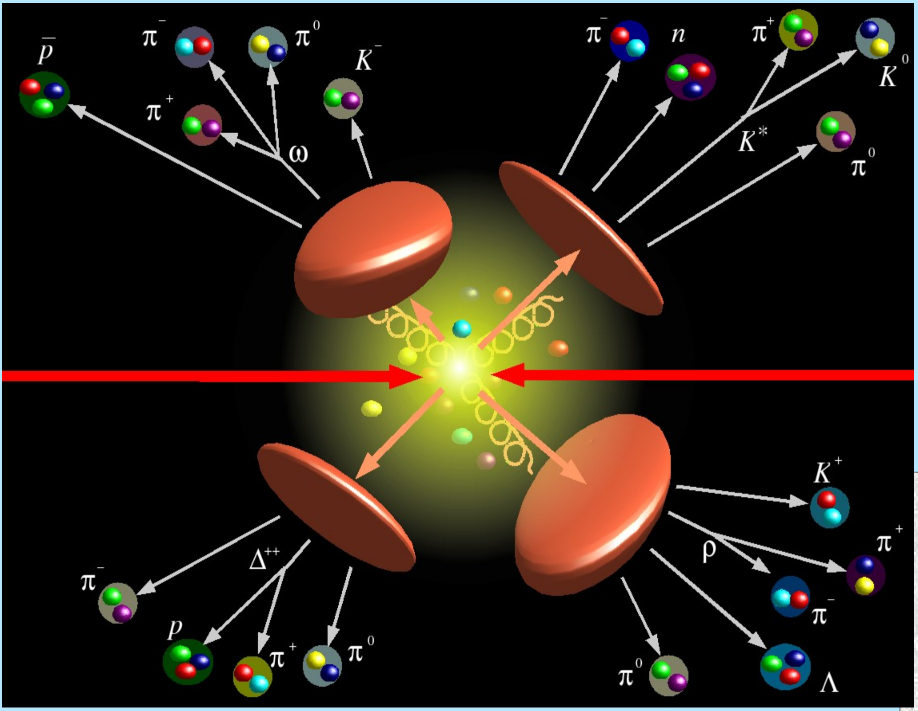
F. Becattini, University of Florence

STATISTICAL MODEL(S): past, present, future

OUTLINE

- Foundations
- Results in heavy ion collisions- debated issues
- Elementary collisions- What is the meaning?
- Conclusions

The basics



Multiple hadron production proceeds from highly excited regions (clusters or fireballs)

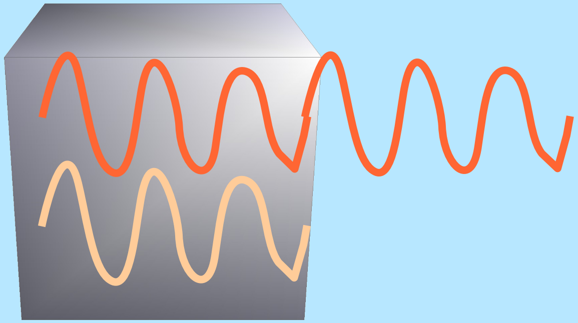
DOGMA:

Every localized multihadronic state compatible with conservation laws is equally likely

Microcanonical ensemble

$$\Omega = \sum_{states} \delta^4 (P - P_{state}) \delta_{Q, Q_{state}} = \sum_{h_V} \langle h_V | \delta^4 (P - \hat{P}) \delta_{Q, \hat{Q}} | h_V \rangle = \sum_{h_V} \langle h_V | P_i | h_V \rangle$$

Beware the difference between a localized $|h_V\rangle$
and an asymptotically free $|f\rangle$ multiparticle state!



$$|1p\rangle_V = \alpha_1|1p\rangle + \alpha_2|2p\rangle + \alpha_3|3p\rangle + \ldots$$

F. B., L. Ferroni, arXiv 0704.1967, EPJC in press

Can we define a probability
for an asymptotic state $|f\rangle$? YES

Define $\boxed{\omega_f \propto \langle f|P_i P_V P_i|f\rangle}$ with $\mathrm{P}_V = \sum_{h_V} |h_V\rangle\langle h_V|$

$$\sum_f \omega_f \propto \mathrm{tr}(P_i P_V P_i) \propto \mathrm{tr}(P_i P_V) = \sum_{h_V} \langle h_V|P_i|h_V\rangle = \Omega$$

It is positive definite and only “conserved” asymptotic states allowed

F. B., “What is the meaning of the statistical hadronization model?”, J. Phys. Conf. Ser. 5 (2005) 175

Interactions: Dashen-Ma-Bernstein theorem

In the thermodynamic limit:

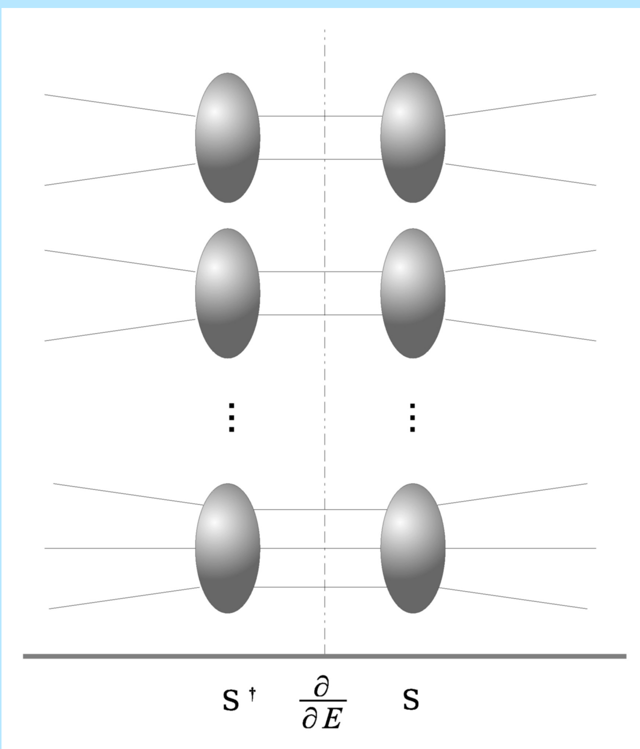
Resonances

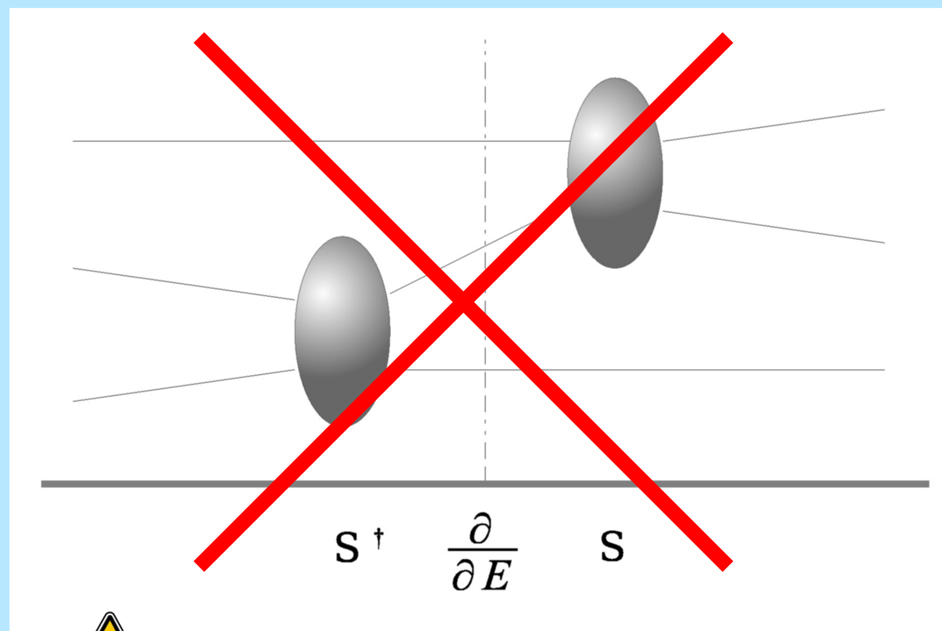
$$Tr\ \delta^4(P-\hat{P}) = Tr\ \delta^4(P-\hat{P}_0) + \frac{1}{4\pi i} Tr\left[S^{-1}\overleftrightarrow{\partial}_E S\,\delta^4(P-\hat{P}_0)\right]$$

Microcanonical

$$Tr\ \exp\left(-\hat{H}/T\right) = Tr\ \exp\left(-\hat{H}_0/T\right) + \frac{1}{4\pi i}\int dE\ \exp\left(-E/T\right) Tr\left[S^{-1}\overleftrightarrow{\partial}_E S\,\delta^4(P-\hat{P}_0)\right]_c$$

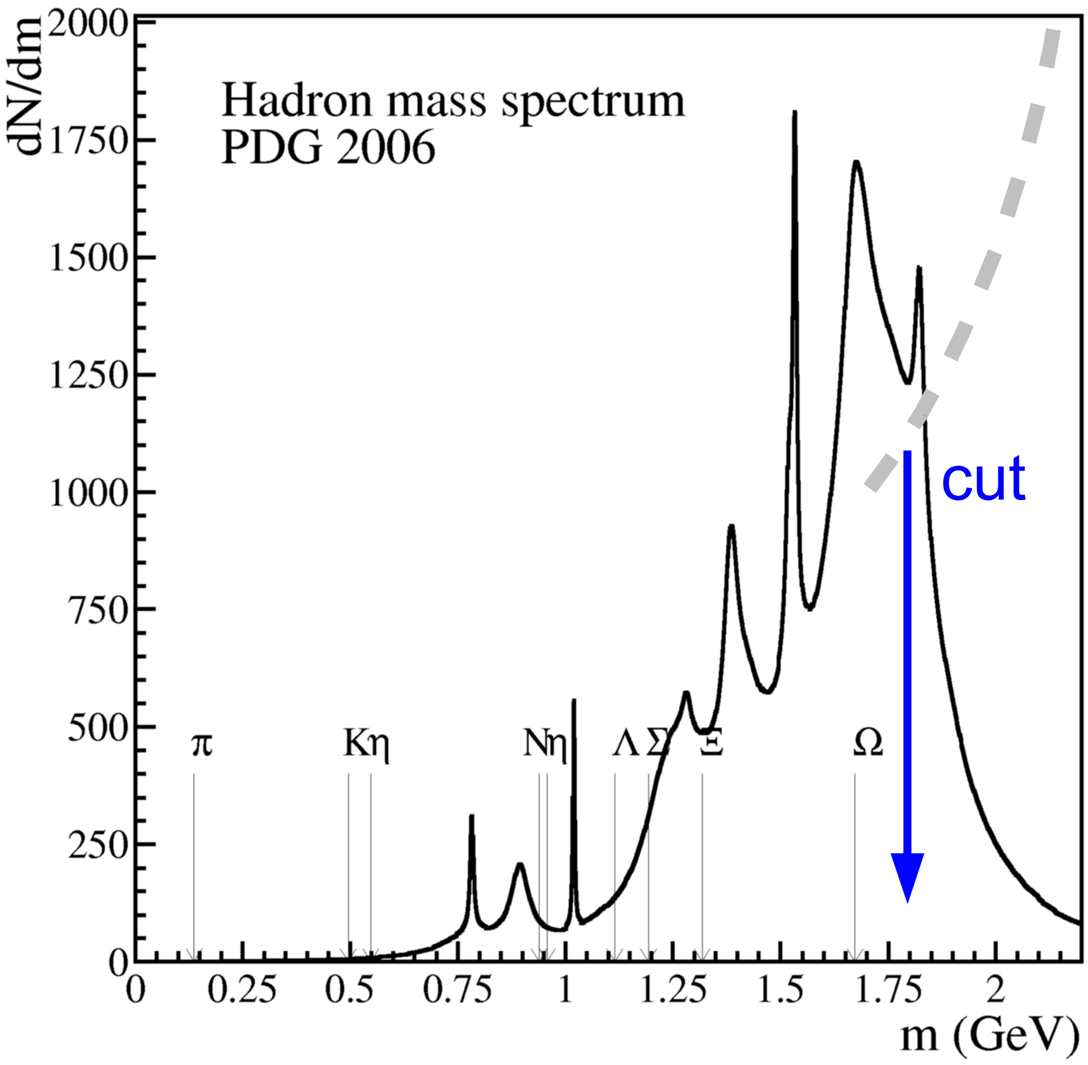
Canonical





usually neglected, but...

Hadron-resonance gas model: neglect the non-resonant part of the S-matrix



Proper assumption for reasonably large temperature

How large T ?

T = O(100) MeV

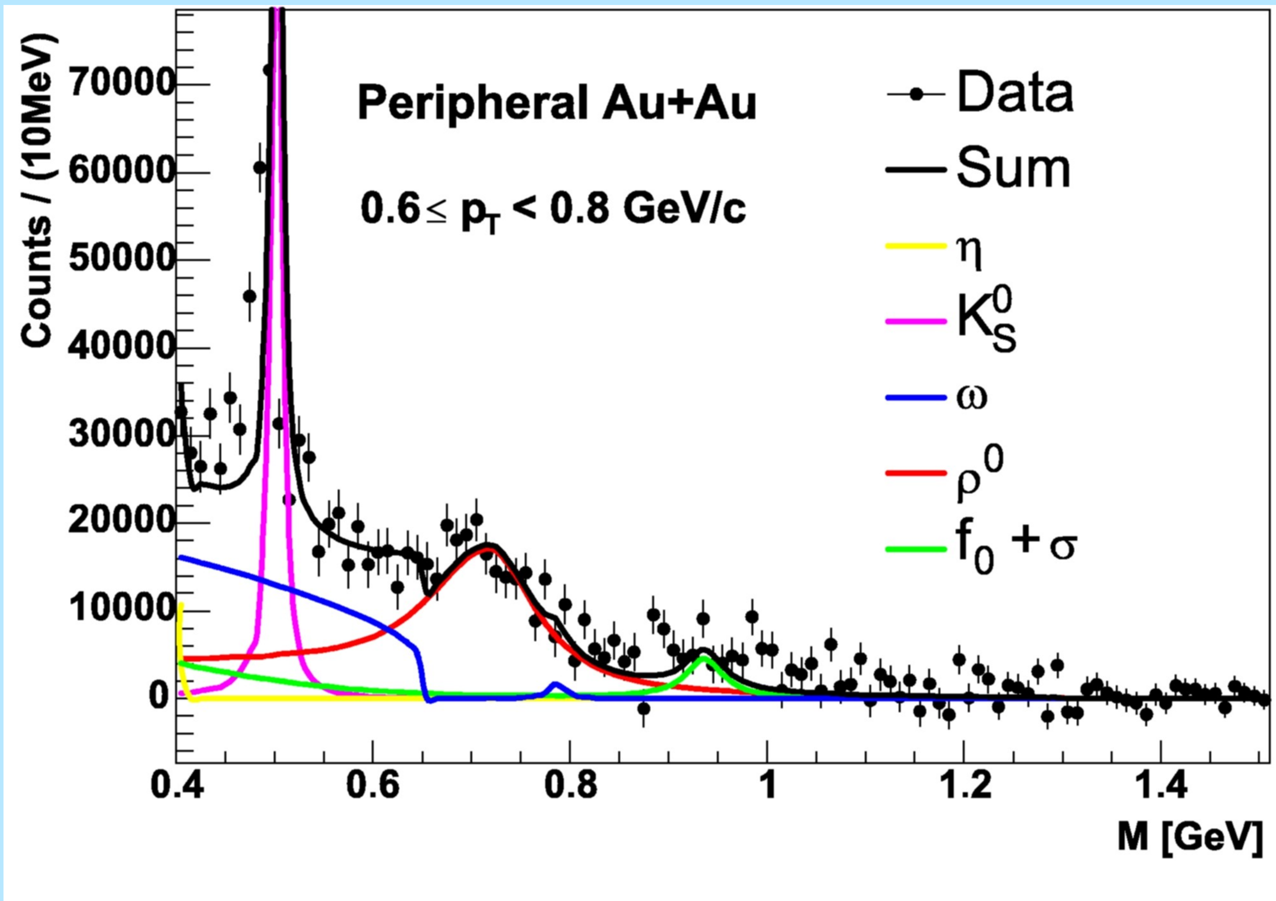
Numerical study still missing!

Invariant mass spectrum in AuAu

$$\frac{dn}{dM} / \frac{d\pm}{dM} Z\, d^3p \frac{p}{\exp[} \frac{1}{\overline{p^2 + M^2} = T] \S\ 1}$$

From DMB theorem

W. Brioniowski et al., Phys. Rev. C 68 (2003) 0349011



Heavy ion collisions

- Particle multiplicities

J. Cleymans, H. Satz, U. Heinz, J. Rafelski, J. Letessier, G. Torrieri, P. Braun-Munzinger, J. Stachel, K. Redlich, N. Xu, W. Brioniowski, W. Florkowski, F. B., J. Manninen, R. Stock, M. Gazdzicki, M. Gorenstein ....

