Multiplicity Fluctuations
Past - Present - Future

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VI-SIM workshop in Bad Liebenzell

H-QM Helmholtz Research School Quark Matter Studies

Outline

Multiplicity Fluctuations in Elementary Particle Collisions

Inspired by Z. Koba, H. B. Nielsen and P. Olesen, Nucl. Phys. B **40** (1972) 317

PHYSICAL REVIEW D VOLUME 7, NUMBER 7 1 APRIL 1973

Evidence for the Systematic Behavior of Charged-Prong Multiplicity Distributions in High-Energy Proton-Proton Collisions*

P. Slattery
Department of Physics and Astronomy, University of Rochester, Rochester, New York 14627
(Received 2 October 1972)

Evidence is presented to support the onset in the 50–303-GeV/c region of incident momentum of the asymptotic prediction of Koba, Nielsen, and Olesen regarding the scaling behavior of the charged-prong multiplicity distribution in proton-proton collisions. Lower-energy data, at 19 and 28.5 GeV/c, are found not to demonstrate this behavior. The impact of this observation on the Mueller-Regge viewpoint is discussed.

One of the simplest and most direct measurements which can be made with a bubble chamber is the determination of charged-particle multiplicities. The present availability of bubble-chamber facilities at Serpukhov, USSR, and at Batavia, USA, has consequently made available for the first time accurate measurements of topological cross sections for very-high-energy proton-proton collisions (50–303 GeV/c). In this article we wish to examine the energy variation of these partial cross sections, and to compare the experimental data with two contrasting asymptotic predictions,

Multiplicity Fluctuations in Elementary Particle Collisions

Inspired by Z. Koba, H. B. Nielsen and P. Olesen, Nucl. Phys. B **40** (1972) 317

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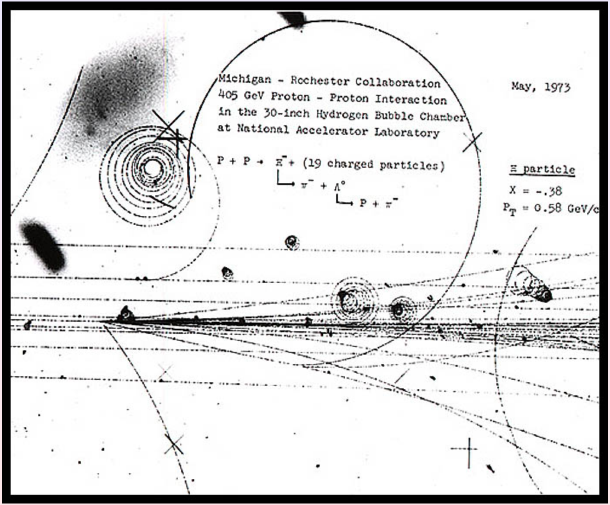
One of the simplest and most direct measurements which can be made with a bubble chamber is the determination of charged-particle multiplicities. The present availability of bubble-chamber

A Bubble Chamber Event



Multiplicity Distributions in elementary collisions

TABLE I. Experimental values of $\langle n \rangle$ and of $\langle n \rangle (\sigma_n / \sigma_{\text{inel}})$ for the reaction $pp \to n$ charged particles at incident momenta of 19, 50, 69, 102, 205, and 303 GeV/c.

n \ p_{lab} (GeV/c)	19 [a]	50	69	102	205	303
			$\langle n \rangle \sigma_n / \sigma_{\text{inel}}$			
2	1.227 ±0.020	1.08 ±0.11	0.940 ±0.068	0.88 ±0.10	0.82 ±0.17	0.50 ±0.14
4	1.783 ±0.027	1.583 ±0.090	1.611 ±0.048	1.589 ±0.091	1.299 ±0.079	1.348 ±0.076
6	0.789 ±0.021	1.345 ±0.084	1.476 ±0.046	1.489 ±0.096	1.622 ±0.096	1.589 ±0.090
8	0.186 ±0.010	0.848 ±0.069	1.013 ±0.038	1.140 ±0.087	1.354 ±0.085	1.504 ±0.086
10	0.0314 ±0.0039	0.344 ±0.037	0.513 ±0.024	0.692 ±0.066	1.031 ±0.074	1.315 ±0.084
12	0.001 50 ±0.000 88	0.081 ±0.017	0.238 ±0.015	0.399 ±0.054	0.801 ±0.064	1.168 ±0.083
14	0.000 50 ±0.000 50	0.036 ±0.011	0.0740 ±0.0079	0.137 ±0.030	0.397 ±0.039	0.605 ±0.058
16		0.0032 ±0.0032	0.0198 ±0.0039	0.037 ±0.016	0.204 ±0.026	0.388 ±0.045
18			0.0023 ±0.0013	0.019 ±0.011	0.071 ±0.016	0.241 ±0.035
20					0.042 ±0.012	0.142 ±0.026
22					0.0129 ±0.0066	0.0189 ±0.0092
24						0.028 ±0.011
26						0.0142 ±0.0085
			$\langle n \rangle$			
	4.018 ±0.022	5.32 ±0.11	5.888 ±0.066	6.38 ±0.12	7.65 ±0.16	8.86 ±0.15

