Tools for String Phenomenology

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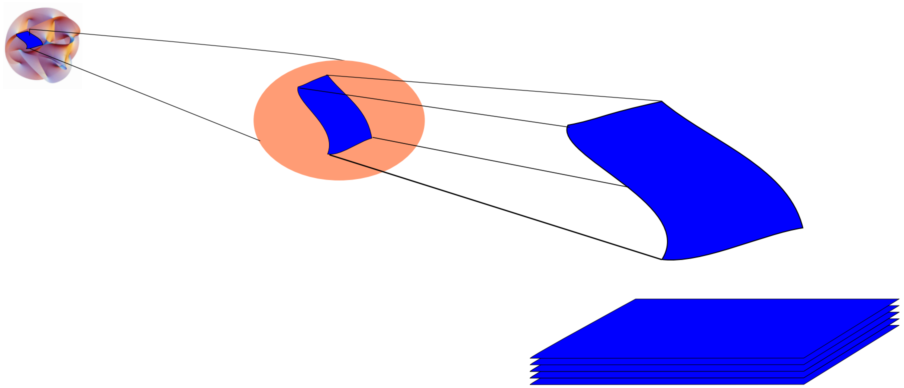


CERN Winter School, February 4-8, 2013

4 Lectures on String Pheno

Lecture 1: Why?
SUSY GUT settings in String Theory

Lecture 2: How?
Higgs Bundles as the Tool for String Phenomenology

Lecture 3: What exactly?
F-theory as a UV completion of Higgs bundles

Lecture 4: Seriously?!
Complete F-theory models and implications

Lecture 1: WHY?

We discussed what frameworks are suitable for a systematic scan of the string theory landscape

- Bottom-up model building
- Gauge dofs localized on branes
- Construction of local models via half-twisted SYM on $S \times \mathbb{R}^{3,1}$
- Working example: $8D$ SYM on complex surface S, aka local F-theory.

$$G \to \tilde{G} \times U(1)$$

$$adj(G) \to \bigoplus_i R_i^{a_i}$$

Then zero modes in the background of such a $\mathcal{L}$ U(1)-bundle are computed from the cohomologies

$$H^i_{\bar{\partial}}(S, \mathcal{L}^{a_i}) \qquad \text{and} \qquad H^i_{\partial}(S, \mathcal{L}^{-a_i})^*$$

Matter

Matter arises from intersection of branes, flavor brane intersected over branes wrapping S over a curve Σ
$\Rightarrow$ from the point of view of the $8D$ SYM, we consider a $6D$ defect theory.

Alternative point of view:
Matter arises from Higgsing a higher rank gauge group.

Matter

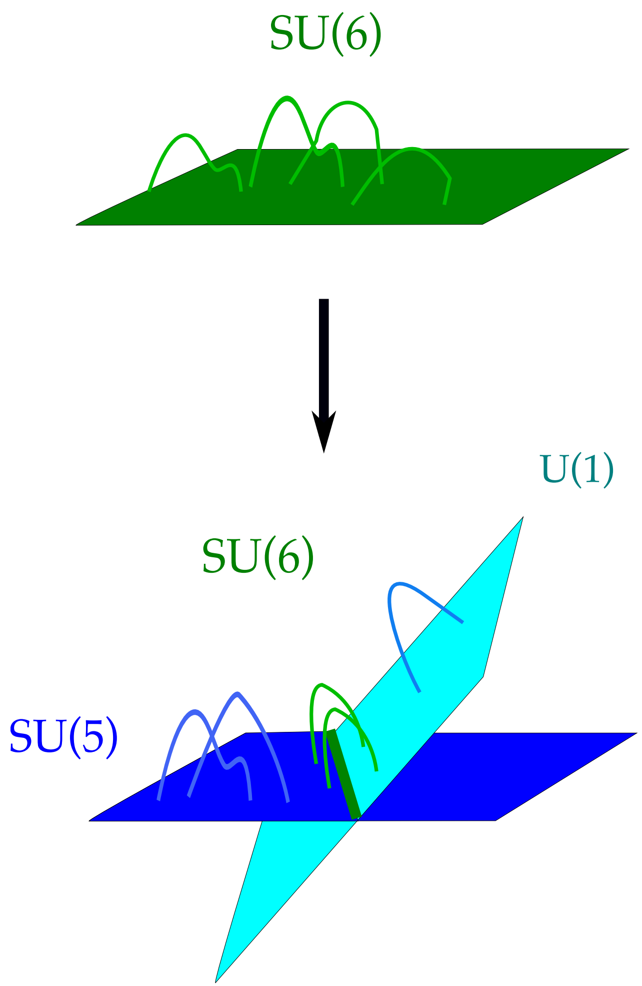
Matter arises from Higgsing a higher rank gauge group.

E.g. **5** and $\bar{\mathbf{5}}$ from $SU(6)$

$$SU(6) \quad \rightarrow \quad SU(5) \times U(1)$$

$$\mathbf{35} \quad \rightarrow \quad \mathbf{24}_0 \oplus \mathbf{1}_0 \oplus \mathbf{5}_6 \oplus \bar{\mathbf{5}}_{-6}$$

Adjoints   Bifundamentals



VEV for adjoint scalar: $\langle \phi^{U(1)} \rangle$

$\Rightarrow$ mass for **5** and $\bar{\mathbf{5}}$

$\Rightarrow$ zero-mode wave-functions localized at intersection $\langle \phi^{U(1)} \rangle = 0$

Similarly: **10** from $SO(10) \rightarrow SU(5) \times U(1)$

Chiral matter: include $U(1)$ background fields for A ("gauge flux")

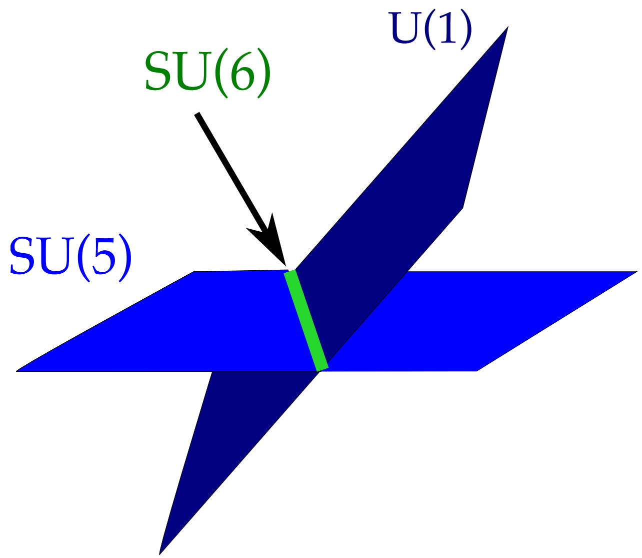
Chiral matter from Curves

Chirality is introduced by switching on gauge bundles on intersecting 7-branes: $U(1)$-bundles $\mathcal{L}_{1,2}$ on S_1 and S_2:

Decompose bifundamentals:

$$G_1 \times G_2 \quad \to \quad H_1 \times H_2 \times U(1)_1 \times U(1)_2$$

$$(\rho^1, \rho^2) \quad \to \quad \bigoplus_j (r_j^1, r_j^2)_{\alpha_j, \beta_j}$$



Zero modes in representation (r_j^1, r_j^2) of $H_1 \times H_2$ with $U(1)$ charges α_j, β_j is

$$N_{(r_j^1, r_j^2)_{\alpha_j, \beta_j}} = h^0(\Sigma, K_\Sigma^{1/2} \otimes \mathcal{L}_1^{\alpha_j}|_\Sigma \otimes \mathcal{L}_2^{\beta_j}|_\Sigma)$$

$K_\Sigma^{1/2}$ = spin-bundle on Σ

Interlude: line bundles on complex curves

$\mathbb{P}^1$: line bundles are classified by degree $n \in \mathbb{Z}$

$$\mathcal{L} = \mathcal{O}(n) : \qquad z^n = \text{ transition functions}$$

Sections:

$$h^0(\mathbb{P}^1, \mathcal{O}(n)) = n + 1, \qquad n \geq 0$$

$$h^0(\mathbb{P}^1, \mathcal{O}(n)) = 0, \qquad n < 0$$

The space of sections is given by polynomials of degree n:
$a_0 + a_1 z + \cdots + a_n z^n$.

Serre duality:

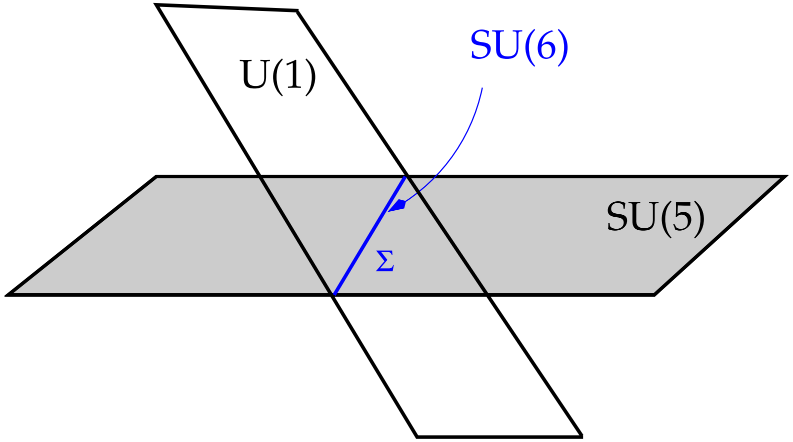
$$h^1(\Sigma, \mathcal{L}) = h^0(\Sigma, \mathcal{L}^* \otimes K_\Sigma)$$

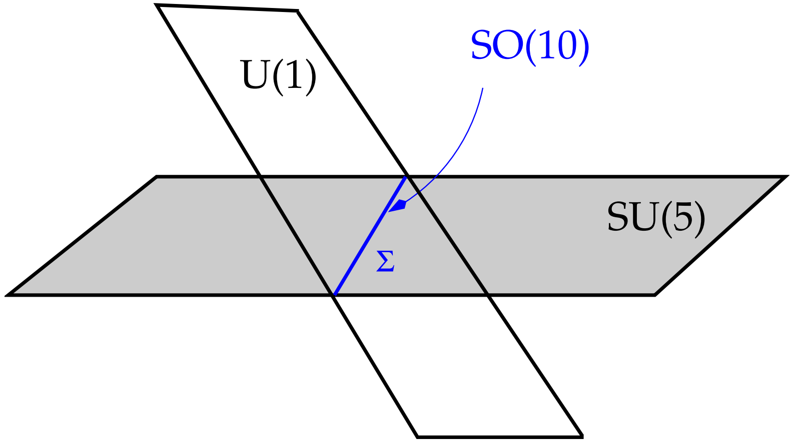
$$K_{\mathbb{P}^1} = \mathcal{O}(-2) \qquad \Rightarrow \qquad h^1(\mathbb{P}^1, \mathcal{O}(n)) = h^0(\mathbb{P}^1, \mathcal{O}(-2-n)) = -n-1\,, \qquad n < 0$$

More generally: on any complex curve, a negative degree bundle has zero holomorphic sections.

Example: $SU(6)$ and $SO(10)$ Enhancements

$G_S = SU(5)$. Consider $\mathbf{\Sigma} = \mathbb{P}^1$, and generate matter $3 \times \bar{\mathbf{5}}$ and $3 \times \mathbf{10}$:





$$SU(6) \longrightarrow SU(5) \times U(1)_a$$

$$\mathbf{35} \longrightarrow \mathbf{24}_0 \oplus \mathbf{1}_0 \oplus \mathbf{5}_{+6} \oplus \bar{\mathbf{5}}_{-6}$$

$$h^0(\mathbb{P}^1, \mathcal{O}(-1) \otimes \mathcal{L}_a^{+6}|_\Sigma) = 0$$

$$h^0(\mathbb{P}^1, \mathcal{O}(-1) \otimes \mathcal{L}_a^{-6}|_\Sigma) = 3$$

$$\Rightarrow \mathcal{L}_a^{-6}|_\Sigma = \mathcal{O}(3)$$

$$SO(10) \longrightarrow SU(5) \times U(1)_b$$

$$\mathbf{45} \longrightarrow \mathbf{24}_0 \oplus \mathbf{1}_0 \oplus \mathbf{10}_{+4} \oplus \overline{\mathbf{10}}_{-4}$$

$$h^0(\mathbb{P}^1, \mathcal{O}(-1) \otimes \mathcal{L}_b^{+4}|_\Sigma) = 3$$

$$h^0(\mathbb{P}^1, \mathcal{O}(-1) \otimes \mathcal{L}_b^{-4}|_\Sigma) = 0$$

$$\Rightarrow \mathcal{L}_b^{4}|_\Sigma = \mathcal{O}(3)$$

Yukawa couplings

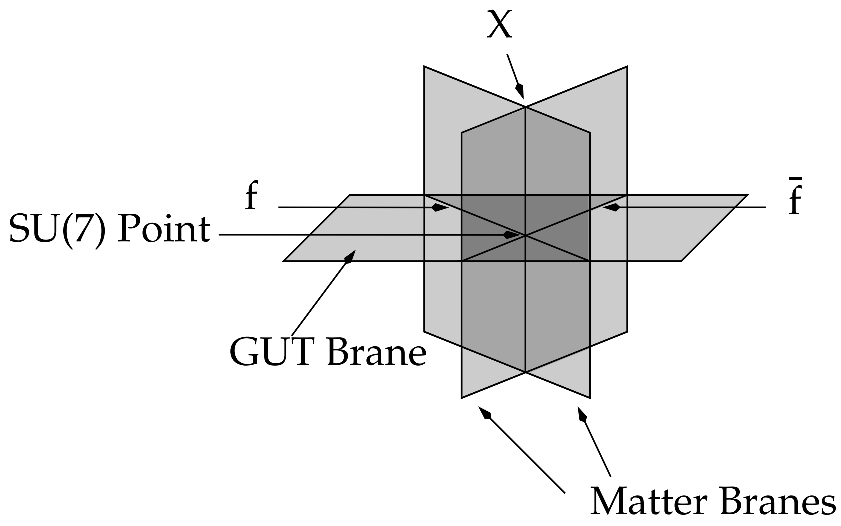
Yukawa couplings from rank 2 enhancement:

$$G_p \to SU(5) \times U(1)_1 \times U(1)_2$$

Examples:

$$SU(7) \to SU(5) \times U(1)_1 \times U(1)_2$$

$$\begin{aligned} \mathbf{48} \to & \left(\mathbf{24}_{0,0} \oplus \mathbf{1}_{0,0} \oplus \mathbf{1}_{0,0}\right) \\ & \oplus \left(\mathbf{5}_{0,+6} \oplus \overline{\mathbf{5}}_{0,-6}\right) \\ & \oplus \left(\mathbf{5}_{-6,0} \oplus \overline{\mathbf{5}}_{+6,0}\right) \\ & \oplus \left(\mathbf{1}_{+6,+6} \oplus \mathbf{1}_{-6,-6}\right) \\ & \Rightarrow \quad W \sim X f \bar{f} \end{aligned}$$



$SO(12)$ and E_6 required for

$$W \supset \lambda_b \, \overline{\mathbf{5}}_H \times \overline{\mathbf{5}}_M \times \mathbf{10}_M + \lambda_t \, \mathbf{5}_H \times \mathbf{10}_M \times \mathbf{10}_M$$

GUT couplings from $SO(12)$ and E_6 Enhancements

Higgs-Matter couplings that are essential for $SU(5)$ GUTs can be generated from two types of enhancements:

$$SO(12) \rightarrow SU(5) \times U(1)_1 \times U(1)_2$$

$$\begin{aligned} \mathbf{66} \rightarrow & \left(\mathbf{24}_{0,0} \oplus \mathbf{1}_{0,0} \oplus \mathbf{1}_{0,0}\right) \\ & \oplus \left(\mathbf{5}_{2,2} \oplus \bar{\mathbf{5}}_{-2,-2}\right) \\ & \oplus \left(\mathbf{5}_{-2,2} \oplus \bar{\mathbf{5}}_{2,-2}\right) \\ & \oplus \left(\mathbf{10}_{0,4} \oplus \overline{\mathbf{10}}_{0,-4}\right) \end{aligned}$$

$$W \sim \mathbf{5} \times \mathbf{5} \times \overline{\mathbf{10}} + \bar{\mathbf{5}} \times \bar{\mathbf{5}} \times \mathbf{10}$$

$$\Rightarrow W \sim \bar{H}_{\bar{\mathbf{5}}} \Phi_{\bar{\mathbf{5}}} \Phi_{\mathbf{10}}$$

$$E_6 \rightarrow SU(5) \times U(1)_1 \times U(1)_2$$

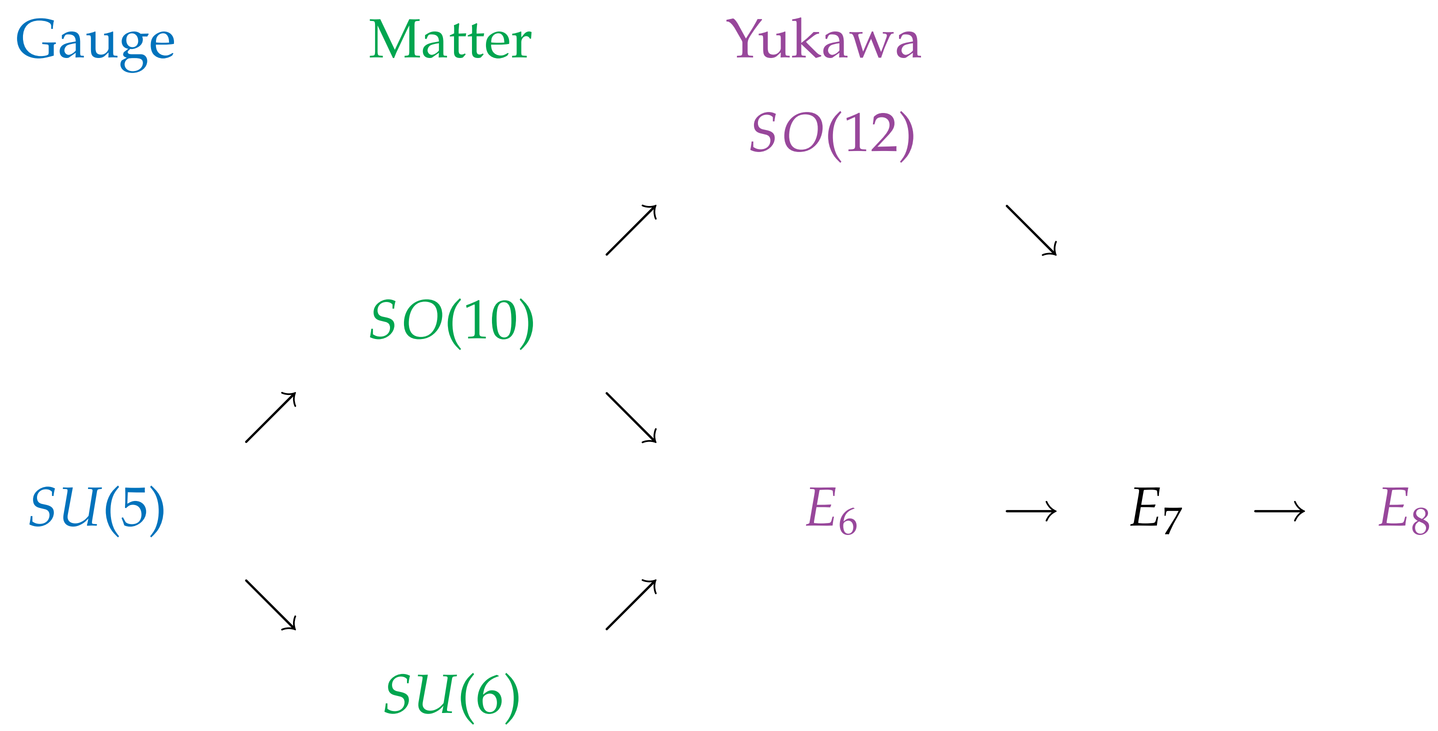
$$\begin{aligned} \mathbf{78} \rightarrow & \left(\mathbf{24}_{0,0} \oplus \mathbf{1}_{0,0} \oplus \mathbf{1}_{0,0}\right) \\ & \oplus \left(\mathbf{5}_{-3,3} \oplus \bar{\mathbf{5}}_{3,-3}\right) \\ & \oplus \left(\mathbf{10}_{-1,-3} \oplus \overline{\mathbf{10}}_{1,3}\right) \\ & \oplus \left(\mathbf{10}_{4,0} \oplus \overline{\mathbf{10}}_{-4,0}\right) \oplus \left(\mathbf{1}_{-5,-3} \oplus \mathbf{1}_{5,3}\right) \end{aligned}$$

