T) = G(\tau,T) - G(\tau = N_t/2,T)$$

Charmonium Correlators vs Reconstructed Correlators





- negative difference for all T
- indications for thermal modifications in the bound state frequency region
- remember: no transport contribution in this channel

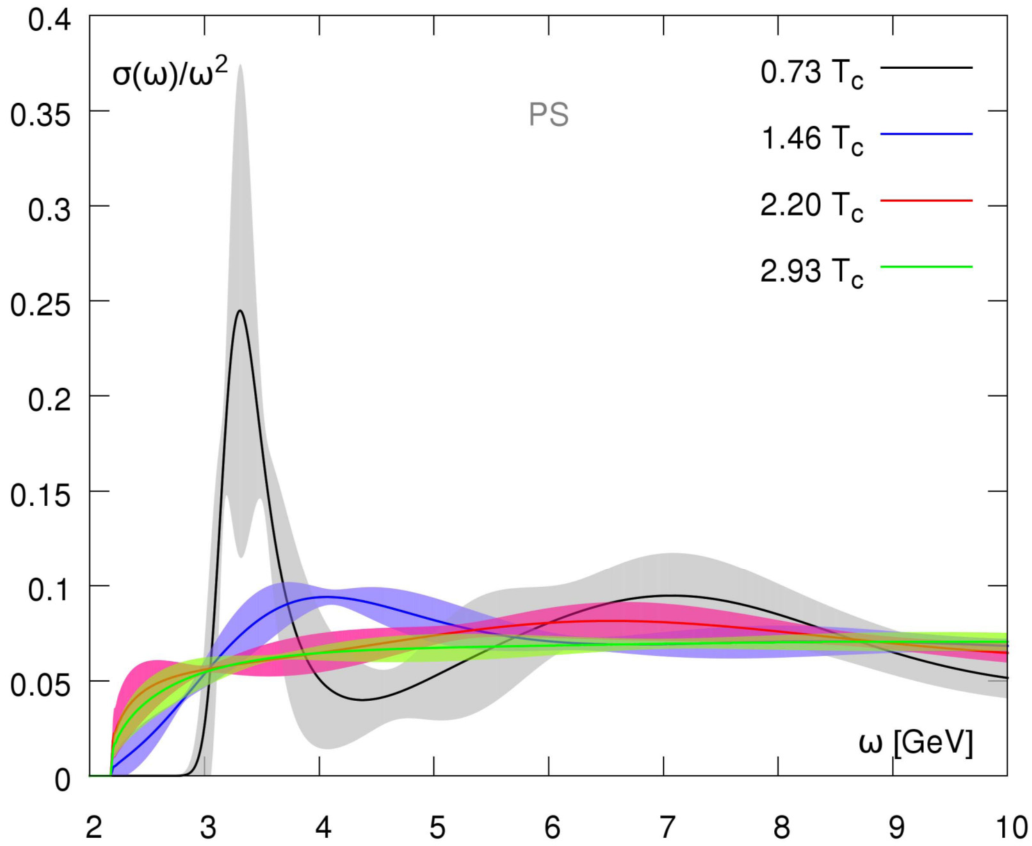
- positive diff. due to small-ω contr.
- positive slope indicates modifications in the bound state frequency region
- remember: small-ω contribution determines transport coefficient

First estimate from fit to vector channel: $2\pi T D \approx 0.6 - 3.4$

Charmonium Spectral function

[H.T.Ding, OK et al., arXiv:1204.4945]

from sophisticated Maximum Entropy Method analysis:



statistical error band from Jackknife analysis

no clear signal for bound states above 1.46 T_c

study of the interesting region closer to T_c on the way!

Charmonium Spectral function

[H.T.Ding, OK et al., arXiv:1204.4945]

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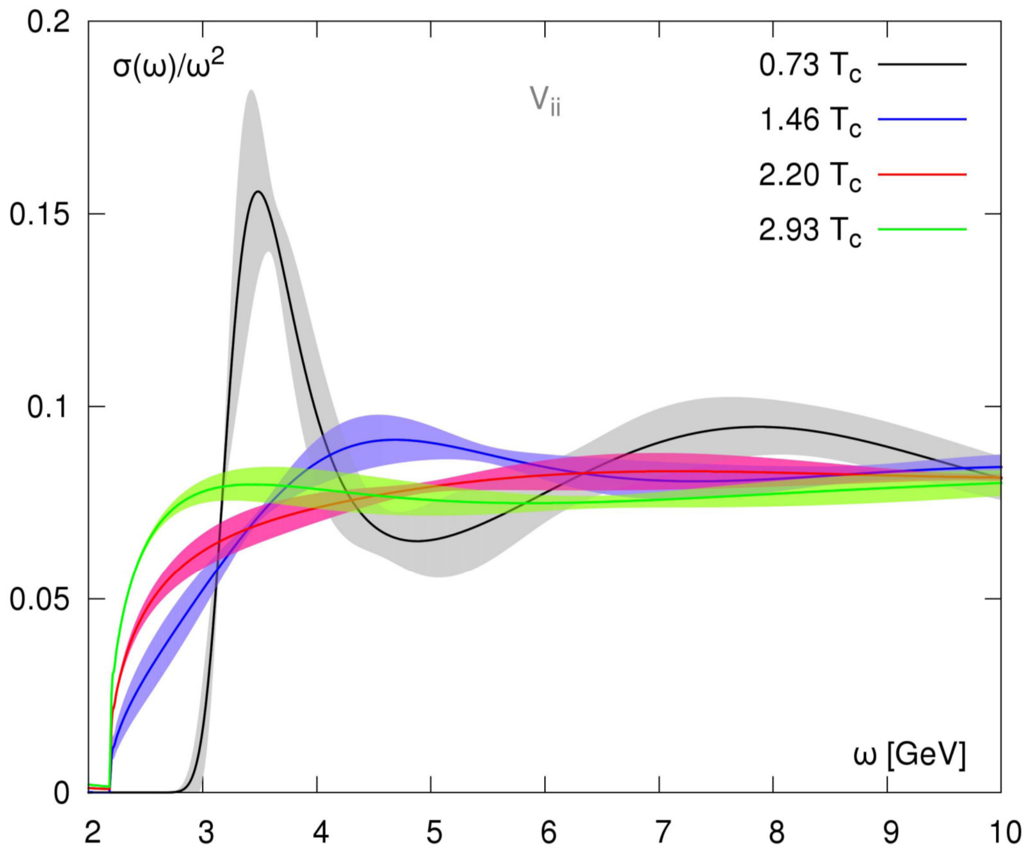

