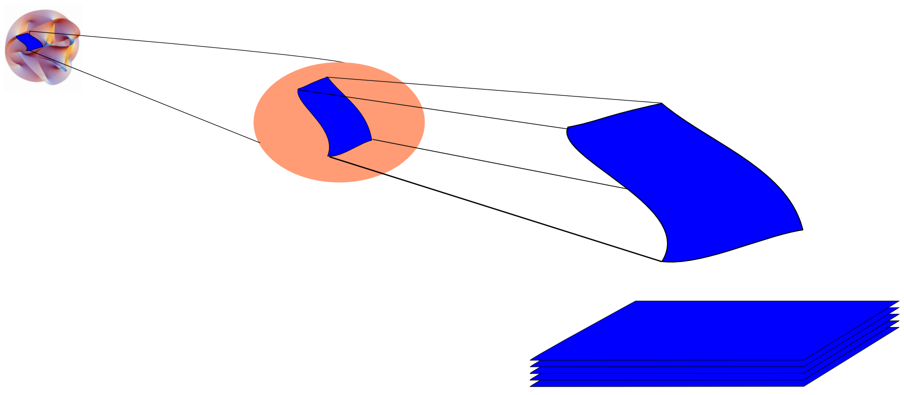
Tools for String Phenomenology

Sakura Schäfer-Nameki
King's College, London



CERN Winter School, February 4-8, 2013

4 Lectures on String Pheno

Lecture 1: Why?
SUSY GUT settings in String Theory

Lecture 2: How?
Higgs Bundles as the Tool for String Phenomenology

Lecture 3: What exactly?
F-theory as a UV completion of Higgs bundles

Lecture 4: Seriously?!
Complete F-theory models and implications

Lecture 4: Seriously?

F-theory, Higgs bundles and Phenomenological implications

Spectral Covers:

Hayashi, Kawano, Tatar, Watari, Donagi, Wijnholt, Marsano, Saulina, SSN, Weigand...

Spectral Cover models and phenomenological properties:

Dolan, Marsano, Saulina, SSN, Dudas, Palti, Heckman, Vafa, Ross, Nilles, Marchesano, Ibanez, Font, ...

Setup so far

Elliptic Calabi-Yau4 compactification, with G-fluxes and degerations of fibers that yield $SU(5)$ GUT

$$y^2 = x^3 + b_5 xy + b_4 z x^2 + b_3 z^2 y + b_2 z^3 x + b_0 z^5$$

Spectral Divisor

$$C_{spectral} : \qquad b_5 xy + b_4 z x^2 + b_3 z^2 y + b_2 z^3 x + b_0 z^5 = 0$$

Limiting behavior close to $x = y = z = 0$: let $t = y/x$ and take limit $t, z \longrightarrow 0$ with $z/t =$ fixed

$$C_{spectral} \quad \longrightarrow \quad t^5 (b_5 + b_4 t + b_3 t^2 + b_2 t^3 + b_0 t^5) = t^5 C$$

where $b_n|_S$ as coefficients.

$\Rightarrow$ direct connection CY4 geometry and Higgs bundle spectral data

$U(1)$ Symmetries

With a $U(1)$ symmetry forbid dimension 5 proton decay operators (and dim 4 by B-L)

$$W_{dim5} = \frac{Q^3 L}{\Lambda}$$

$\Lambda > 10^{27}$ GeV.

Related issue: μ-problem (for natural EWSB $\mu \sim$ O(100)GeV, but why?! Superpotential coupling), fobid treelevel

$$W_\mu = \mu H_u H_d$$

Both problem can be addressed with one $U(1)$ (assuming the $U(1)$ is compatible with Yukawa couplings)

$$\Rightarrow \qquad U(1)_{PQ}: \qquad q_{H_u} + q_{H_d} \neq 0$$

In Spectral cover:
$\mathcal{C} = \mathcal{C}^{(3)}\mathcal{C}^{(2)}$ (3+2 split with monodromy group $S_3 \times \mathbb{Z}_2$) realizes $U(1)_{PQ}$.

Recall: Pitfalls of GUT models: Exotica

- Additional GUT gauge bosons need to be lifted:

$$SU(5) \to SU(3) \times SU(2) \times U(1)_Y$$

$$\mathbf{24} \to (\mathbf{8},\mathbf{1})_0 \oplus (\mathbf{1},\mathbf{3})_0 \oplus (\mathbf{1},\mathbf{1})_0 \oplus (\mathbf{3},\overline{\mathbf{2}})_{-5} \oplus (\overline{\mathbf{3}},\mathbf{2})_{+5}$$

Gauge Fields Exotics

- Higgs triplets

$$SU(5) \to SU(3) \times SU(2) \times U(1)_Y$$

$$\mathbf{5}_H \to (\mathbf{1},\mathbf{2})_{+1/2} \oplus (\mathbf{3},\mathbf{1})_{-1/3}$$

H_u Exotics

Breaking the GUT group in F-theory

$$SU(5) \rightarrow SU(3) \times SU(2) \times U(1)_Y$$

1. Wilson lines: flat connections
   $\Rightarrow$ not all 4-dimensional manifolds S allow for these
   ([Marsano, Clemens, Pantev, Raby] S=Enriques, 5 H_u and H_d)
   $\Rightarrow$ approach is contrary to initial goal to extract generic features