$$W \sim \mathbf{5} \times \mathbf{10} \times \mathbf{10} + \bar{\mathbf{5}} \times \overline{\mathbf{10}} \times \overline{\mathbf{10}}$$

$$\Rightarrow W \sim H_{\mathbf{5}} \Phi_{\mathbf{10}} \Phi_{\mathbf{10}}$$

Local $SU(5)$ GUT

Necessary to get all the matter and Yukawas:



$\Rightarrow$ Unified description in terms of E_8 gauge theory

Lecture 2: HOW?

[Donagi, Wijnholt], [Hayashi, Kawano, Tatar, Watari], [Marsano, Saulina,S-N]

Matter

Matter arises from Higgsing a higher rank gauge group.

E.g. **5** and $\bar{\mathbf{5}}$ from $SU(6)$

$$SU(6) \quad \rightarrow \quad SU(5) \times U(1)$$

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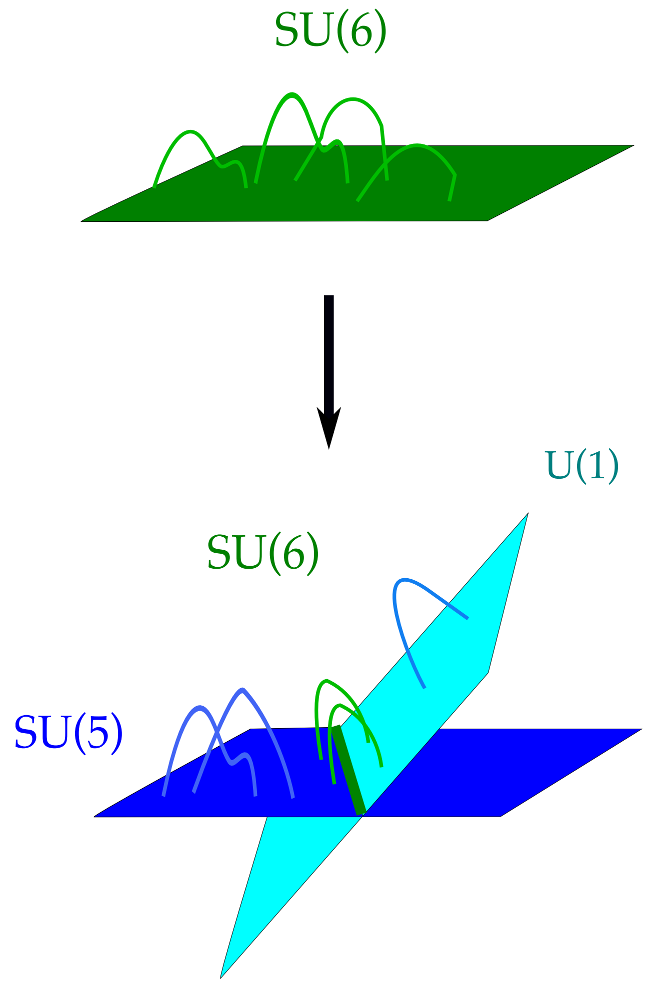
Adjoints Bifundamentals



VEV for adjoint scalar: $\langle \phi^{U(1)} \rangle$

$\Rightarrow$ mass for **5** and $\bar{\mathbf{5}}$

$\Rightarrow$ zero-mode wave-functions localized at intersection $\langle \phi^{U(1)} \rangle = 0$

More on Matter: Higgs bundle perspective

$$SU(6) \quad \rightarrow \quad SU(5) \times U(1)$$

$$\mathbf{35} \quad \rightarrow \quad \mathbf{24}_0 \oplus \mathbf{1}_0 \oplus \mathbf{5}_6 \oplus \bar{\mathbf{5}}_{-6}$$

Adjoints Bifundamentals

VEV for adjoint scalar: $\langle \phi^{U(1)} \rangle$.

EoM for 8d fermions $\mathbf{5}$ and $\bar{\mathbf{5}}$:

$$\omega \wedge D_A \psi_\alpha + \frac{i}{2}[\overline{\phi}, \chi_\alpha] = 0$$
$$\bar{D}_A \chi_\alpha - [\overline{\phi}, \psi_\alpha] = 0$$

$\langle \phi^{U(1)} \rangle$ acts as mass for fermions, and we get massless fermions localized precisely at

$$\text{Matter locus}: \qquad \langle \phi^{U(1)} \rangle = 0$$

Yukawa couplings

Yukawa couplings from rank 2 enhancement:

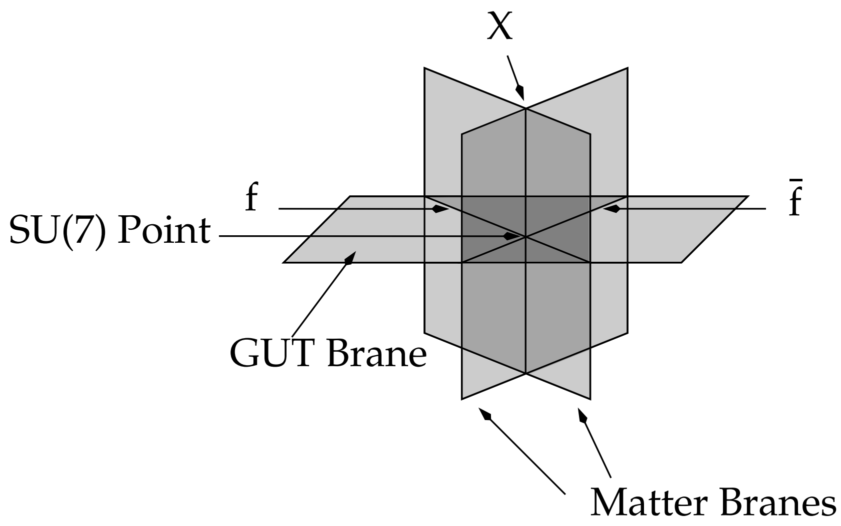
$$G_p \xrightarrow{(\phi^{U(1)_1},\phi^{U(1)_2})} SU(5)\times U(1)_1\times U(1)_2$$