statistical error band from Jackknife analysis

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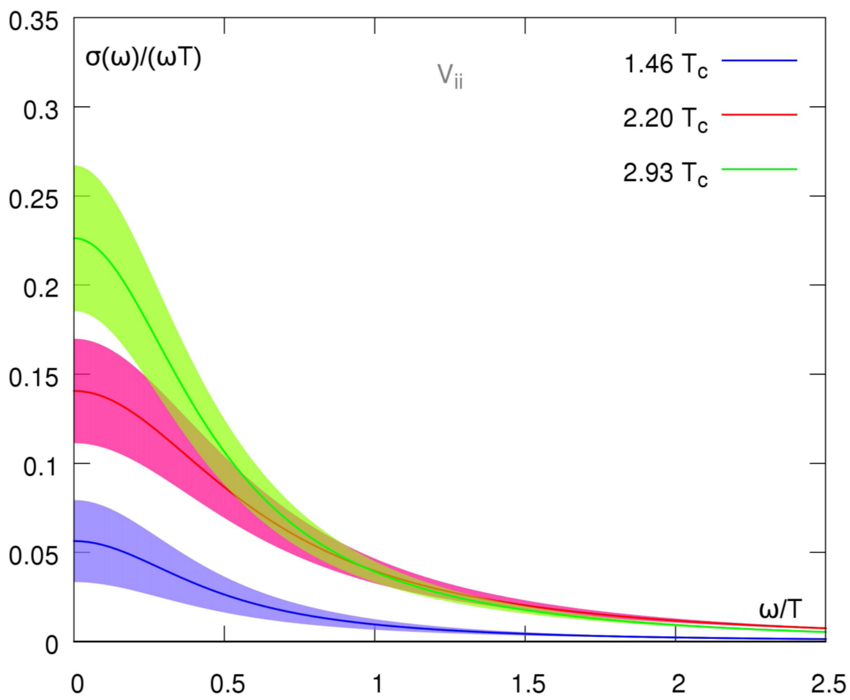
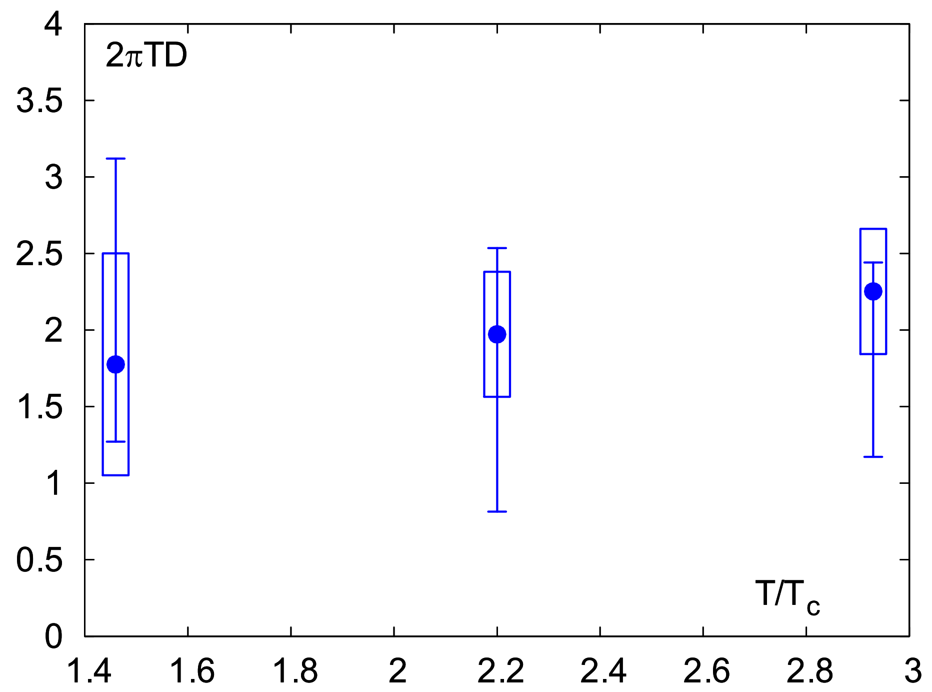
study of the interesting region closer to T_c on the way!

Charmonium Spectral function – Transport Peak



[H.T.Ding, OK et al., arXiv:1204.4945]



$$D = \frac{\pi}{3\chi_{00}} \lim_{\omega \to 0} \frac{\rho_{ii}(\omega, \vec{p}=0, T)}{\omega T}$$

Perturbative estimate (α_s~0.2, g~1.6):

LO: $2\pi TD \simeq 71.2$
NLO: $2\pi TD \simeq 8.4$

[Moore&Teaney, PRD71(2005)064904,
Caron-Huot&Moore, PRL100(2008)052301]

Strong coupling limit:

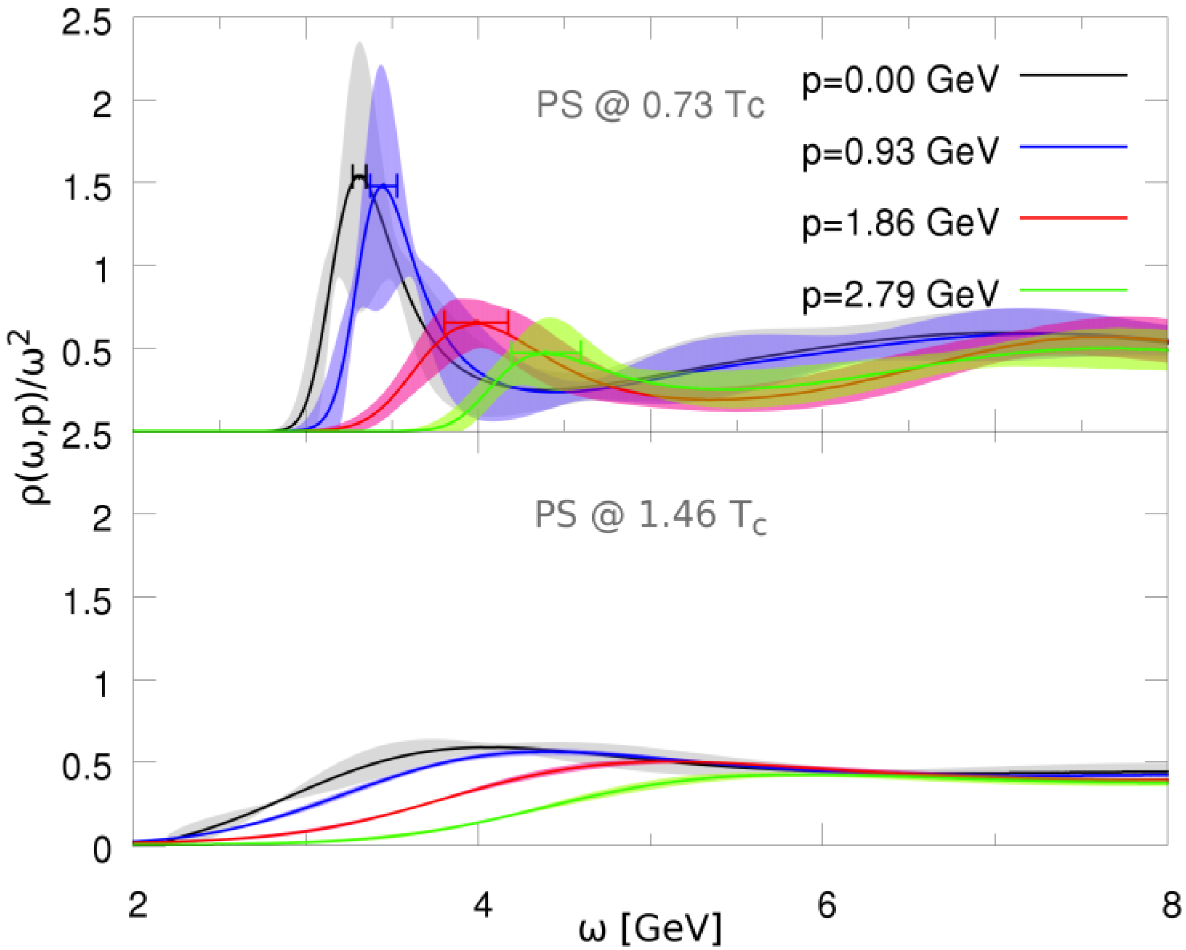
$2\pi TD = 1$

[Kovtun, Son & Starinets,JHEP 0310(2004)064]

Charmonium Spectral function – momentum dependence

[H.T.Ding, arXiv:1210.5442]

preliminary results for the momentum dependence:



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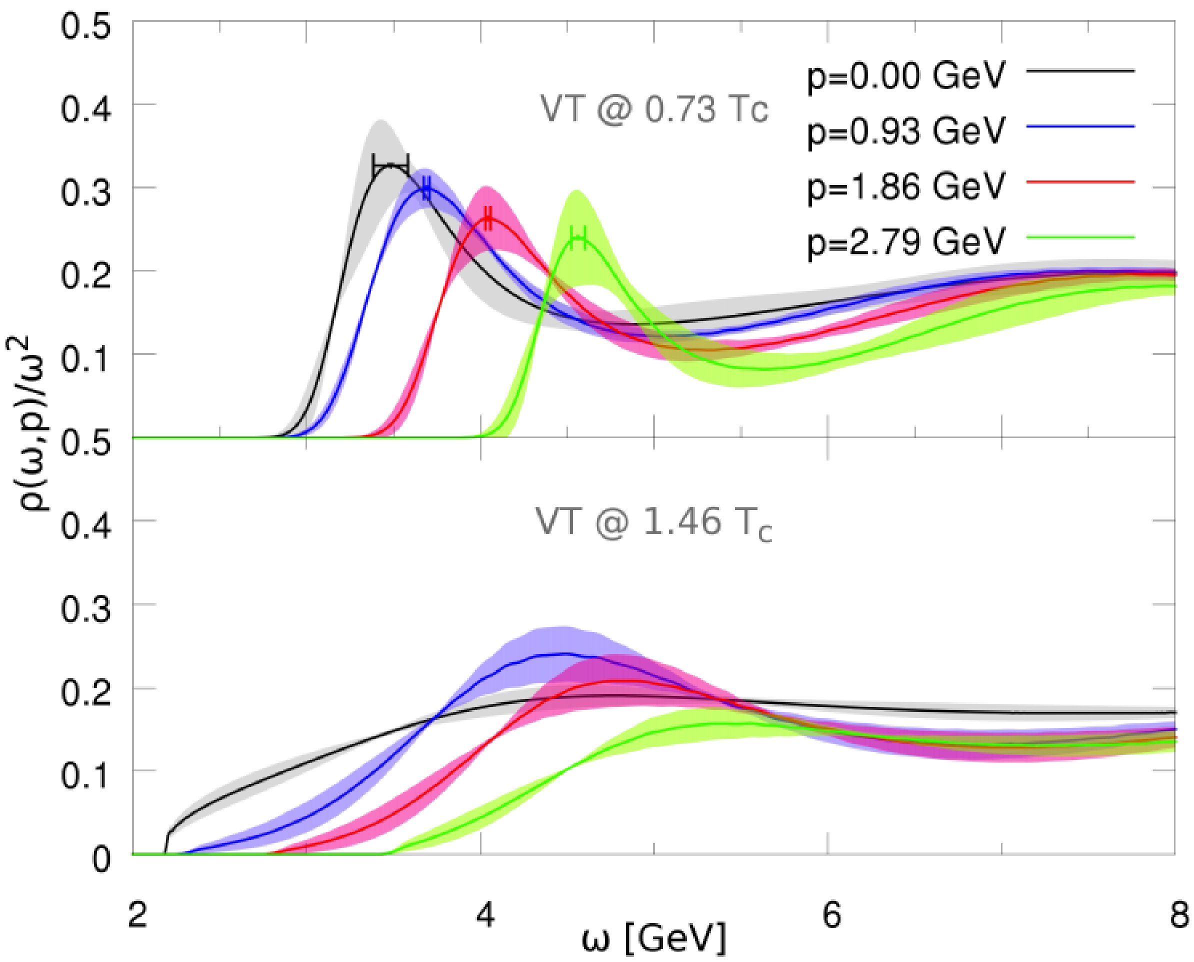


Bottomonium - Lattice NRQCD

[G.Aarts et al., JHEP11(2011)103]