[a] Not plotted in Fig. 1.

P. Slattery, Phys. Rev. D **7**, 2073 (1973)

Multiplicity Distributions in elementary collisions

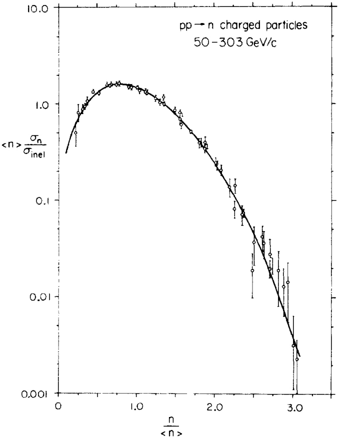


FIG. 1. Plot of $\langle n\rangle(\sigma_n/\sigma_{\rm inel})$ vs $(n/\langle n\rangle)$ for the reaction $pp \to n$ charged particles at incident momenta of 50, 69, 102, 205, and 303 GeV/c. The individual data points are tabulated in Table I. The curve is an empirical fit to these data; the functional form employed is presented in the text [Eq. (2)].

$$\sigma_n/\sigma_{\rm inel} \xrightarrow[s\to\infty]{} \frac{1}{\langle n\rangle}\psi\left(\frac{n}{\langle n\rangle}\right) . \qquad (1)$$

Here, σ_n is the partial cross section for the reaction $pp \to n$ charged particles, $\sigma_{\rm inel}$ is the total inelastic pp cross section (throughout this paper σ_2 does not include the elastic channel), $\langle n\rangle$ is the average number of charged particles produced at a particular value of squared center-of-mass energy s, and ψ is an energy-independent function.

In Fig. 1 we examine this prediction in the 50–303-GeV/c range of incident momenta by plotting $\langle n\rangle(\sigma_n/\sigma_{\rm inel})$ versus $n/\langle n\rangle$ for these data. The fact that a smooth curve may be drawn through all of the data points with a satisfactory χ^2 is a dramatic indication that the semi-inclusive scaling concept is experimentally valid in this range of incident momentum. The particular parametrization of the function $\psi(n/\langle n\rangle)$ which yields the curve shown in the figure is given by the formula

$$\psi(z=n/\langle n\rangle) = (3.79z + 33.7z^3 - 6.64z^5 + 0.332z^7)e^{-3.04z} . \qquad (2)$$

P. Slattery, Phys. Rev. D **7**, 2073 (1973)
Eq.(1) from Z. Koba, H. B. Nielsen and P. Olesen, Nucl. Phys. B **40** (1972) 317.

The Concept of Similarity of Distributions

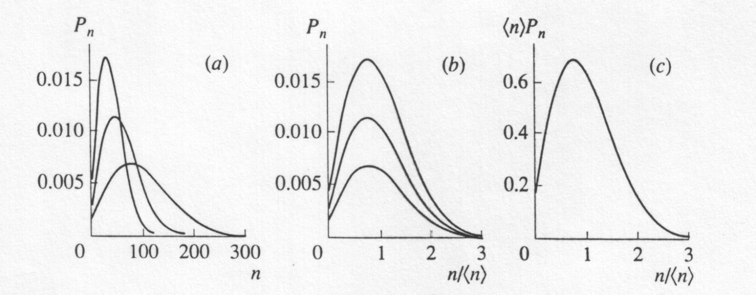
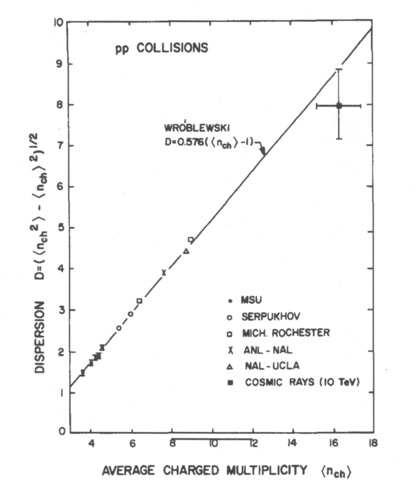


Fig. 1. Definition of the concept of similarity of continuous functions (KNO scaling). Normalized functions (*a*) are similar if upon a linear contraction of each of them along the horizontal direction in proportion to some of its horizontal dimensions—for example, $\langle n \rangle$ (*b*)—and a linear extension along the vertical direction in the same proportion (*c*) they coincide at each point.