- Transverse momentum spectra

W. Brioniowski, W. Florkowski (single freeze-out model)

- Lineshape of resonances

W. Brioniowski, W. Florkowski

- Fluctuations and correlations

M. Gorenstein, V. Begun, M. Gazdzicki, M. Hauer

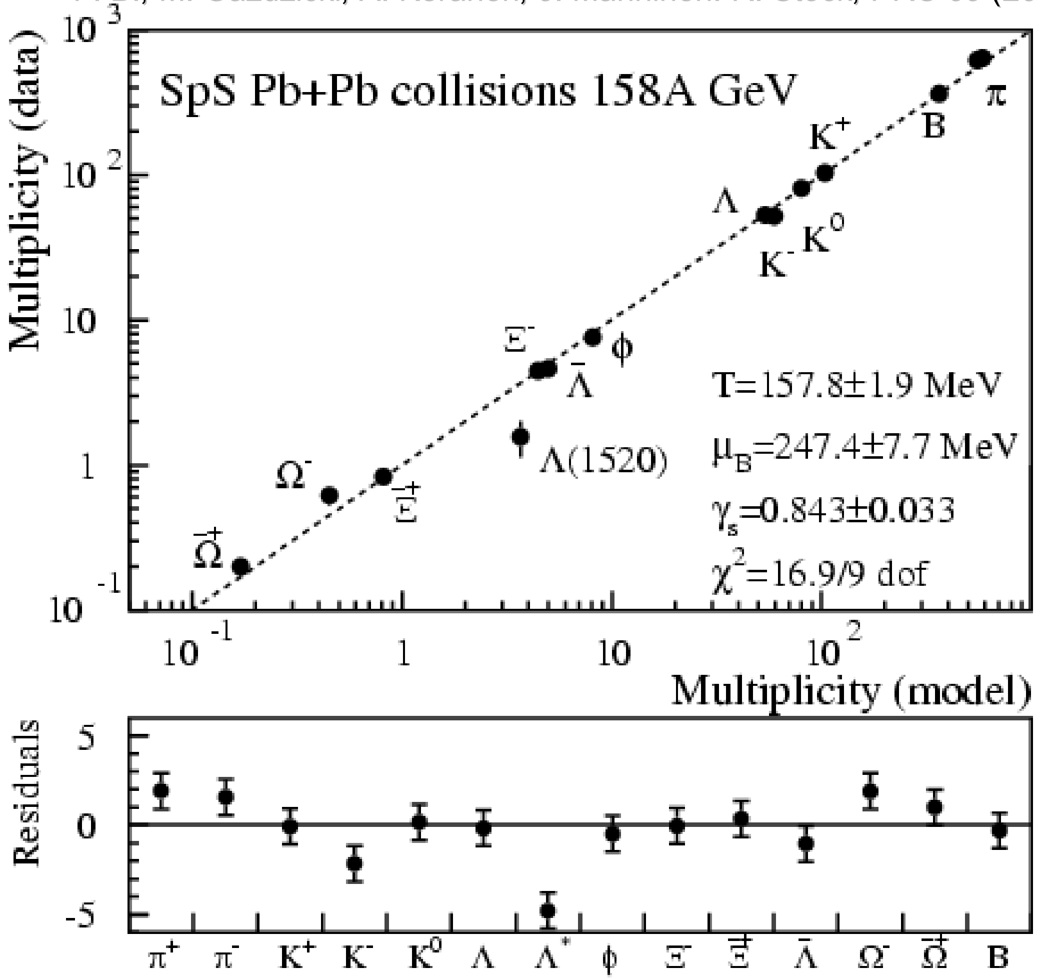
- Production of J/psi and heavy flavoured hadrons

M. Gazdzicki, M. Gorenstein, P. Braun-Munzinger, J. Stachel, A. Kostyuk, F.B....

Fit to full phase space multiplicities by NA49 with T, μ_B, V, γ_S

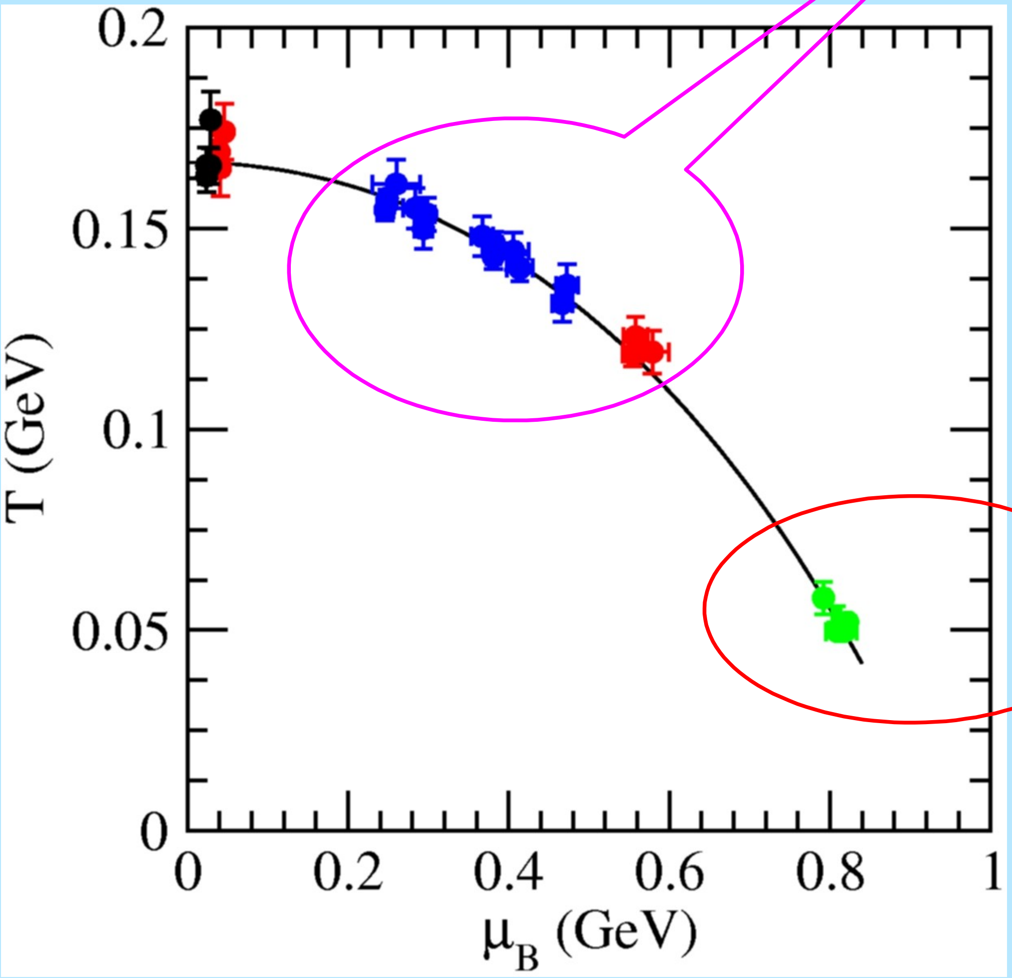
$$\langle n_j \rangle = (2J_j+1) V \frac{\gamma_S^{n_s}}{(2\pi)^3} \int d^3p \exp[-\sqrt{p^2+m_j^2}/T] \exp[\boldsymbol{\mu} \cdot \boldsymbol{q_j}/T]$$

F. B., M. Gazdzicki, A. Keranen, J. Manninen. R. Stock, PRC 69 (2004) 024905



Chemical freeze-out curve

J. Cleymans et al. Phys.Rev.C73:034905,2006.



Points taken from

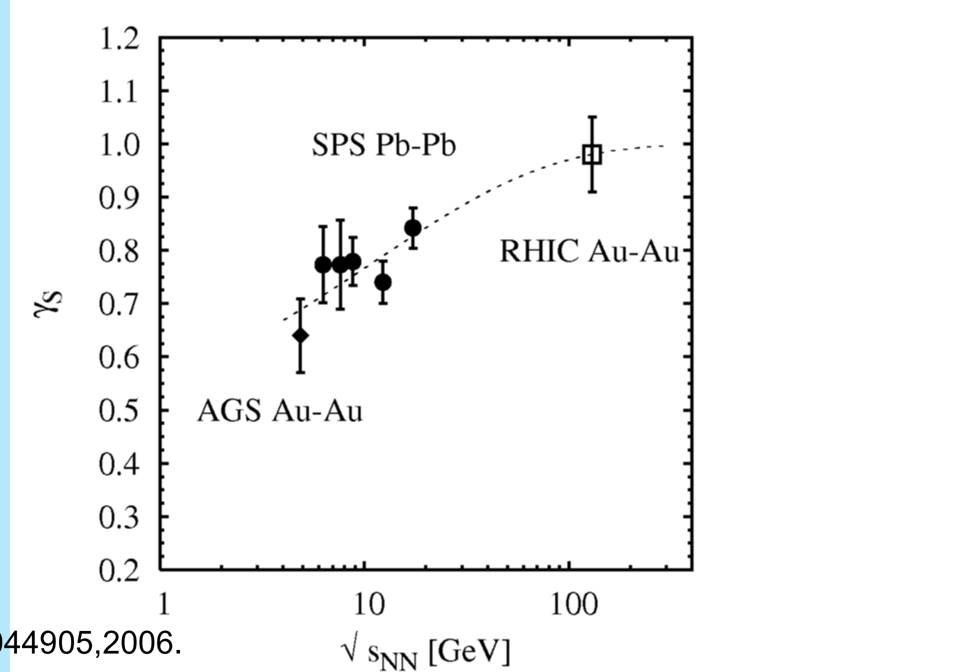
F.B., J. Manninen, M. Gazdzicki, Phys.Rev.C73:044905,2006.

?

Don't know if the HRG model still applies!

Debated or unsettled issues

- Equilibrium suppression factors: γ_S, γ_q
- Full phase space vs midrapidity
- Ratios vs yields



F.B., J. Manninen, M. Gazdzicki, Phys.Rev.C73:044905,2006.

Midrapidity vs full phase space

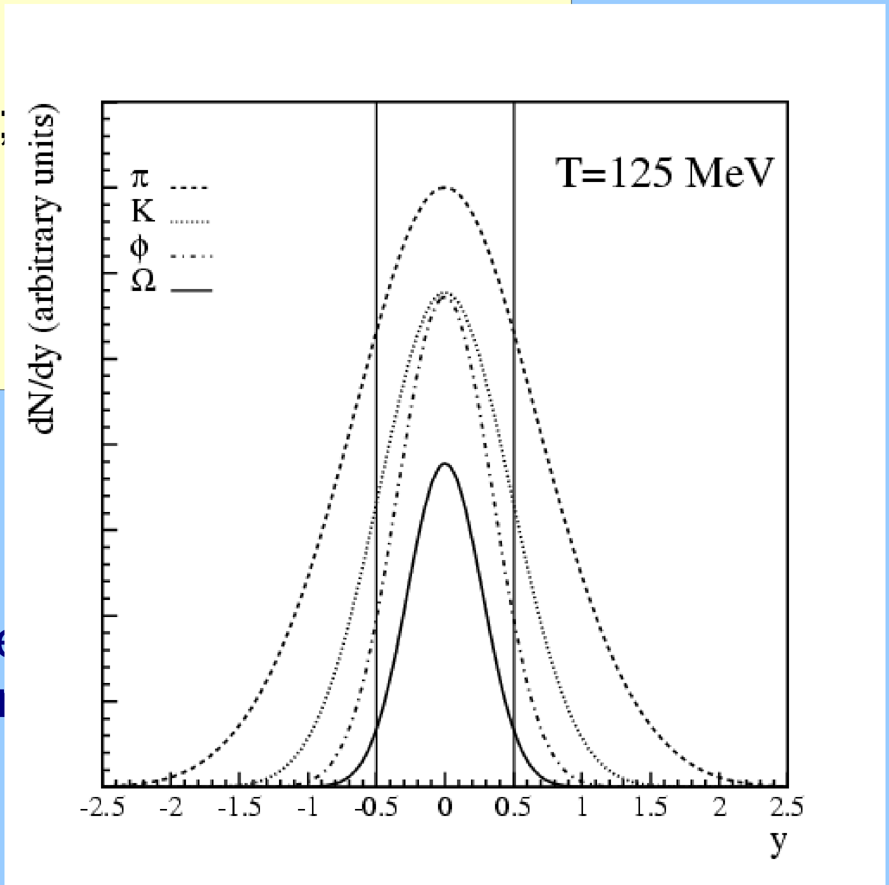
- Midrapidity densities yield exact properties of the central fireball only if there is a sufficiently large window of invariance around midrapidity (RHIC - LHC)

$$\frac{dN}{dy} = \int_{-\infty}^{+\infty} dY\, \rho(Y)\, \frac{dn}{dy}(\mu(Y); T(Y); y - Y)$$

$$= \int_{-\infty}^{+\infty} dy'\, \rho(y - y')\, \frac{dn}{dy}(\mu(y - y');$$

$$\simeq \rho(y) \int_{-\infty}^{+\infty} dy'\, \frac{dn}{dy}(\mu(y); T(y); y')$$

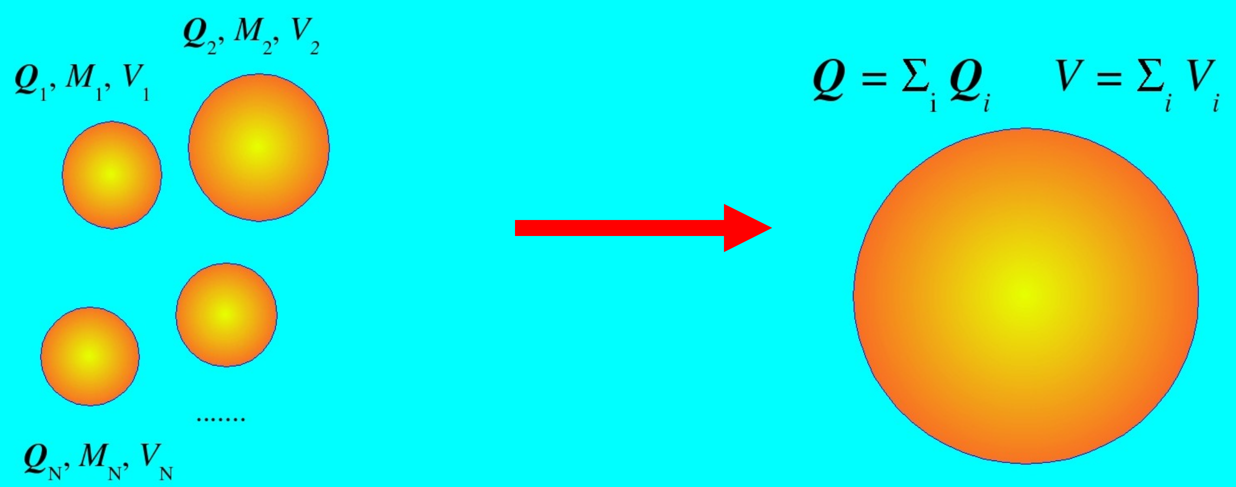
$\Rightarrow$ $\sigma \gg \sigma_{fireball}$



- If the rapidity distribution is not wide e
enhance heavier (i.e. strange) hadron

Full phase space: *possible* equivalence with a global cluster

$$\langle n_j \rangle_i (Q_i, P_i, V_i) \equiv \langle n_j \rangle_i (Q_i, P_i^2, V_i)$$



Much larger cluster !