In non-relativistic QCD the Lagrangian is expanded in terms of v=|**p**|/M

$$\mathcal{L} = \mathcal{L}_0 + \delta\mathcal{L}$$

with

$$\mathcal{L}_0 = \psi^\dagger \left(D_\tau - \frac{\mathbf{D}^2}{2M} \right) \psi + \chi^\dagger \left(D_\tau + \frac{\mathbf{D}^2}{2M} \right) \chi$$

and

$$\begin{aligned}
\delta\mathcal{L} = &-\frac{c_1}{8M^3} \left[\psi^\dagger (\mathbf{D}^2)^2 \psi - \chi^\dagger (\mathbf{D}^2)^2 \chi \right] \\
&+ c_2 \frac{ig}{8M^2} \left[\psi^\dagger \left(\mathbf{D} \cdot \mathbf{E} - \mathbf{E} \cdot \mathbf{D} \right) \psi + \chi^\dagger \left(\mathbf{D} \cdot \mathbf{E} - \mathbf{E} \cdot \mathbf{D} \right) \chi \right] \\
&- c_3 \frac{g}{8M^2} \left[\psi^\dagger \boldsymbol{\sigma} \cdot \left(\mathbf{D} \times \mathbf{E} - \mathbf{E} \times \mathbf{D} \right) \psi + \chi^\dagger \boldsymbol{\sigma} \cdot \left(\mathbf{D} \times \mathbf{E} - \mathbf{E} \times \mathbf{D} \right) \chi \right] \\
&- c_4 \frac{g}{2M} \left[\psi^\dagger \boldsymbol{\sigma} \cdot \mathbf{B} \psi - \chi^\dagger \boldsymbol{\sigma} \cdot \mathbf{B} \chi \right]
\end{aligned}$$

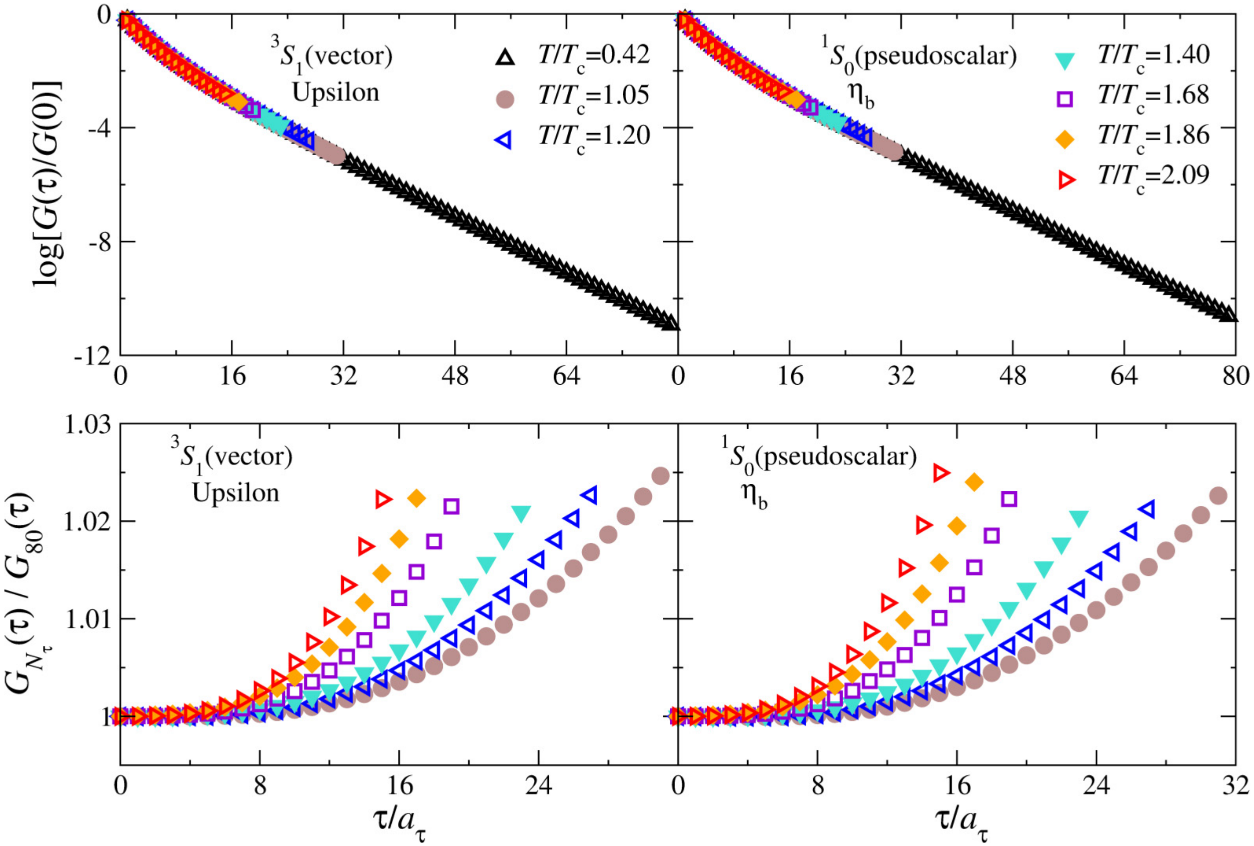
which is correct up to order O(v^4) [G.T.Bodwin,E.Braaten,G.P.Lepage, PRD 51 (1995) 1125]

Bottomonium - Lattice NRQCD

[G.Aarts et al., JHEP11(2011)103]

correlation function calculated as an initial value problem → no costly matrix inversion

no restriction by thermal boundary conditions



N_s	N_τ	T(MeV)	T/T_c	N_{cfg}
12	80	90	0.42	250
12	32	230	1.05	1000
12	28	263	1.20	1000
12	24	306	1.40	500
12	20	368	1.68	1000
12	18	408	1.86	1000
12	16	458	2.09	1000