A. I. Golokhvastov, Phys. Atom. Nucl. **64** (2001) 84

Scaling of Dispersions



A. Wroblewski, Acta Phys. Polon. B **4**, 857 (1973).
Figure actually found in:
J. Whitmore,Phys. Rept. **10**, 273 (1974).

KNO scaling

$$P(n) = \frac{\sigma_n}{\sigma_{inel.}} \rightarrow \frac{1}{\langle n \rangle} \psi \left(\frac{n}{\langle n \rangle} \right)$$

implies scaling of moments

$$\langle n^q \rangle \rightarrow c_q \langle n \rangle^q, \quad q = 2, 3, 4, \cdots$$

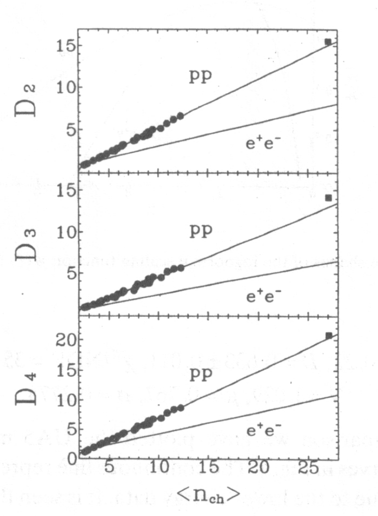
which implies scaling of Dispersions

$$D_q \equiv \left(\langle n^q \rangle - \langle n \rangle^q \right)^{1/q} = \langle n \rangle \times const$$

Scaled Variance

$$\omega \equiv \frac{(D_2)^2}{\langle n \rangle} = \frac{\langle n^2 \rangle - \langle n \rangle^2}{\langle n \rangle}$$

Scaling of Dispersions



M. Gazdzicki, R. Szwed, G. Wrochna and
A. K. Wroblewski, Mod. Phys. Lett. A **6**, 981 (1991).

KNO-G scaling

$P(n) = \int_{\tilde{n}}^{\tilde{n}+1} P(\tilde{n})$, where

$P(\tilde{n}) = \frac{1}{\langle\tilde{n}\rangle}\psi(\frac{\tilde{n}}{\langle\tilde{n}\rangle})$ and $\langle\tilde{n}\rangle \approx \langle n\rangle - 1$

A. I. Golokhvastov, Sov. J. Nucl. Phys. **27**, 430 (1978) [Yad. Fiz. **27**, 809 (1978)]

Wroblewski relation

follows from KNO-G scaling

$$D_2 = 0.576\,(\langle n\rangle - 1)$$

R. Szwed and G. Wrochna, Z. Phys. C **29**, 255 (1985)

Log-Normal Scaling Function

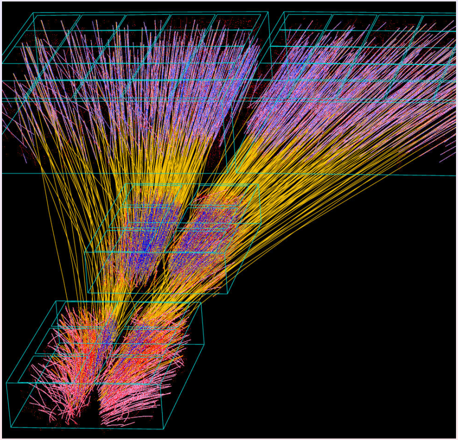
$$\Psi(z) = \frac{N}{\sqrt{2\pi}\sigma}\,\frac{1}{z+c}\,\exp\left[-\frac{[\ln(z+c)-\mu]^2}{2\sigma^2}\right]$$

R. Szwed and G. Wrochna, Z. Phys. C **47** (1990) 449

Multiplicity Fluctuations in Heavy Ion Collisions

- Enhanced fluctuations are one of the main proposed signals for a possible phase transition of QGP matter to hadronic matter or even a possible critical point.
- Unlike for pp multiplicity distributions, data is only available for limited geometric acceptance.
- Unlike in pp collision, the number of interacting nucleons fluctuates in A+A collisions.
- Possibly there are a few ‘unexpected‘ things to learn?