Equivalence holds only if the probabilities *w* are statistical

F. B., G. Passaleva, Eur. Phys. J. C 23 (2002) 551

Rapidity widths

- Top SPS for pions:

single fireball T=125 MeV $\sigma_{th} = 0.8$

actual $\sigma \sim 1.3$

- RHIC 200 for pions:

$\sigma \sim 2.0$

$\overline{X}/X$ stable over 2 units of rapidity

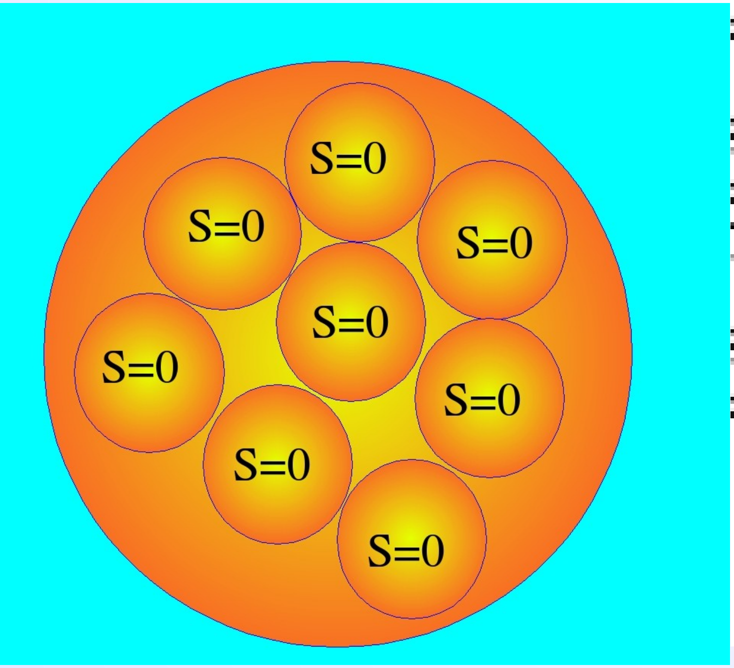
Conclusion: midrap possible from RHIC onwards,at SPS full phase space more appropriate.

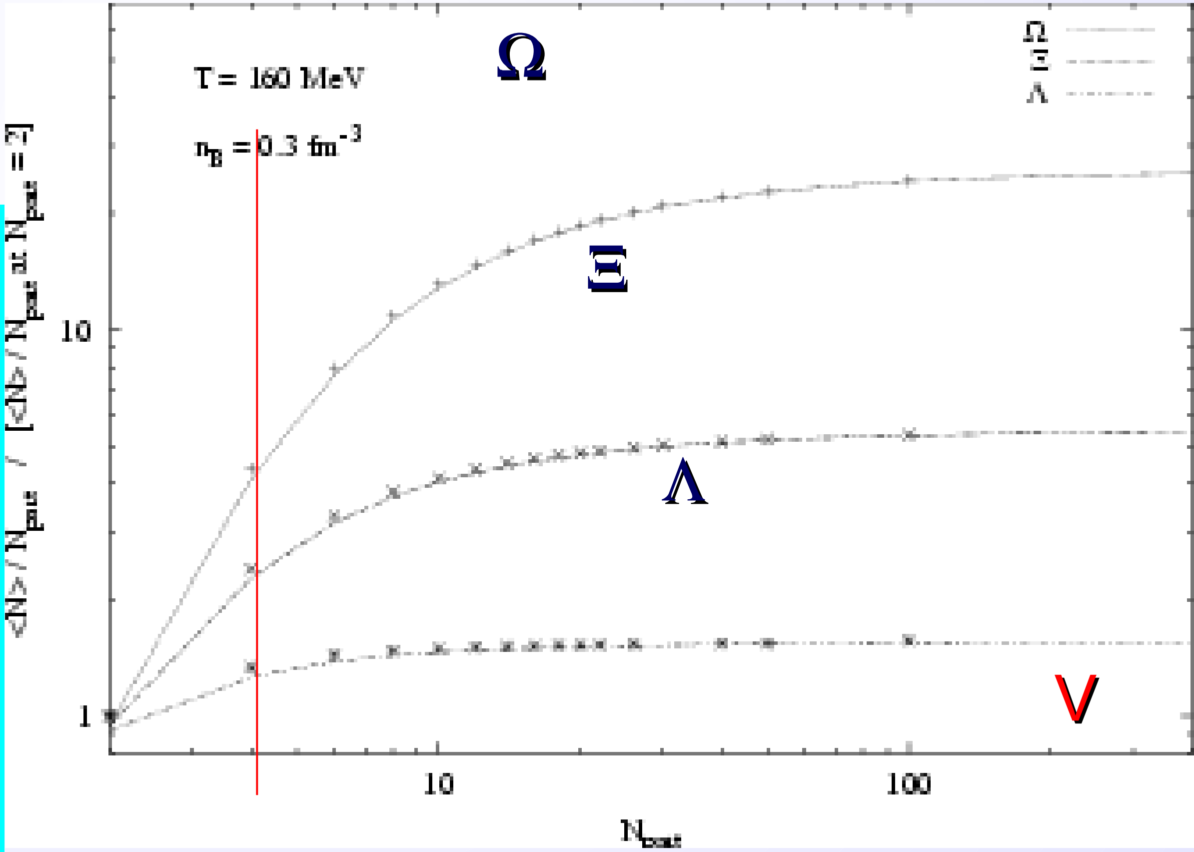
Ongoing studies: W. Broniowski, B. Biedron, 0709.0126 and PRC75 054905 (2007)
F.B., J. Cleymans, hep-ph 0701029

Strangeness *canonical* suppression

There is no γ_S $\gamma_S < 1$ is an effect of *local strangeness neutrality*

S. Hamieh et al., Phys. Lett. B 486, 61





It does not apply to ϕ meson

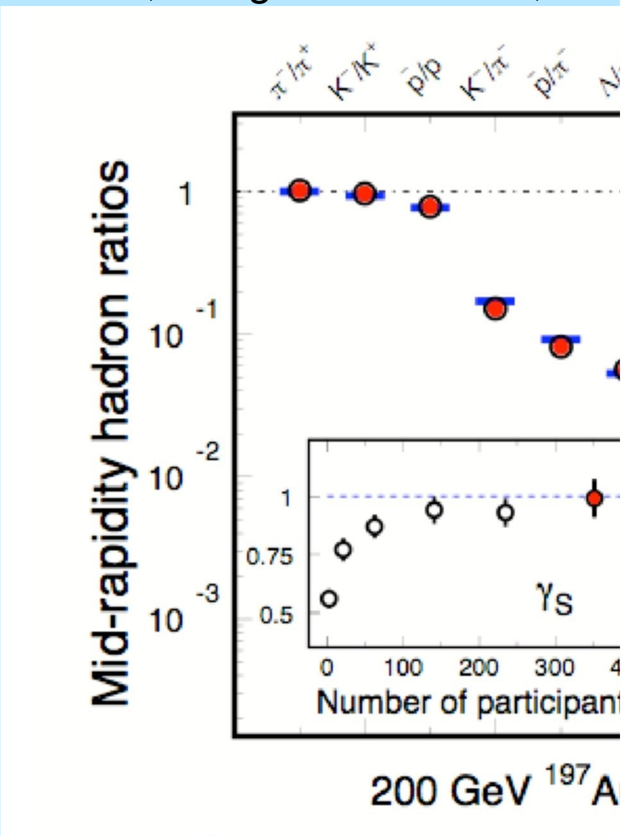
Pointed out very early

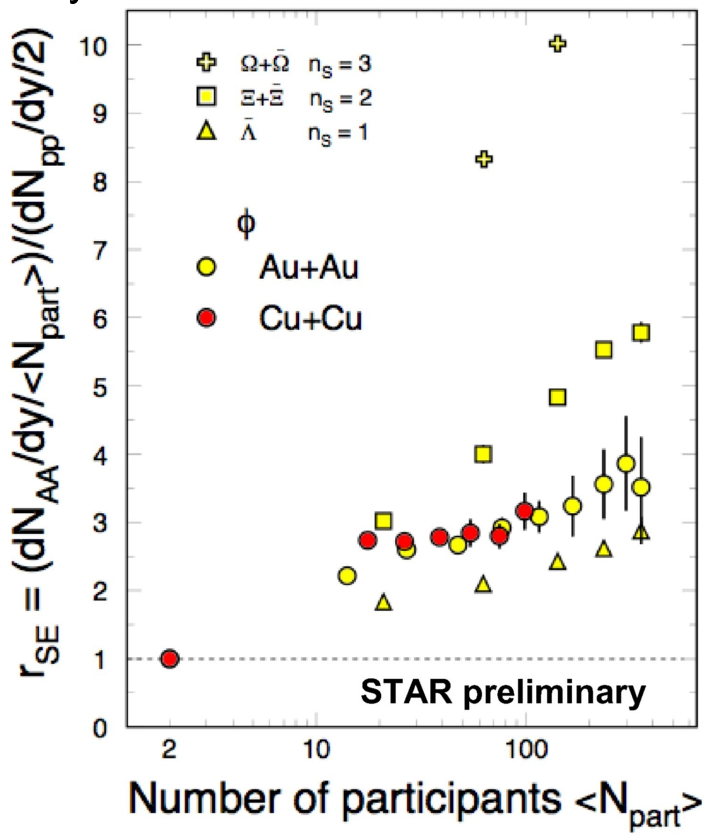
F. B., M. Gazdzicki, J. Sollfrank, Eur. Phys.J. C 5 (1998) 143
J. Sollfrank et al., “Canonical strangeness enhancement”, Nucl. Phys. A638 (1998) 339c, proc. QM97

re-stated in

F. B., A. Keranen, J. Manninen, M. Gazdzicki, R. Stock, Phys. Rev. C 69 (2004) 024905

Nu Xu, talk given at CPOD, GSI, July 07





200 GeV collisions

- The multi-strange baryons productions Ξ, Ω are enhanced in heavy ion collisions compared to that of in p+p collisions

- The ϕ-meson productions are also enhanced, but may be with different trends

⇨

The enhancements are NOT due to CE suppression!

The role of γ_q, γ_S, m_h?

STAR:
- *PRL.* ***98*** *(2007) 062301 (nucl-ex/0606014)*
- *nucl-ex/ 0703033*
- *nucl-ex/ 0705.2511*

Ratios vs yields

Fit to ratios are generally biased F.B. arXiv 0707.4154

Selecting N or N-1 ratios from N measurements of yields involves an information loss which gives rise to a biased and inefficient estimator

The bias is not easy to assess and depends on the sample

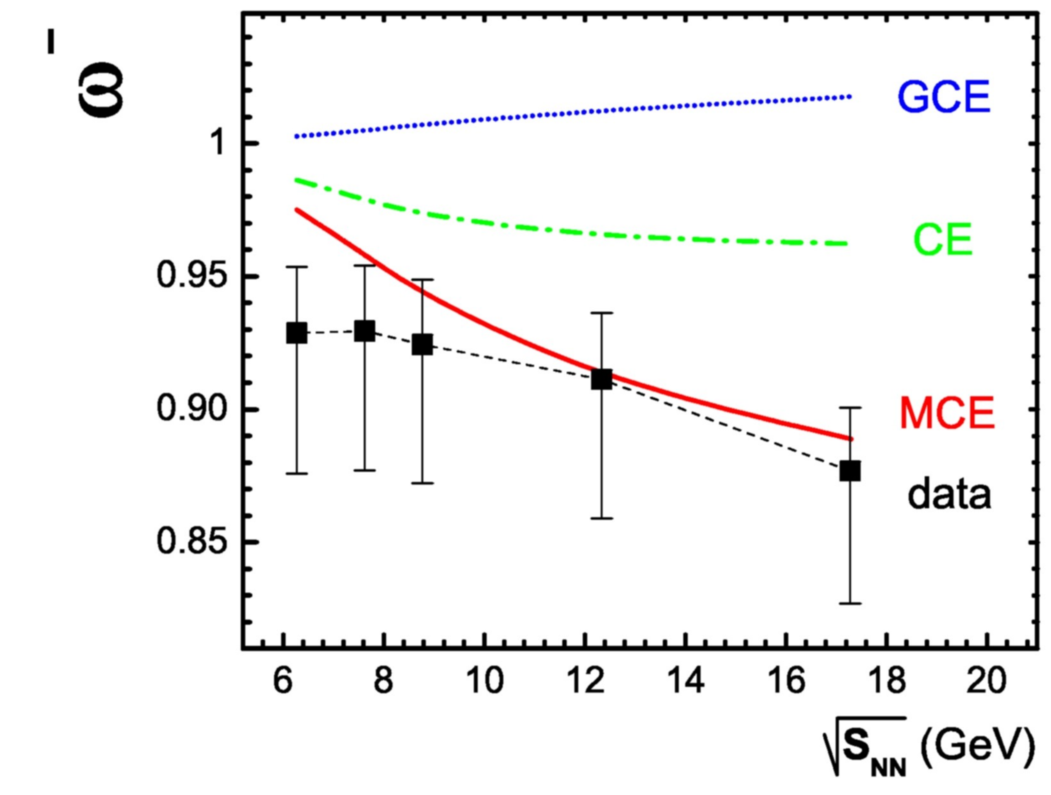
RATIOS CAN BE DANGEROUS AND SHOULD NOT BE USED UNLESS QUOTED BY THE EXPERIMENT

The role of conservation laws: fluctuations

In the thermodynamic limit, fluctuations still affected by conservation laws (not so for first moments)

V. Begun, M. Gazdzicki, M. Gorenstein, O. Zozulya Phys. Rev. C 70 (2004) 034901

V. Begun et al. Phys. Rev. C 76 (2007) 024902



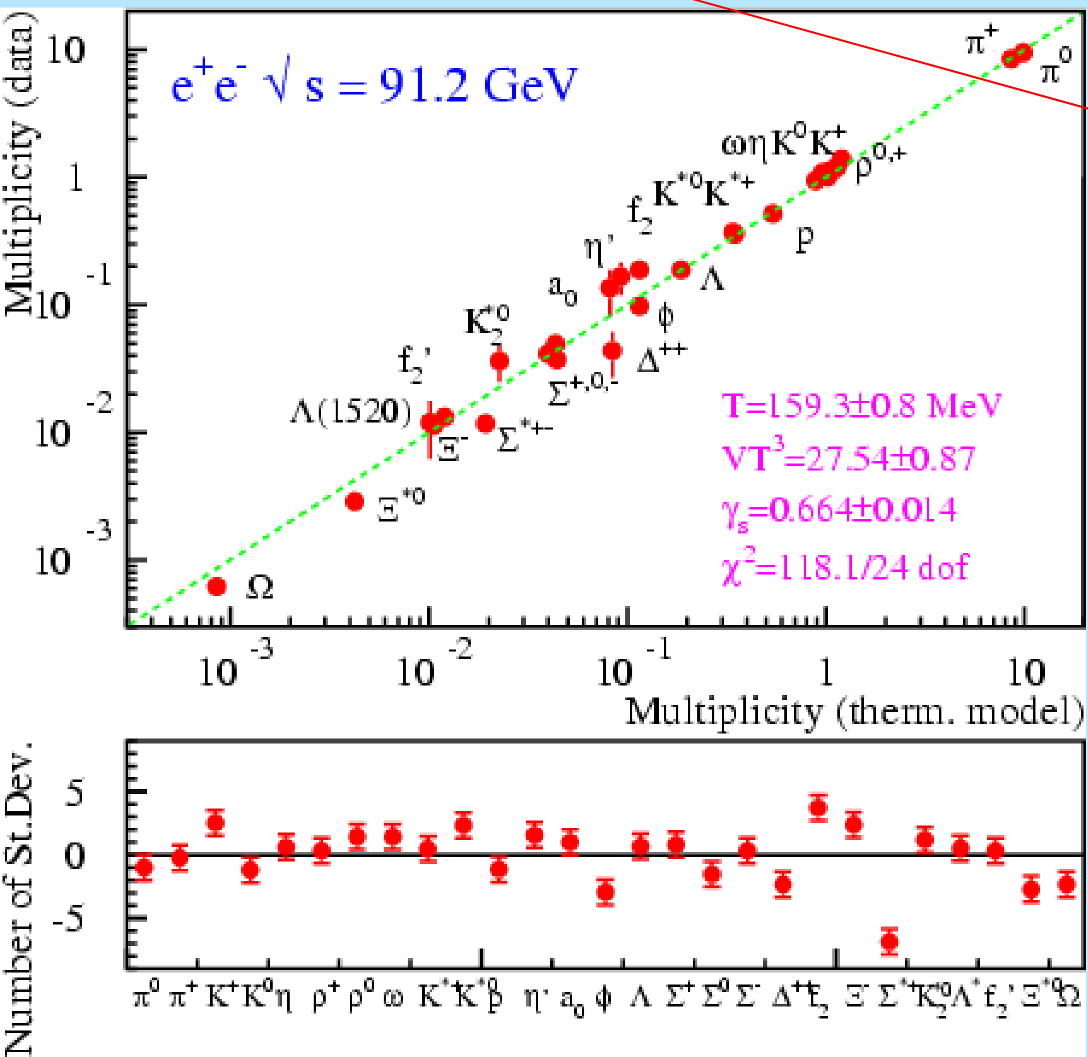
Statistical model works in e^+e^-

F. B., Z. Phys. C69 (1996) 485

$$\langle n_j \rangle = (2J_j+1) V \frac{\gamma_S^{n_s}}{(2\pi)^3} \int d^3p \exp[-\sqrt{p^2+m_j^2}/T] \frac{Z(\mathbf{Q}-\mathbf{q}_j)}{Z(\mathbf{Q})}$$