2. Hypercharge flux [Buican, Malyshev, Morrison, Verlinde, Wijnholt]
   $\Rightarrow$ background gauge field for $U(1)_Y$ (cunningly constructed not to mass up the hypercharge gauge boson) removes Higgs triplets and XY bosons $(\mathbf{3}, \overline{\mathbf{2}})_{-5} \oplus (\overline{\mathbf{3}}, \mathbf{2})_{+5}$
   $\Rightarrow$ Generic construction independent of S

Hypercharge GUT breaking

Requirement: $U(1)_Y$ gauge boson remains massless!
Consider $\mathcal{L}_Y$ background flux. In F-theory CS coupling

$$\int_{Y \times \mathbb{R}^{1,3}} C_4 \wedge G_4 \wedge G_4$$

Expanding $G = (F_Y + c_1(\mathcal{L}_Y)) \wedge \omega_Y$ and $C_4 = C_2^i \wedge \omega^i$, $\omega^i \in H^2(B_3, \mathbb{Z})$

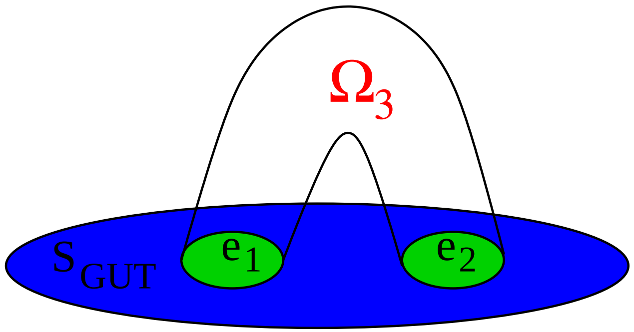
generates mass term for F_Y : $$\left[\mathrm{Tr}(T_Y^2) \int_S c_1(\mathcal{L}_Y) \wedge i^* \omega^i\right] \int_{\mathbb{R}^{1,3}} C_2^i \wedge F_Y$$

This can be avoided if

$$\int_S c_1(\mathcal{L}_Y) \wedge i^* \omega^i = 0 \qquad \forall \omega^i \in H^2(B_3, \mathbb{Z})$$

$U(1)_Y$ remains massless if there is a 3-chain Ω_3 in B_3 whose boundary is the dual inside S of $c_1(\mathcal{L}_Y)$

$$\partial \Omega_3 = e_1 \cup (-e_2)$$



Can we realize any $U(1)$ symmetry that forbids

$$W_{\mu} \sim \mu H_u H_d \quad \text{and} \quad W_{\text{dim5}} \sim \frac{1}{\Lambda} Q^3 L$$

in a local F-theory GUTs with hypercharge flux GUT breaking?

Can we realize any $U(1)$ symmetry that forbids

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in a local F-theory GUTs with hypercharge flux GUT breaking?

No!

- Classification of all $U(1)$ symmetries that can be realized in local F-theory models
- Any local F-theory model of this kind has non-GUT exotics
  $\Rightarrow$ quasi-chiral (f_q and $\bar{f}_q$)
  $\Rightarrow$ Mass-term will break $U(1)$
  $\Rightarrow$ Unification?

Anomalies

F_Y restriction on $\Sigma_{\mathbf{10}}$ and $\Sigma_{\bar{\mathbf{5}}}$ generates chiral spectrum
$\Rightarrow$ Require $G_{MSSM}^2 \times U(1)$ mixed anomaly cancellation for additional $U(1)$
[Marsano]
$\Rightarrow$ Additional constraints from $U(1)_Y \times U(1)^2$ anomalies [Palti]
(these imply additional constraints on the spectrum we'll discuss in a moment)

Constraints from Anomalies

Consider $G_{MSSM}^2 \times U(1)$ mixed anomaly cancellation for additional $U(1)$ imply

$$\sum_{\mathbf{10}_i \text{ multiplets}} q_i \, (F_Y)|_{\mathbf{10}_i} = \sum_{\bar{\mathbf{5}}_i \text{ multiplets}} q_i \, (F_Y)|_{\bar{\mathbf{5}}_i}$$

Consequences of these anomaly relations:

- Minimal $SU(5)$ GUT
  $\Rightarrow$ the only $U(1)$ compatible is $U(1)_\chi$ ($B - L$ and Y)
  $\Rightarrow q_{H_u} + q_{H_d} = 0$
- If $U(1)$ forbids W_μ and $W_{\text{dim } 5}$, i.e. $q_{H_u} + q_{H_d} \neq 0$
  $\Rightarrow$ there are always non-GUT exotics
  $\Rightarrow$ Can lift these by $U(1)$-charged GUT-singlet X

$$W_{\text{ex}} = \lambda X f_{\text{ex}} \bar{f}_{\text{ex}}$$

Survey

[Dolan, Marsano, Saulina, SSN]

Parametrization of non-GUT exotics:

$SU(5)$ origin	Exotic Multiplet	Degeneracy
$\mathbf{10} \oplus \overline{\mathbf{10}}$	$(\mathbf{1},\mathbf{1})_{+1} \oplus (\mathbf{1},\mathbf{1})_{-1}$	$M+N$
	$(\mathbf{3},\mathbf{2})_{+1/6} \oplus (\overline{\mathbf{3}},\mathbf{2})_{-1/6}$	M
	$(\overline{\mathbf{3}},\mathbf{1})_{-2/3} \oplus (\mathbf{3},\mathbf{1})_{+2/3}$	$M-N$
$\overline{\mathbf{5}} \oplus \mathbf{5}$	$(\overline{\mathbf{3}},\mathbf{1})_{+1/3} \oplus (\mathbf{3},\mathbf{1})_{-1/3}$	K
	$(\mathbf{1},\mathbf{2})_{-1/2} \oplus (\mathbf{1},\mathbf{2})_{+1/2}$	$K-L$

Then all models that have realization in local models:

Model Class	Exotic Spectra		Dim 5
I	$N-L=1$		XQ^3L/Λ^2
II	$N-L=2$	$K \geq M$	X^2Q^3L/Λ^3
III	$L=2$	$M=N=0$	$X^{\dagger 2}Q^3L/\Lambda^4$
IV	$N-L=1$	$K-L=M$	XQ^3L/Λ^2

Unification

Apart from consistency with anomalies, phenomenological requirements (no dim 5 proton decay, μ-term), we need to ensure consistency with unification.

Additional non-GUT exotics contribute to 1-loop β-functions as

$$\delta b_1 = 3M + K + \frac{1}{5}(-2N - 3L)$$
$$\delta b_2 = 3M + K - L$$
$$\delta b_3 = 3M - N + K\,.$$

In particular, non-universality is measured by $q_{H_u} + q_{H_d} = q_X \Delta$, where

$$\Delta = N - L = \delta b_2 - \delta b_3 = \frac{1}{6}(5\delta b_1 + 3\delta b_2 - 8\delta b_3)$$

Unification versus Proton Decay

We parametrized the models by $q_{H_u} + q_{H_d} = q_X \Delta$, where

$$\Delta = N - L = \delta b_2 - \delta b_3 = \frac{1}{6}(5\delta b_1 + 3\delta b_2 - 8\delta b_3)$$

- Unification: Δ measures disruption from unification
- Proton decay: the following coupling is allowed

$$\frac{1}{\Lambda}\int d^2\theta \left(\frac{X}{\Lambda}\right)^{\Delta} Q^3 L$$

$\Rightarrow$ Intrinsic tension: Unification versus protection from Proton Decay

$\Rightarrow$ Requires analysis of RG contributions (high and low-scale thresholds)

Combined RG

General parametrization of corrections: Deviation of M_U and α^{-1} from 1-loop value $M_U^{(0)}$ and $\alpha_3(m_Z)^{-1,(1-loop)}$

$$\alpha_3(m_Z)^{-1} = \alpha_3(m_Z)^{-1,(1-loop)} + \delta$$

$$\frac{1}{2\pi}\ln\frac{M_U}{m_Z} = \frac{1}{2\pi}\ln\frac{M_U^{(0)}}{m_Z} + \delta\left(\frac{1}{2\pi}\ln\frac{M_U}{m_Z}\right)$$

	2-loop	Low Thresh	High Thresh	Exotic
δ	$\delta^{(2-loop)} = -0.8197$	$\frac{19}{28\pi}\ln\frac{m_{SUSY}}{m_Z}$	$\frac{3}{14\pi}\ln\frac{M_{\text{Bulk}}}{M_U^{(0)}} + \Delta^{(finite)}$	$\frac{9}{14\pi}(L-N)\ln\frac{M_{\text{Bulk}}}{M_{\text{Exotic}}}$
$\delta\left(\frac{1}{2\pi}\ln\frac{M_U}{m_Z}\right)$	$\delta_{M_U}^{(2-loop)} = 0.07446$	$-\frac{25}{168\pi}\ln\frac{m_{SUSY}}{m_Z}$	$\delta_{M_U}^{(finite)} - \frac{19}{28\pi}\ln\frac{M_{\text{Bulk}}}{M_U^{(0)}}$	$\frac{N-L}{28\pi}\ln\frac{M_{\text{Bulk}}}{M_{\text{Exotic}}}$

$\Rightarrow$ Δ^{finite} is only geometry (of S)-dependent quantity

$\Rightarrow$ Can make this consistent with gauge couplings at weak scale

F-theory Input: Recap

Any model with $U(1)_Y$ flux GUT breaking and $U(1)$ induced protection from PD and μ-problem
$\Rightarrow$ non-GUT exotics parametrized by

$SU(5)$ origin	Exotic Multiplet	Degeneracy
	$(\mathbf{1},\mathbf{1})_{+1} \oplus (\mathbf{1},\mathbf{1})_{-1}$	$M+N$
$\mathbf{10} \oplus \overline{\mathbf{10}}$	$(\mathbf{3},\mathbf{2})_{+1/6} \oplus (\overline{\mathbf{3}},\mathbf{2})_{-1/6}$	M
	$(\overline{\mathbf{3}},\mathbf{1})_{-2/3} \oplus (\mathbf{3},\mathbf{1})_{+2/3}$	$M-N$
$\overline{\mathbf{5}} \oplus \mathbf{5}$	$(\overline{\mathbf{3}},\mathbf{1})_{+1/3} \oplus (\mathbf{3},\mathbf{1})_{-1/3}$	K
	$(\mathbf{1},\mathbf{2})_{-1/2} \oplus (\mathbf{1},\mathbf{2})_{+1/2}$	$K-L$

Then all Spectral Cover Models fall into the following classification:

Model Class	Exotic Spectra		Dim 5
I	$N-L=1$		XQ^3L/Λ^2
II	$N-L=2$	$K \geq M$	X^2Q^3L/Λ^3
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IV	$N-L=1$	$K-L=M$	XQ^3L/Λ^2

What to do with the exotics?