A NA49 event

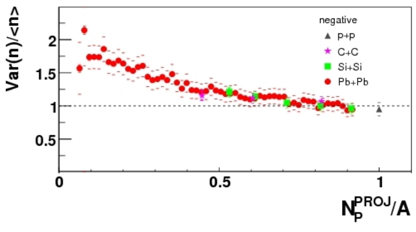


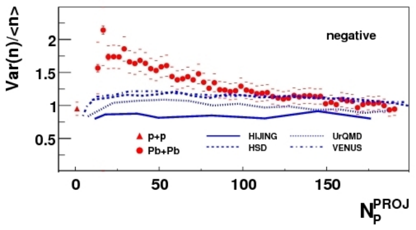
Fixed Target experiments

In this context some important features are:

- mostly forward acceptance
- calorimeter to measure projectile spectators
- however one cannot measure target spectators

Centrality Dependence of NA49 fluctuation data at 158AGeV





C. Alt *et al.* [NA49 Collaboration], Phys. Rev. C **75**, 064904 (2007)

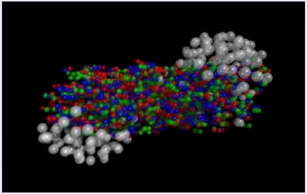
Centrality and Acceptance

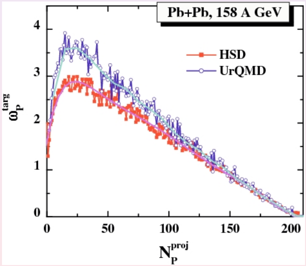
- fixed number of projectile participants
- target participants cannot be measured
- acceptance $1.1 < y_{c.m.} < 2.6$

Apparently unexpected:

- Strong increase towards peripheral collisions seen in Pb+Pb data
- Not reproduced by models!

Fluctuation of N_{part} in Transport Models

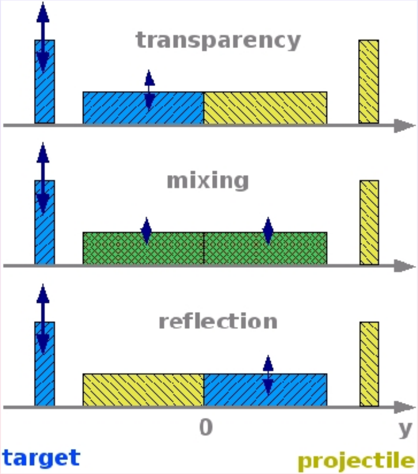




V. P. Konchakovski, S. Haussler, M. I. Gorenstein, E. L. Bratkovskaya, M. Bleicher and H. Stoecker, Phys. Rev. C **73**, 034902 (2006)

- average number of target and projectile participants should be equal, $\langle N_{part}^{targ} \rangle \approx \langle N_{part}^{proj} \rangle$
- BUT: only number of projectile participants is fixed experimentaly, and number of target participant fluctuates considerable (in transport simulations)
- This 'trivial' contribution can be minimized only for the sample of most central events.
- Why is similar behaviour not seen in transport simulations with NA49 accpetance?

Transport Models are too ‘transparent‘

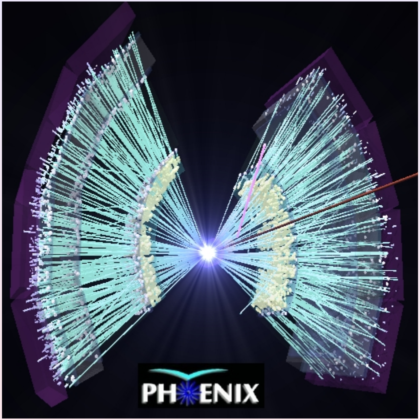


Fluctuations in the target hemnisphere do not move across to the projectile hemnisphere.

There seems to be a significant amount of ‘mixing‘ of target and projectile matter in data.

