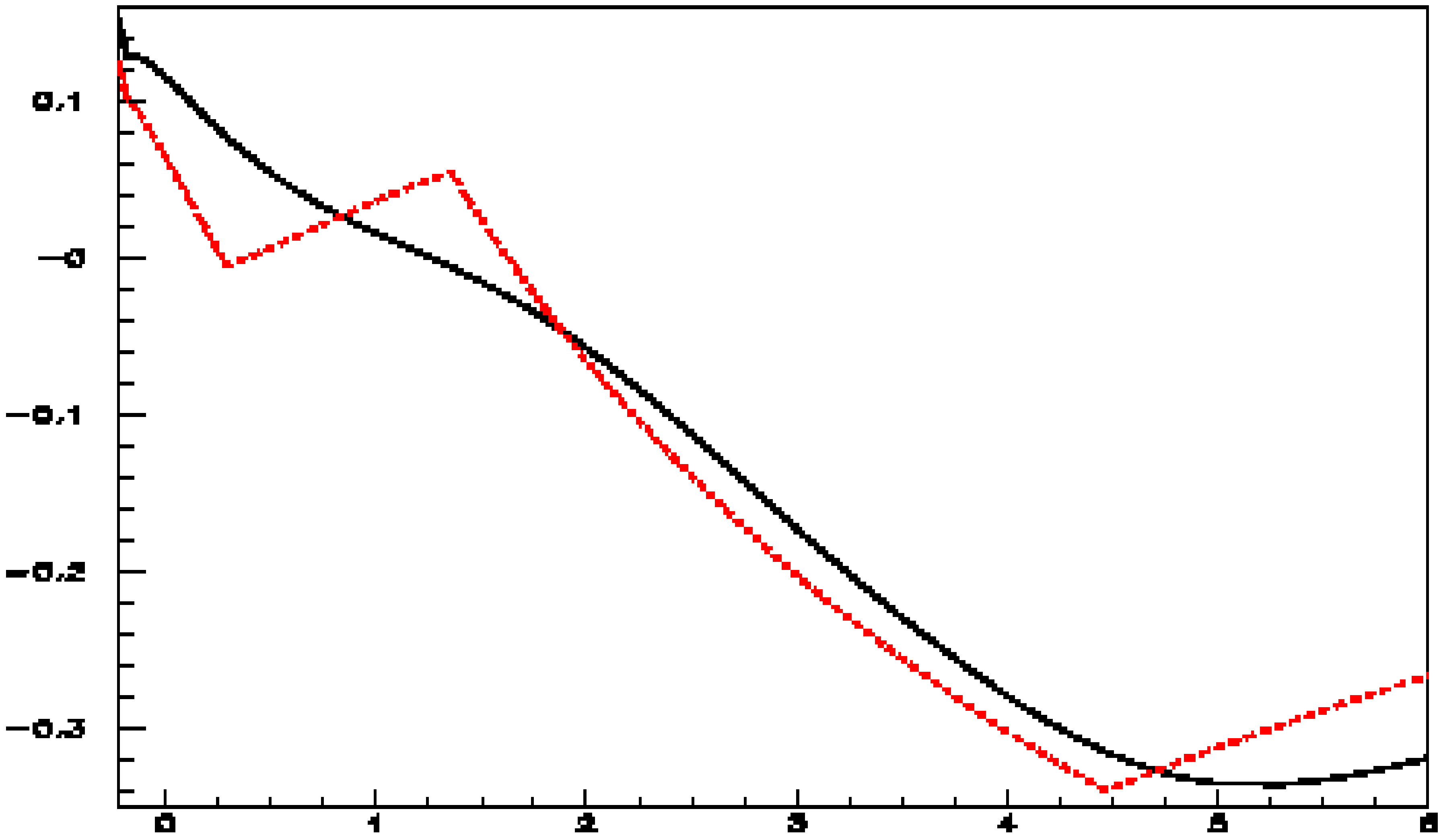
Six Flavour PDF scheme

- Motivations
- DGLAP equations
- Modifying DGLAP equations (intuitive approximation: CFNS)
- Derivation of LO massive splitting functions
- Possible extension to NLO case.
- CFNS massless comparison

Motivations

- Present observable (structure functions , cross sections) theoretical predictions are essentially based on Massless PDFs
- Schemes induce better and better modeling of the observables but they induce also discontinuities which may lead to doubts on the whole methodology
- Heavy quarks do exist before they get a pdf even if in some instances some of their observables may be built out of light partons



$x=.1$ $F_3^{e^- \to \nu X}$ versus $\log_{10}(Q^2)$ black CFNS full massless

Constraints on the method

- Have always six flavors but with heavy quarks contributions increasing with Q^2
- Modify splitting functions … to satisfy heavy quarks kinematic constraints and continuity
- Do all that coherently, satisfying sumrules and recovering massless situation for $Q^2 \to \infty$

DGLAP equations

- The electron proton reaction is :
  $e(l) + p(p) \rightarrow e(l-q) + X(l+q)$
  Quadrimomenta are within ().
  $Q^2 = Sxy = -q^2$ $x = Q^2/(2pq)$ $y = pq/pl$ $S = (l+p)^2$
  $W^2 = (p+q)^2 = M^2 + Q^2(1/x - 1)$ $t = \ln(Q^2)$
- Parton o may be kicked out of the target if $W > 2m_o$ where m_0 is the parton *o* mass. This translate in a kinematical limit $x < l_o$ with
  $l_o^{-1} = 1 + 4\, m_0^2/Q^2$
- Light partons fulfill always this condition but heavy ones do only for $Q^2 \rightarrow \infty$

Parton distributions, functions of x and Q^2 ,are noted by the name of the parton species $p=g,d,\bar{d},u,\bar{u},s,\bar{s},c,\bar{c},b,\bar{b},t,\bar{t}$ … and for the quarks $d^{\pm} = d \pm \bar{d}$, …

DGLAP equations are:

$\partial o/\partial t = \Sigma_i P_{oi} \otimes i$

P are splitting kernels also functions of x and Q^2 ,$\otimes$ stands for the convolution integral ,i and o run on the 1+2 N_f partons species.

N_f is the number of active flavors.
Usually it is the number of quark families with $Q^2 > m^2_0$

DGLAP + integro differentiel subsystem

$$\partial g/\partial t = P_{gg} \otimes g + \Sigma_r P_{gr} \otimes r^+$$
$$\partial q^+/\partial t = P_{qg} \otimes g + \Sigma_r P_S \otimes r^+ + P^+_{NS} \otimes q^+$$

Momentum fractions $p = \int_o^1 x\, p(x)\, dx$

$$\partial g/\partial t = P_{gg}\, g + \Sigma_r P_{gr}\, r^+$$
$$\partial q^+/\partial t = P_{qg}\, g + \Sigma_r P_S\, r^+ + P^+_{NS}\, q^+$$

Modifying DGLAP equations

- The idea is to modify the kernels in order to satisfy simultaneously the 3 kinematical constraints
  $x < l_i$, $x < x_o$, $x < l_o$
  P_{oi} is the change to parton *o* at Bjorken x radiated by parton *i* at x/t.
  Problematic cases are for *o* heavier than *i* like for c -> b

- Replace : P_{oi} *by* $P_{oi} \otimes \delta_o$ with $\delta_o = \delta(x - I_o)$
- Or replace $P(x)$ by $P(\xi)$ with $\xi = x/I_o$
- Consequences: $\partial g/\partial t = P_{gg}\ g + \Sigma_{gr}\ P_{gr} r^+$
  $\partial q^+/\partial t = I_q (P_{qg}\ g + \Sigma_r P_S r^+) + P^+_{NS}\ q^+$
  With $I_q = \int x \delta_q \, dx$
- This leads to $\boldsymbol{N_f = 3 + I_c + I_b + I_t}$
  Sum of the phase spaces of the quarks
- N_f has to be used consistently in all the P
  Also in the β function (β_0 appears in P_{gg})

Modified DGLAP equations

- **DGLAP becomes**
  $\partial g/\partial t = P_{gg} \otimes g + \Sigma_{gr} P_{gr} \otimes r^+$
  $\partial q^+/\partial t = \delta_q \otimes (P_{qg} \otimes g + \Sigma_r P_S \otimes r^+) + P^+_{NS} \otimes q^+$
- Sum of quark equations shows that g and Σ decouple from the others
- Non singlets cannot be defined as before but one may use Σ_{light} to define them. Light's are still decoupled and heavy's become decoupled when they are fully active.

- For $l_q \to 0$ corresponding equation get decoupled (Appelquist-Carazzone theorem)
- Several equivalent linear combinations of the new equations may be used
- This (intuitive) method in the following will be referred as CFNS

Coefficient functions

- Structure functions are obtained by:
  $F^{a}{}_{o} = \Sigma_i C_{io} \otimes i$
  C are the coefficient functions
- This has exactly the same structure as the DGLAP equations and is nothing more than a change of scheme ($\overline{MS}$ to DIS for F_2) so it is natural to use the same procedure than for the DGLAP equations .

Derivation of LO massive structure functions

- Uses method of G.Altarelli and G.Parisi (1977)
- $\boldsymbol{F} = e^2_h \alpha_s \xi\, \mathbf{C^0_g} \otimes g$ is the α_s order massive structure function S.Riemersma et al. PLB 347 where $\xi = Q^2/m^2_h$,t= $\ln(\xi)$
- $\partial \boldsymbol{F} / \partial t = e^2_h \alpha_s \partial (\xi\, \mathbf{C^0_g}) / \partial t \otimes g$
- $\boldsymbol{F} = e^2_h h^+$
- $\partial \boldsymbol{F} / \partial t = e^2_h \alpha_s \boldsymbol{P_{hg}^0} \otimes g$ From massive DGLAP
- $\boldsymbol{P_{hg}^0} = (\xi\, \mathbf{C^0_g}) / \partial t$

- At LO the only reaction is g $\rightarrow$ h $\bar{h}$ but not only heavy quarks are produced also gluon disappear . This has to be taken care of by a change in $\boldsymbol{P_{gg}^{0}}$ of β_0 the coefficient of its δ function: It has to be such that the momentum sum rule is satisfied.
- It imply as in CFNS a new definition of N_f

P_{hg}^{0} *Calculation*

- Define: $\eta = \xi(z^{-1} -1)-1$,$u = 4z\xi^{-1}$,$\beta = (1-u/(1-z))^{-1}$,$L=\log((1+ \beta)/(1-\beta)$
- $\xi\, \mathbf{C^0_{Tg}} = 2\pi\, T_f\, z[-2\{(1-2z)^2+(1-z)u\}\beta + \{2(1-z)^2 +2z^2 +2(1-z)u-u^2\}L]$
- $\rightarrow 2\pi\, T_f\, z\, \{2(1-z)^2 +2z^2\}t$ for $\xi \rightarrow \infty$
- $\partial\, \xi\, \mathbf{C^0_{Tg}})/ \partial t = 2\pi\, T_f\, z[2\{ (1-z)u\}\beta -2\{(1-2z)^2 +(1-z)u\}(1- \beta^2)/ \beta - \{2(1-z)u-2u^2\}L + \{2(1-z)^2 +2z^2 +2(1-z)u-u^2\})/ \beta]$
- $\rightarrow 2\pi\, T_f\, z\, \{2(1-z)^2 +2z^2\}$ for $\xi \rightarrow \infty$