gauge configurations from n_f=2 dynamical Wilson fermion action

$a_s \simeq 0.162$ fm
$1/a_t \simeq 7.35$ GeV
a_s/a_τ=6 anisotropic lattice

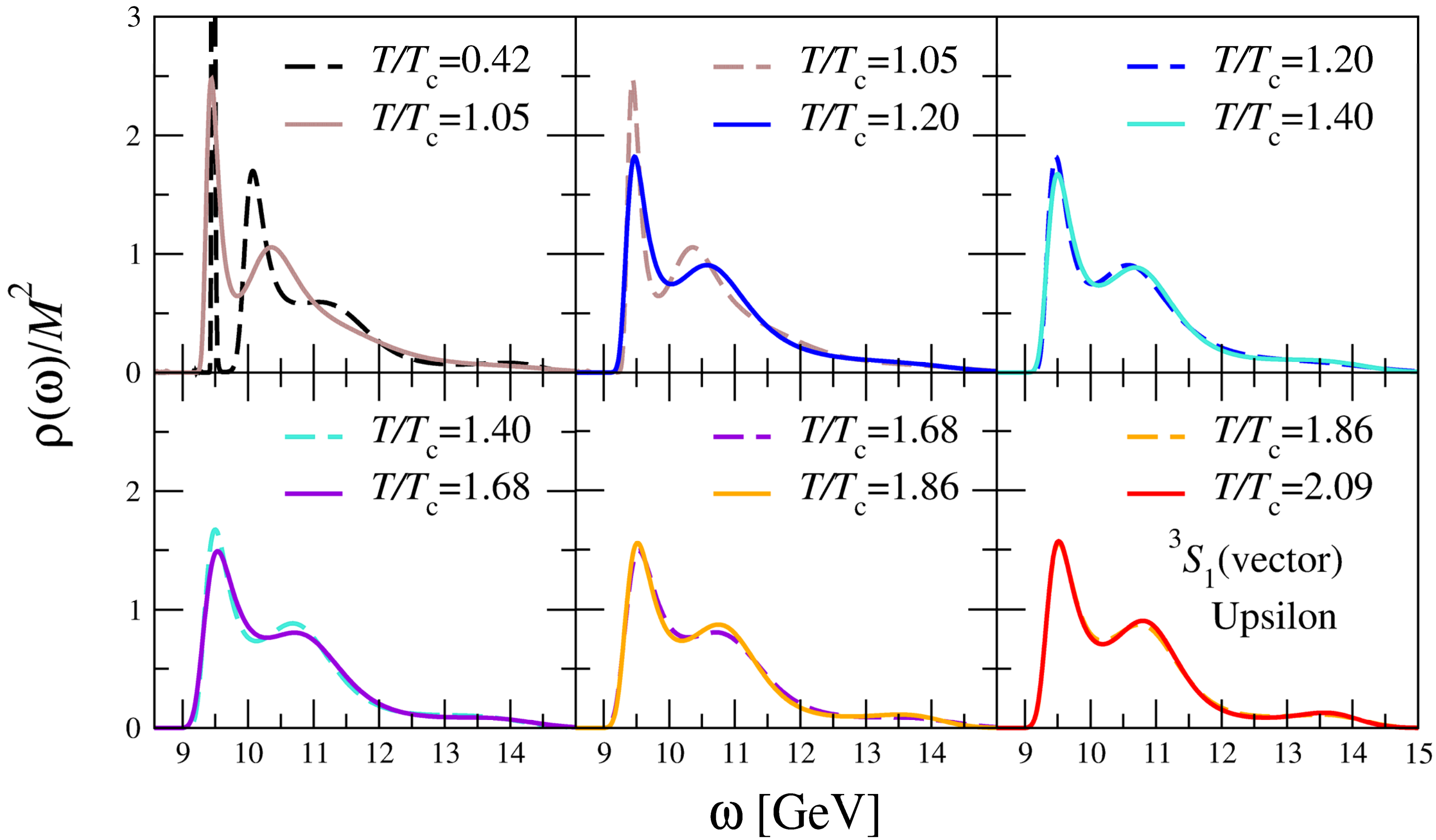
Bottomonium - Lattice NRQCD

[G.Aarts et al., JHEP11(2011)103]

Kernel is T-independent, contributions at ω < 2M absent

no small-ω contribution → no information on transport properties

$$G(\tau) = \int_{-2M}^{\infty} \frac{d\omega'}{2\pi} \exp(-\omega'\tau)\rho(\omega') \quad , \quad \omega' = \omega - 2M$$



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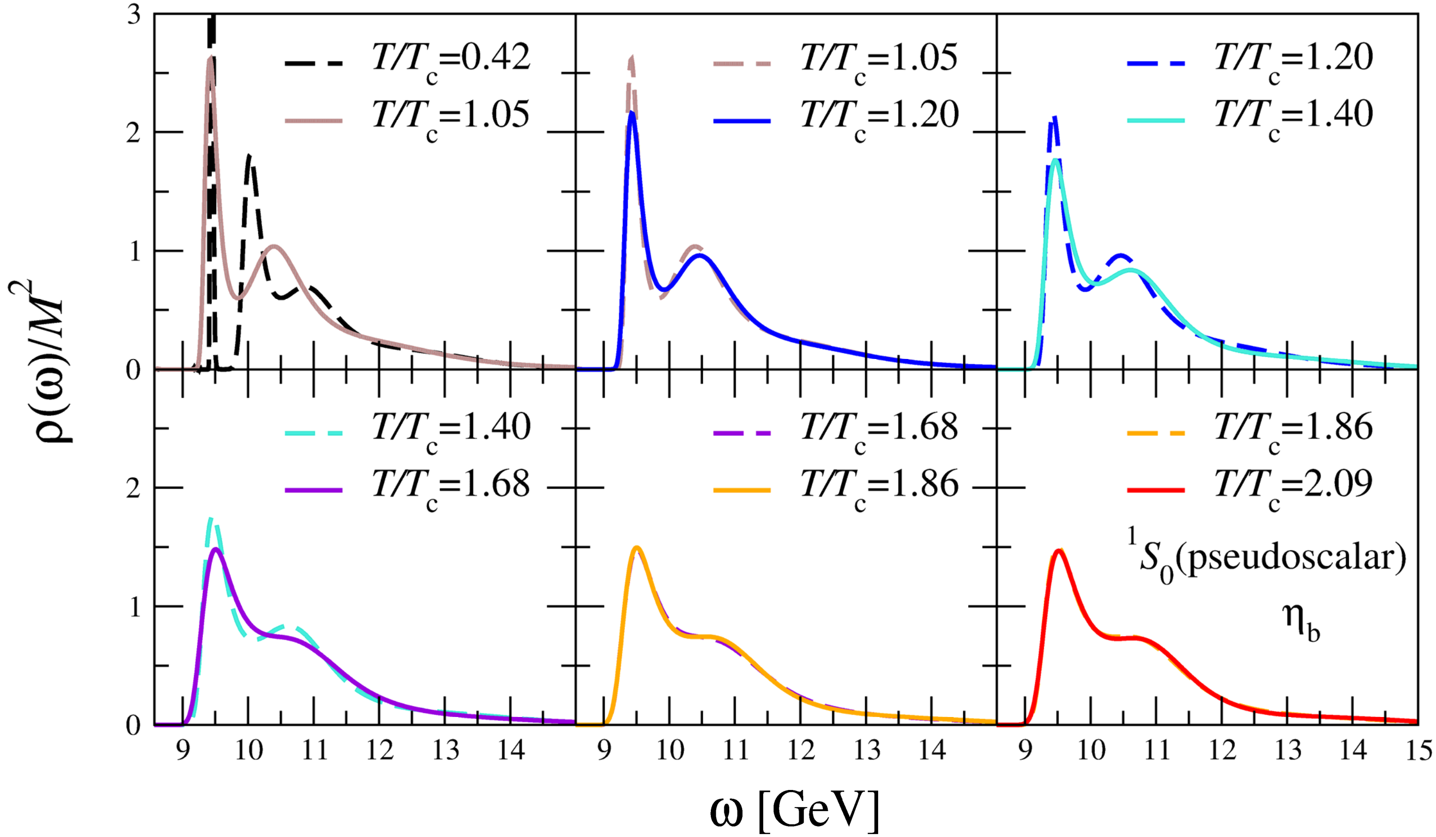