M. Gazdzicki and M. I. Gorenstein,
Phys. Lett. B **640**, 155 (2006)

A PHENIX event

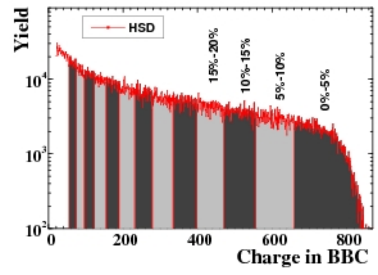


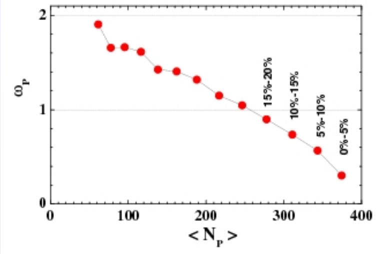
Collider experiments

In this context some important features are:

- mostly acceptance around mid-rapidity
- they have calorimeters to measure spectator nucleons of both colliding ions
- however, since one cannot measure beam fragments, the measurement of N_{part} is rather unprecise

Fluctuation of N_{part} at RHIC





Centrality definition

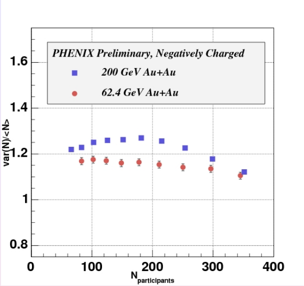
Beam-Beam-Counters (BBC) measure charged particle multiplicity in the pseudo-rapidity range $3.0 < |\eta| < 3.9$

Participant fluctuations

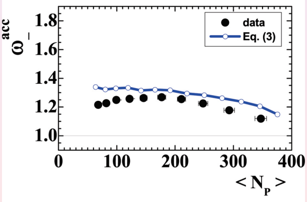
- cannot be neglected!
- but are also not necessarily the same in data and HSD simulations!

V. P. Konchakovski, M. I. Gorenstein and E. L. Bratkovskaya, arXiv:0704.1831 [nucl-th]

PHENIX Multiplicity Fluctuation Data



J. T. Mitchell, J. Phys. Conf. Ser. **27**, 88 (2005)



V. P. Konchakovski, M. I. Gorenstein and E. L. Bratkovskaya, arXiv:0704.1831 [nucl-th]

Independent source model

$$\omega = \omega^{NN} + n\,\omega_P$$

- ω^{NN} mult. fluc. of N+N collisions
- n average multiplicity from one sources
- ω_P fluctuation of number of sources

‘acceptance scaling‘ with q

$$\omega^{acc} = 1 - q + q\,\omega^{NN} + q\,n\,\omega_P$$

Centrality dependence

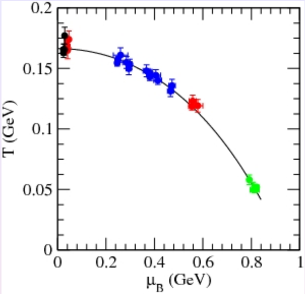
is different from that at SPS!

Possibly only due to different methods for centrality determination ??

A Short Summary on Centrality

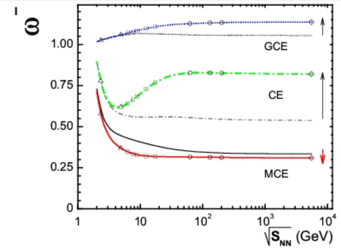
- N_{part} fluctuations are a dominant source of multiplicity fluctuations!
- Even at fixed number of projectile participants - the number of target participants can still vary considerably.
- For the study of multiplicity fluctuations only the sample of most central collisions (about 1%) should be used.
- For purely technical reasons centrality selection is done in different ways in fixed target and collider experiments.

Resonance Gas Multiplicity Fluctuations



- GCE : no conservation laws enforced
- CE : only charge (B,S,Q) conservation
- MCE : energy and charge fixed

Model parameters follow the chemical freeze-out line for central Pb+Pb (Au+Au) collisions of
J. Cleymans, H. Oeschler, K. Redlich, and S. Wheaton, Phys. Rev. C **73**, 034905 (2006)
F. Becattini, J. Manninen, and M. Gaździcki, Phys. Rev. C **73**, 044905 (2006)

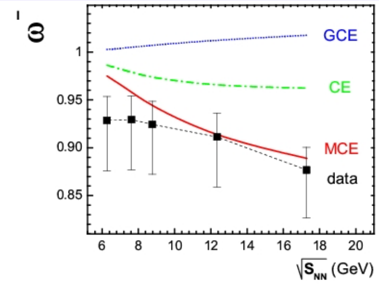


V. V. Begun, M. Gazdzicki, M. I. Gorenstein, M.H., V. P. Konchakovski, and B. Lungwitz, Phys. Rev. C **76** (2007) 024902

Fluctuations are different in different ensembles

V. V. Begun, M. Gazdzicki, M. I. Gorenstein and O. S. Zozulya, Phys. Rev. C **70**, 034901 (2004)
F. Becattini, A. Keranen, L. Ferroni and T. Gabbriellini, Phys. Rev. C **72**, 064904 (2005)

Comparison of Resonance Gas to NA49 Data



B. Lungwitz, AIP Conf. Proc. **892**, 400 (2007)

Agreement with Data is surprising!