C_{Lh}^{0} Longitudinal coefficient function

- $\partial(\xi\, C^0_{Lg})/\partial t = 8\pi\, T_f\, z^2[(1-z)(1-\beta^2)/\beta - u(L-1-/\beta)]$ $\rightarrow 0$ for $\xi \rightarrow \infty$
- Using $\partial F_L/\partial t = e^2_h\, \alpha_s\, C^0_{Lh} \otimes P_{hg}^0 \otimes g$ and $C^0_{Th}=1$
- one find $C^0_{Lh}=[\partial(\xi\, C^0_{Tg})/\partial t]^{-1} \otimes \partial(\xi\, C^0_{Lg})/\partial t$
- Where $[\partial(\xi\, C^0_{Tg})/\partial t]^{-1} \otimes [\partial(\xi\, C^0_{Tg})/\partial t] = \delta(z-l_h)$
- Notice using F_2 instead of F_T will give a different result for the two coefficient functions (and N_f would not be monotonous in Q^2)

About resumation

- To get this result (equivalent to a resumation) a derivation has been made followed by an integration but by the mean of an integro-differential equation.
- This probably has a connection with the procedure for Feynman graph of propagator derivation with respect to a parameter followed by an integration introducing and unknown constant
- Like for light DGLAP the constant is the initial pdfs

Extension to NLO case

- α_s^2 order massive structure function also exist S.Riemersma et al. PLB 347 and it is tempting to use the same method but things are much more involved.
- For F_T and F_L there is one C^1_g but two C^1_q D^1_q coefficients functions with different charge factors
- There is 2 splitting functions P_{hg}^1 ,P_{hq}^1 and 4 coefficient functions to be determined by the 4 C^1 . This is analogous to the LO case where C^0_{Th}= 1 has been imposed.

- **C^1** ‘s are partly tabulated in PLB347 which render their derivation and extrapolation to high ξ difficult .
- **C^n** may contain factors up to t^{n+1} which may induce higher logarithmic divergences

cfns massless comparison

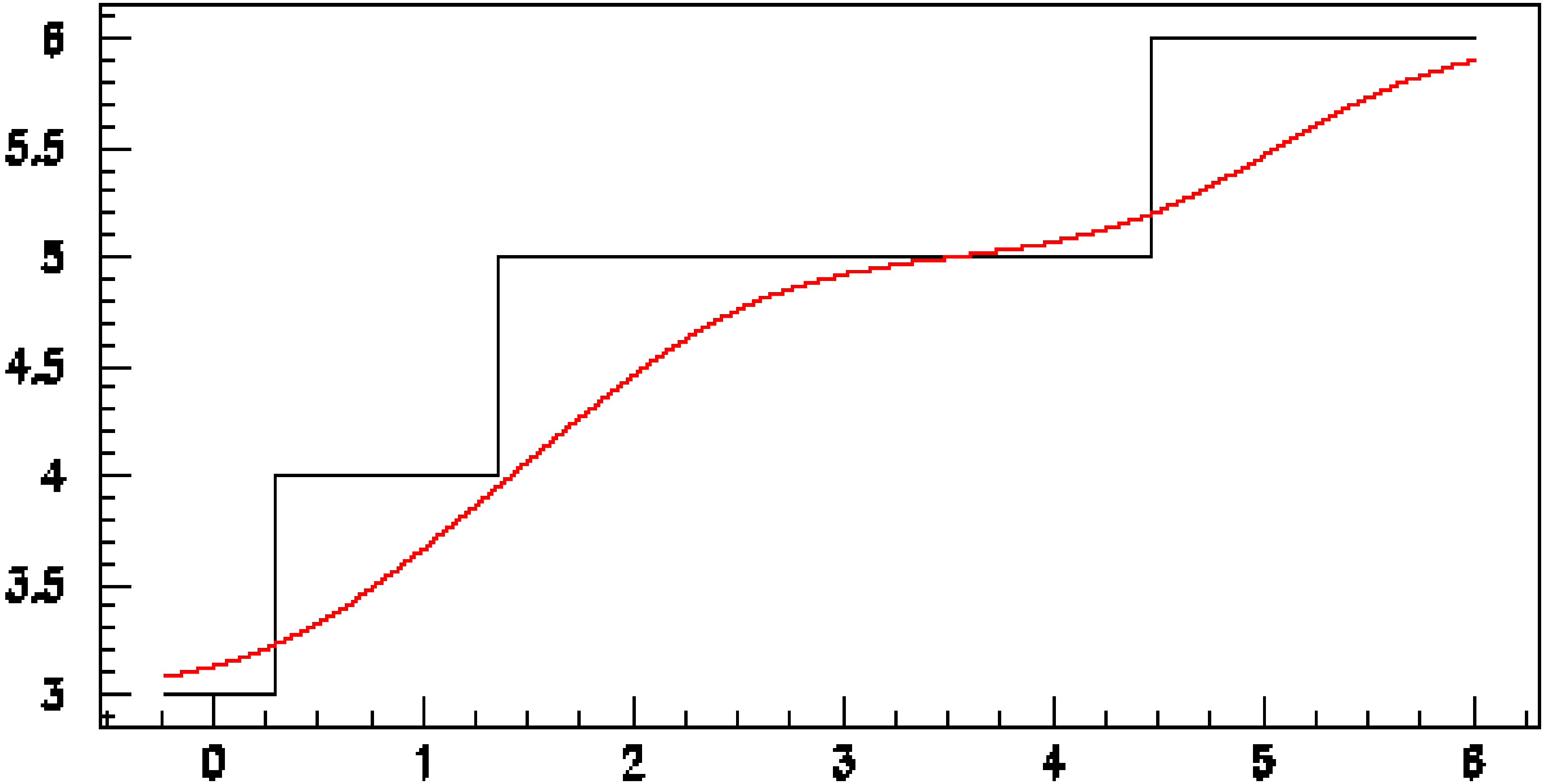
- I am of course aware that any comparison should be done at the NNLO level between this new scheme and the other outstanding schemes at least at the pdf level and probably even more, at the structure function level.
- But for that two evolution codes are needed both running at NNLO, one accepting the usual schemes and one built for the new scheme

Kinematical range used

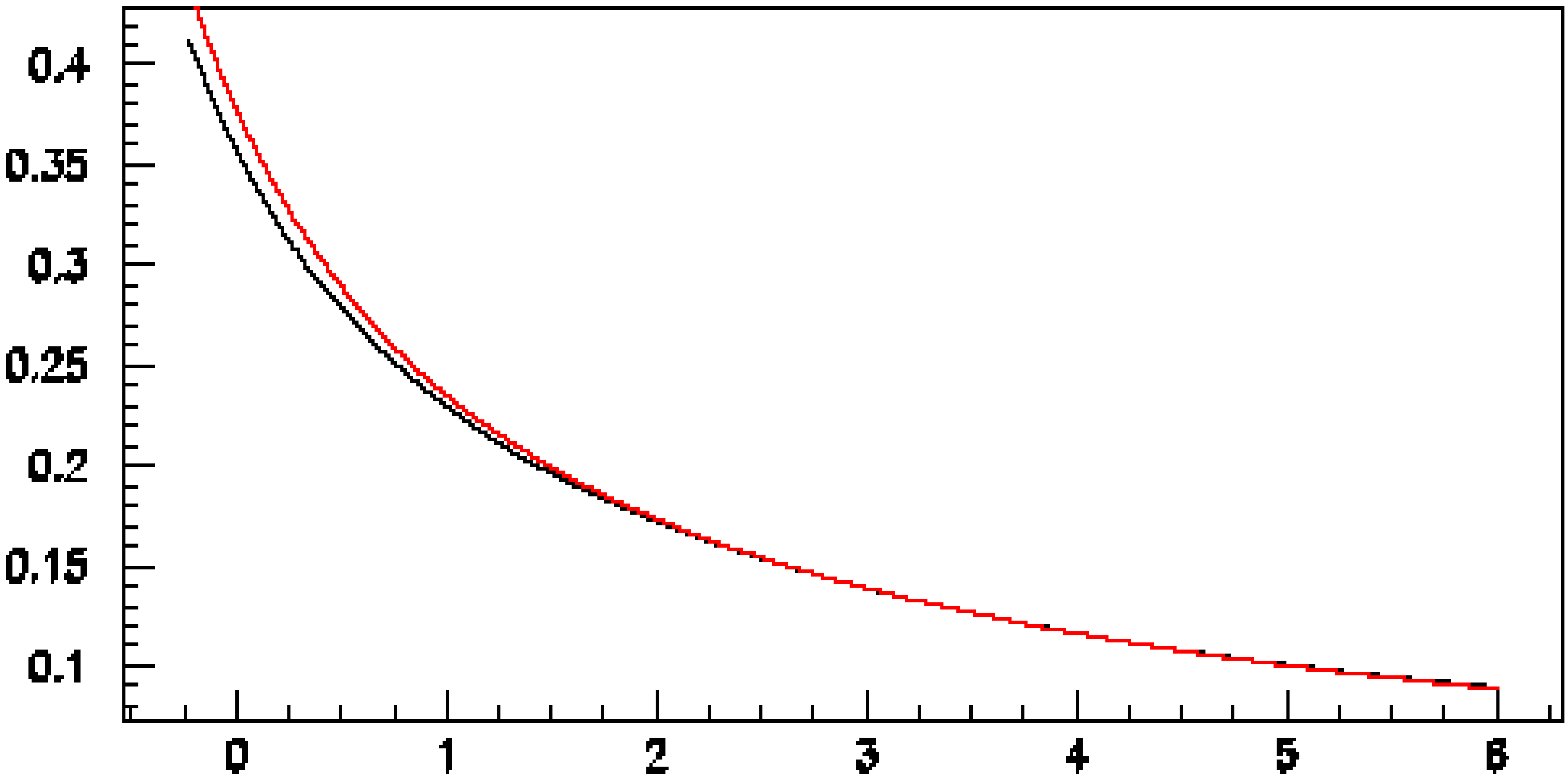
- Very often the start of evolution Q^2_{in} is chosen just below the charm mass squared in order to define pdf inputs only for the light partons. But for if heavy quarks are present when the kinematical range permits and in order to have only light quarks present at input, I have used Q^2_{in}=.6 Gev^2 low enough to justify neglecting all the heavy quarks at input. Needless to say that at so low a Q^2 predictivity is completely absent but it is a parameter less way to get a sensible charm when out of the non perturbative region.