Chemical factor

Extra strangeness suppression



- A very good fit (taking into account the introduced assumptions)
- Only 3 parameters needed to reproduce more than 20 particles

F.B., Nucl. Phys. A 702 (2002) 336

String model parameters

Parameter	Name	Default	Range gen.	Fit Result		
				Value	stat.	sys.
Λ_{QCD}	PARJ(81)	0.29	0.25 - 0.35	0.297	± 0.004	$^{+0.007}_{-0.008}$
Q_0	PARJ(82)	1.0	1.0 - 2.0	1.56	± 0.11	$^{+0.21}_{-0.15}$
a	PARJ(41)	0.3	0.1 - 0.5	0.417	± 0.022	$^{+0.011}_{-0.015}$
b	PARJ(42)	0.58	0.850	optimized		
σ_q	PARJ(21)	0.36	0.36 - 0.44	0.408	± 0.005	$^{+0.004}_{-0.004}$
$P({}^1S_0)_{ud}$	-	0.5	0.3 - 0.5	0.297	± 0.021	$^{+0.102}_{-0.011}$
$P({}^3S_1)_{ud}$	-	0.5	0.2 - 0.4	0.289	± 0.038	$^{+0.004}_{-0.026}$
$P({}^1P_1)_{ud}$	-	0.	see text	0.096		
$P(other\ P\ states)_{ud}$	-	0.	see text	0.318		
γ_s	PARJ(2)	0.30	0.27 - 0.31	0.308	± 0.007	$^{+0.004}_{-0.036}$
$P({}^1S_0)_s$	-	0.4	0.3 - 0.5	0.410	± 0.038	$^{+0.026}_{-0.013}$
$P({}^3S_1)_s$	-	0.6	0.2 - 0.4	0.297	± 0.021	$^{+0.020}_{-0.004}$
$P(P\ states)_s$	-	0.	see text	0.293		
ϵ_c	PARJ(54)	-	variable	-0.0372	± 0.0007	$^{+0.0011}_{-0.0012}$
$P({}^1S_0)_c$	-	0.25	0.26			
$P({}^3S_1)_c$	-	0.75	0.44	adj. to data		
$P(P\ states)_c$	-	0.	0.3			
ϵ_b	PARJ(55)	-	variable	-0.00284	± 0.00005	$^{+0.00012}_{-0.00010}$
$P({}^1S_0)_b$	-	0.25	0.175			
$P({}^3S_1)_b$	-	0.75	0.525	adj. to data		
$P(P\ states)_b$	-	0.	0.3			
$P(qq)/P(q)$	PARJ(1)	0.1	0.08 - 0.11	0.099	± 0.001	$^{+0.005}_{-0.002}$
$[P(us)/P(ud)]/\gamma_s$	PARJ(3)	0.4	0.593	adj. to data		
$P(qq1)/P(qq0)$	PARJ(4)	0.05	0.07	adj. to data		
extra baryon supp.	PARJ(19)	0.	0.5	adj. to data, only for uds		
extra η supp.	PARJ(25)	1.0	0.65	0.65 ± 0.06		
extra η' supp.	PARJ(26)	1.0	0.23	0.23 ± 0.05		

Table 49: Parameter settings and fit results for JETSET 7.4 PS with default decays

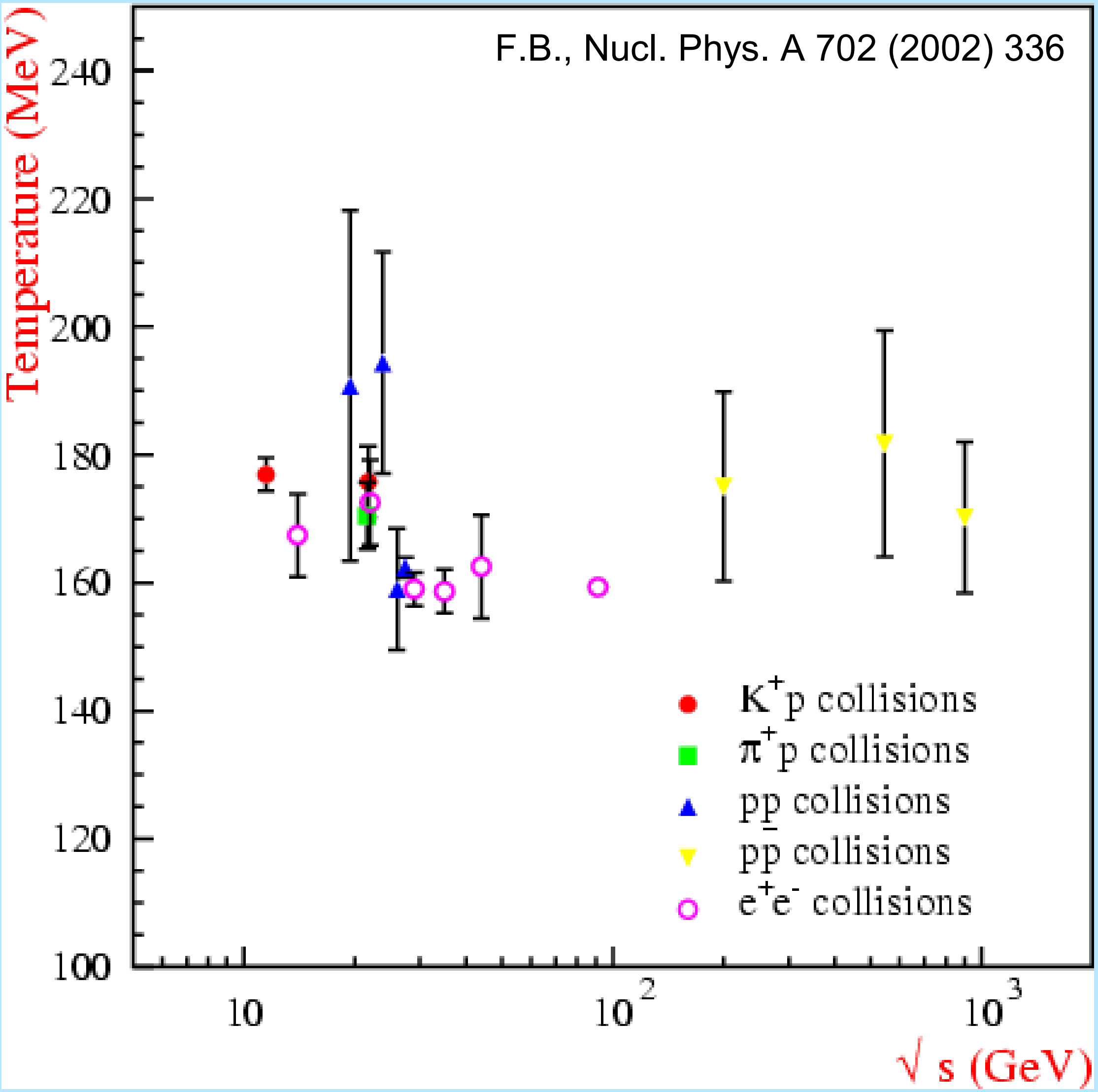
From: Delphi collaboration, CERN-PPE 96-120

Compilation of heavy flavoured hadron production abundances in e^+e^- at $\sqrt{s}$=91.2 GeV

F.B., J. Phys. G 23 (1997) 1933

Hadron	Prediction 1	Prediction 2	Measured	Pull
D^+	0.0926	0.0923	0.087±0.008 (ADO)	-0.67(-0.70)
D^0	0.233	0.233	0.227±0.012 (ADO)	-0.50(-0.50)
D_s	0.0579	0.0563	0.066±0.010 (O)	+0.81(+0.97)
D^{*+}	0.108	0.108	0.0880±0.0054 (ADO)	-3.7 (-3.7)
D_s^+/c-jet	0.103	0.0981	0.128±0.027 (A)	+0.92(+1.1)
D_1/c-jet	0.0347	0.0363	0.038±0.009 (A)	+0.37(+0.19)
D_2^*/c-jet	0.0471	0.0495	0.135±0.052 (A)	+1.7(+1.6)
D_{s1}/c-jet	0.00536	0.00544	0.016±0.0058 (O)	+1.8(+1.8)
B^0/b-jet	0.412	0.412	0.384±0.026 (AD)	-1.1(-1.1)
B^*/B	0.692	0.692	0.747±0.067 (ADLO)	+0.82(+0.82)
B^*/b-jet	0.642	0.639	0.65 ±0.06 (D)	+0.13(+0.17)
B_s/b-jet	0.106	0.101	0.122±0.031 (A)	+0.52(+0.68)
$B^{**}_{u,d}$/b-jet	0.206	0.213	0.26 ±0.05 (DO)	+1.0(+0.90)
B^{**}/B	0.251	0.259	0.27 ±0.06 (A)	+0.32(+0.18)
B_s^{**}/b-jet	0.021(0.011)	0.022(0.011)	0.048±0.017 (D)	+1.6(+1.6)
B_s^{**0}/B^+	0.026(0.013)	0.026(0.013)	0.052±0.016 (O)	+1.6(+1.6)
Λ_c^+	0.0248	0.0264	0.0395±0.0084 (O)	+1.7(+1.6)
b-baryon/b-jet	0.0717	0.0764	0.115 ±0.040 (A)	+1.1(+0.97)
$(\Sigma_b + \Sigma_b^*)$/b-jet	0.0404	0.0437	0.048 ±0.016 (D)	+0.48(+0.27)
$\Sigma_b/(\Sigma_b^* + \Sigma_b)$	0.411	0.410	0.24 ±0.12 (D)	-1.4(-1.4)

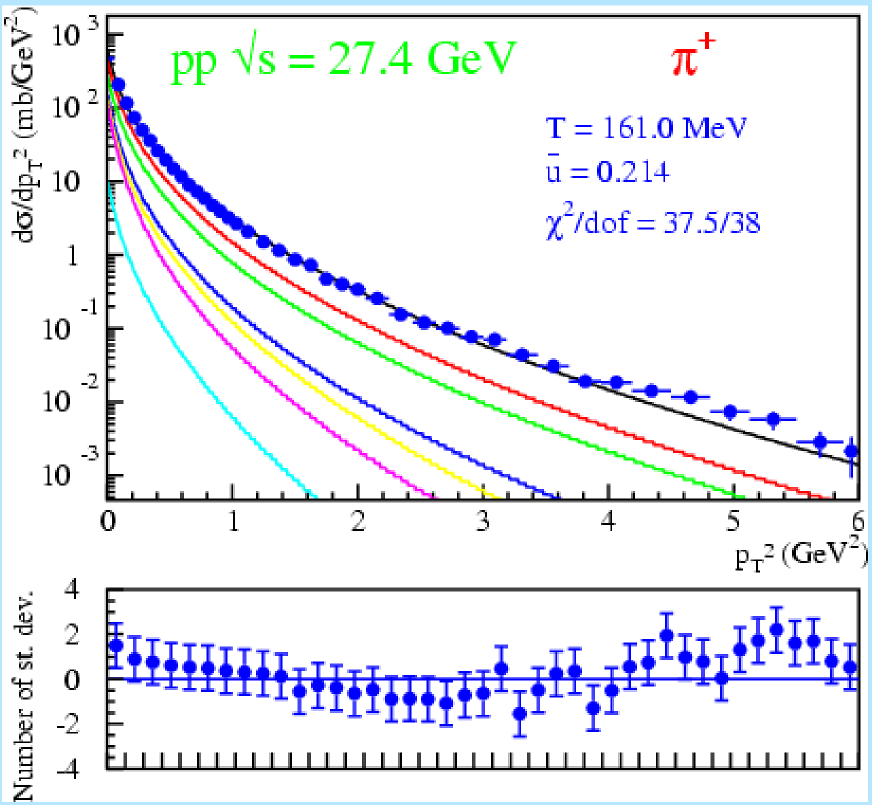
Temperature in elementary collisions: ~ 160 MeV at high energy

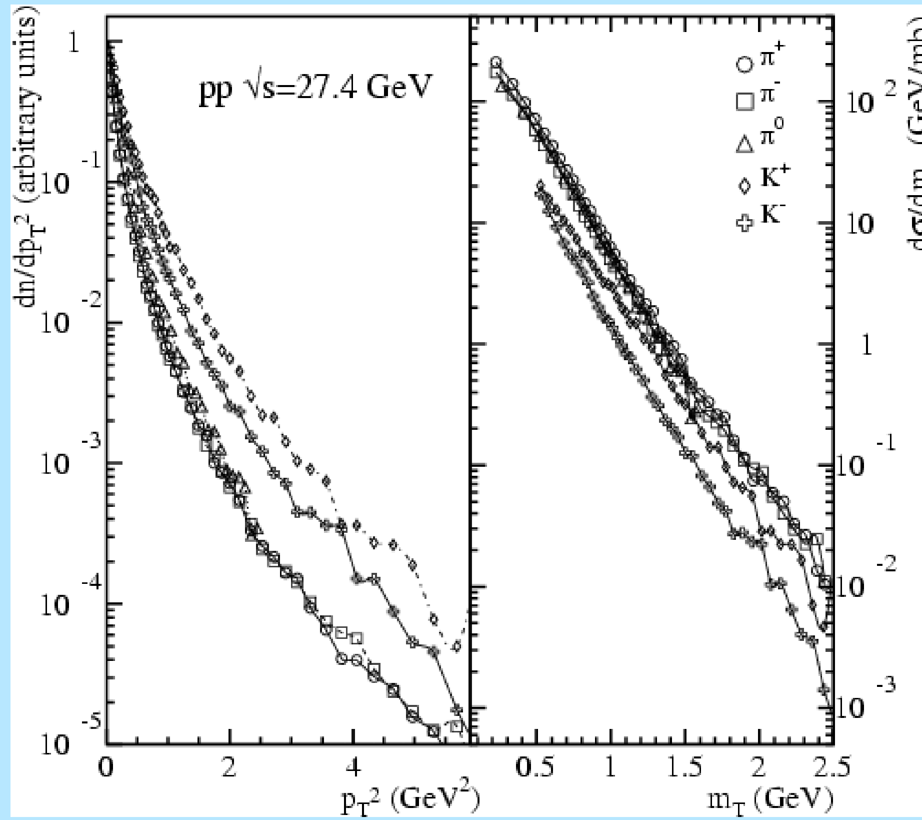


Analysis of p_T spectra in hh

F.B., G. Passaleva, Eur. Phys. J. C 23 (2002) 551

- Consistent fits with two parameters: *T* and $\langle u \rangle_T$
- m_T scaling





Why?

This would appear to be a compelling case for the production of a Quark Gluon Plasma. The problem is that the fits done for heavy ions to particle abundances work even better in e^+e^- collisions. One definitely expects no Quark Gluon Plasma in e^+e^- collisions. There is a deep theoretical question to be understood here: How can thermal models work so well for non-thermal systems? Is there some simple saturation of phase space?