Especially since we made some strong assumptions/approximations

do we see the effect of conservation laws on fluctuations?

Experimental Acceptance

- changes from 4% at 20AGeV to about 16% at 158AGeV
- has been taken into account via an 'uncorrelated particle' approximation

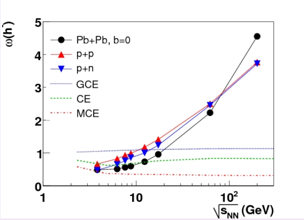
Other Choices for Parameters

For reviews see also:
A. Andronic, P. Braun-Munzinger and J. Stachel, Nucl. Phys. A **772** (2006) 167.
J. Letessier and J. Rafelski, arXiv:nucl-th/0504028.

- lead to VERY similar results
- with the notable exception of γ_q models, however due to energy conservation still $\omega < 1$

V.V.Begun, M.Gazdzicki, M.I.Gorenstein, M.H., V.P.Konchakovski, and B.Lungwitz, Phys. Rev. C **76** (2007) 024902

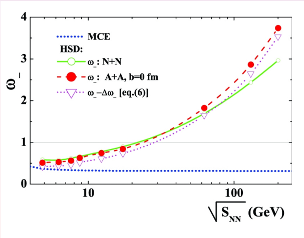
Energy Dependence in Transport Models



UrQMD

HRG

B. Lungwitz and M. Bleicher, arXiv:0707.1788, Phys. Rev. C, in print



HSD

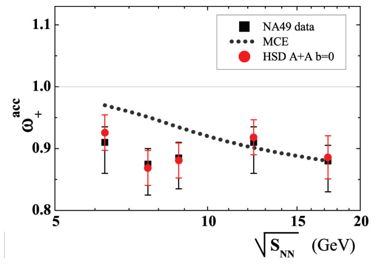
HRG

V. P. Konchakovski, M. I. Gorenstein and E. L. Bratkovskaya, Phys. Lett. B **651**, 114 (2007)

- In both transport models the scaled variance is similar in A+A and p+p collisions
- In particular $\omega \propto \langle N \rangle$ (Wroblewski relation)
- ω increases monotonically with $\sqrt{s_{NN}}$

- In the SPS energy range both relativistic microscopic transport models and MCE HRG are below $\omega < 1$.
- However for RHIC energies they differ by a factor of 10.
- Fluctuations in transport models do not ‘thermalize‘?!

Energy Dependence of NA49 Data



Both HSD and MCE HRG are in good agreement with NA49 multiplicity fluctuation data for (1%) most central Pb+Pb collisions.

Larger experimental acceptance should allow to distinguish equilibrium and non-equilibrium models.

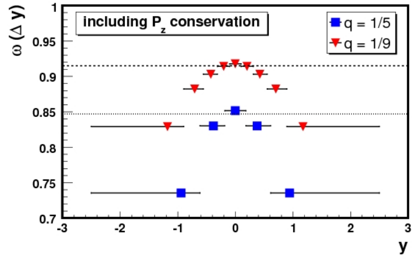
V. P. Konchakovski, M. I. Gorenstein and E. L. Bratkovskaya, Phys. Lett. B **651**, 114 (2007)

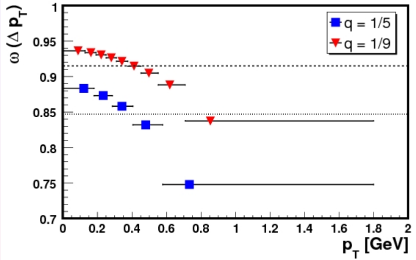
A Short Summary on Energy Dependence

- Transport models show similar behavior of ω in A+A and p+p collisions.
- In comparison to the above the thermal model shows a rather flat dependence of the scaled variance on collision energy.
- Both transport models and MCE formulation of HRG are in good agreement with NA49 data.
- Present NA49 data does not allow for a conclusive distinction between models.

Momentum Cuts in Micro-Canonical Ensemble

Boltzmann pion gas at $T = 160 MeV$ and zero charge density.