Data sample for fit

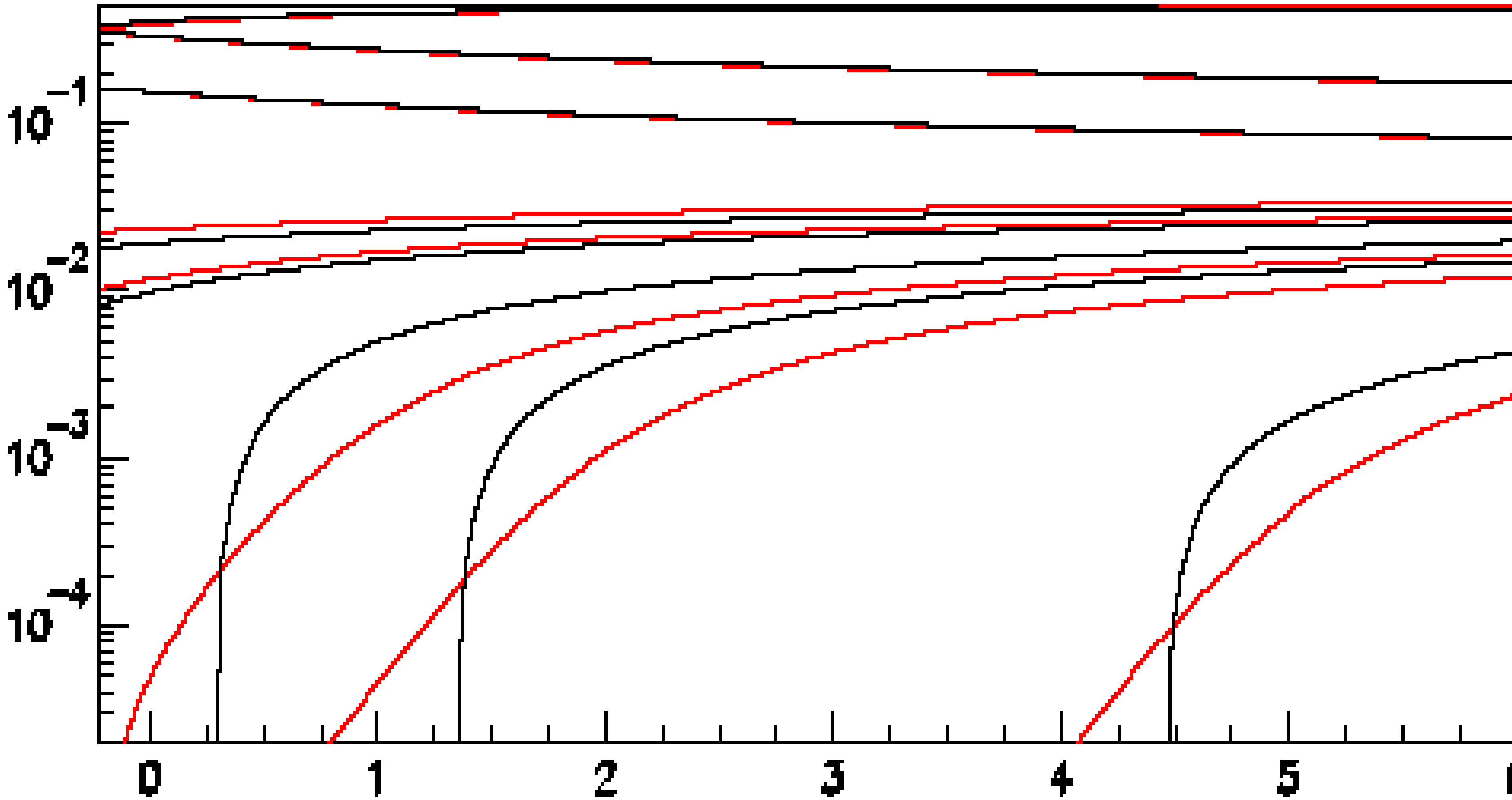
- A data sample made of about 1900 F2 or cross section measurements extracted from NMC BCDMS on protons and H1 preliminary is used to fit the input pdfs independently for CFNS and for massless scheme.
- The fitted Pdfs are G U_v D_v $\bar{U}=\bar{D}=1.6\,\bar{S}$
- The aim of this exercise is to show what kind of new features might be seen and how far they extend away from the transition points of the other sheme.



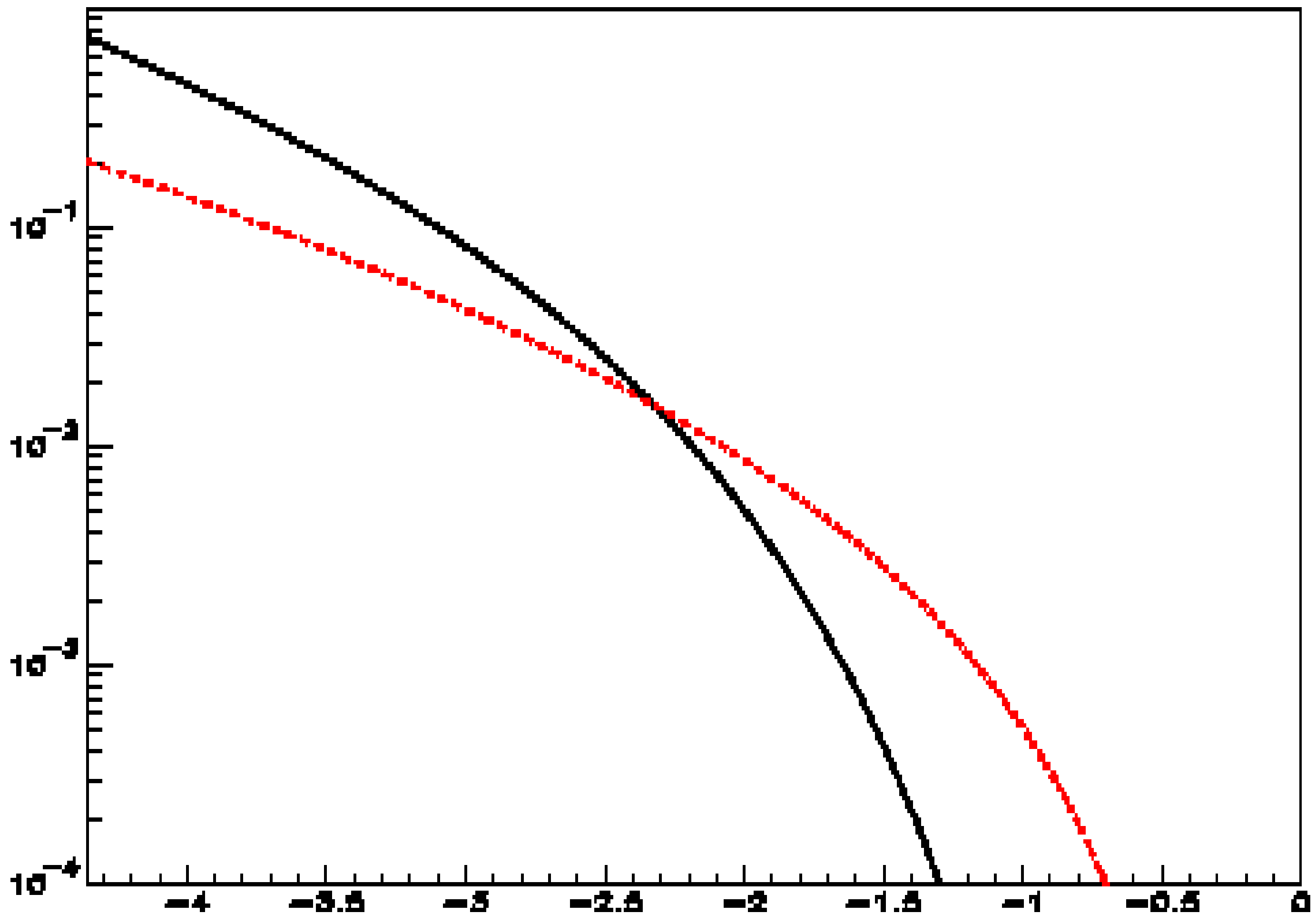
N_f $(\log_{10}Q^2)$ black massless red massive



$\alpha_s(\log_{10}Q^2)$ black massless red massive



Momentum fractions($\log_{10}Q^2$) of g, u_{val}, d_{val} $\bar{u}=\bar{d}, \bar{s}, \bar{c}, \bar{b}, \bar{t}$
black massless red massive



At Q^2 =45000 where cfns and massless momentum fractions are equal

top log(x) distribution black massless and red cfns

QCD representation of pdfs and kernels

- Pdfs are represented by n vectors in the vectorial spce defined by the base
- Use an x network such that $x_{i+1} = s\ x_i$ $x_n = 1$ $s > 1$ and base functions such that $\phi_{j-1}(x)=\phi_j(sx)$
- Kernels are upper triangular band matrices $K=\Sigma_{k=0}^{n}\ m_k\ B_k$ where B_k matrix elements are $B_k(i,j) = \delta_{i+k,j}$ with $k>0$
- $B_i\ B_j = B_j\ B_i = B_{i+j}$
- For the kernels following operations are easy and fast: vector multiplication, kernel multiplication, inversion, square root, exponentiation.

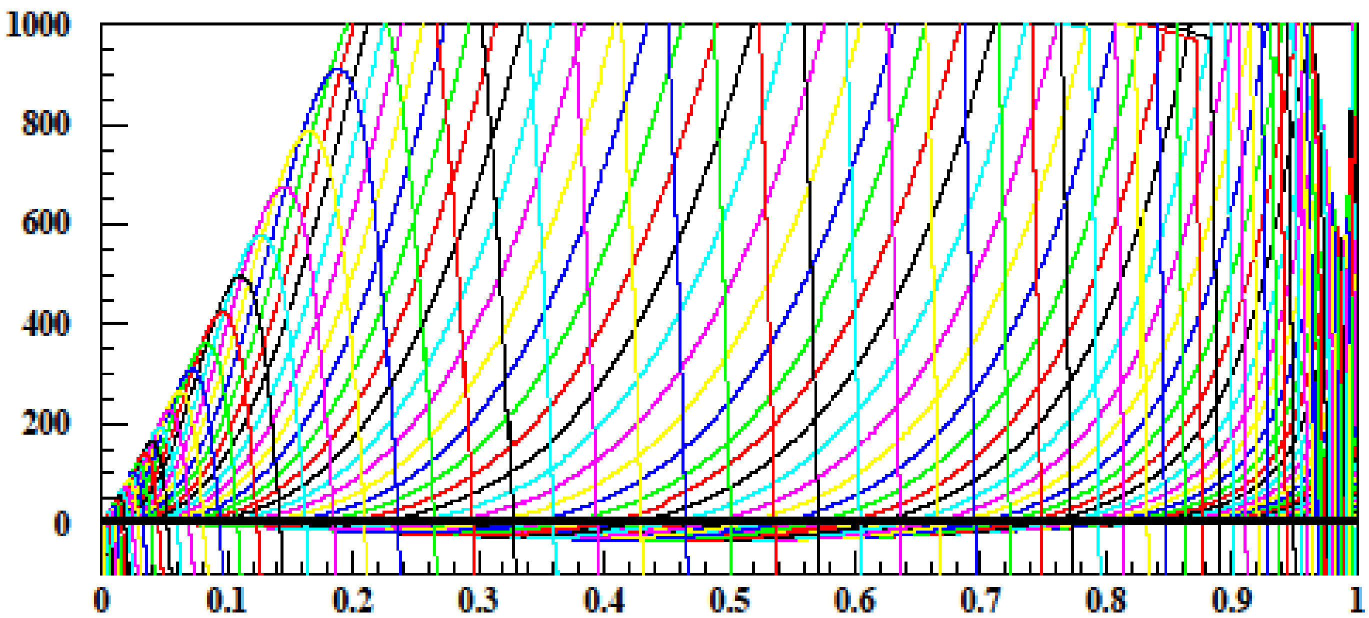
References

- G. Altarelli anf G. Parisi Nucl.Phys B128(1977) 298-318
- S. Riemersma and all PLB 347 (1995) 143-151