Examples:

$$SU(7) \longrightarrow SU(5)\times U(1)_1\times U(1)_2$$

$$\mathbf{48} \longrightarrow \left(\mathbf{24}_{0,0}\oplus \mathbf{1}_{0,0}\oplus \mathbf{1}_{0,0}\right)$$
$$\oplus \left(\mathbf{5}_{0,+6}\oplus \overline{\mathbf{5}}_{0,-6}\right)$$
$$\oplus \left(\mathbf{5}_{-6,0}\oplus \overline{\mathbf{5}}_{+6,0}\right)$$
$$\oplus \left(\mathbf{1}_{+6,+6}\oplus \mathbf{1}_{-6,-6}\right)$$

$$\Rightarrow \qquad W\sim Xf\bar{f}$$



Local $SU(5)$ GUT

Necessary to get all the matter and Yukawas:

$$
\begin{array}{ccccccccc}
\text{Gauge} & & \text{Matter} & & \text{Yukawa} & & & & \\
 & & & & SO(12) & & & & \\
 & & & \nearrow & & \searrow & & & \\
 & & SO(10) & & & & & & \\
 & \nearrow & & \searrow & & & & & \\
SU(5) & & & & E_6 & \longrightarrow & E_7 & \longrightarrow & E_8 \\
 & \searrow & & \nearrow & & & & & \\
 & & SU(6) & & & & & &
\end{array}
$$

$\Rightarrow$ Unified description in terms of E_8 gauge theory

E_8

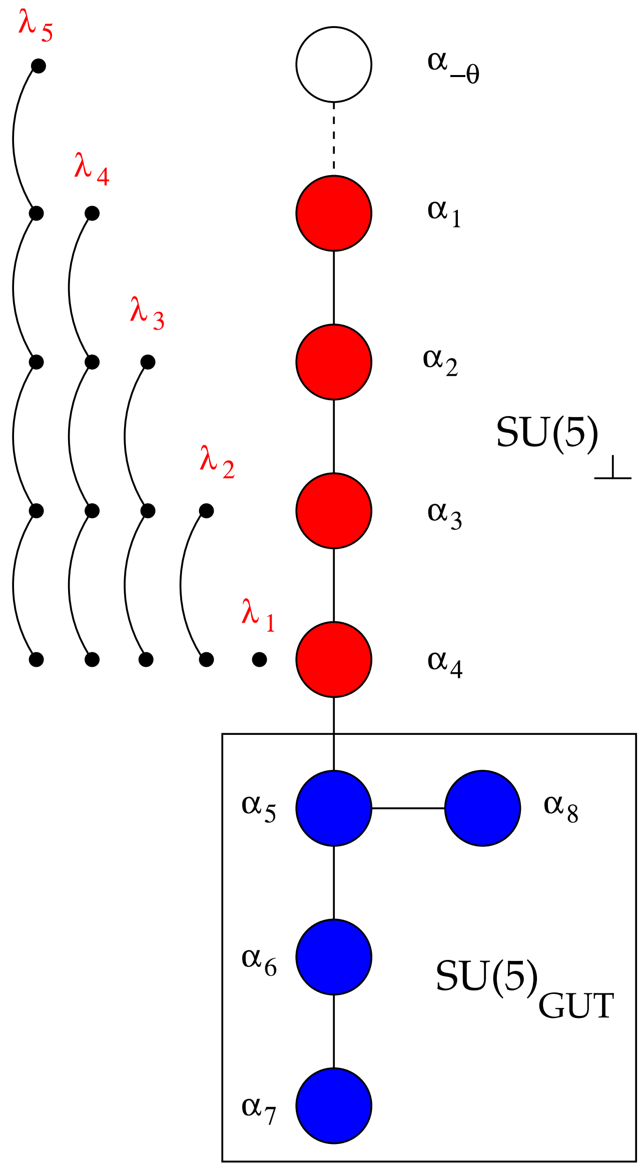
Consider starting point to be E_8 gauge theory, which is Higgsed to $SU(5)_{GUT}$ (or alternative GUT group $\subset E_8$)

$$E_8 \quad \rightarrow \quad SU(5)_\perp \times SU(5)_{\text{GUT}}$$

$$\mathbf{248} \quad \rightarrow \quad (\mathbf{24}, \mathbf{1}) + (\mathbf{1}, \mathbf{24}) + (\overline{\mathbf{10}}, \mathbf{5}) + (\overline{\mathbf{5}}, \overline{\mathbf{10}}) + (\mathbf{10}, \overline{\mathbf{5}}) + (\mathbf{5}, \mathbf{10})$$

broken via

$SU(5)_\perp$ adjoint vev $\langle \Phi \rangle \sim \text{diag}\,(\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5)$



$SU(5)$ from Higgsing E_8

$$E_8 \xrightarrow{\Phi} SU(5)_\perp \times SU(5)_{\mathrm{GUT}}$$

$$\mathbf{248} \longrightarrow (\mathbf{24},\mathbf{1}) + (\mathbf{1},\mathbf{24}) + (\overline{\mathbf{10}},\mathbf{5}) + (\overline{\mathbf{5}},\overline{\mathbf{10}}) + (\mathbf{10},\overline{\mathbf{5}}) + (\mathbf{5},\mathbf{10})$$

$SU(5)_\perp$ adjoint Higgs field VEV

$$\langle \Phi^{SU(5)_\perp} \rangle \sim \mathrm{diag}\,(\lambda_1, \lambda_2, \lambda_3\, \lambda_4, \lambda_5)$$

As before: mass for fermions in $SU(5)_{GUT}$ are set by VEV

$$m \sim \langle \Phi^{SU(5)_\perp} \rangle$$

$$\Leftrightarrow \text{ Matter localizes at zeroes of } \langle \Phi^{SU(5)_\perp} \rangle$$

Example:

$(\mathbf{5},\mathbf{10})$: $\quad$ $\mathbf{10}$ localizes at $\lambda_i = 0$

$(\mathbf{10},\overline{\mathbf{5}})$: $\quad$ $\overline{\mathbf{5}}$ localizes at $\lambda_i + \lambda_j = 0$

$\Rightarrow$ Matter loci are characterized in terms of vanishing loci of eigenvalues of $\langle \Phi^{SU(5)_\perp} \rangle$