From: L. Mc Lerran, Lectures "The QGP and the CGC", hep-ph 0311028

References

- F. Becattini, U. Heinz, Z.Phys C 76, 269 (1997)
- U. Heinz, Nucl. Phys A 661 140 (1999)
- H. Satz, Nucl. Phys. Proc. Suppl. 94, 204 (2001)
- J. Hormuzdiar, S. D. H. Hsu and G. Mahlon, Int. J. Mod. Phys. E 12, 649 (2003)
- R. Stock, Phys. Lett. B 456, 277 (1999)
- Y. Dokshitzer, Acta Phys. Polon. B 36 361 (2005)
- A. Bialas, Phys. Lett. B 466 301 (1999)
- V. Koch, Nucl. Phys. A 715 108 (2003)
- L. McLerran, arXiv:hep-ph/0311028
- F. Becattini, J. Phys. Conf. Ser. 5 175 (2005)
- D. Kharzeev, arXiv:hep-ph/0511354

PROBLEM:

- The thermodynamic-like production of different species cannot be the result of (local) post-hadronization equilibration through hadronic collisions in pp and e+e-

- Also in heavy ions simulations of post-hadronization rescattering show that there is little chance of equilibrating particle abundances through inelastic binary collisions

→ HADRONS SHOULD BE BORN AT EQUILIBRIUM (Hagedorn 1970)

Hadronization as Hawking radiation

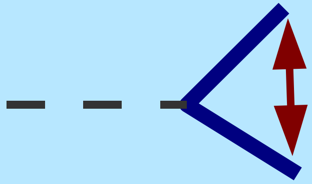
D. Kharzeev, Eur. Phys. J. A 29 (2006) 83
D. Kharzeev. Nucl. Phys. A 774 (2006) 315
H. Satz, hep-ph/0612151
P. Castorina, D. Kharzeev, H. Satz, Eur. Phys. J. C52 (2007) 187

Analogy between Hawking-Unruh radiation and thermal partonic-hadronic emission

No information transfer from inside event horizon
The Hawking-Unruh radiation can only be thermal

black hole $T = \frac{1}{8\pi G M}$ uniformly acc. $T = \frac{a}{2\pi}$

$\bar{q}\,q$ pair(s) from e+e- annihilation decelerated by the string tension σ (~ 1 GeV/fm)



$$T = \sqrt{\frac{\sigma}{2\pi}} \simeq 177 \quad MeV$$

Options

The thermodynamical-like behaviour is only mimicked by data. It is a property of hadronization and should be called "*phase space dominance*"

J. Hormuzdiar, S. D. H. Hsu and G. Mahlon, Int. J. Mod. Phys. E 12, 649 (2003)

*The temperature **is not** a real temperature*

weak dependence on kinematics

$$\Gamma_{\{N_j\}} \propto \frac{1}{(2\pi)^{3N}} \prod_j \frac{(2J_j+1)^{N_j}}{N_j!} \int \frac{d^3 p_1}{2\varepsilon_1} \dots \frac{d^3 p_N}{2\varepsilon_N} |M_{if}|^2 \delta^4\left(P - \sum_{i=1}^{N} p_i\right)$$

$$\langle n \rangle_j = \frac{\alpha}{(2\pi)^3} \int \frac{d^3 p}{2\epsilon_j} e^{-\beta \epsilon_j}$$

Genuine statistical equilibrium within a finite volume

$$\Omega_{\{N_j\}} = \frac{V^N}{(2\pi)^{3N}} \prod_j \frac{(2J_j+1)^{N_j}}{N_j!} \int d^3 p_1 \dots d^3 p_N \delta^4\left(P - \sum_{i=1}^{N} p_i\right)$$

$$\langle n \rangle_j = \frac{V}{(2\pi)^3} \int d^3 p \, e^{-\beta \epsilon_j}$$

F. B., J. Phys. Conf. Ser. 5 175 (2005)

How to probe a genuine statistical model ?

Need to test exclusive channel rates

$$\frac{BR_{\{N_j\}}}{BR_{\{M_j\}}} = \frac{\Omega_{\{N_j\}}}{\Omega_{\{M_j\}}}$$

More sensitive to the integration measure ($V\, d^3p$ vs $d^3p/2\varepsilon$) because information is not integrated away

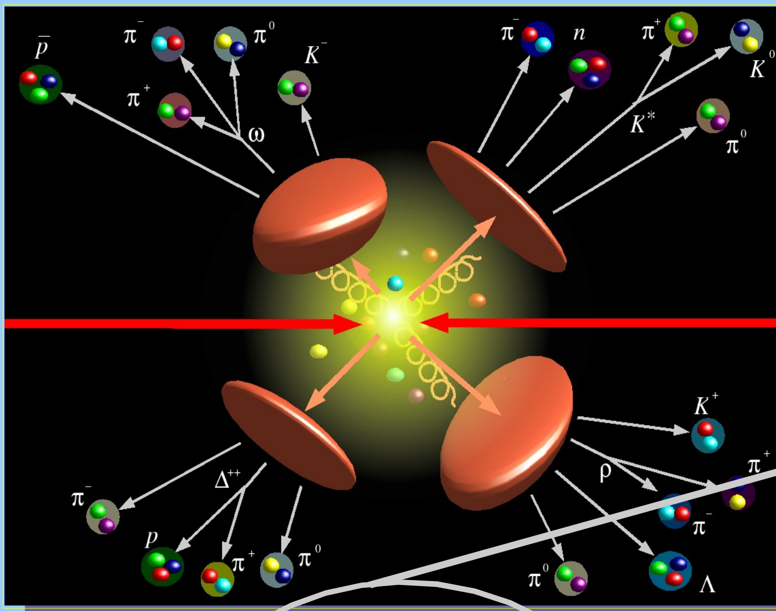
Data available at low energy ($\sqrt{s} < 3$ GeV)

Need <u>full</u> microcanonical calculations

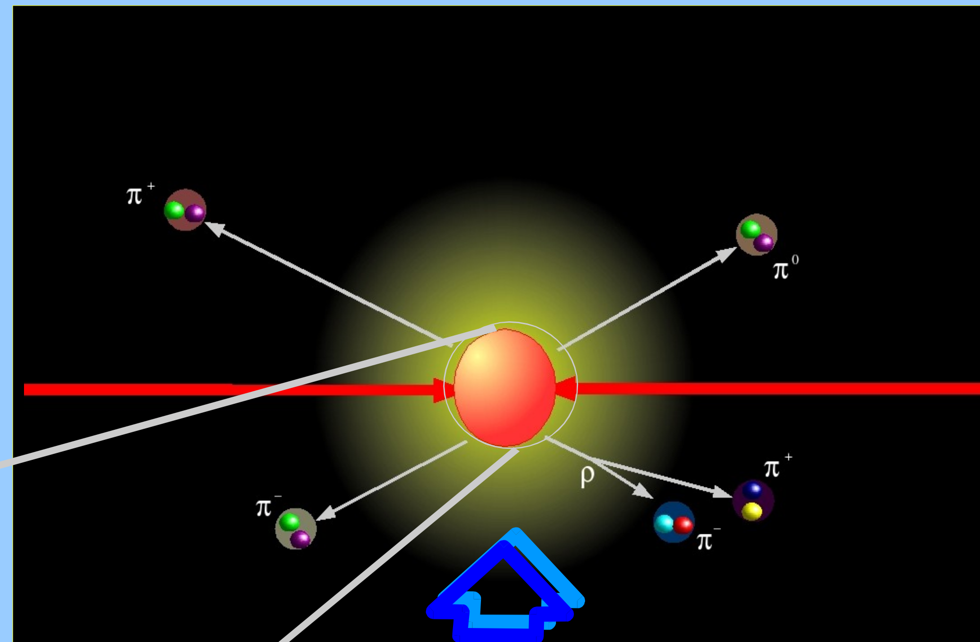
long and ongoing project: F.B., L. Ferroni, Eur. Phys. J. C35, 243; Eur. Phys. J. C38, 225; L. Ferroni, Ph.D thesis Dec. 2006; F. B., L. Ferroni, arXiv 07041967 and 0707.0793, Eur. Phys. J. C in press; F. B., L. Ferroni in preparation

Statistical hadronization at high and low energy

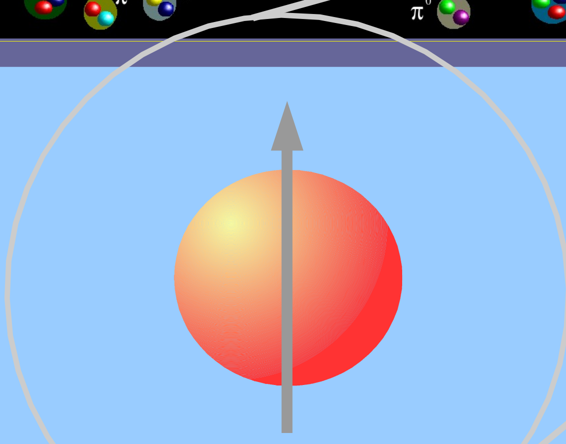
high $\sqrt{s}$ >~ 10 GeV



low $\sqrt{s}$ <~3 GeV



single cluster at rest in the c.m.



P,C, Π, I, I_3, J, λ

Most general
Microcanonical
Ensemble

The microcanonical weight of a channel

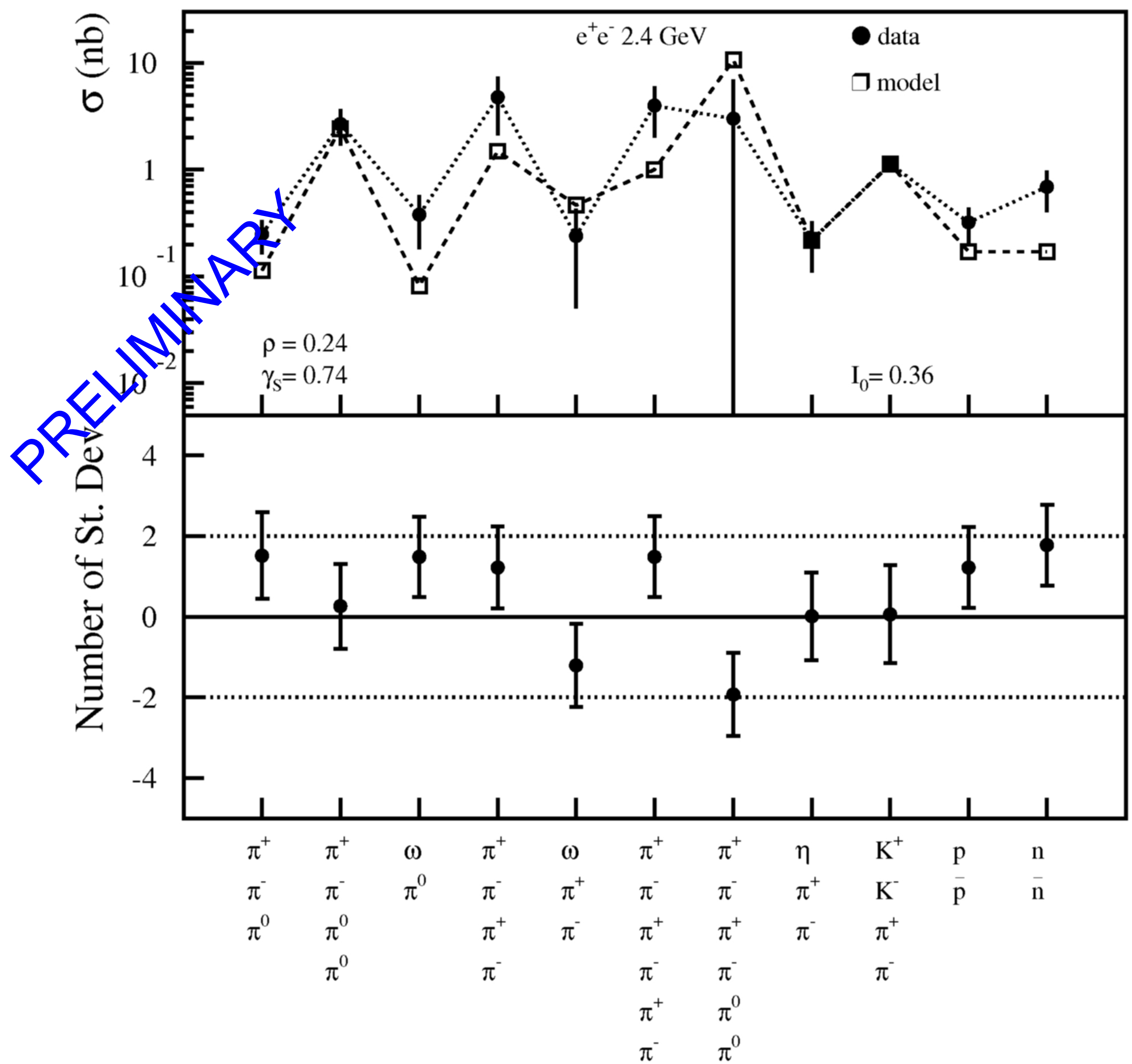
$$\Omega_{\{N_j\}} = \sum_{\boldsymbol{\rho}} \left[\prod_{j=1}^{K} \chi(\rho_j)^{b_j} \right] \frac{1}{8\pi} \int_0^{4\pi} \mathrm{d}\psi \left[\prod_{j=1}^{k} \frac{1}{N_j!} \prod_{n_j=1}^{N_j} \int \mathrm{d}^3 \mathrm{p}_{n_j} \right]$$

$$\times \delta^4 \left(P - \sum_{n=1}^{N} p_n \right) \sin\frac{\psi}{2} \sin\left[\left(J + \frac{1}{2} \right) \psi \right] \prod_{j=1}^{K} \left[\prod_{l_j=1}^{L_j} \left[\frac{\sin[(S_j + \frac{1}{2}) l_j \psi]}{\sin(\frac{l_j \psi}{2})} \right]^{h_{l_j}(\rho_j)} \right]$$

$$\times \left(\prod_{j=1}^{K} \prod_{l_j=1}^{L_j} F_V^{(s)}(\mathbf{p}_{\rho_j(l_j)} - \mathsf{R}_3^{-1}(\psi)\mathbf{p}_{l_j}) + \Pi\Pi_f \prod_{j=1}^{K} \prod_{l_j=1}^{L_j} F_V^{(s)}(\mathbf{p}_{\rho_j(l_j)} + \mathsf{R}_3^{-1}(\psi)\mathbf{p}_{l_j}) \right)$$

$$\times \left(\mathcal{I}_{\boldsymbol{\rho}}^{\{N_j\}}(I, I_3) \prod_{j=1}^{K} \prod_{l_j=1}^{L_j} \delta_{\alpha_{\rho_j(l_j)} \alpha_{l_j}} + \chi_C^0 \chi_C \overline{\mathcal{I}}_{\boldsymbol{\rho}}^{\{N_j\}}(I, I_3) \prod_{j=1}^{K} \prod_{l_j=1}^{L_j} \delta_{-\alpha_{\rho_j(l_j)} \alpha_{l_j}} \right)$$

Includes : Energy-momentum, Angular momentum, Isospin, Parity, C-parity, B, Q, S conservation



PRELIMINARY
e+e- 2.4 GeV
data
model
σ (nb)
ρ = 0.24
γS= 0.74
I0= 0.36
Number of St. Dev.
π+ π- π0
π+ π- π0 π0
ω π0
π+ π- π+ π-
ω π+ π-
π+ π- π+ π- π+ π-
π+ π- π+ π- π0 π0
η π+ π-
K+ K- π+ π-
p p̄
n n̄

Outlook

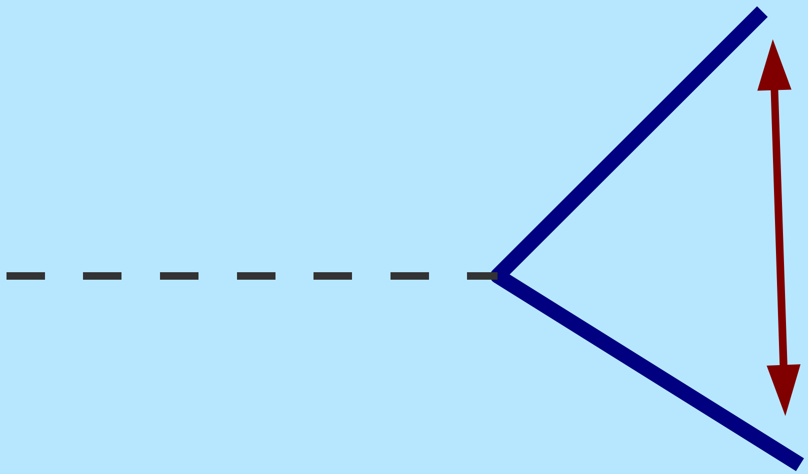
- Hadronization process shows clear statistical behaviour from e+e- at low energy to heavy ion collisions at high energy, with a critical T~160 MeV
- The origin of it is still unclear. Interesting ideas, further investigations ongoing
- Interesting developments in relativistic statistical mechanics (micro, fluctuations)

Heavy ions: early partonic thermalization over a large volume (on the hadronic size scale) – proper parton deconfinement

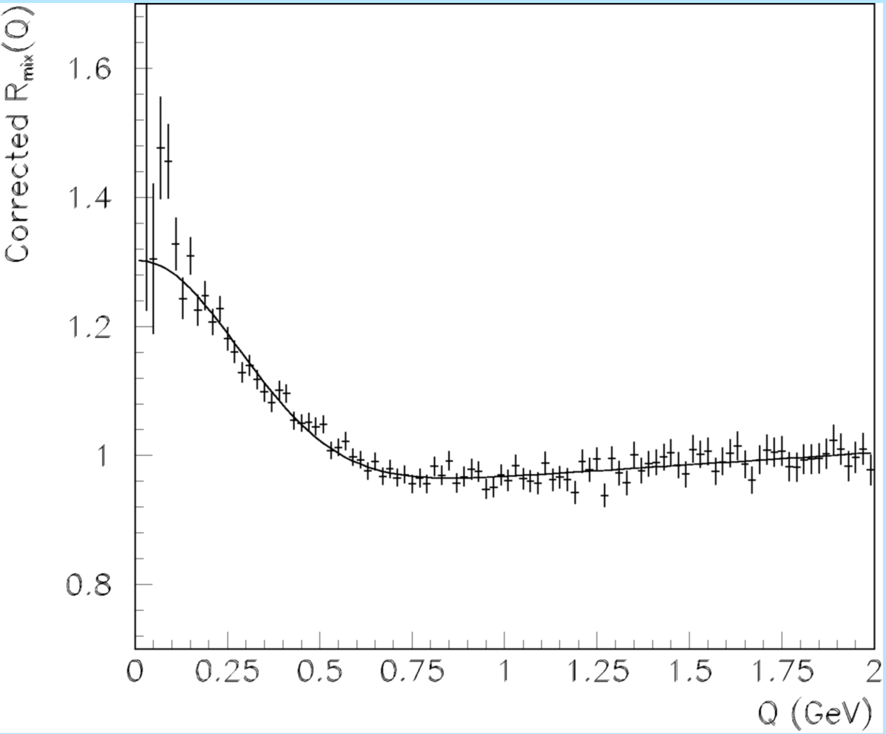
Elementary collisions: late, pre-hadronization statistical equilibrium over hadronic-sized clusters