- FIN

NLO technical difficulty

- C^1 's are partly tabulated in PLB347 which render their derivation and extrapolation to high ξ difficult . Below is shown the second derivative of C^0_{Tg} versus z from ξ =0.04 to ξ =10^{11}



- At NLO there is 2 splitting functions P_{hg}^{1}, P_{hq}^{1} and 2 coefficient functions to be determined by the 4 C^{1}. This is analogous to the LO case where $C^{0}_{Th} = 1$ has been imposed.
- The best choice would be to replace in the equation $F = \alpha_s \xi\, C^{0}_{p} \otimes \mathbf{p} + \alpha^2_s \xi\, C^{1}_{p} \otimes \mathbf{p}$ where **p** is a vector in the parton pdf space by $Z \otimes \mathbf{p}_{DIS}$. **Z** is a matrix depending upon the massless C^{1}.
- In this way DIS splitting kernels will be found which may be reverted back to $\overline{M}\ \overline{S}$ by defining (arbitrarily) a $\mathbf{Z}_{massive}$ going to $\mathbf{Z}_{massless}$ for $\xi \rightarrow \infty$
- C^{n} contains factors t^{n+1} which induce divergences. They are cancelled by derivation only for t. But the C^{1} seem to have no t^{2} terms as seen below

- In this way DIS splitting kernels will be found which may be reverted back to $\overline{\mathrm{M}}\ \overline{\mathrm{S}}$ by defining (arbitrarily) a $\mathbf{Z}_{\text{massive}}$ going to $\mathbf{Z}_{\text{massless}}$ for $\xi \rightarrow \infty$

Extension to NLO case

- α_s^2 order massive structure function also exist S.Riemersma et al. PLB 347 and it is tempting to use the same method but things are much more involved.
- The best choice would be to replace in the equation $\boldsymbol{F} = \alpha_s \xi\, \mathbf{C}^{\mathbf{0}}{}_{\mathbf{p}} \otimes \mathbf{p} + \alpha^2{}_s \xi\, \mathbf{C}^{\mathbf{1}}{}_{\mathbf{p}} \otimes \mathbf{p}$ where **p** is a vector in the parton pdf space by $\mathbf{Z} \otimes \mathbf{p}_{\mathrm{DIS}}$. **Z** is a matrix depending upon the massless $\boldsymbol{C}^{1}$.

- $\mathbf{C^1}$'s are partly tabulated in PLB347 which render their derivation and extrapolation to high ξ difficult . Below is shown the second derivative of $\mathbf{C^0_{Tg}}$

Derivation of LO massive splitting functions

Uses method inspired by G.Atarelli and G.Parisi
Asymptotic Freedom in Parton language N.P. B126(1977)

$\mathcal{F} = e_h^2 \xi \alpha_s C_g^0 \otimes g$ is the α_s order massive structure function
S.Riemersma et al. PLB 347 (1995) 143-151

$\frac{d\mathcal{F}}{dt} = e_h^2 \alpha_s \frac{d(\xi C_g^0)}{dt} \otimes g$ (keeping α_s fixed)

where $\xi = \frac{Q^2}{m_h^2}$, $t = \ln(\xi)$

On the other hand
$F = e_h^2 q_h$

$\frac{dF}{dt} = e_h^2 \alpha_s \bar{\mathcal{P}}_{hg}^0 \otimes g$

From this one may deduce:
$\bar{\mathcal{P}}_{hg}^0 = \frac{d(\xi C_g^0)}{dt}$

Where all the derivatives are taken at z fixed.

At LO the only reaction is $g \to h\bar{h}$

but not only heavy quarks are produced but also gluons disappear.

This has to be taken care of by a change in $\mathcal{P}_{gg}$ of β_0

the coefficient of the δ function : It has to be such that the momentum

sumrule is satisfied. It imply a calculable modification of the flavor number N_f

P_{gg} calculation

$$\xi c^{(0)}_{T,g}(\eta,\xi) = \frac{\pi}{2} T_f 4z\left[-2\left\{(1-2z)^2+(1-z)u\right\}\beta + \left\{2(1-z)^2+2z^2+2(1-z)u-u^2\right\}L\right]$$

$$\to \frac{\pi}{2} T_f 4z[2(1-z)^2+2z^2]t \text{ for } \xi \to \infty$$

Using $u = 4z\xi^{-1}$, $\beta = (1-\frac{u}{1-z})^{-1}$, $L = \ln\frac{1+\beta}{1-\beta}$

$$\frac{\partial \xi c^{(0)}_{T,g}(\eta,\xi)}{\partial t} = \frac{\pi}{2} T_f 4z\left[2(1-z)u\beta - 2\left\{(1-2z)^2+(1-z)u\right\}\frac{1-\beta^2}{2\beta} + \left\{-2(1-z)u+2u^2\right\}L + \left\{2(1-z)^2 + 2z^2+2(1-z)u-u^2\right\}/\beta\right]$$

Longitudinal coefficient function

SHOP.4

$$\xi c^{(0)}_{L,g}(\eta,\xi) = \frac{\pi}{2} T_f 4^2 z^2 \left[2\beta(1-z) - uL \right]$$

$$\frac{\xi c^{(0)}_{T,g}(\eta,\xi)}{\partial u} = \frac{\pi}{2} T_f 4^2 z^2$$

SHOP.6

0.0.1 Longitudinal structure function

In contrast to the massless case massive quarks exhibit a longitudinal structure function even at leading order so it is necessary to define coefficient functions at this order cf([?]):
$c^0_{T,h} = \delta(1-x), c^0_{L,h}$ (= 0,for massless case).
Using

$$\frac{d\mathcal{F}_L}{dt} = e_h^2 a_s \frac{d(\xi C^0_{Lg})}{dt} \otimes g \qquad (7)$$

One find

$$c^0_{L,h} = [\frac{d(\xi C^0_{Tg})}{dt}]^{-1} \otimes \frac{d(\xi C^0_{Lg})}{dt} \qquad (8)$$

Where the inverted kernel $\otimes$ the kernel itself gives the heavyside function H(l-x)

About ressumation

SHOP.5

Notice that to get this result (equivalent to a ressumation) a derivation has been made followed by an integration but by the mean of an integro- differential equation. This probably has a connexion with the procedure for Feynmann graph of propagator derivation with respect to non integrated parameters followed by an integration introducing an unknown constant. Here the constant is the set of initial pdfs.

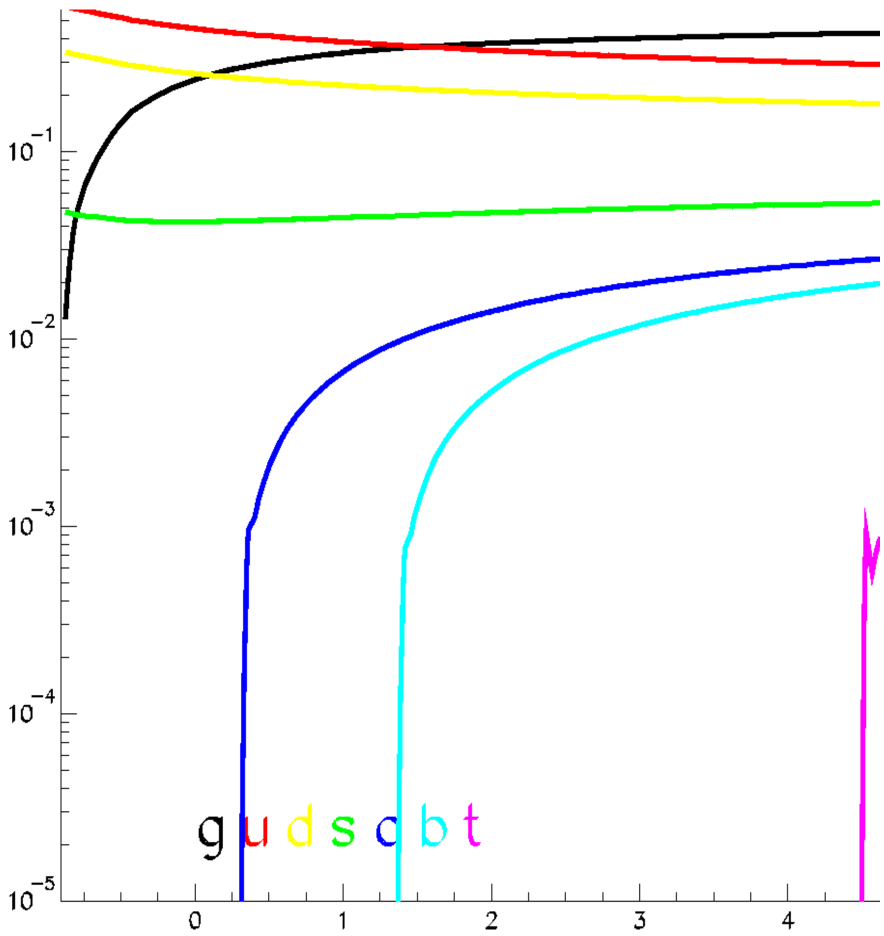
Theoretical arguments

- It is important to note moreover that the ideas presented here are not new:
- $\xi = x/l_0$ is the scaling variable used in H. Georgi and H.D. Politzer Phys Rev D 15,7 (1976) and many other papers
- H. Georgi and H.D. Politzer use anomalous dimensions variable with Q^2. Anomalous dimensions leading to splitting functions their arguments should hold here.
  For this they advocate $l_o = Q^2/(Q^2+2mQ^2)$