Either try to lift them, or make them useful for something (signature).

One obvious application:

Use them as messengers in gauge mediated susy breaking

Can such a model fly?

SUSY-breaking 101

Breaking SUSY spontaneously in the visible sector leads to sum rules for the masses of superpartners and SM particles, which would lead to light superpartners that are not observed.

General parametrization of SUSY breaking terms:
Constraint: should not reintroduce quadratic divergences
⇒Soft SUSY breaking terms: 178-parameter space.

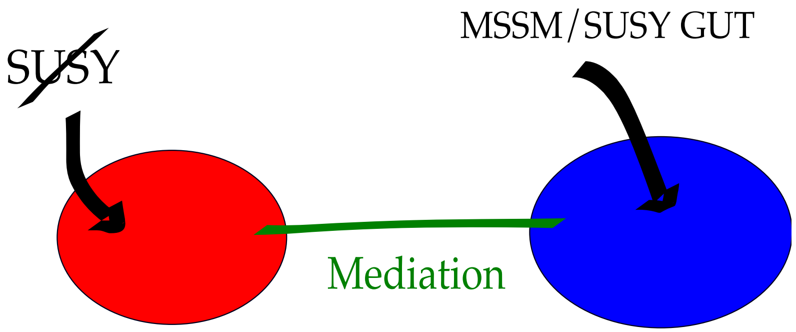
Approach taken historically:
"universal" mass parameters, e.g. universal gaugino (superpartners of gauge bosons), universal scalar (superpartners of fermions) masses. No conceptual reason behind this

⇒ LHC is chopping away parameter space of these "universal" models

SUSY-breaking 101

Spontaneous SUSY breaking has to occur in a hidden sector

$\Rightarrow$ SUSY breaking is mediated to visible sector by a messenger sector



- gravity: $1/M_{Planck}$ operators generate soft terms
- gauge interactions: messenger fields charged under MSSM/GUT gauge group couple to both visible and SUSY breaking sector

GMSB with non-GUT exotic Messengers

$$W = F_X X + \lambda_X X f \bar{f} + W_{SU(5)GUT}$$

Forbid μ-term and dim 4 proton decay at tree-level by $U(1)$

	X	$\mathbf{10}_M$	$\bar{\mathbf{5}}_M$	$\mathbf{5}_H$	$\bar{\mathbf{5}}_H$
$U(1)$	-2	1	-1	-2	0

X picks up VEV, μ dynamically generated by Giudice-Masiero type mechanism

$$\frac{1}{M_{\rm GUT}} \int d^4\theta X^\dagger H \overline{H} \quad \Rightarrow \quad \mu \sim \frac{F_X}{M_{\rm GUT}}$$

$\Rightarrow F_X \sim 10^{19}$ GeV^2, i.e. high-scale GMSB, Sweetspot SUSY/local F-theory

GMSB gaugino masses:

$$M^i_{1/2}(M_{\rm Mess}) = \delta b_i \left(\frac{\alpha_i(M_{\rm Mess})}{4\pi}\right)\left(\frac{F_X}{M_{\rm Mess}}\right)$$

2-loop squark and slepton masses:

$$m^2_Q(M_{\rm Mess}) = \sum_{\text{relevant } i} \frac{c_i\, \delta b_i\, \alpha_i(\mu)^2}{8\pi^2}\left|\frac{F_X}{M_{\rm Mess}}\right|^2$$

Beta-function shifts from exotics:

$$\begin{aligned}
\delta b_1 &= 3M - \frac{2}{5}N + K - \frac{3}{5}L \\
\delta b_2 &= 3M + K - L \\
\delta b_3 &= 3M - N + K\,.
\end{aligned}$$

Gaugino Mass Relations

Standard universal gaugino mass relations

$$M_1 : M_2 : M_3 \simeq 1 : 2 : 6$$

gets replaced by

$$M_1 : M_2 : M_3 \simeq 1 : 2 \left(\frac{3M + K - L}{3M - 2N/5 + K - 3L/5} \right) : 6 \left(\frac{3M + K - N}{3M + K - 2N/5 - 3L/5} \right)$$