Geometry and Higgs bundle

Higgs field $\Phi^{SU(5)_\perp}$ breaks E_8 to $SU(5)$, function on the surface S_{GUT}.
Consider its spectral data:

$$\mathcal{C} = b_5 + b_4 s + b_3 s^2 + b_2 s^3 + b_0 s^5 = \det(s - \langle \Phi^{SU(5)_\perp} \rangle) = 0$$

where s is a section of K_S. More precisely:

$$\mathcal{C} \subset \mathbb{P}^1(K_S \oplus \mathcal{O}_S)$$

Equivalently

$$b_m \sim \mathrm{Tr}\Phi^m b_0$$

$\Rightarrow$ Geometric data b_m specify Higgs bundle spectral data.

$$b_n(\lambda_i) = b_0\, P_n(\lambda_i)$$

$$b_5 \sim b_0 \lambda_1 \lambda_2 \lambda_3 \lambda_4 \lambda_5 \,, \qquad b_4 \sim b_0 \sum_{i<j<k<l} \lambda_i \lambda_j \lambda_k \lambda_l$$

$$b_3 \sim b_0 \sum_{i<j<k} \lambda_i \lambda_j \lambda_k \,, \qquad b_2 \sim b_0 \sum_{i<j} \lambda_i \lambda_j \,, \qquad b_1 = \sum_i \lambda_i = 0$$

Matter and Yukawa loci in the Spectral Cover

$$\mathcal{C}\ :\quad b_5 + b_4 s + b_3 s^2 + b_2 s^3 + b_0 s^5 = 0$$

Various enhancement loci translate into

$SO(10)$:	**10** matter	$0 = b_5 \sim \prod_i \lambda_i$
$SU(6)$:	$\bar{\mathbf{5}}$ matter	$0 = P = b_0 b_5^2 - b_2 b_3 b_5 + b_3^2 b_4 \sim \prod(\lambda_i + \lambda_j)$
$SO(12)$:	Bottom:	$0 = b_5 = b_3$: $(\lambda_i + \lambda_j) + (\lambda_k + \lambda_l) + (\lambda_m) = 0$
E_6 :	Top:	$0 = b_5 = b_4$: $(\lambda_i) + (\lambda_j) + (-\lambda_k - \lambda_l) = 0$
E_8 :		$0 = b_2 = b_3 = b_4 = b_5$

Spectral Cover

Spectral cover 5-fold cover of S_{GUT}:

$$\mathcal{C}\ :\quad b_5 + b_4 s + b_3 s^2 + b_2 s^3 + b_0 s^5 = 0$$

Higgses

$$E_8 \longrightarrow SU(5)_{GUT} \times U(1)^4$$

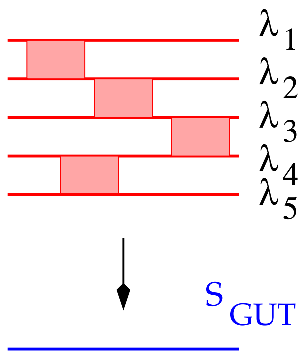
where each $U(1)$ is labeled by λ_i.

However:

$$b_n(\lambda_i) = b_0\, P_n(\lambda_i)$$

$\Rightarrow$ $\lambda_i(b_n)$ has branch-cuts

$\Rightarrow$ generically sheets are connected and we don't get any additional $U(1)$

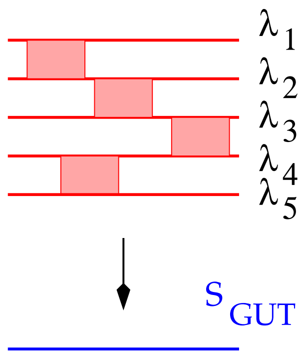


Spectral Cover, Monodromies and $U(1)$s

Spectral cover 5-fold cover of S_{GUT}:

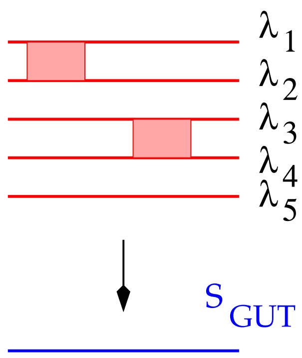
$$\mathcal{C}\ :\ \ b_5 + b_4 s + b_3 s^2 + b_2 s^3 + b_0 s^5 = 0$$

Monodromy $G \subset S_5$ acts on $\mathcal{C}$



$$\Rightarrow \qquad \mathcal{C} = \prod_i \mathcal{C}^{(i)} \qquad \text{factors into orbits of } G_{\text{mono}}$$

- G_{mono} = transitive subgroup of S_5:
  only invariant combination is $\sum_{i=1}^{5} \lambda_i = 0$
  $\Rightarrow$ no gauged $U(1)$



- λ_i in reducible representation of G_{mono}:
  $\mathcal{C}$ factors into N components
  $\Rightarrow (N-1)$ gauged $U(1)$s

Possible Monodromy Groups

Monodromy groups $G \subset S_5$ and orbits of λ_i i.e. $\mathbf{10}$'s:

$\mathbf{10}$ orbits	# of $U(1)'s$	Monodromy Group
$(\lambda_1)(\lambda_2)(\lambda_3)(\lambda_4)(\lambda_5)$	4	id
$(\lambda_i\lambda_j)(\lambda_k)(\lambda_l)(\lambda_m)$	3	$\mathbb{Z}_2$
$(\lambda_i\lambda_j)(\lambda_k\lambda_l)(\lambda_m)$	2	$\mathbb{Z}_2 \times \mathbb{Z}_2$
$(\lambda_i\lambda_j\lambda_k)(\lambda_l)(\lambda_m)$	2	$S_3\,,\ \mathbb{Z}_3$
$(\lambda_i\lambda_j\lambda_k)(\lambda_l\lambda_m)$	1	$S_3 \times \mathbb{Z}_2\,,\ \mathbb{Z}_3 \times \mathbb{Z}_2$
$(\lambda_i\lambda_j\lambda_k\lambda_l)(\lambda_m)$	1	$S_4\,,\ D_4\,,\ \mathbb{Z}_4\,,\ \text{Klein}_4$
$(\lambda_1\lambda_2\lambda_3\lambda_4\lambda_5)$	0	S_5

For $4+1$: $D_4\,,\ \mathbb{Z}_4\,,\ \text{Klein}_4$ gives different orbits for $\overline{\mathbf{5}}$ i.e. $\lambda_i + \lambda_j$.