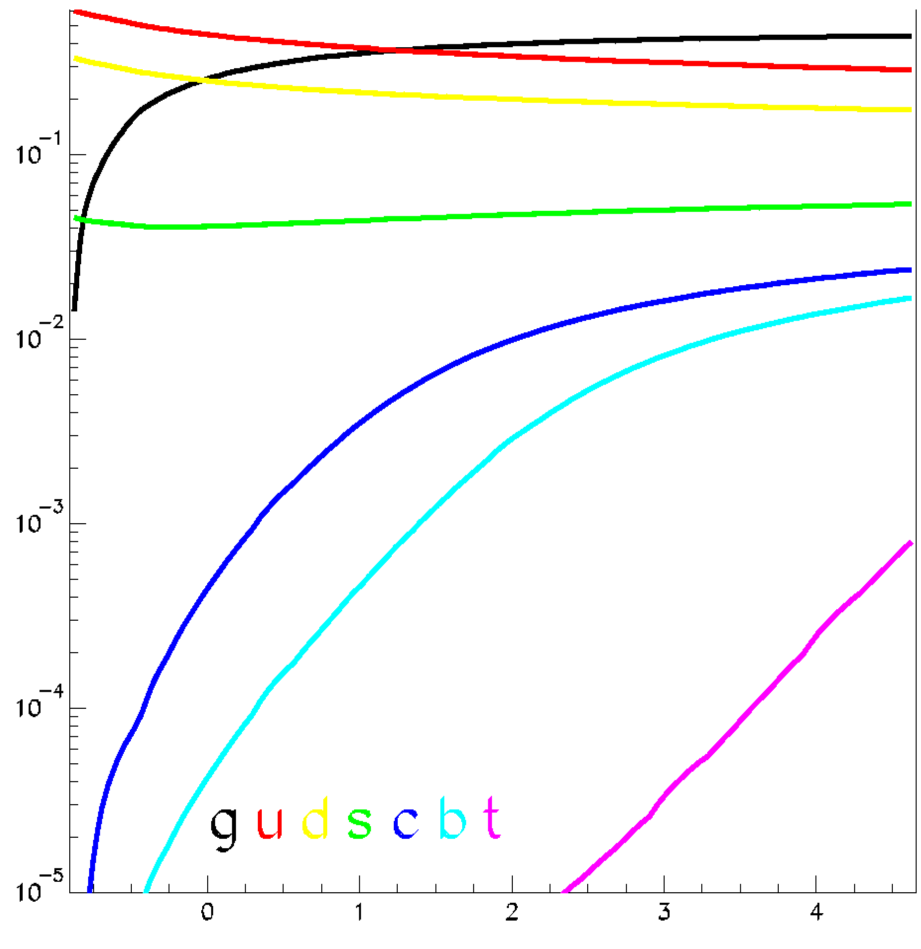
- They present a β_o variable with Q^2.
  Here it is $l_o = Q^2/(Q^2+5mQ^2)$
- S.Brodsky et al arXiv:hep-ph/9906324 have N_f order dependent
- Last reference to this is D.D. Dietrich arXiv:0908.1364 [hep-th]
- As already stated the procedure leading to satisfaction of the kinematical constraints as been used lately for coefficient functions in GM-VFNS schemes
  R.S. Thorne arXiv:1006.5925 [hep-th]

Momentum fraction

massless



cfns



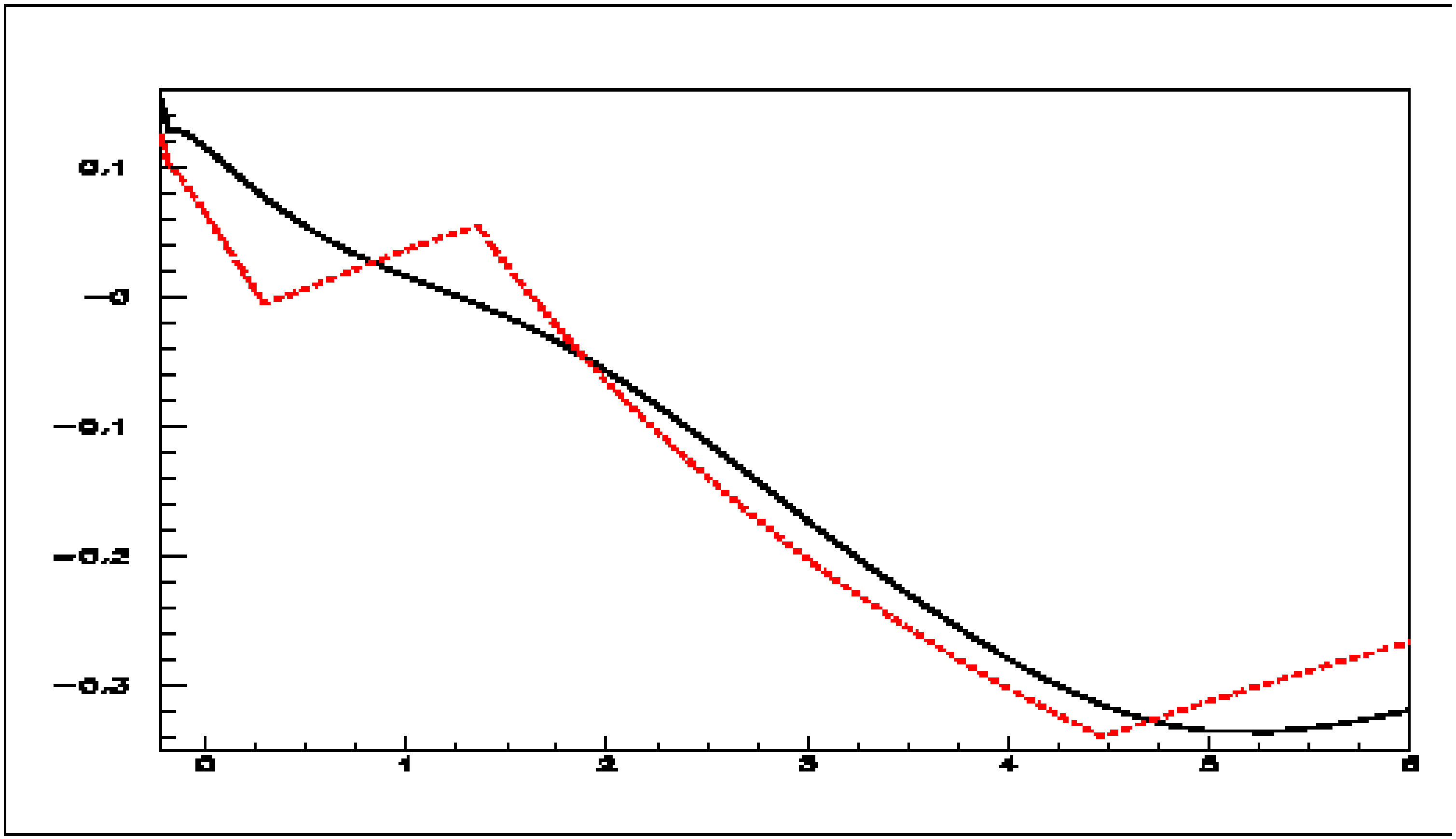
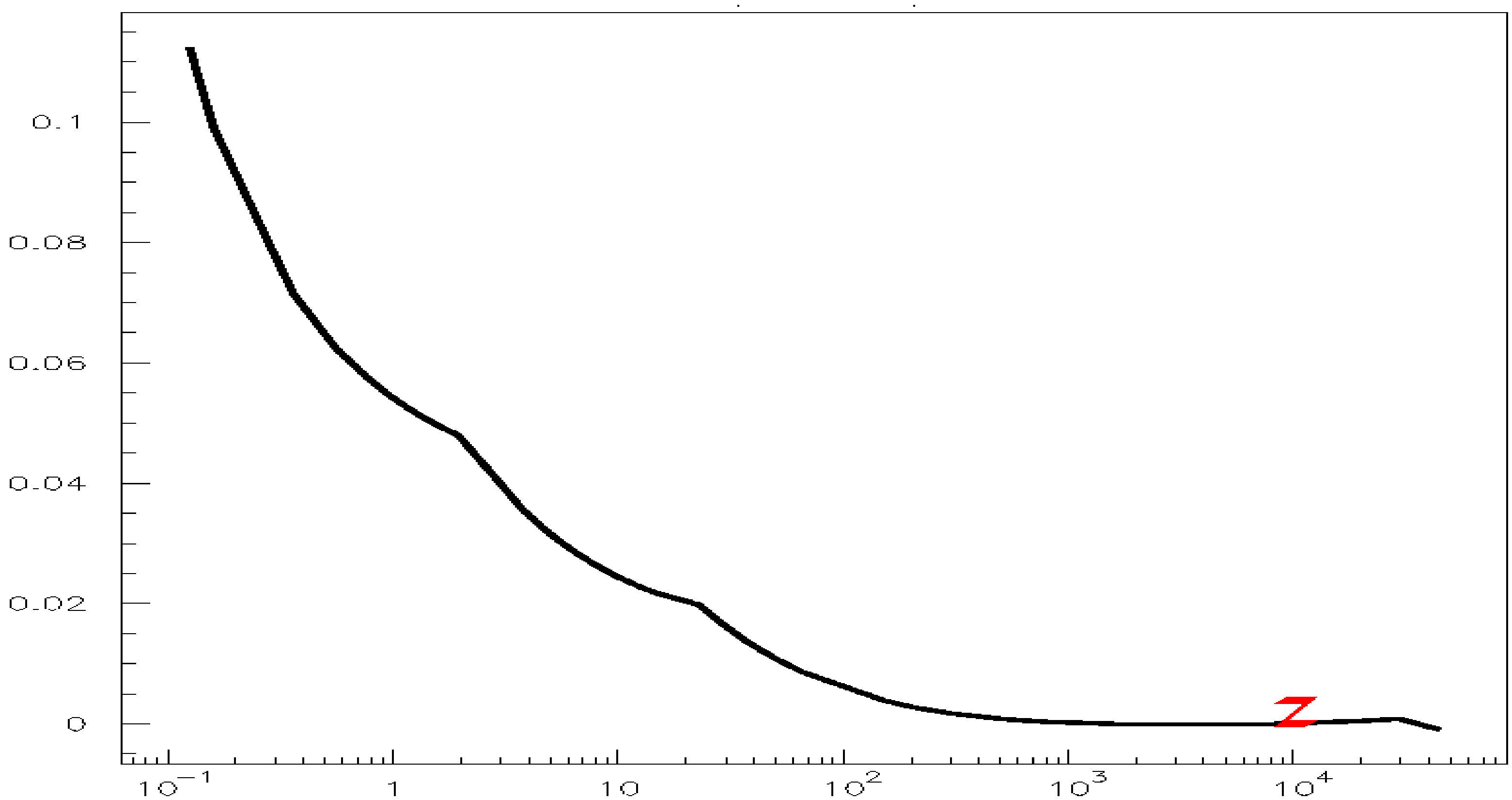


Figure 1: $\mathcal{F}_3^{e^- p \to \nu X}$ versus $\log 10 Q^2$ cfns: black full, massless: red dotted.

OLD (dis_keep)

- Notice also how small are the differences between massless and continuous behavior for α_s .
- Apart numerical differences (which may be eventually cured by a judicious choice of ϕ) the outcome is similar for the various l_0 used

α_s cfns massless relative difference



Cfns potential weak points

- It essentially assumes that DGLAP equations and all its components should exist even if they are calculable only when quarks are light or quasi light, so it interpolates coherently between those cases and full scale range.
- Other schemes have also their approximations
- I am not able to decide what is the best
- Renormalization group should decide and I am not able to look into it.

Cfns strong points

- It does not mix up different α_s orders as do mixed schemes.
- Heavy quarks participate to the evolution when they start to appear, that is at the beginning and at very low x and even at leading order in α_s.
- There is no internal and external partons, only internals.