$\bar{q}$ q pair(s) from e+e- annihilation decelerated by the strin

tension σ (~ 1 GeV/fm)



$$T = \sqrt{\frac{\sigma}{2\pi}} \simeq 177 \quad MeV$$

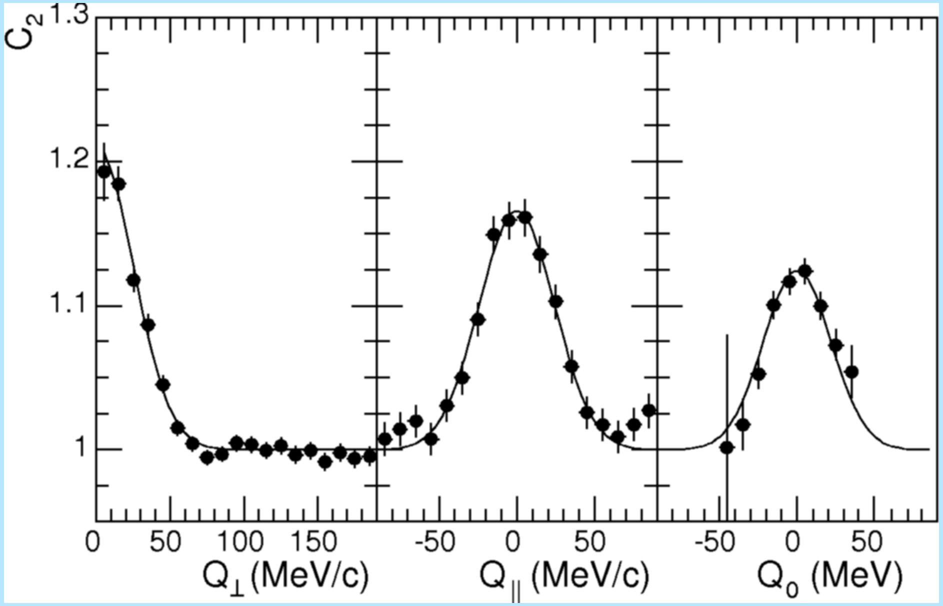


e+e- collisions at $\sqrt{s}$ = 91.2 GeV

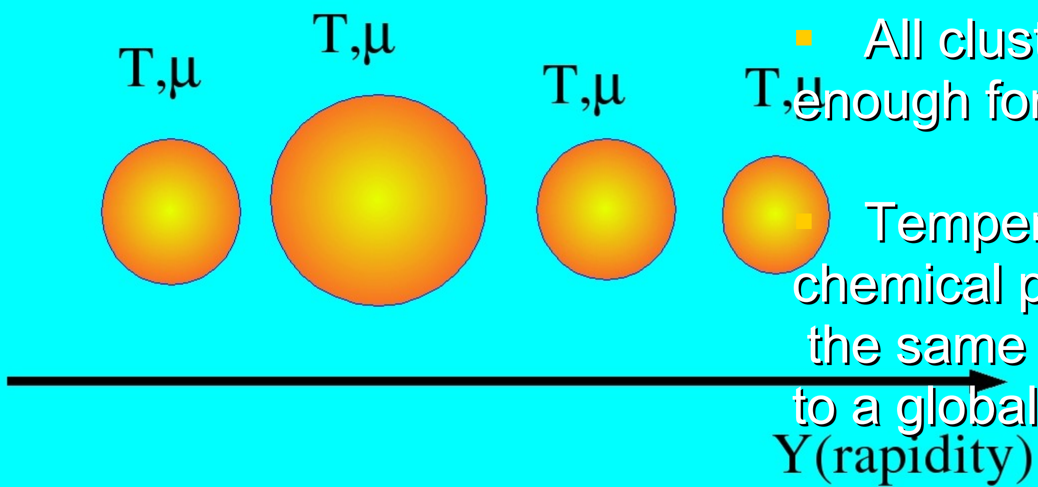
ALEPH coll., Phys. Rep. 294 (1998) 1

Pb-Pb collisions at 158 GeV/c

NA49 coll., Phys. Lett. B 382 (1996) 181



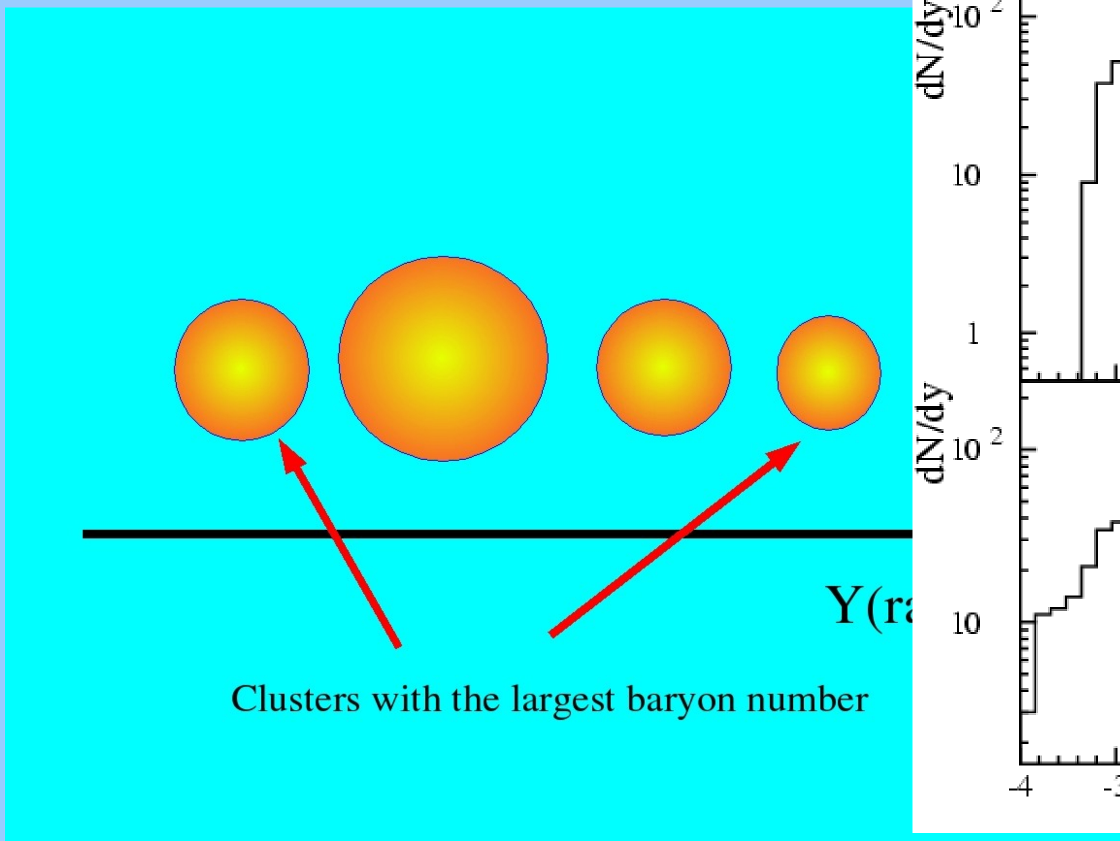
A different approach (in HIC)

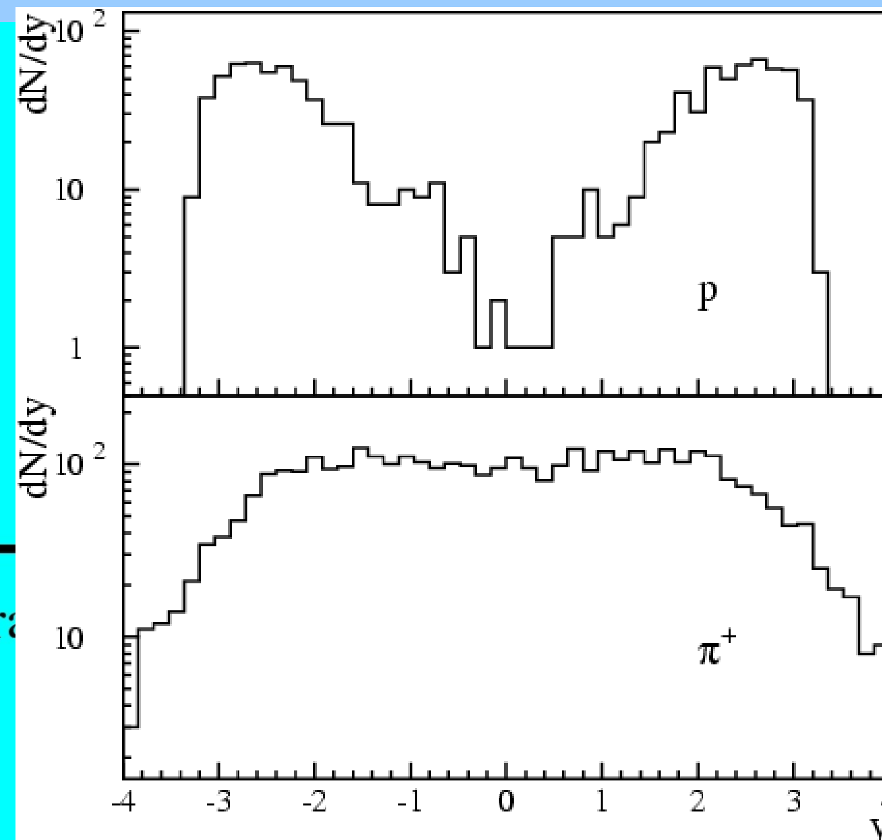


More restrictive

- All clusters large enough for GC to apply
- Temperature and chemical potentials must be the same for the equivalence to a global fireball to apply

Equivalence may only apply to Lorentz-invariant observables





Since single cluster's average multiplicities are independent of its momentum, clusters can be ordered in rapidity without affecting the overall multiplicities

Phase space dominance is not trivial

In principle, $|M_{if}|^2$ may depend on $m_1^2, m_2^2, \ldots, m_{12}^2, m_{13}^2, \ldots$
as well as on $I_1, I_2, \ldots, I_1 \cdot I_2, I_1 \cdot I_3, \ldots,$

Example:

$$|M_{if}|^2 \propto (\alpha^3 M)^N \prod_{i=1}^{N} f(\alpha m_i) g(I_i)$$

quite restrictive: again, only one scale α and factorization

$$\langle n_j \rangle = \frac{(2J_j+1)(\alpha^3 M)}{(2\pi)^3} g(I_j) f(\alpha m_j) \int \frac{d^3 p}{2\varepsilon} \exp(-\beta \sqrt{p^2 + m_j^2})$$

The thermal-like behaviour can be easily distorted at ANY scale of Multiplicity (just take $g(I)=AI^2+C$ or $f(\alpha m)=(\alpha m)^5$)

Derive the statistical features within other models

A. Bialas, Ph

W. Florkowsk

Fluctuation of the string te

exponential shape, e.g. of

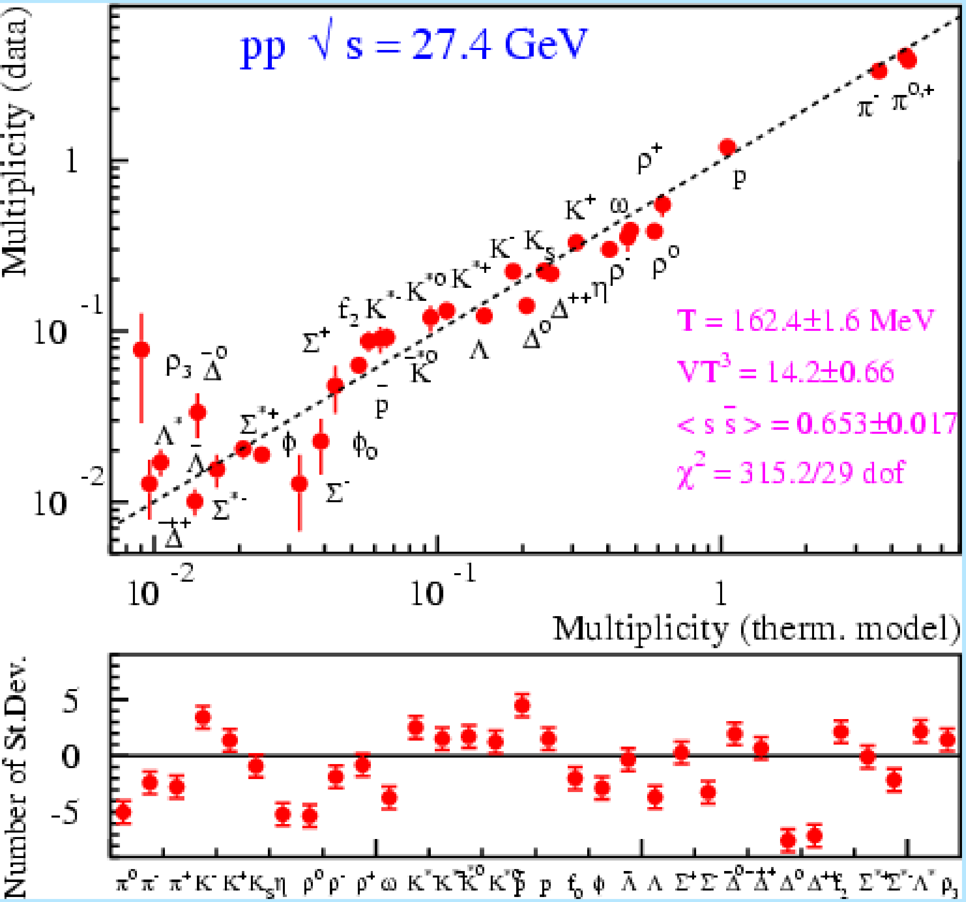
Parameter	Name	Default	Range gen.	Fit Result		
				Value	stat.	sys.
Λ_{QCD}	PARJ(81)	0.29	0.25 - 0.35	0.297	± 0.004	$^{+0.007}_{-0.008}$
Q_0	PARJ(82)	1.0	1.0 - 2.0	1.56	± 0.11	$^{+0.21}_{-0.15}$
a	PARJ(41)	0.3	0.1 - 0.5	0.417	± 0.022	$^{+0.011}_{-0.015}$
b	PARJ(42)	0.58	0.850	optimized		
σ_q	PARJ(21)	0.36	0.36 - 0.44	0.408	± 0.005	$^{+0.004}_{-0.004}$
$P(^1S_0)_{ud}$	-	0.5	0.3 - 0.5	0.297	± 0.021	$^{+0.102}_{-0.011}$
$P(^3S_1)_{ud}$	-	0.5	0.2 - 0.4	0.289	± 0.038	$^{+0.004}_{-0.026}$
$P(^1P_1)_{ud}$	-	0.	see text	0.096		
$P(other\ P\ states)_{ud}$	-	0.	see text	0.318		
γ_s	PARJ(2)	0.30	0.27 - 0.31	0.308	± 0.007	$^{+0.004}_{-0.036}$
$P(^1S_0)_s$	-	0.4	0.3 - 0.5	0.410	± 0.038	$^{+0.026}_{-0.013}$
$P(^3S_1)_s$	-	0.6	0.2 - 0.4	0.297	± 0.021	$^{+0.020}_{-0.004}$
$P(P\ states)_s$	-	0.	see text	0.293		
ϵ_c	PARJ(54)	-	variable	-0.0372	± 0.0007	$^{+0.0011}_{-0.0012}$
$P(^1S_0)_c$	-	0.25	0.26			
$P(^3S_1)_c$	-	0.75	0.44	adj. to data		
$P(P\ states)_c$	-	0.	0.3			
ϵ_b	PARJ(55)	-	variable	-0.00284	± 0.00005	$^{+0.00012}_{-0.00010}$
$P(^1S_0)_b$	-	0.25	0.175			
$P(^3S_1)_b$	-	0.75	0.525	adj. to data		
$P(P\ states)_b$	-	0.	0.3			
$P(qq)/P(q)$	PARJ(1)	0.1	0.08 - 0.11	0.099	± 0.001	$^{+0.005}_{-0.002}$
$[P(us)/P(ud)]/\gamma_s$	PARJ(3)	0.4	0.593	adj. to data		
$P(qq1)/P(qq0)$	PARJ(4)	0.05	0.07	adj. to data		
extra baryon supp.	PARJ(19)	0.	0.5	adj. to data, only for uds		
extra η supp.	PARJ(25)	1.0	0.65	0.65 ± 0.06		
extra η' supp.	PARJ(26)	1.0	0.23	0.23 ± 0.05		