Refined structure from Spectral Cover

Example: $G = \mathbb{Z}_4 =< (1234) >$:

Decomposition of λ_i under $G = C_4$:

$$\{\lambda_i\} \rightarrow \mathbf{10}^{(1)}_{\lambda_1,\lambda_2,\lambda_3,\lambda_4} + \mathbf{10}^{(2)}_{\lambda_5}$$

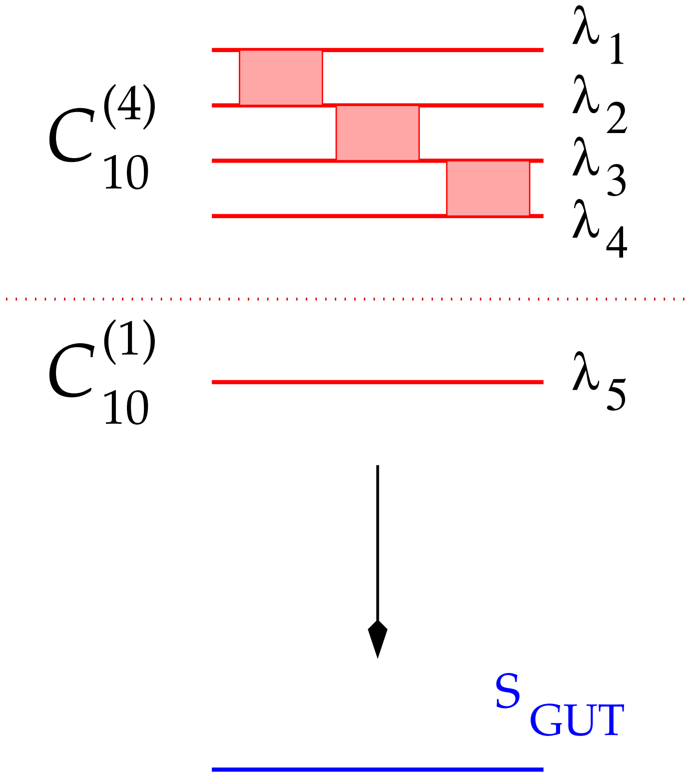
$$\{\lambda_i + \lambda_j\} \rightarrow \overline{\mathbf{5}}^{(1)}_{\lambda_1+\lambda_2,\cdots} + \overline{\mathbf{5}}^{(2)}_{\lambda_1+\lambda_3,\cdots} + \overline{\mathbf{5}}^{(3)}_{\lambda_1+\lambda_5,\cdots}$$

E_6 and $SO(12)$ points yield:

$$\mathbf{10}^{(1)} \times \mathbf{10}^{(1)} \times \mathbf{5}^{(1)}, \quad \mathbf{10}^{(1)} \times \mathbf{10}^{(1)} \times \mathbf{5}^{(2)}$$

$$\mathbf{10}^{(2)} \times \overline{\mathbf{5}}^{(1)} \times \overline{\mathbf{5}}^{(1)}, \quad \mathbf{10}^{(2)} \times \overline{\mathbf{5}}^{(2)} \times \overline{\mathbf{5}}^{(2)},$$

But: $\mathbf{10}^{(2)} \times \overline{\mathbf{5}}^{(1)} \times \overline{\mathbf{5}}^{(2)}$ forbidden.



We can assign $U(1)$ charges to get all the allowed couplings. But there is no assignment of charges that excludes $\mathbf{10}^{(2)} \times \overline{\mathbf{5}}^{(1)} \times \overline{\mathbf{5}}^{(2)}$. This requires knowledge of the monodromies.

Simple constraints from absence of proton decay

Any $4+1$ factorization does in fact not fobid

$$\text{dim 5 proton decay:} \qquad \frac{Q^3 L}{\Lambda}$$

In fact to construct a model that forbids this

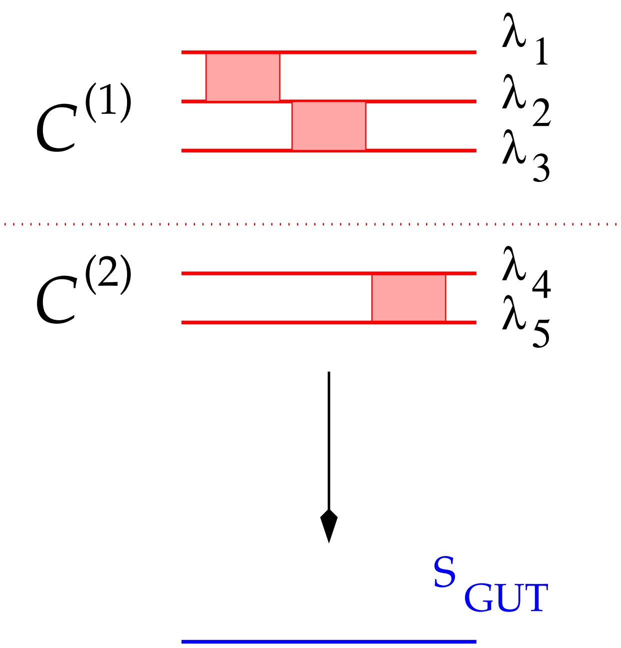
$$U(1)_{PQ}: \qquad q_{H_u} + q_{H_d} \neq 0$$

In combination with Yukawas this implies absence of dim 5 operator.

How to generate such models?

Example Model: 3+2 factorization

$$\mathcal{C} = \mathcal{C}^{(1)}\mathcal{C}^{(2)} : \qquad (a_0 s^3 + a_1 s^2 + a_2 s + a_3)(e_0 s^2 + e_1 s + e_2) = 0\,.$$



$$b_5 = a_3 e_2$$

$$b_4 = a_3 e_1 + a_2 e_2$$

$$b_3 = a_3 e_0 + a_2 e_1 + a_1 e_2$$

$$b_2 = a_2 e_0 + a_1 e_1 + a_0 e_2$$

$$b_0 = a_0 e_0$$

$$b_1 = 0 \qquad \Rightarrow a_1 e_0 + a_0 e_1 = 0$$

Consider e.g.