- It should be better at the small x due to evolution
- It covers the charm-bottom region where they are both opening up which is not yet the case in usual schemes

Parton distributions are noted by the name of the parton species $p = g, d, \bar{d}, u, \bar{u}, \ldots$ and for the quarks $d^{\pm} = d \pm \bar{d}, \ldots$ are also introduced.

DGLAP equations read :

$$\frac{\partial o(Q^2)}{\partial ln(Q^2)} = \sum_i \mathcal{P}_{oi} \otimes i(Q^2)$$

$\mathcal{P}$ are splitting functions, i and o run on the $1 + 2\mathcal{N}_f$ partons species. $\mathcal{N}_f$ refers to the number of active flavors and is the main problem of the overall approach: Usually $\mathcal{N}_f (= N_f$ integer) is taken as the number of quark families such that $Q^2 > m_o^2$.

DGLAP equations separate into two independent subsystems when making use of q^+ and q^- ,q^+ one is:

$$\frac{\partial g(Q^2)}{\partial ln(Q^2)} = \mathcal{P}_{gg} \otimes g(Q^2) + \sum_{q=d}^{t} \mathcal{P}_{gq} \otimes q^+(Q^2)$$

$$\frac{\partial q^+(Q^2)}{\partial ln(Q^2)} = \tilde{\mathcal{P}}_{qg} \otimes g(Q^2) + \mathcal{P}_{NS}^+ \otimes q^+(Q^2) + \sum_{r=d}^{t} \tilde{\mathcal{P}}_S^+ \otimes r^+(Q^2)$$

Where

$$\mathcal{P}_{NS}^{\pm} = \mathcal{P}_{qq}^V \pm \mathcal{P}_{q\bar{q}}^V,\ \mathcal{P}_S^{\pm} = \mathcal{P}_{qq}^S \pm \mathcal{P}_{q\bar{q}}^S,\ \tilde{\mathcal{P}}_{qg} = \frac{\mathcal{P}_{qg}}{\mathcal{N}_f},\ \tilde{\mathcal{P}}^{\pm}{}_S = \frac{\mathcal{P}_S^{\pm}}{\mathcal{N}_f}.$$

Kernels $\mathcal{P}$ are polynomials in $a_s = \frac{\alpha_s}{4\pi}$ and $\mathcal{N}_f$ as follows:

$$
\begin{array}{lllllll}
\mathcal{P}_{gg} = a_s(\mathcal{P}_{gg}^{00} & +\mathcal{N}_f\mathcal{P}_{gg}^{01}) & +a_s^2(\mathcal{P}_{gg}^{10} & +\mathcal{N}_f\mathcal{P}_{gg}^{11}) & +a_s^3(\mathcal{P}_{gg}^{20} & +\mathcal{N}_f\mathcal{P}_{gg}^{21} & +{\mathcal{N}_f}^2\mathcal{P}_{gg}^{22}\ ) \\
\mathcal{P}_{qg} = a_s & \mathcal{N}_f\mathcal{P}_{qg}^{01} & +a_s^2 & \mathcal{N}_f\mathcal{P}_{qg}^{11} & +a_s^3 & (\mathcal{N}_f\mathcal{P}_{qg}^{21} & +{\mathcal{N}_f}^2\mathcal{P}_{qg}^{22}\ ) \\
\mathcal{P}_{gq} = a_s\mathcal{P}_{gq}^{00} & & +a_s^2(\mathcal{P}_{gq}^{10} & +\mathcal{N}_f\mathcal{P}_{gq}^{11}) & +a_s^3(\mathcal{P}_{gq}^{20} & +\mathcal{N}_f\mathcal{P}_{gq}^{21} & +{\mathcal{N}_f}^2\mathcal{P}_{gq}^{22}\ )
\end{array}
$$

$$
\begin{array}{lllllll}
\mathcal{P}_{qq}^{V} = a_s\mathcal{P}_{qq}^{V00} & & +a_s^2(\mathcal{P}_{qq}^{V10} & +\mathcal{N}_f\mathcal{P}_{qq}^{V11}) & +a_s^3(\mathcal{P}_{qq}^{V20} & +\mathcal{N}_f\mathcal{P}_{qq}^{V21} & +{\mathcal{N}_f}^2\mathcal{P}_{qq}^{V22}) \\
\mathcal{P}_{q\bar{q}}^{V} = & & a_s^2(\mathcal{P}_{q\bar{q}}^{V10} & +\mathcal{N}_f\mathcal{P}_{q\bar{q}}^{V11}) & +a_s^3(\mathcal{P}_{q\bar{q}}^{V20} & +\mathcal{N}_f\mathcal{P}_{q\bar{q}}^{V21} & +{\mathcal{N}_f}^2\mathcal{P}_{q\bar{q}}^{V22}) \\
\mathcal{P}_{S}^{+} = & & a_s^2(& \mathcal{N}_f\mathcal{P}_{S}^{+11}) & +a_s^3(& \mathcal{N}_f\mathcal{P}_{S}^{+21} & +{\mathcal{N}_f}^2\mathcal{P}_{S}^{+22}) \\
\mathcal{P}_{S}^{-} = & & & & a_s^3(& \mathcal{N}_f\mathcal{P}_{S}^{-21} & +{\mathcal{N}_f}^2\mathcal{P}_{S}^{-22})
\end{array}
$$

$\mathcal{P}_{gg}^{01}$ existence comes from the β_0 term in $\mathcal{P}_{gg}$.

Momentum sum rule

Sumrules

The total parton momentum is:
$\int_0^1 \Pi x dx = 1$ **with** $\Pi = g + \sum_{q=d}^{t} q^+$
This imply that derivative of $\int_0^1 \Pi x dx$ **with respect of** $ln(Q^2)$ **is null for any set of partons.**
From that derives easily the following set of properties of the kernel integrals which will be noted $\mathcal{Q} = \int_0^1 \mathcal{P} x dx$

$$\mathcal{Q}_{gg} + \mathcal{Q}_{qg} = 0$$
$$\mathcal{Q}_{gq} + \mathcal{Q}^+_{NS} + \mathcal{N}_f \mathcal{Q}^+_S = 0$$

Which has to be valid for any value of a_s **and** $\mathcal{N}_f$

Modifying DGLAP equations

The idea is to modify the kernels in order to satisfy simultanously the three kinematical constraints $x_o < l_o$,$x_i < l_i$, $x_o < x_i$

$\mathcal{P}_{oi}$ **is the change to outcoming parton** o **at Bjorken** x **radiated by incoming parton** i **at Bjorken** $\frac{x}{t}$**. Problematic cases are when parton** o **is heavier than parton** i **like for** $c \rightarrow b$ **Replacing** $\mathcal{P}$ **by** $\mathcal{K}$ **, the problematic changing terms are:**

$$\int_x^1 \mathcal{K}_{oi}(\frac{x}{z}) i(z) \frac{dz}{z}$$

Requesting this term to be null for $x \geq l_0$ means that $\mathcal{K}(u) = 0$ for $u \geq l_o$ which is satisfied by

$$\mathcal{K}_{oi} = \mathcal{P}_{oi} \otimes \delta_{oi}$$

With the definition (for $l_o \leq 1$)

$$\delta_{oi} = \phi_{oi}\delta(x - l_o)$$

The only constraint so far on ϕ is that it goes to 0 for $Q^2 \to 0$ and to 1 for $Q^2 \to \infty$.

Note that the effect of the δ function is to replace $\mathcal{P}(x)$ by $\mathcal{P}(\xi)$ with $\xi = \frac{x}{l_o}$

With this modification the q^+ subsystem becomes:

$$\frac{\partial g(Q^2)}{\partial ln(Q^2)} = \mathcal{P}_{gg} \otimes \delta_{gg} \otimes g(Q^2) + \sum_{q=d}^{t} \mathcal{P}_{gq} \otimes \delta_{gq} \otimes q^+(Q^2)$$

$$\frac{\partial q^+(Q^2)}{\partial ln(Q^2)} = \tilde{\mathcal{P}}_{qg} \otimes \delta_{qg} \otimes g(Q^2) + \mathcal{P}^+_{NS} \otimes \delta^{NS}_{qq} \otimes q^+(Q^2) + \sum_{r=d}^{t} \tilde{\mathcal{P}}^+_{S} \otimes \delta_{qr} \otimes r^+(Q^2)$$

With the help of $\int_0^1 x\delta(x-l)dx = l$ and the use of Q relations given by the usual DGLAP, the momentum sumrule determines completly the modifications to do for heavy quark h:
$\delta_{hr} = l_h \delta(x - l_h)$ **for any** $r \in u, ..., t$
and also $\mathcal{N}_f = \sum_{q=d}^{t} l_q$
All the others δ_{oi} do not need to exist.

Note that for $l_q \to 0$ the corresponding DGLAP equation will get decoupled and the kinematical constraint automatically verified.
In fact other solutions may be found implying correlated changes in the δ_{oi}.
An example will be given in the last part of this talk.

Modifying α_s and coefficients functions

As seen above the momentum sumrule leads to a specific non integer value of $\mathcal{N}_f$ and as a consequence also for β_0 and by extension to the full set of β governing the α_s running. It is also natural that the copling constant depends on flavor activity and not only on flavor number.
Anyhow α_s and parton evolution are linked by renormalisation group theory.

As it is the structure functions and not the parton distributions which are observable one has to find also a procedure to modify the coefficient functions.

The change parton distribution $\rightarrow$ structure function has exactly the same structure that the one of DGLAP equations:

$$\frac{\partial o(Q^2)}{\partial ln(Q^2)} \rightarrow F_o$$

$$\mathcal{P} \rightarrow \mathcal{C}$$

This change is in fact nothing more than a change of scheme, an example is going from $\bar{M}S$ to DIS for F_2.
For F_1 and F_3 the schemes are unnamed but they still exist.

From this one may infer that coefficient functions have to be modified in the same way that splitting functions.
In fact it is even the importance of kinematic constraints stressed by one of the R.Thorne papers and its use in the lastest schemes which lead me to do this work.

Charged current
Exactly the same procedure will be used, the phase space only will change using:

$$l_o^{-1} = 1 + \frac{m_o^2}{Q^2}$$

System decoupling

$\Sigma_L = \sum_{q=d}^{s} q$ $\qquad l_{LN} = l - \frac{\Sigma_L}{3}$

$\phi_q = l_q \delta(x - l_q)$ $\qquad \phi = \sum_{q=d}^{t} \phi_q$

Subscript LN is used do distinguich these non singlets from the usual ones which may not be used here with flavor number varying continously. With these one get the following subsystem:

$$\begin{aligned}
\frac{\partial g(Q^2)}{\partial ln(Q^2)} &= \mathcal{P}_{gg} \otimes g(Q^2) + \mathcal{P}_{gq} \otimes \Sigma(Q^2) \\
\frac{\partial \Sigma(Q^2)}{\partial ln(Q^2)} &= \phi \otimes \bar{\mathcal{P}}_{qg} \otimes g(Q^2) + (\mathcal{P}_{NS} + \phi \otimes \bar{\mathcal{P}}_S) \otimes \Sigma(Q^2) \\
\frac{\partial l_{LN}(Q^2)}{\partial ln(Q^2)} &= \mathcal{P}_{NS} \otimes l_{LN}(Q^2) \\
\frac{\partial h(Q^2)}{\partial ln(Q^2)} &= \phi_h \otimes \bar{\mathcal{P}}_{qg} \otimes g(Q^2) + \mathcal{P}_{NS} \otimes h(Q^2) + \phi_h \otimes \bar{\mathcal{P}}_S \otimes \Sigma(Q^2)
\end{aligned}$$

Evolving the seven distributions g,Σ,c,b,t,d,u the full system may be recovered using:

$\Sigma_L = \Sigma - c - b - t$ $\qquad d_{LN} + u_{LN} + s_{LN} = 0$

Notice that this formulation is not unique: any linear combination of the seven equations with coefficients independent Q^2 could be used.