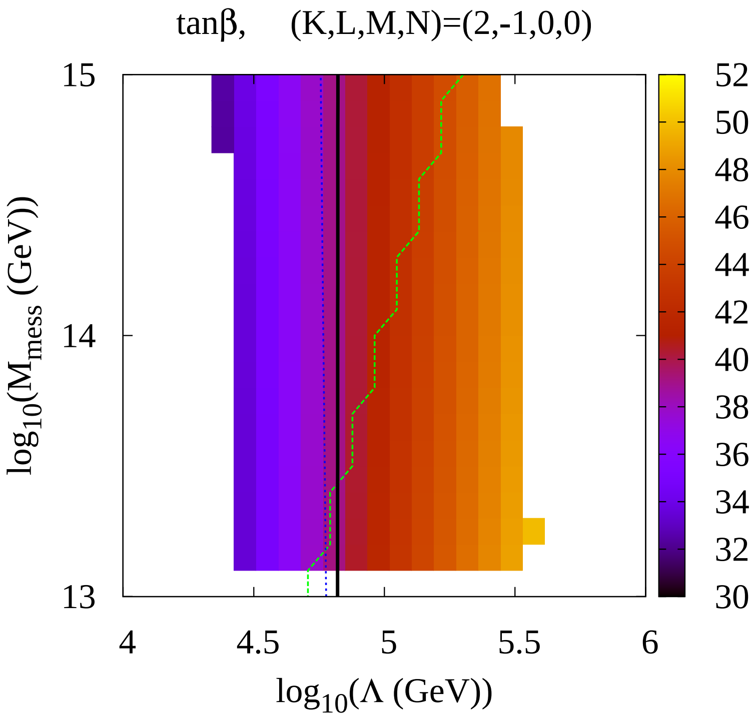
Benchmark Models

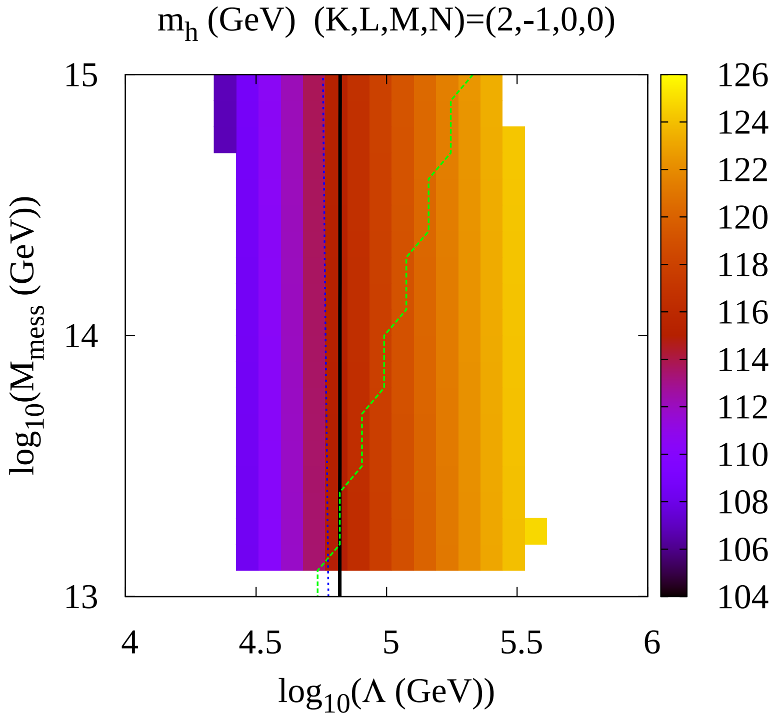
Benchmark flux choices:

L	Benchmark Model	(K, L, M, N)	M_{Mess}	Λ	$m_{\tilde{g}}$
-2	$\text{NonUniv}_{L=-2}$	$(0, -2, 1, 0)$	10^{14}GeV	8×10^4GeV	1686GeV
-1	$\text{NonUniv}_{L=-1}^{\text{SmallFlux}}$	$(2, -1, 0, 0)$	10^{14}GeV	9×10^4GeV	1307GeV
$+2$	$\text{NonUniv}_{L=2}$	$(3, 2, 0, 0)$	10^{14}GeV	7×10^4GeV	1489GeV
-1	$\text{NonUniv}_{L=-1}^{\text{LargeFlux}}$	$(6, -1, 7, 0)$	10^{14}GeV	1.3×10^4GeV	2337GeV
0	mGMSB5	$(3, 0, 0, 0)$	10^{10}GeV	7×10^4GeV	1503GeV