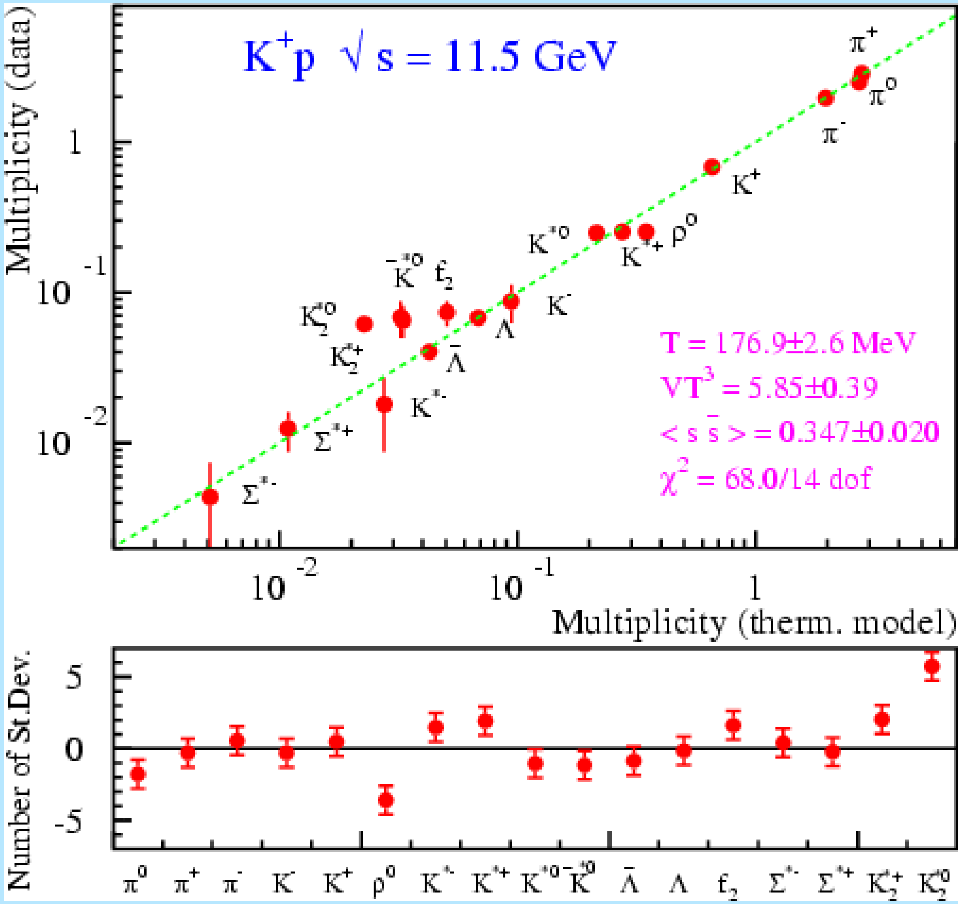
Table 49: Parameter settings and fit results for JETSET 7.4 PS with default decays

From: Delphi collaboration, CERN-PPE 96-120

Hadronic collisions

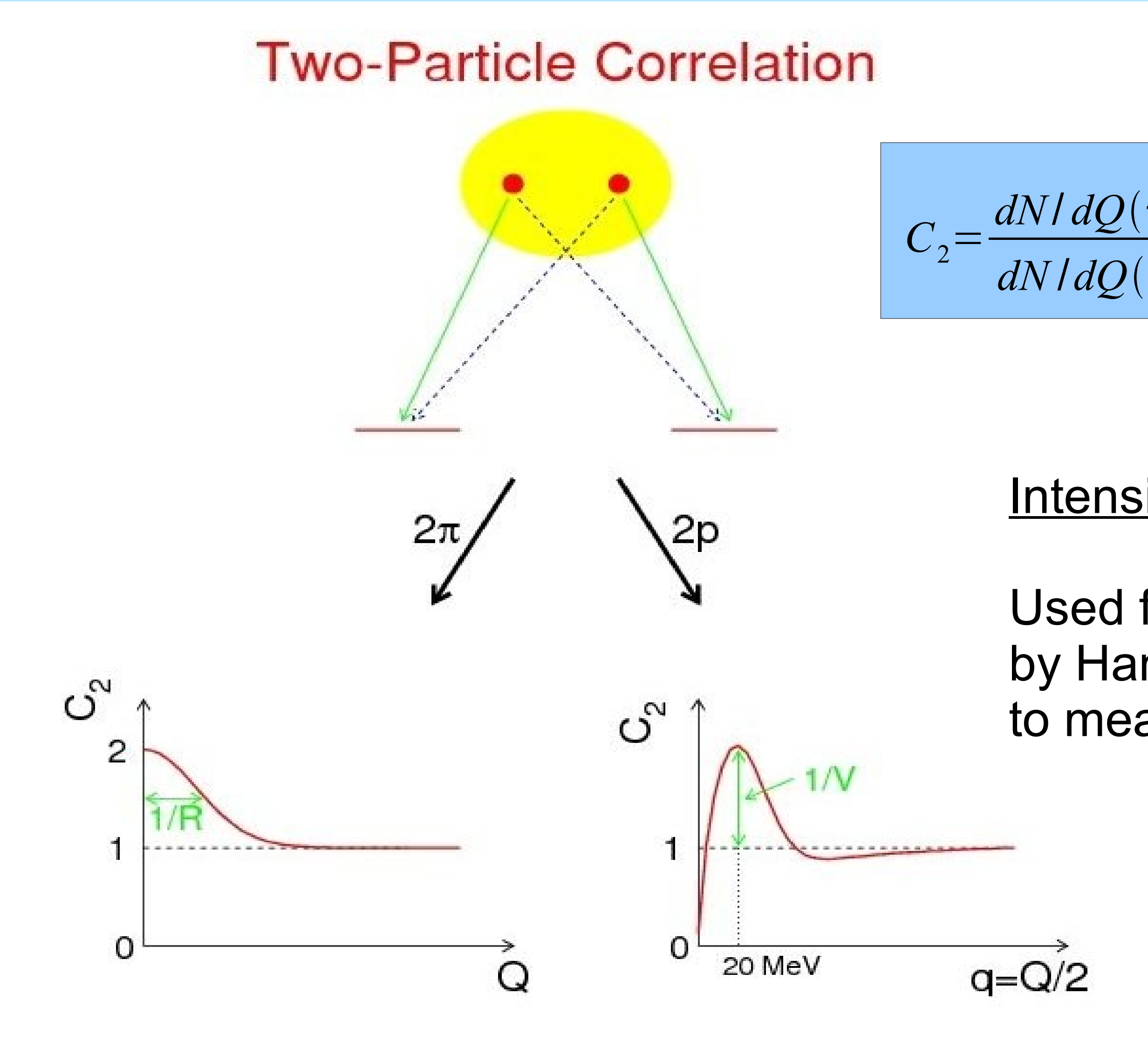
F. B., U. Heinz, Z. Phys. C76 (1997) 269





F.B., G. Passaleva, Eur. Phys. J. C 23 (2002) 551

Quantum statistics correlations provide evidence of the finite size of the source



$$C_2 = \frac{dN/dQ(\pi^+\pi^+)}{dN/dQ(\pi^+\pi^-)} = \quad [1+\lambda \exp(-R^2 Q^2)]$$

<u>Intensity interpheromentry</u>

Used first in astrophysics
by Hanbury-Brown and Twiss
to measure radii of stars

Interacting hadron gas

At sufficiently large temperatures, effective reduction to a non-interacting ideal gas of hadrons and resonances

Dashen Ma Bernstein theorem Phys. Rev. 187 (1969) 345

Non Relativistic

$$V \to \infty$$

$$Tr\,\delta(E-\hat{H}) = Tr\,\delta(E-\hat{H}_0) + \frac{1}{4\pi i} Tr[\delta(E-\hat{H}_0) S^{-1} \overleftrightarrow{\partial}_E S]$$

Relativistic generalization

$$Tr\,\delta^4(P-\hat{P}) = Tr\,\delta^4(P-\hat{P}_0) + \frac{1}{4\pi i} Tr[\delta^4(P-\hat{P}_0) S^{-1} \overleftrightarrow{\partial}_E S]$$

Is it possible to define a probability of an asymptotic free state | *f* > ?

YES

Define

$$p_f \propto \langle f | P_i P_V P_i | f \rangle \quad \text{with} \quad \mathrm{P}_V = \sum_{h_V} | h_V \rangle \langle h_V |$$

$$\sum_f p_f \propto \mathrm{tr}(P_i P_V P_i) \propto \mathrm{tr}(P_i P_V) = \sum_{h_V} \langle h_V | P_i | h_V \rangle = \Omega$$

All p_f are positive definite as:

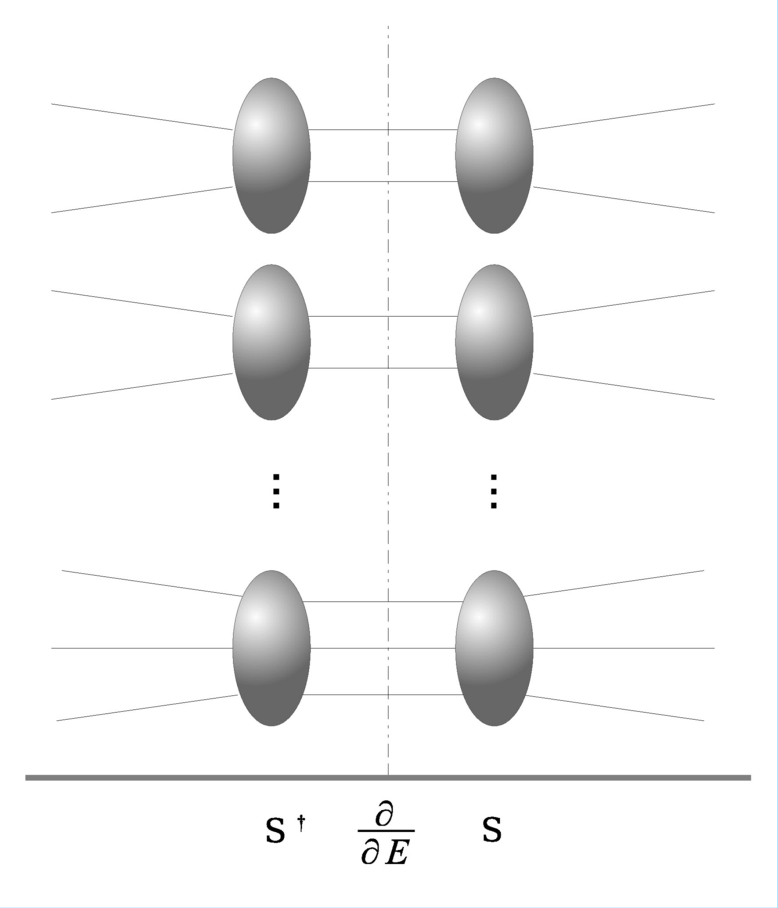
$$\langle f | P_i P_V P_i | f \rangle = \sum_{h_V} | \langle f | P_i | h_V \rangle |^2$$

In the SHM, the cluster is described by the mixture of states:

$$W = \sum_{h_V} P_i | h_V \rangle \langle h_V | P_i = P_i P_V P_i$$

How to get to the ideal hadron resonance gas?

- Cluster decomposition of the S-matrix
- Consider symmetric diagrams
- Neglect non-resonant background



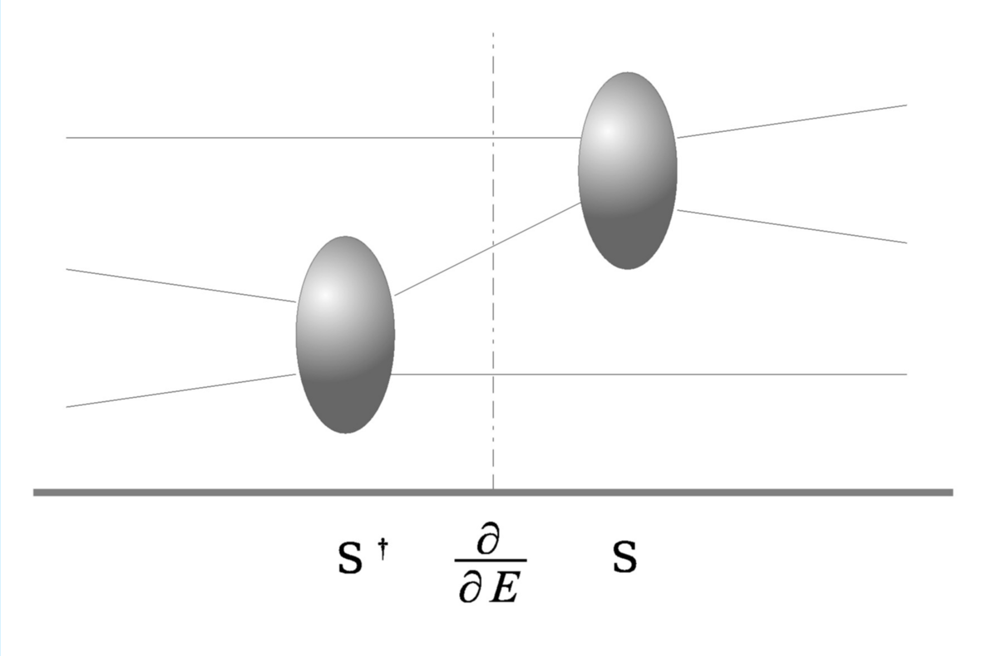
$$S^{-1}\partial_E S|_X \rightarrow \sum_k (2J_k+1)\frac{V}{(2\pi)^3}\int dM_k \int d^3P_k\, BW(M_k)\, BR(k\rightarrow X_k)\, \delta^4(P-\sum_i p_i)$$

Hadron-gas resonance model tested on the lattice

Confirmed in many respects!

F. Karsch et al., Phys. Lett. B 571 (2003) 67; Eur. Phys. J. C 29 (2003) 549

WARNING!



Hadron-resonance gas
does not include contribution
of non-symmetric diagrams
which survive in the thermodynamicl limit

The microcanonical ensemble
and its partition function

F.B., “What is the meaning of the statistical hadronization model?”