System decoupling

$\Sigma_L = \sum_{q=d}^{s} q$ $\qquad$ $l_{LN} = l - \frac{\Sigma_L}{3}$

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$$
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\frac{\partial g(Q^2)}{\partial ln(Q^2)} &= \mathcal{P}_{gg} \otimes g(Q^2) + \mathcal{P}_{gq} \otimes \Sigma(Q^2) \\
\frac{\partial \Sigma(Q^2)}{\partial ln(Q^2)} &= \phi \otimes \bar{\mathcal{P}}_{qg} \otimes g(Q^2) + \left(\mathcal{P}_{NS} + \phi \otimes \bar{\mathcal{P}}_S\right) \otimes \Sigma(Q^2) \\
\frac{\partial l_{LN}(Q^2)}{\partial ln(Q^2)} &= \mathcal{P}_{NS} \otimes l_{LN}(Q^2) \\
\frac{\partial h(Q^2)}{\partial ln(Q^2)} &= \phi_h \otimes \bar{\mathcal{P}}_{qg} \otimes g(Q^2) + \mathcal{P}_{NS} \otimes h(Q^2) + \phi_h \otimes \bar{\mathcal{P}}_S \otimes \Sigma(Q^2)
\end{aligned}
$$

Evolving the seven pdfs g,Σ,c,b,t,d_{LN},u_{LN} the full system may be recovered using:

$\Sigma_L = \Sigma - c - b - t$ $\qquad$ $d_{LN} + u_{LN} + s_{LN} = 0$

Notice that this formulation is not unique: any linear combination of the seven equations with coefficients independent Q^2 could be used.

Implementation (in new QCDFIT)

- It is a program which works in x space
- It includes an optimizing interface (Minuit).
- it accept a variety of input distributions.
- It has a variety of outputs: Pdfs, cross section for lepto production,Drell-Yang mechanism ...
- It pre-calculate the full evolution
- It use an x grid linearly spaced in logx and a $\log Q^2$ grid approximately in α_s
- Pdf -> one dimensional x array for a given Q^2

- Kernels have a matrix representation but due to their splitting or parton branching nature they are upper triangular band matrices with $M_{ij}=m_{i-j}$ with $i \geq j$ and so are also one dimensional arrays for a given Q^2.
- Integration of the renormalization group equation is made numerically as its parameters are functions of N_f and so of Q^2

The transport matrices defined by

$$o(Q^2_{j+1}) = \sum_i \mathcal{T}(o,i) \otimes i(Q^2_j)$$

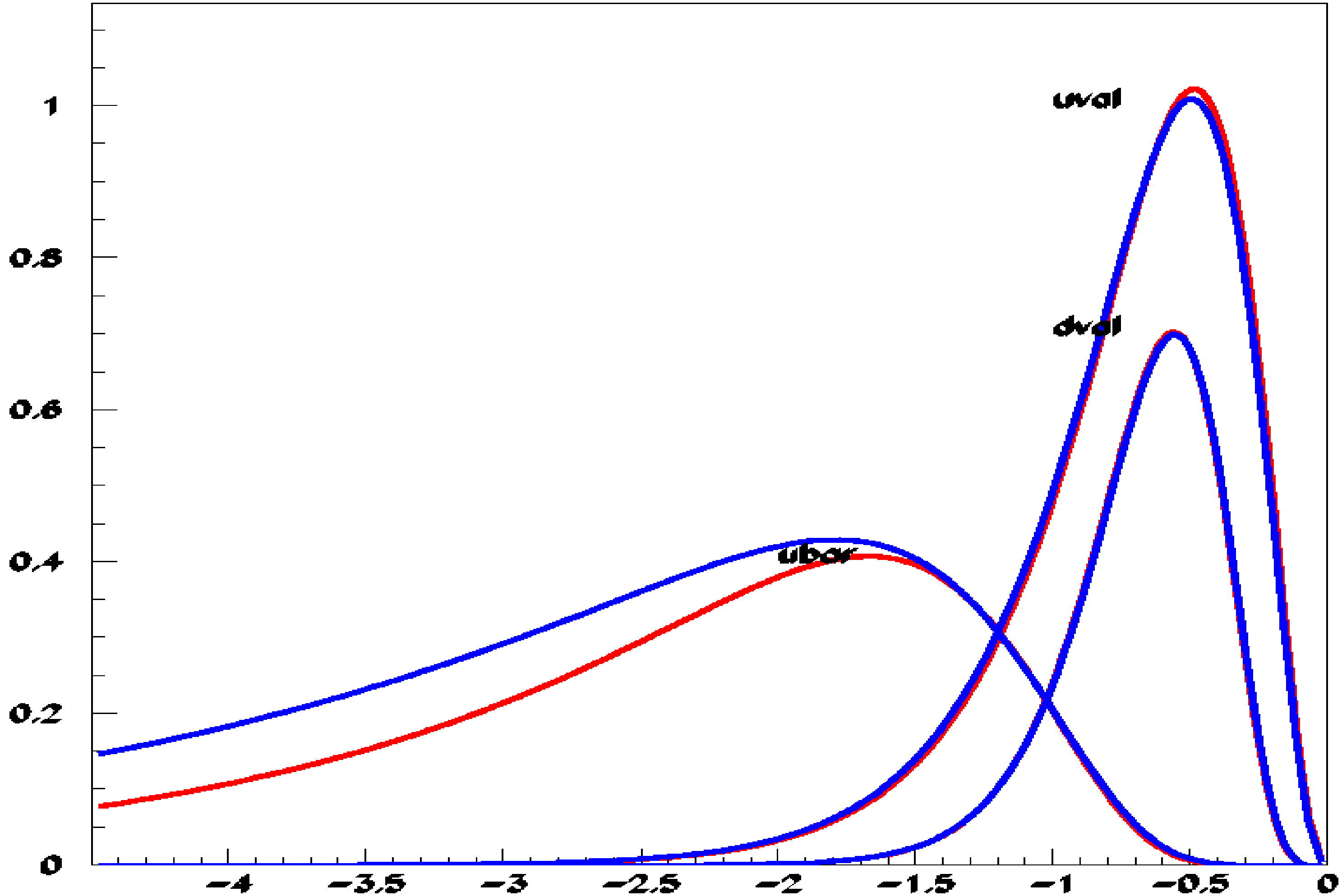
are obtained by integration of the subsystems using

$$\mathcal{T}(o,i) = \prod_{Q^2_j}^{Q^2_{j+1}} \otimes \left(1 + \frac{\partial^2 o}{\partial i \partial ln(Q^2)} \delta ln(Q^2)\right)$$

In the product $\delta ln(Q^2)$ has to be small enough to vary only according to rounding errors when increasing the number of Q^2 nodes.

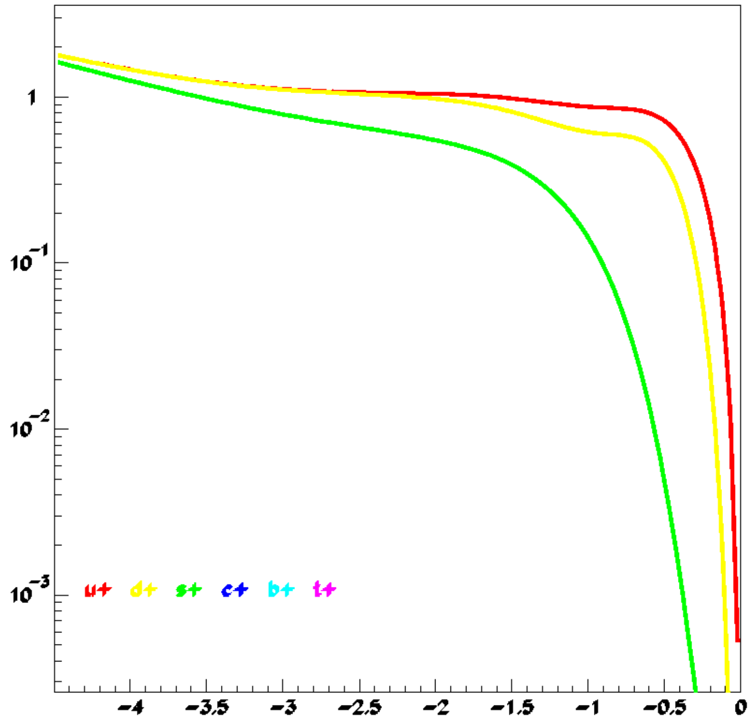
- My first concern was also to check as soon as possible my ideas by building a transport matrix integration valid at the same time for CFNS and for constant flavor evolution