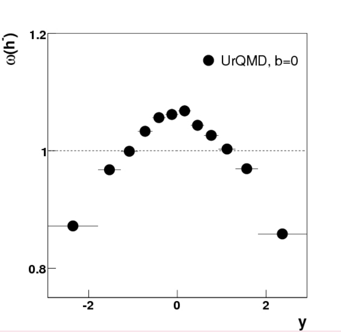


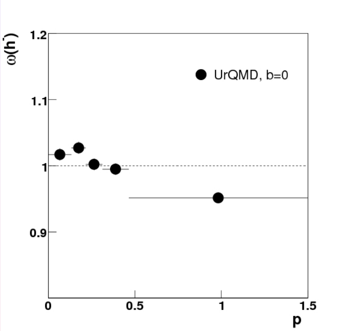
- Each bin contains same fraction of total yield
- Bars indicate size of the bin

Energy and momentum conservation lead to suppressed multiplicity fluctuations at high $|y|$ and p_T.

Momentum Cuts in UrQMD

UrQMD simulation of central Pb+Pb collision at b=0





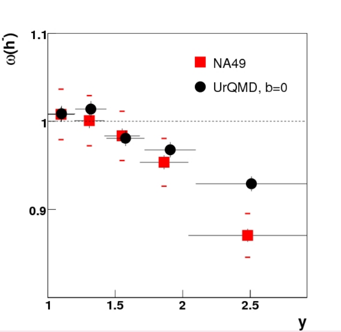
Construction of bins is the same as before.

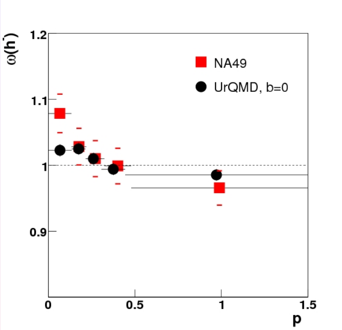
MCE suppression of fluctuations also in non-equilibrium systems?

B. Lungwitz and M. Bleicher, arXiv:0707.1788 [nucl-th], Phys. Rev. C, in print

Momentum Cuts in NA49 Data

UrQMD vs. NA49 158AGeV Pb+Pb data





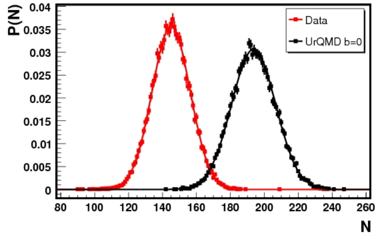
Rapidity and transverse momentum dependence also seen in data!

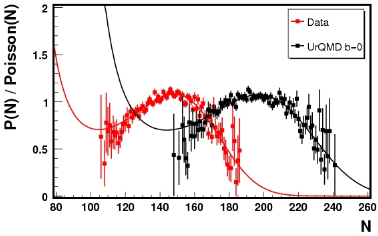
MCE effects are of similar magnitude as proposed enhancement due to a phase transition / critical point!

B. Lungwitz, talk given at Workshop on Critical Point and Onset Deconfinement and private communication

Comparison of UrQMD to NA49 Distribution Data

Multiplicity Distribution of negatively charged particles for most central (1%) Pb+Pb collision at 158 AGeV, both Data and UrQMD simulation, acceptance $1 < y_\pi < y_{beam}$.



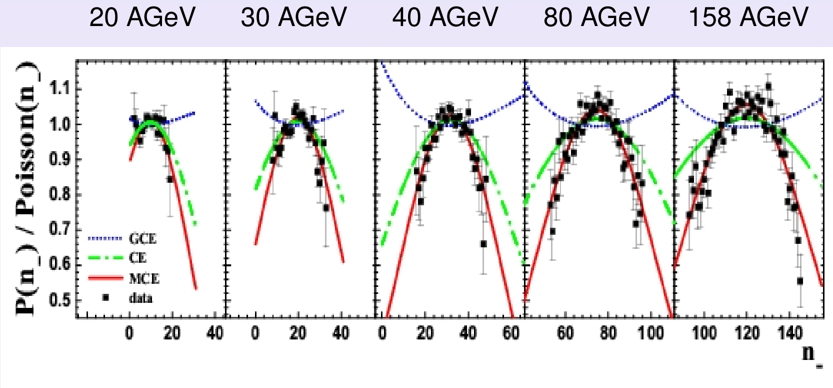


B.Lungwitz and M.Bleicher, private communication

UrQMD overpredicts yields here by 33% but ω agrees within 1%

Both UrQMD and data well fitted by Gaussians !