Lattice cut-off effects – free spectral functions

[G.Aarts et al., JHEP11(2011)103]

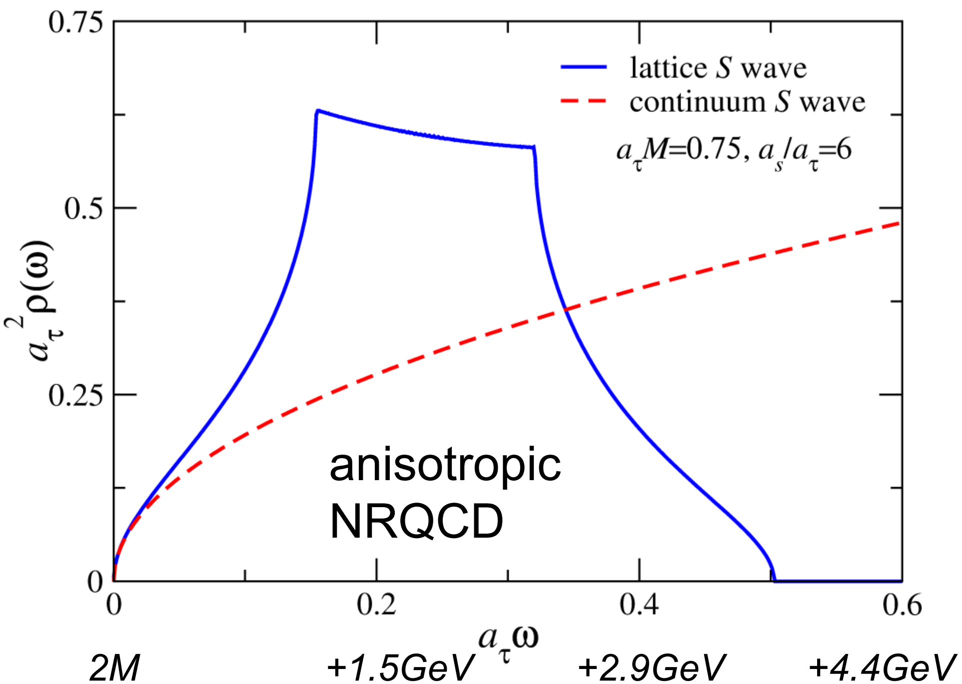
gauge configurations from n_f=2
dynamical Wilson fermion action

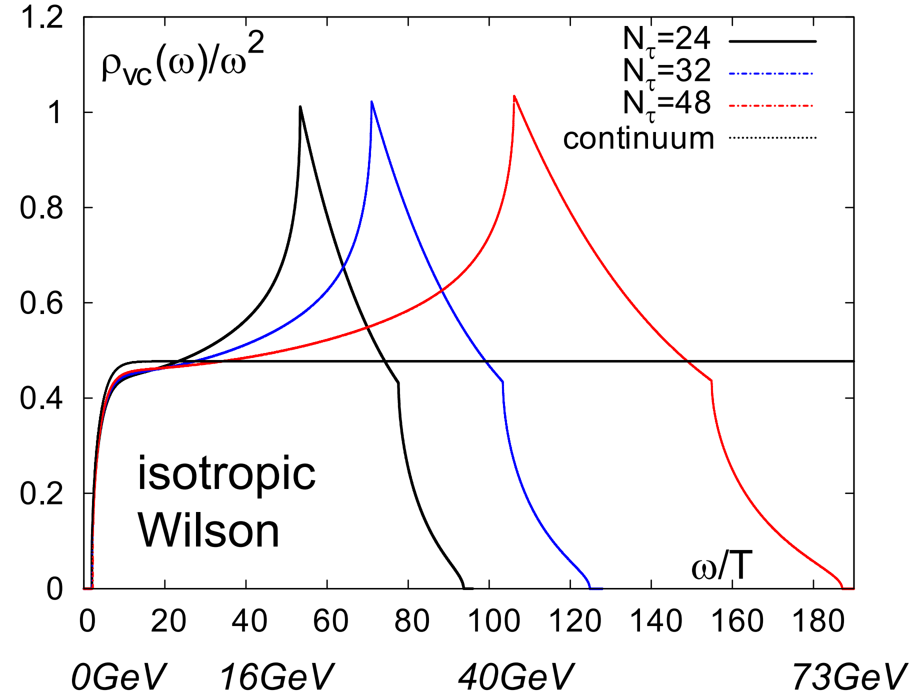
$a_s \simeq 0.162$ fm
$1/a_t \simeq 7.35$ GeV

[H.T.Ding, OK et al., arXiv:1204.4945]

gauge configurations from
quenched action

$a \simeq 0.01$ fm
$1/a \simeq 19$ GeV





cut-off effects and energy resolution determined by spatial lattice spacing

no continuum limit in NRQCD, $a_s M \gg 1$

only small energy region accessible

continuum limit straight forward, but expensive

transport properties accessible

[see also F.Karsch et al., PRD68 (2003) 014504]