Fitted Pdf

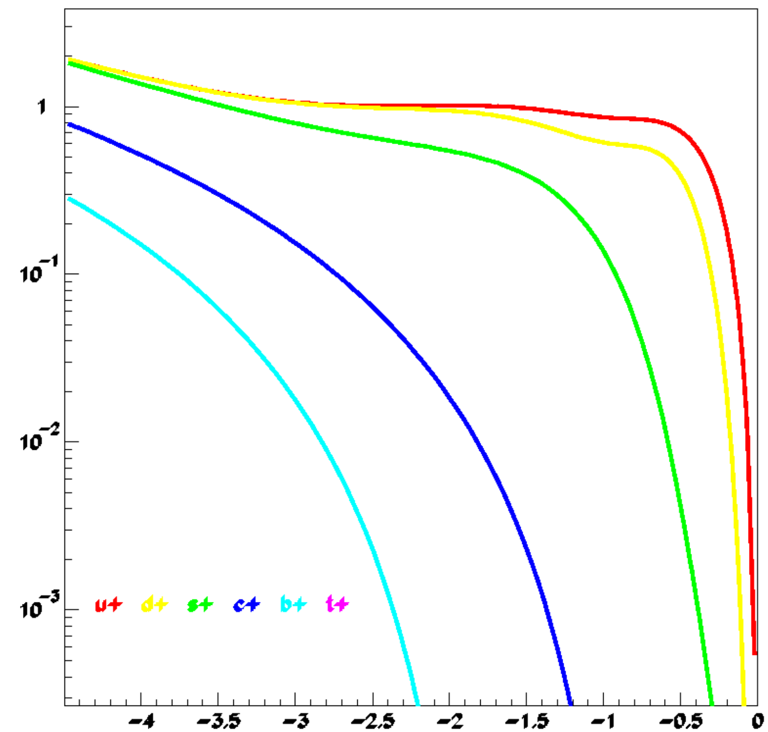


Charm mass^2

massless

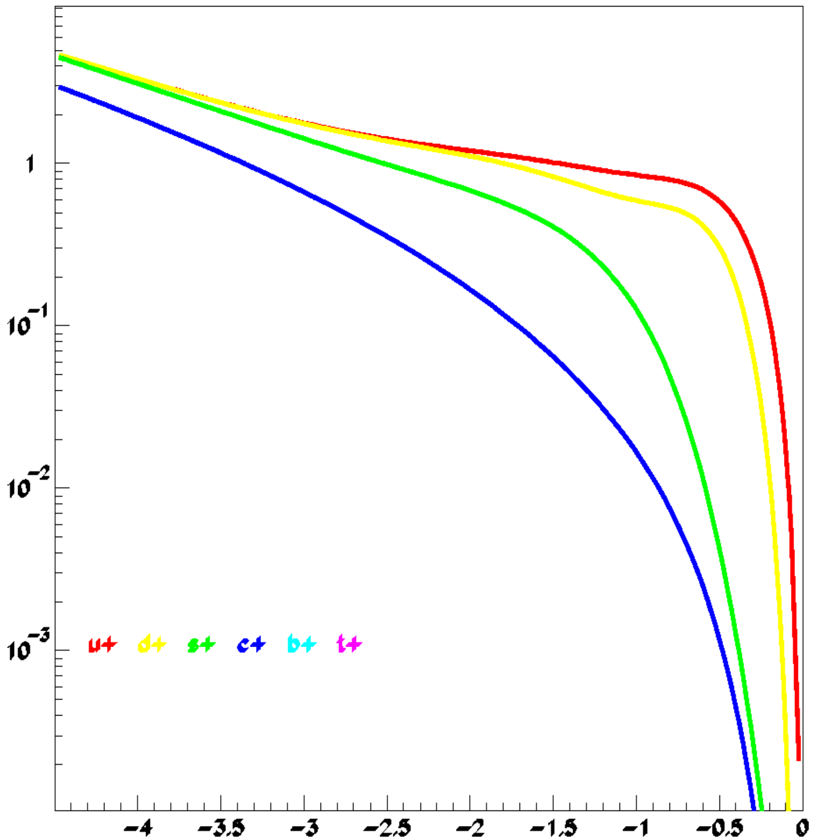


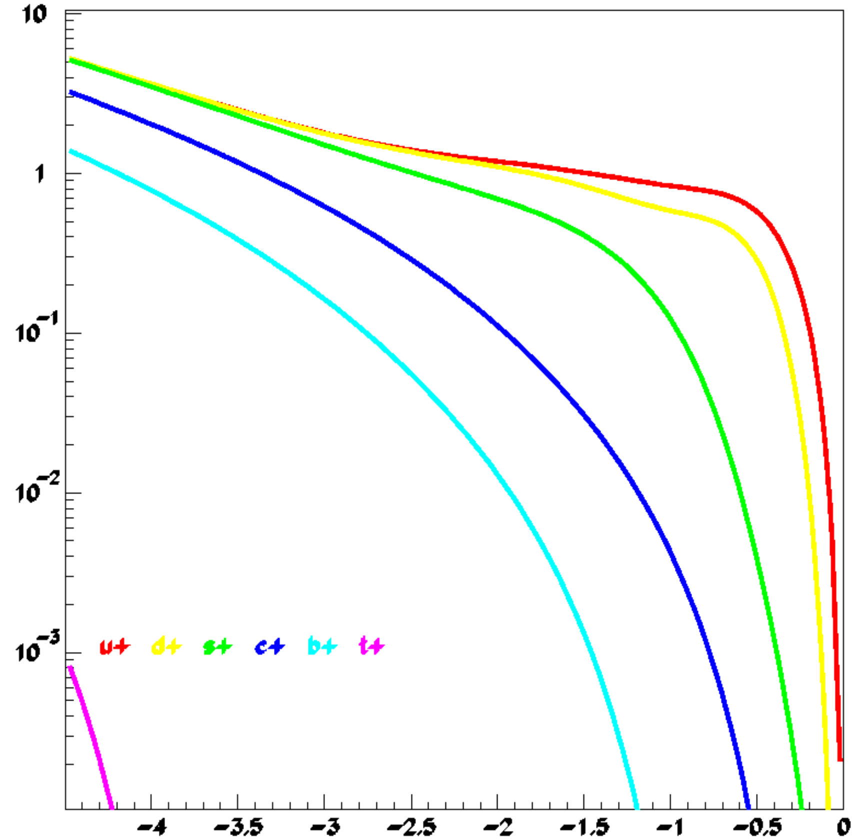
cfns



Beauty mass^2

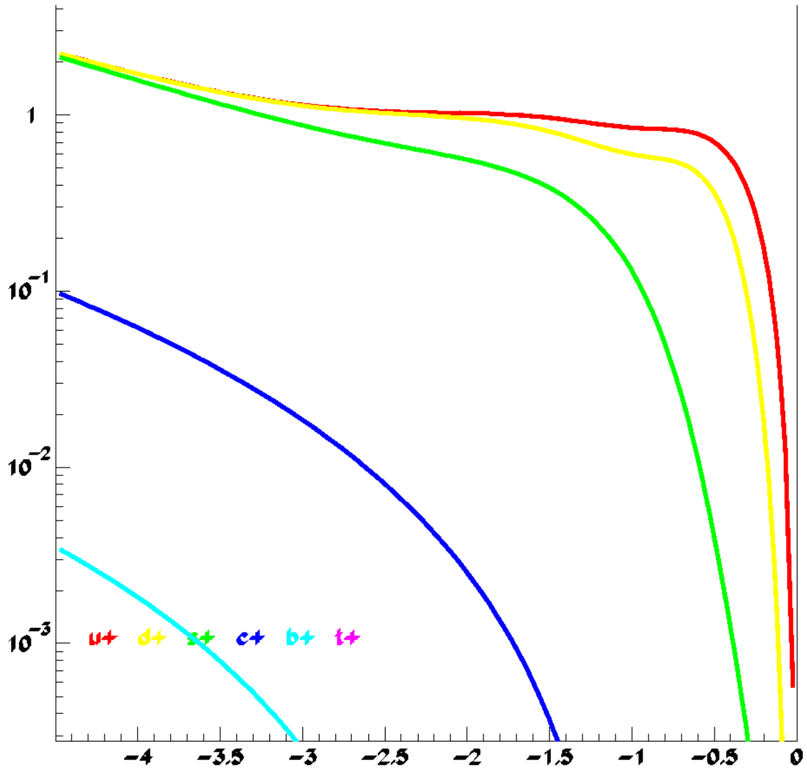
massless | cfns



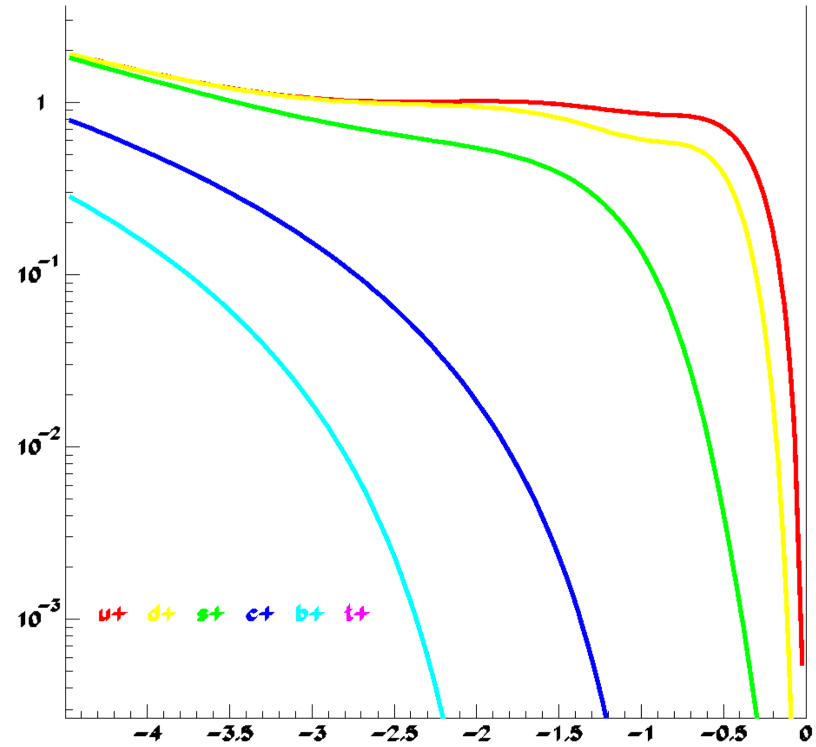


Charm mass^2

cfns2

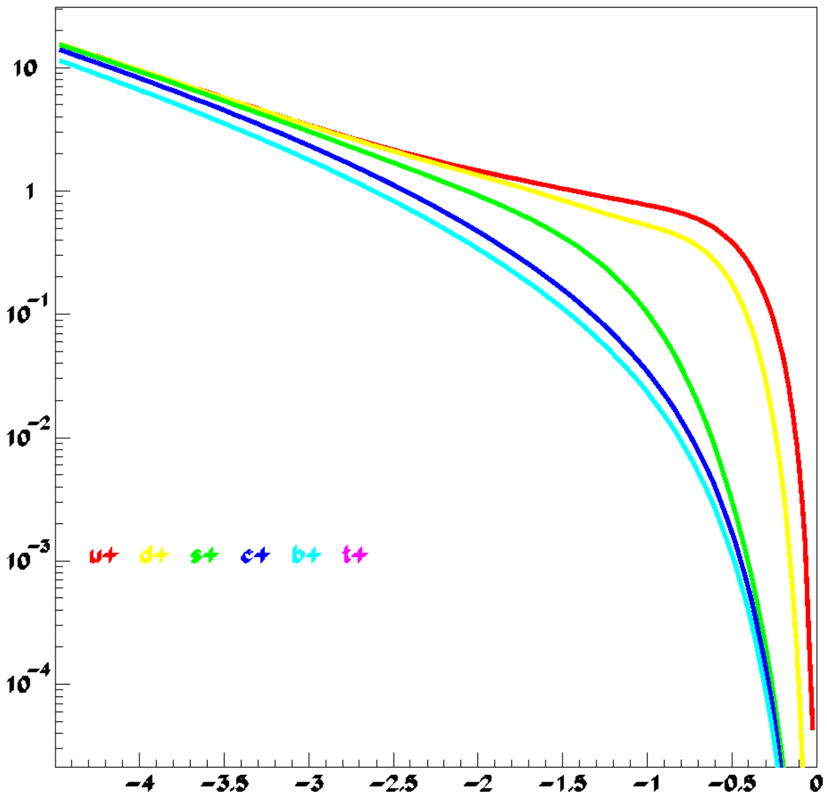


cfns



Top mass^2

massless



cfns