Parameter Scan: tan β and Higgs mass

Benchmark $(K, L, M, N) = (2, -1, 0, 0)$

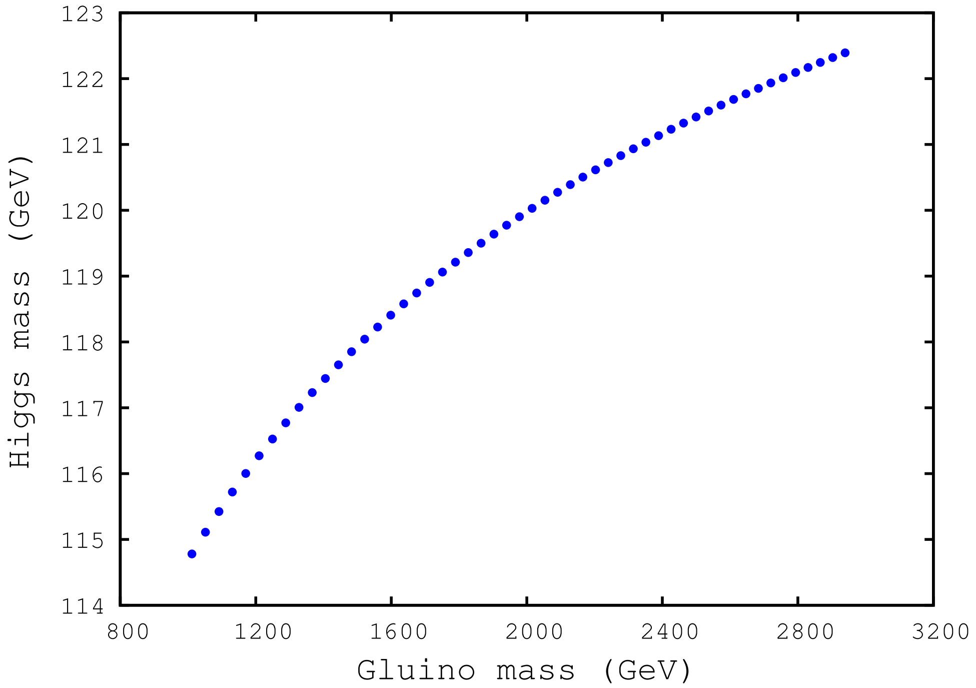




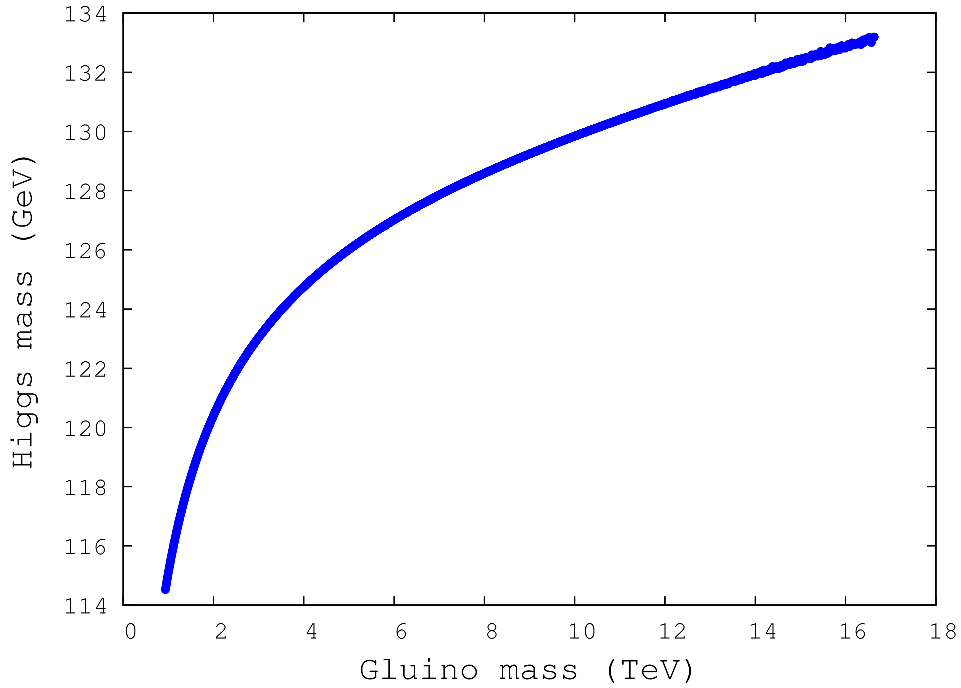
Note: $\Lambda = F_X / M_{\text{Mess}}$

Note: high gluino masses, so not reachable with LHC so far.