Comparison of HRG to NA49 Distribution Data



B. Lungwitz *et al.* [NA49 Collaboration], PoS C **FRNC2006**, 024 (2006)
V. V. Begun, M. Gazdzicki, M. I. Gorenstein, M.H, V. P. Konchakovski and B. Lungwitz, Phys. Rev. C **76**, 024902 (2007)

Conclusion

- A lot of good data is now available and has been studied
- More effort is need in order to understand trivial and not so trivial effects in data
- Data in larger acceptance, for different ion sizes, and at different energies would be very helpful
- A systematic study of momentum space dependence should be carried out
- A more detailed description of phase transitions and their effect on fluctuations is needed

Fluctuation data carries quite a lot of information about dynamics!

For supplying plots and references

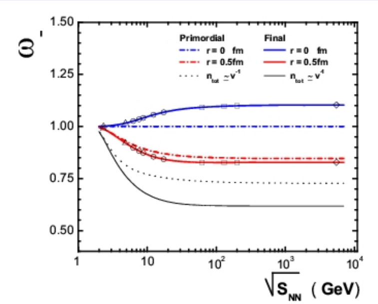
Thanks to

- Benjamin
- Marcus
- Marek
- Giorgio, and
- Volodya

Fluctuations and Interaction

Van der Waals Gas

model repulsive interactions between hadrons



- suppression of densities can be removed by rescaling the system volume
- suppression of fluctuations is qualitatively different
- could be a first step towards a simple model with a phase transition

M.I. Gorenstein, M. Gaździcki, W. Greiner, Phys. Rev. C **72**, 024909 (2005)

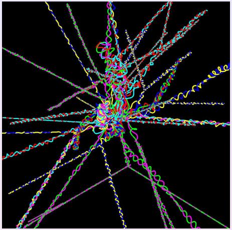
Maybe only of academical interest,

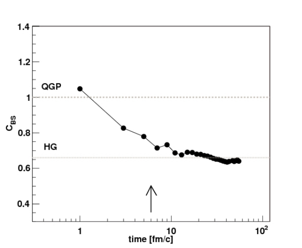
but would that hold true in transport theory?

D.H. Rischke, M.I. Gorenstein, H. Stöcker, W. Greiner, Z.Phys.C**51** 485-490, 1991
M. I. Gorenstein, M.H. and D. O. Nikolajenko, Phys. Rev. C **76**, 024901 (2007)

Fluctuations and the QGP

Does the signal survive hadronization?





S. Haussler, S. Scherer and M. Bleicher, arXiv:hep-ph/0702188.

The qMD model

- treats quarks and anti-quarks as classical point-like objects
- interaction via long-range color potential

M. Hofmann, M. Bleicher, S. Scherer, L. Neise, H. Stoecker and W. Greiner, Phys. Lett. B **478**, 161 (2000)

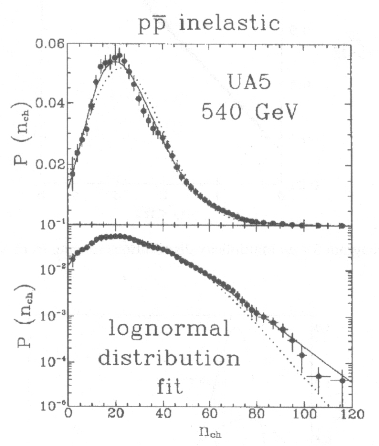
Baryon-strangeness correlations

Different degrees of freedom in QGP and HRG

- $C_{BS} = \frac{\langle B \cdot S \rangle - \langle B \rangle \langle S \rangle}{\langle S^2 \rangle - \langle S \rangle^2}$
- QGP : $C_{BS} \approx 1$
- HRG : $C_{BS} \approx 0.66$

V. Koch, A. Majumder and J. Randrup, Phys. Rev. Lett. **95**, 182301 (2005)

The KNO scaling function



M. Gazdzicki, R. Szwed, G. Wrochna and
A. K. Wroblewski, Mod. Phys. Lett. A **6**, 981 (1991).

Koba-Nielsen-Olesen Scaling Function

$$\Psi(z) = \frac{N}{\sqrt{2\pi}\sigma} \frac{1}{z+c} \exp\left[-\frac{[\ln(z+c)-\mu]^2}{2\sigma^2}\right]$$

R. Szwed and G. Wrochna, Z. Phys. C **47** (1990) 449

And as soon as it was found

Already scaling violation!