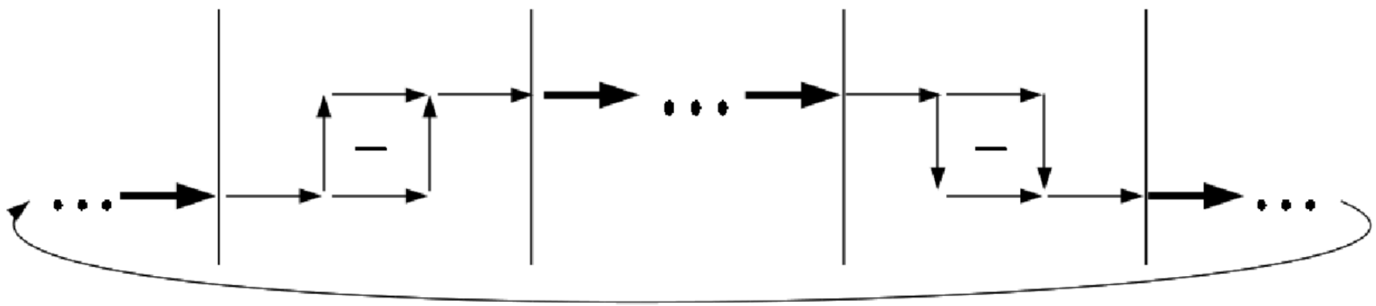
Heavy Quark Momentum Diffusion Constant

[A.Francis,OK,M.Laine,J.Langelage, arXiv:1109.3941]

Heavy Quark Effective Theory (HQET) in the large quark mass limit

leads to a (pure gluonic) “color-electric correlator”

[J.Casalderrey-Solana, D.Teaney, PRD74(2006)085012, S.Caron-Huot,M.Laine,G.D. Moore,JHEP04(2009)053]



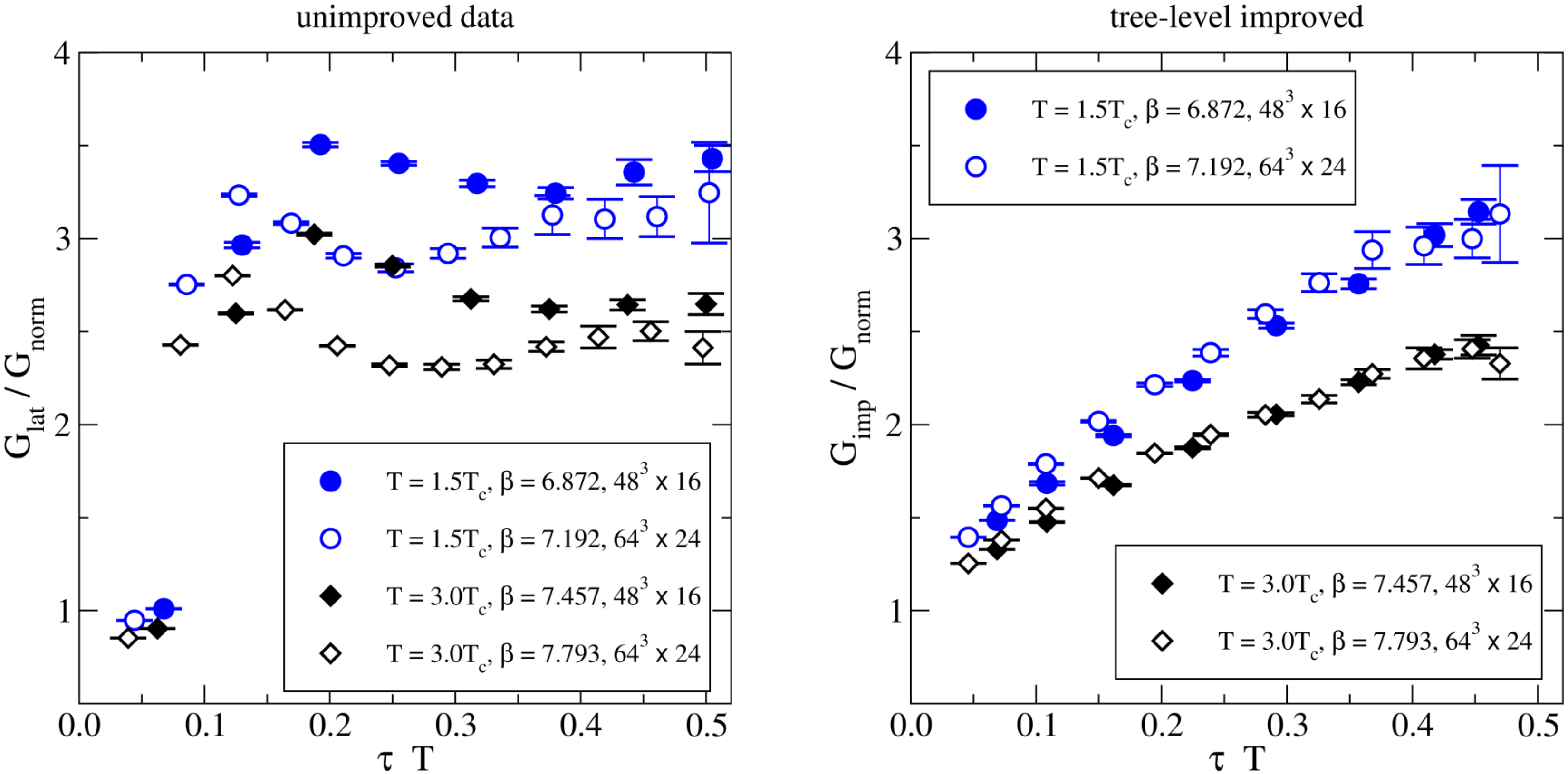
$$G_{\mathrm{E}}(\tau) \equiv -\frac{1}{3}\sum_{i=1}^{3} \frac{\left\langle \mathrm{Re\,Tr}\left[U(\tfrac{1}{T};\tau)\, gE_i(\tau,\mathbf{0})\, U(\tau;0)\, gE_i(0,\mathbf{0})\right]\right\rangle}{\left\langle \mathrm{Re\,Tr}[U(\tfrac{1}{T};0)]\right\rangle}$$

Heavy quark (momentum) diffusion:

$$\kappa = \lim_{\omega\to 0} \frac{2T\rho_{\mathrm{E}}(\omega)}{\omega} \quad , \quad D = \frac{2T^2}{\kappa}$$

Heavy Quark Momentum Diffusion Constant

[A.Francis,OK,M.Laine,J.Langelage, arXiv:1109.3941]



due to the gluonic nature of the operator, signal is extremely noisy

→ multilevel combined with link-integration techniques used to improve the signal

→ tree-level improvement (right figure) to reduce discretization effects

[similar studies by H.B.Meyer, New J.Phys.13 (2011) 035008 and D.Banerjee,S.Datta,R.Gavai,P.Majumdar,PRD85(2012)014510]