$$a_0 = \alpha e_0\,, \qquad a_1 = -\alpha e_1$$

Matter

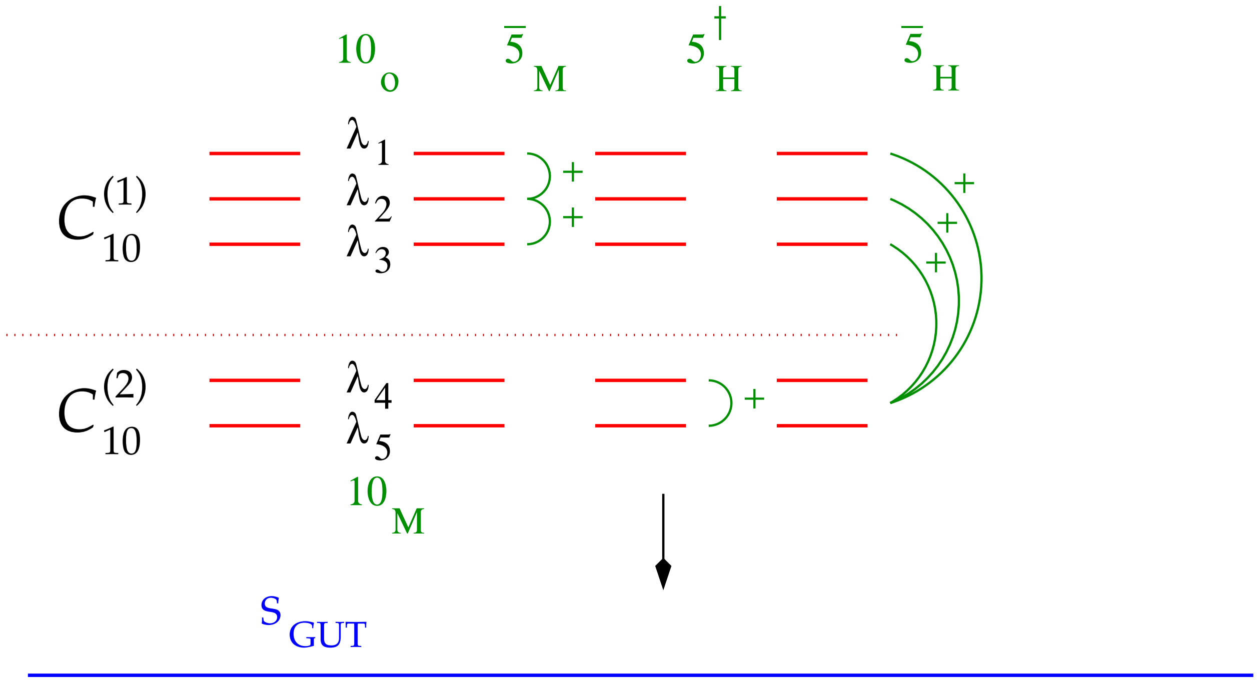
Matter	Spectral Cover Origin	Weights	$U(1)$ Charge
$\mathbf{10}^{(1)}$	$\mathcal{C}^{(1)}$	λ_i	-2
$\mathbf{10}^{(2)}$	$\mathcal{C}^{(2)}$	λ_a	$+3$
$\overline{\mathbf{5}}^{(1)}$	$\mathcal{C}^{(1)} - \mathcal{C}^{(1)}$	$\lambda_i + \lambda_j$	-4
$\overline{\mathbf{5}}^{(2)}$	$\mathcal{C}^{(2)} - \mathcal{C}^{(2)}$	$\lambda_a + \lambda_b$	$+6$
$\overline{\mathbf{5}}^{(1)(2)}$	$\mathcal{C}^{(1)} - \mathcal{C}^{(2)}$	$\lambda_i + \lambda_a$	$+1$

(1)

Matter

$$\Sigma_{\mathbf{10}}: \qquad b_5 = a_3 e_2 = 0$$

$$\Sigma_{\bar{\mathbf{5}}}: \qquad P = P_1 P_2 P_3 = 0$$



Matter	$U(1)_{PQ}$
$\mathbf{10}_M$	$+3$
$\mathbf{5}_H$	-6
$\bar{\mathbf{5}}_H$	$+1$
$\bar{\mathbf{5}}_M$	-4
$\mathbf{10}_{\text{other}}$	-2

Spectral Cover Fluxes and Chirality

So far we just translated the matter and Yukawa loci into the language of the spectral cover.

Recall

$$\mathcal{C} \subset \mathbb{P}^1(K_S \oplus \mathcal{O}_S) \xrightarrow{\pi} S$$

with homogeneous coordinates $[U, V] \in \mathbb{P}^1$, where $U \in H^0(\mathcal{O}(1) \otimes K_S)$ and $V \in H^0(\mathcal{O}(1))$.

When writing $\mathcal{C} : b_5 + b_4 s + b_3 s^2 + b_2 s^3 + b_0 s^5 = 0$, $s = \frac{U}{V}$ is a section of K_S.

In this framework: Chiral matter?

$\Rightarrow$ Higgs bundle is data Φ and a gauge field A, which corresponds to an $(SU(N))$ vector bundle V over S.

$\Rightarrow$ Construction of V from line bundle over $\mathcal{C}$

Spectral cover flux for $SU(N)$

Given spectral cover $\mathcal{C} \xrightarrow{p_{\mathcal{C}}} S$, and a line bundle

$$\mathcal{L} \longrightarrow \mathcal{C}$$

$$\Rightarrow \qquad V = p_{\mathcal{C}*}\mathcal{L} \longrightarrow S$$

is an $SU(n)$ vector bundle over S if

$$c_1(\mathcal{L}) \in H^{1,1}(\mathcal{C}, \mathbb{Z}) \qquad \text{and} \qquad c_1(p_*\mathcal{L}) = 0 \quad \text{tracelessness}$$

Main idea:

construct $H^{1,1}$ classes from curves in $\mathcal{C}$ (dual to $(1,1)$ forms).

Spectral cover flux for $SU(N)$

Let σ = hyperplane class of $\mathbb{P}^1$. Consider curves

$$\mathcal{C} \cdot \pi^* \Sigma \qquad \text{and} \qquad \mathcal{C} \cdot \sigma$$

Suitable linear combination of these are both properly quantized and satisfy $c_1 = 0$. For example

$$\gamma = (N\sigma - \pi^*(\Sigma_N)) \cdot \mathcal{C}$$

where Σ_N =curve at $s = 0$ in $\mathcal{C}$.
For $N = 5$ this is $b_5 = 0$ which is just the **10** matter curve.

Chirality: Restriction to matter curves gives chirality: $\gamma.\Sigma_{matter}$

$\Rightarrow$ $\mathcal{C}$ can be used to construct fluxes
$\Rightarrow$ For $\mathcal{C} = \mathcal{C}^{(n)}\mathcal{C}^{(m)}$ more choices as we can satisyfy $c_1(p_*\mathcal{L}) = 0$ by e.g. $(n\mathcal{C}^{(m)} - m\mathcal{C}^{(n)}) \cdot \sigma$

Summary Lecture 2:

$8d$ SYM on $S \times \mathbb{R}^{1,3}$ with Higgs bundle (A, Φ)

- Spectral cover $\mathcal{C}$ = "eigenvalue data of Φ"
- $\mathcal{C}$ Branched covering over S
- Construct bundle $V \to S$ from line bundle $\mathcal{L} \to S$

Approach is not limited to $8d$ SYM

- Heterotic string in 10d on elliptic CY3 has spectral cover construction
- Proposals by M. Wijnholt to study in the same veine:
  9d SYM (Type I')
  7d SYM (M-theory on G_2)

$\Rightarrow$ For the remaining lectures we'll develop the string theoretic realization of the Higgs bundles for 8d

$\Rightarrow$ F-theory

Next:

Lecture 3: F-theory: from Higgs Bundles to Geometry