Higgs versus gluino mass

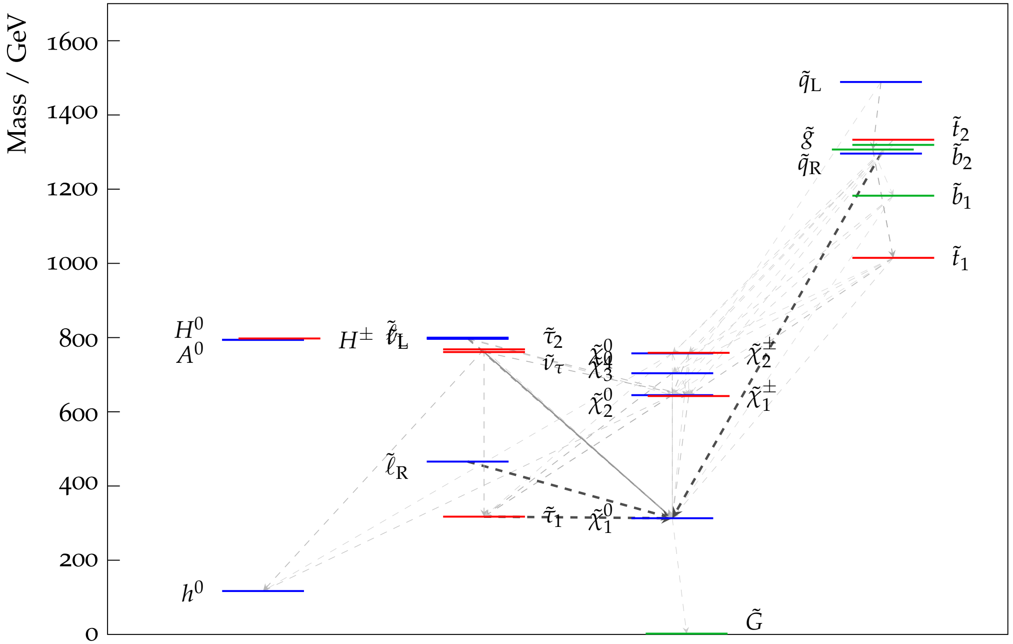


$(K, L, M, N) = (2, -1, 0, 0)$

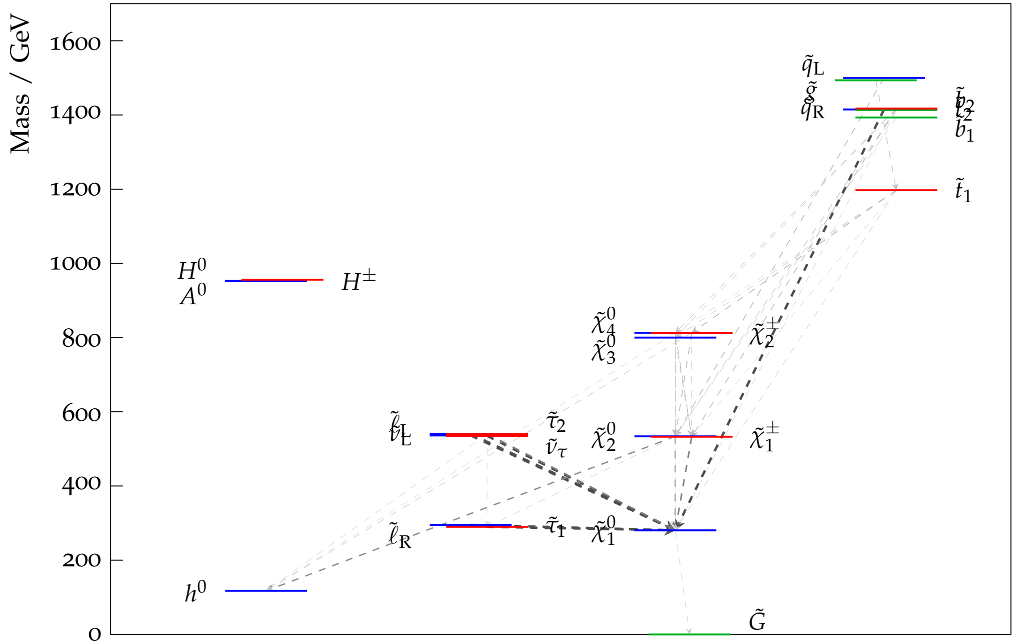


$(K, L, M, N) = (1, 0, 0, -1)$

Comparison with mGMSB Lookalike



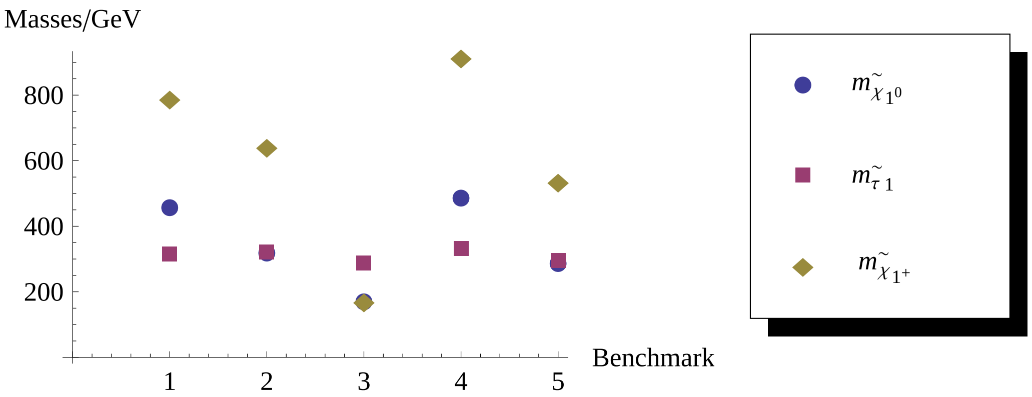
$(K, L, M, N) = (2, -1, 0, 0)$

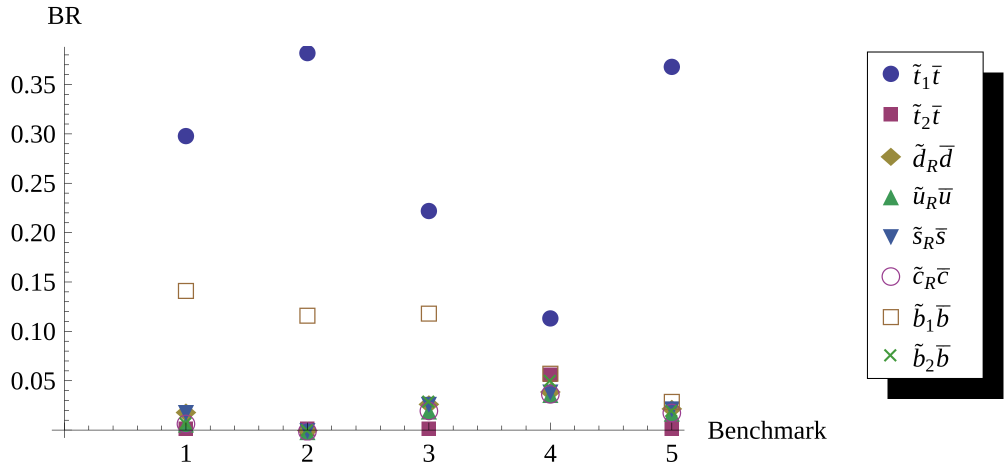


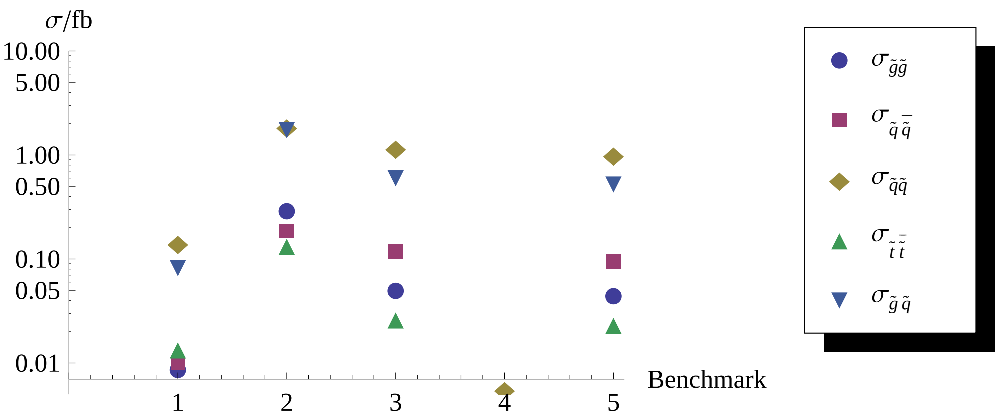
mGMSB $(2, 0, 0, 0)$

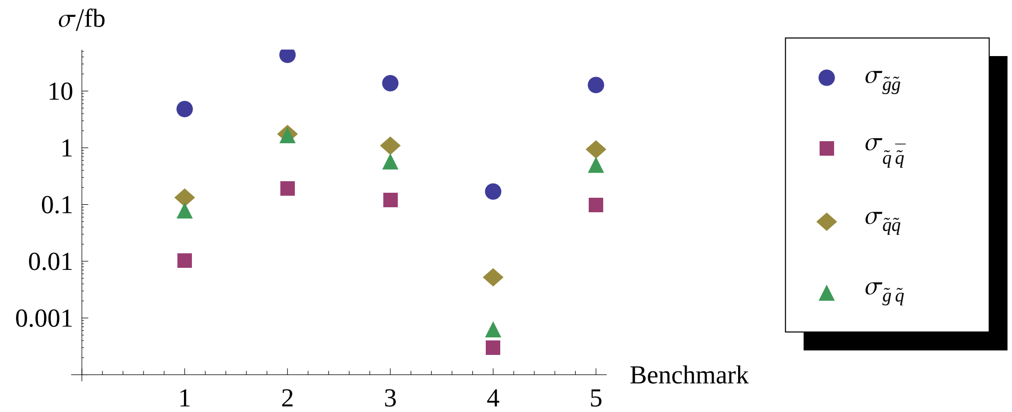
$(\delta b_1, \delta b_2, \delta b_3) = (2.6, 3, 2)$ for non-GUT model.

NLSPs and BR for Benchmarks









Methods to distinguish from mGMSB Lookalike?

- Models have very similar features (spectra, production cross sections, large $\tan\beta$)
- Reconstrution by studying channels

$$\tilde{q}_L \to \chi_2^0 q \to \tilde{l}_R^{\pm} l^{\mp} q \to \chi_1^0 l^{\pm} l^{\mp} q$$

Model	m_{ll}	m_{llq}^{edge}	m_{llq}^{thr}	m_{lq}^{high}	m_{lq}^{low}
mGMSB	138.7	1126	306	1102	396
Non-GUT $L = -1$	330.2	1011	550	856	688

$m_{ll} = (m^2_{\tilde{\chi}_0^1} - m^2_{\tilde{\ell}_R})(m^2_{\tilde{\ell}_R} - m^2_{\tilde{\chi}_0^2})/m^2_{\tilde{\ell}_R}$

= kinetic invariant for dilepton channel $\chi_2^0 \to \tilde{l}^{\pm} l^{\mp} \to \chi_1^0 l^{\pm} l^{\mp}$

Additional Refinements

- Flavor: Madrid group
- Variations with D3-branes: Harvard group
- M-theory/F-theory connection: Munich/Penn
- Non-perturbative effects
- Global consistency, geometries and fluxes: Munich, Heidelberg, "Techies", Penn

⇒ Immense progress and much more to explore

Short: Dominant SUSY breaking mechanism? Kaehler potentials? Natural SUSY?
Long: Extensions of systematic studies to M/G2, Type I'...!