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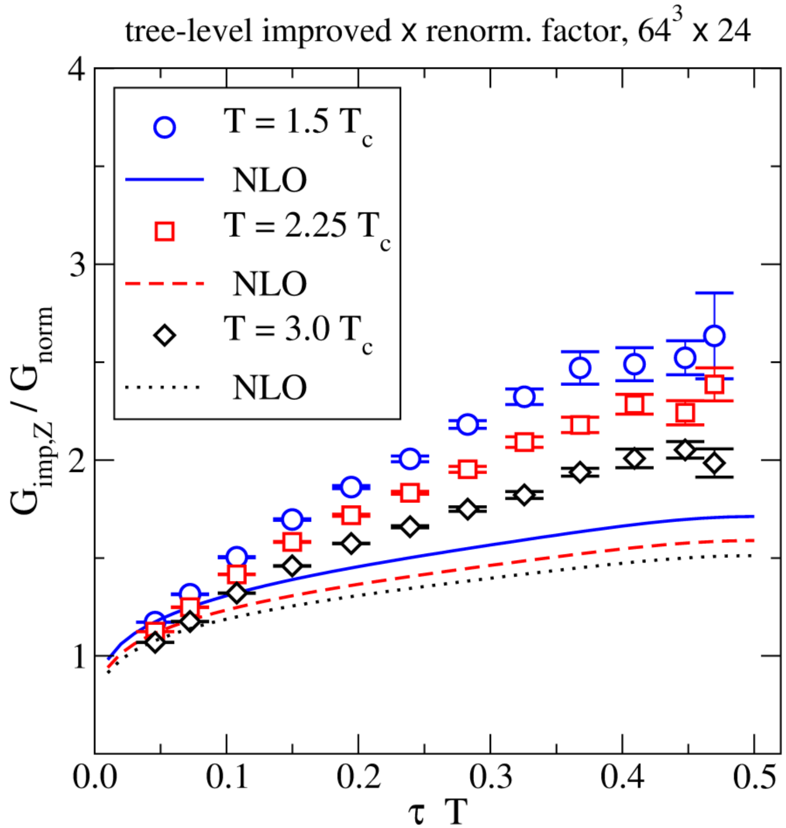

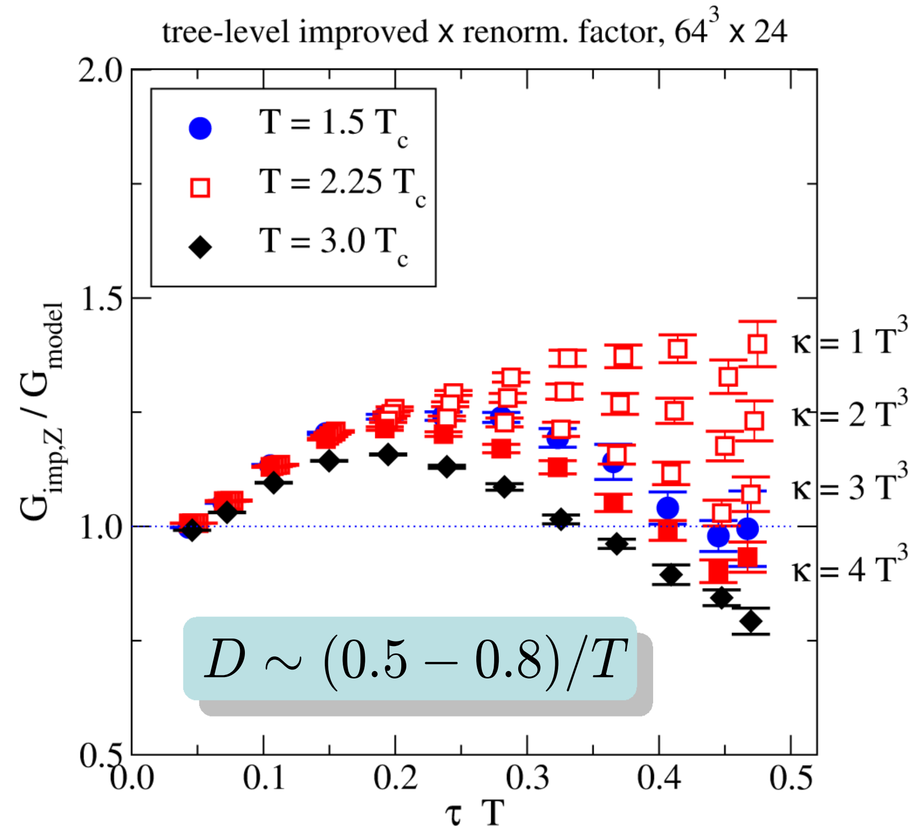

Model spectral function: transport contribution + NLO [Y.Burnier et al. JHEP 1008 (2010) 094)]

$$\rho_{\text{model}}(\omega) \equiv \max\left\{\rho_{\text{NLO}}(\omega), \frac{\omega\kappa}{2T}\right\}$$

$$G_{\text{model}}(\tau) \equiv \int_0^\infty \frac{d\omega}{\pi}\rho_{\text{model}}(\omega)\frac{\cosh\left(\frac{1}{2}-\tau T\right)\frac{\omega}{T}}{\sinh\frac{\omega}{2T}}$$

Still large uncertainties but very promising

→ thermodynamic+continuum limit needed

→ more constraints on the spectral function

→ other operators and observables from EFT?

Conclusions

Charmonium:

[F.Karsch, E.Laermann, S.Mukherjee, P.Petreczky, arXiv:1203.3770]:

"… the change in the behavior of the charmonium screening masses around $T=1.5T_c$ is likely due to the melting of the meson states."

[H.T.Ding, OK et al., arXiv:1204.4945]:

Detailed knowledge of the **vector correlation function** at various T in quenched QCD

→ **continuum extrapolation** of correlation function still needed!

Results so far depend on MEM analysis → Ansätze more difficult due to m_q dependence

→ **Heavy quark diffusion constant:** $2\pi DT \approx 2$

→ **No signs for bound states at and above 1.46 T_c**

Conclusions

Effective Field Theory on the Lattice:

[FASTSUM Collaboration, G.Aarts et al., JHEP11(2011)103]:

NRQCD study of bottomonium:

survival of ground state till 2 T_c

melting of excited states above T_c

qualitative agreement with CMS results

[A.Francis,OK,M.Laine,J.Langelage, arXiv:1109.3941 and
Banerjee et al., PRD85(2012)014510]:

HQET operator for Heavy Quark Momentum Diffusion constant

first results promising

Need to understand systematic uncertainties in both approaches in more detail