J. Phys. Conf. Ser. 5 (2005) 175, hep-ph 0410403

The usual definition:

$$\Omega = \sum_{states} \delta^4 (P - P_{state})$$

Can be generalized as:

$$\Omega = \sum_{h_V} \langle h_V | P_i | h_V \rangle$$

P_i projector on the cluster’s initial state

$| h_V >$ multihadronic state *within* the cluster

canonical:

$$Z = \sum_{h_V} \langle h_V | \exp(-H/T) | h_V \rangle$$

Full microcanonical ensemble

Conservation=projection onto an irreducible state

F.B., L. Ferroni, Eur. Phys. J. C 35 (2004) 243

Decompose the projector:

$$P_i = P_{P,J,\lambda,\pi} P_\chi P_{I,I_3} P_Q$$

- *P* 4-momentum
- *J* spin
- λ helicity
- π parity
- χ C-parity
- **Q** abelian charges
- I, I_3 isospin

The projectors on 4-momentum, spin-helicity and parity factorize if *P=(M,0)*

$$P_{P,J,\lambda,\pi} = \frac{1}{(2\pi)^4} \int d^4x \, e^{iP\cdot x} \exp(-i\hat{P}\cdot x)\,(2J+1) \int dR \, D^J(R)^{\lambda\,*}_{\lambda}\, U(R) \frac{I + \pi U(\Pi)}{2}$$

$$P_{I,I_3} = \int d\mu(g)\,\mathrm{tr}\, D(g^{-1})\, U(g)$$

Full microcanonical ensemble

Projection onto localized states

In principle, projection P_V should be made on localized field states:

$$P_{V_{1-particle}} = \int_V D\psi \,|\psi\rangle\langle\psi| \quad \text{with} \quad |\psi\rangle = \otimes_x |\psi(x)\rangle$$

If $V^{1/3} > \lambda_C$ (at most 1.4 fm), it is a good approximation to identify localized N-particle states with asymptotic N-particle states

In general this is not true

Rate of a multi-hadronic channel $\{N_j\}=(N_1,\ldots,N_K)$

F.B., L. Ferroni, Eur. Phys. J. C38 (2004) 225

For *non-identical particles:*

$$\Gamma_{\{N_j\}} \propto \Omega_{\{N_j\}} = \frac{V^N}{(2\pi)^{3N}} \prod_{i=1}^{N} (2J_i+1) \int d^3p_1 \ldots d^3p_N \, \delta^4\left(P - \sum_{i=1}^{N} p_i\right)$$

For identical particles: cluster decomposition

$$\Omega_{\{N_j\}} = \int d^3p_1 \ldots d^3p_N \, \delta^4(P-P_f) \boxed{\prod_j \sum_{\{h_{n_j}\}}} \frac{(\mp 1)^{N_j+H_j} (2J_j+1)^{H_j}}{\prod_{n_j=1}^{N_j} n_j^{h_{n_j}} h_{n_j}!} \prod_{l_j=1}^{H_j} F_{n_{l_j}}$$

$$H_j = \sum_{n_j=1}^{N_j} h_{n_j} \qquad N_j = \sum_{n_j=1}^{N_j} n_j h_{n_j} \qquad F_{n_l} = \prod_{i_l=1}^{n_l} \frac{1}{(2\pi)^3} \int_V d^3x \, \exp\left[ix\cdot\left(p_{i_l} - p_{c_l(i_l)}\right)\right]$$

Generalization of the expression in M. Chaichian, R. Hagedorn, Nucl. Phys. B92 (1975) 445 which holds only for large *V*

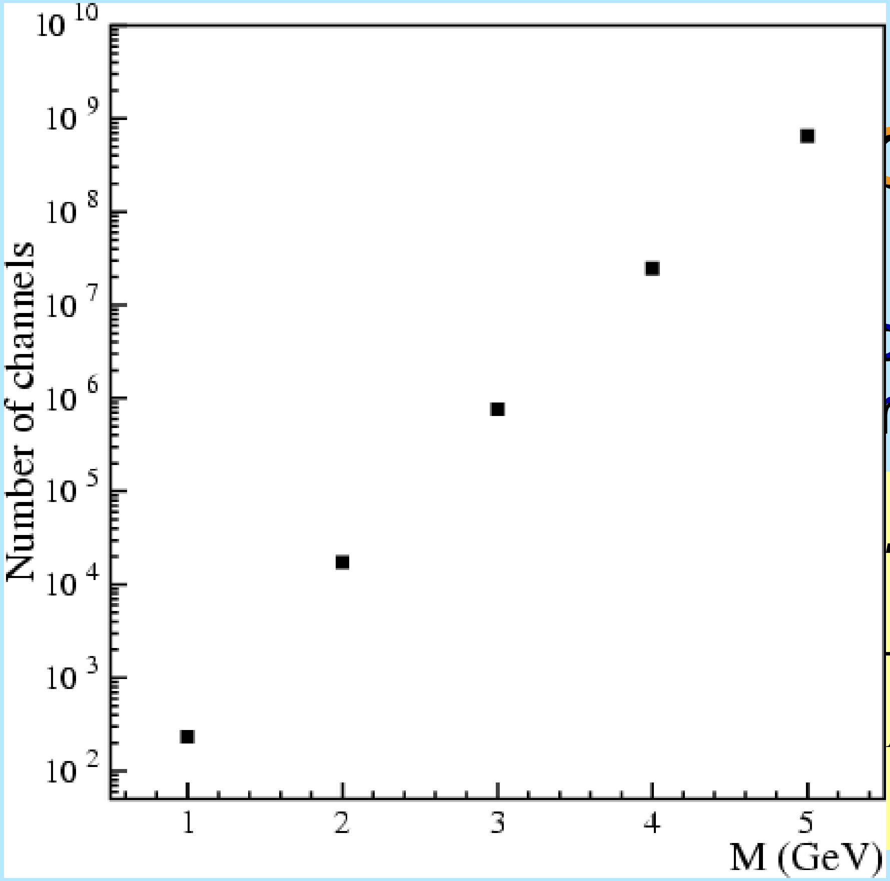
partitions

Recent work on microcanonical ensemble

- V. Begun, M. Gorenstein, A. Kostyuk, A. Zozulya: Phys. Rev. C 71 (2005) 054904
- V. Begun, M. Gorenstein, A. Kostyuk, A. Zozulya: nucl/th 0505069

Inequivalence between micro and Grand-canonical ensemble with regard to fluctuations (talks in this conference)

- F.B., A. Keranen, L. Ferroni, T. Gabbriellini: nucl/th 0507039
- F. B., L. Ferroni: Eur. Phys. J. C 35 (2004) 243; Eur. Phys. J. C 38 (2004) 225
- F. M. Liu, J. Aichelin, K. Werner, M. Bleicher: Phys. Rev. C 69 (2004) 054002
- K. Bugaev, J. Elliott, L. Moretto, L. Phair: nucl/th 0504010; hep/ph 0504011



...dron gas ensemble

...ossible

...rically via numerical integrations

$$\frac{\Omega_{\{N_j\}}\delta_{Q,\sum_j N_j q_j}}{\ldots_{\{N_j\}}\delta_{Q,\sum_j N_j q_j}}$$

Main difficulty: size

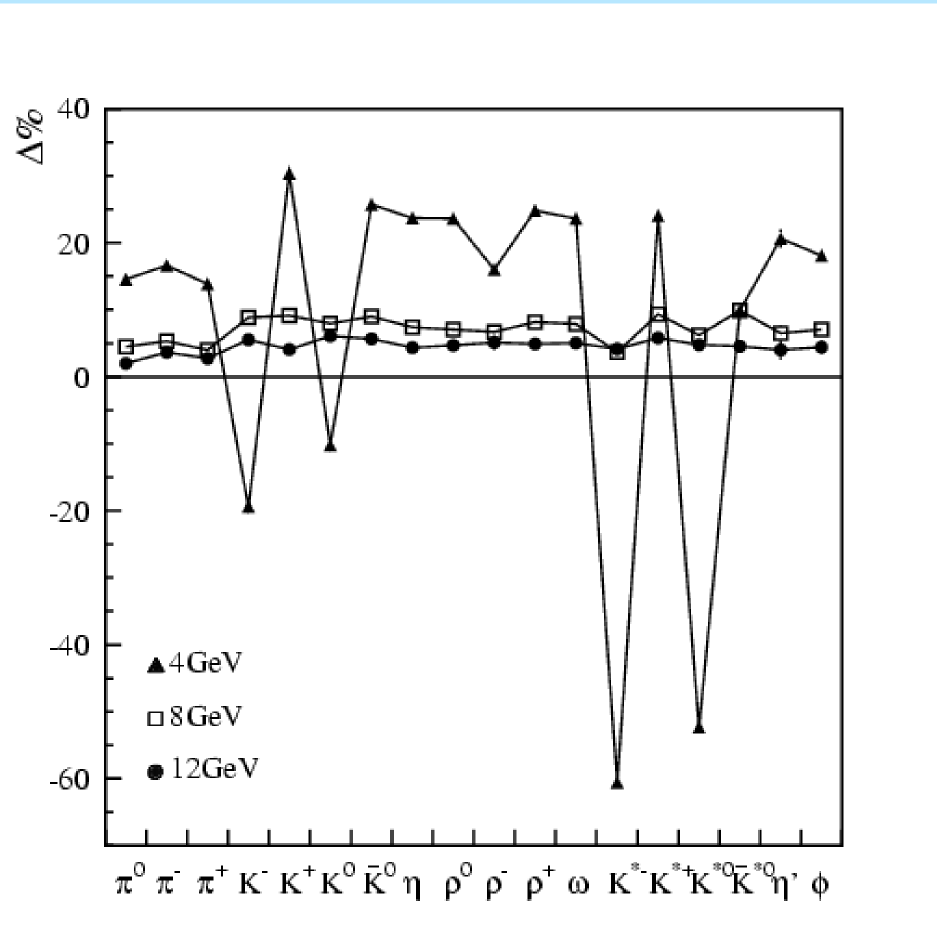
271 light-flavoured species in the hadron-resonance gas give rise to a huge number of channels *{N}*

→ Monte-Carlo methods

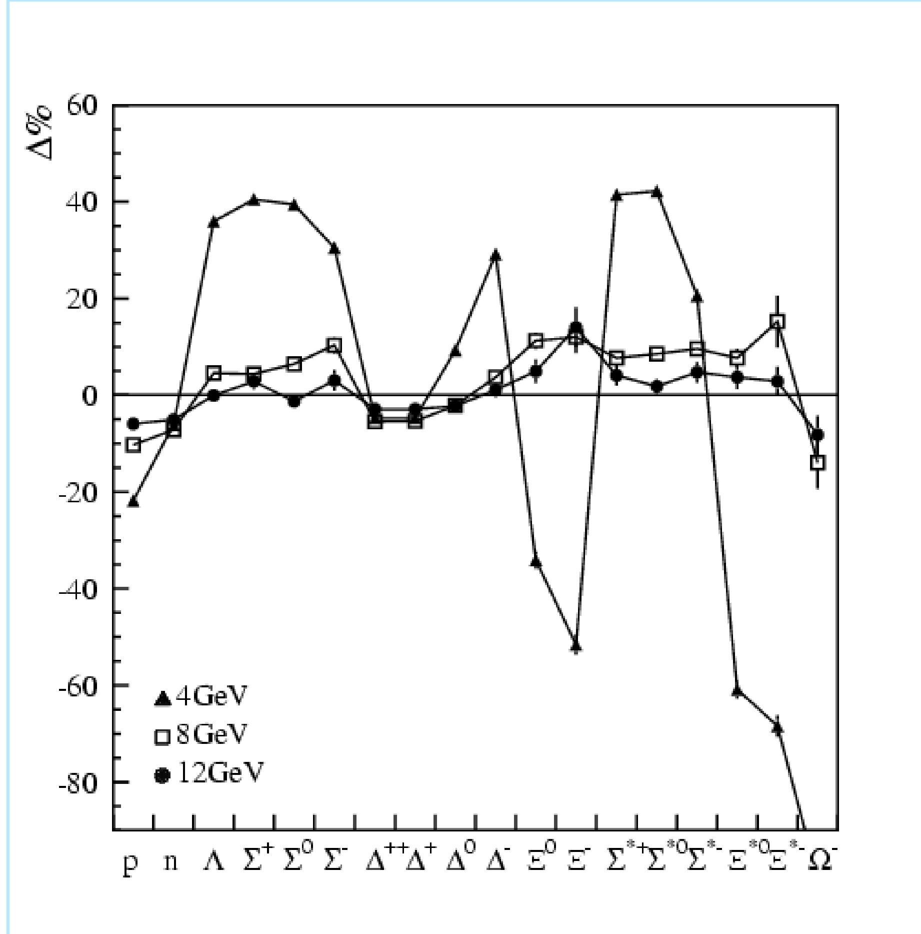
Comparison between μC and C hadron multiplicities

pp-like cluster, $M/V = 0.4\ GeV/fm^3$

Mesons



Baryons



Work (still) in progress: a Monte-Carlo event generator

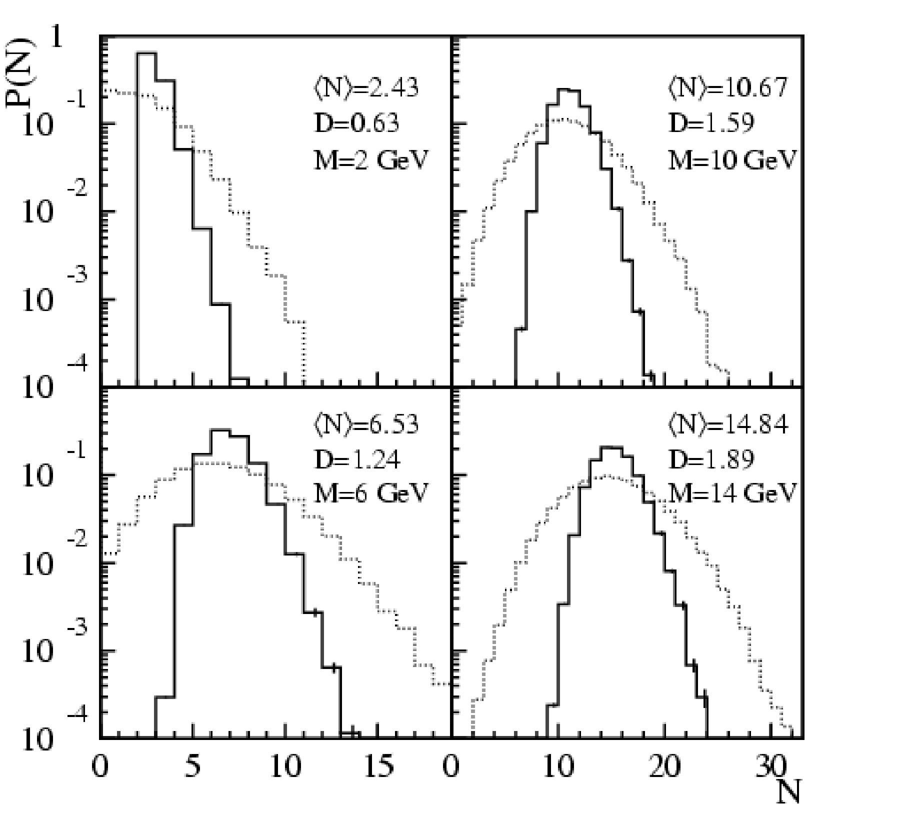
1. Versatile tool to test the model against more complex observables
2. Pragmatic approach: the Statistical Hadronization Model works well, why not using it ?

- To be matched to parton shower or other clustering models (HERWIG).

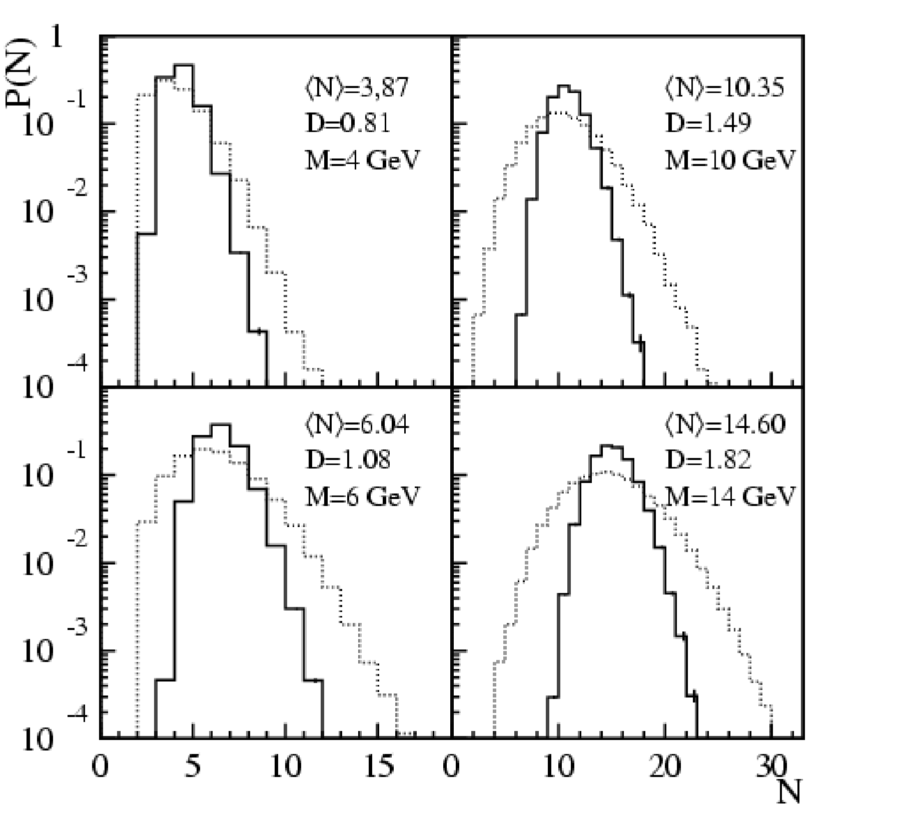
Comparison between μC and C hadron multiplicity distributions

Inequivalence between C and μC in the thermodynamic limit

Q=0 cluster, *M/V=0.4* GeV/fm^3

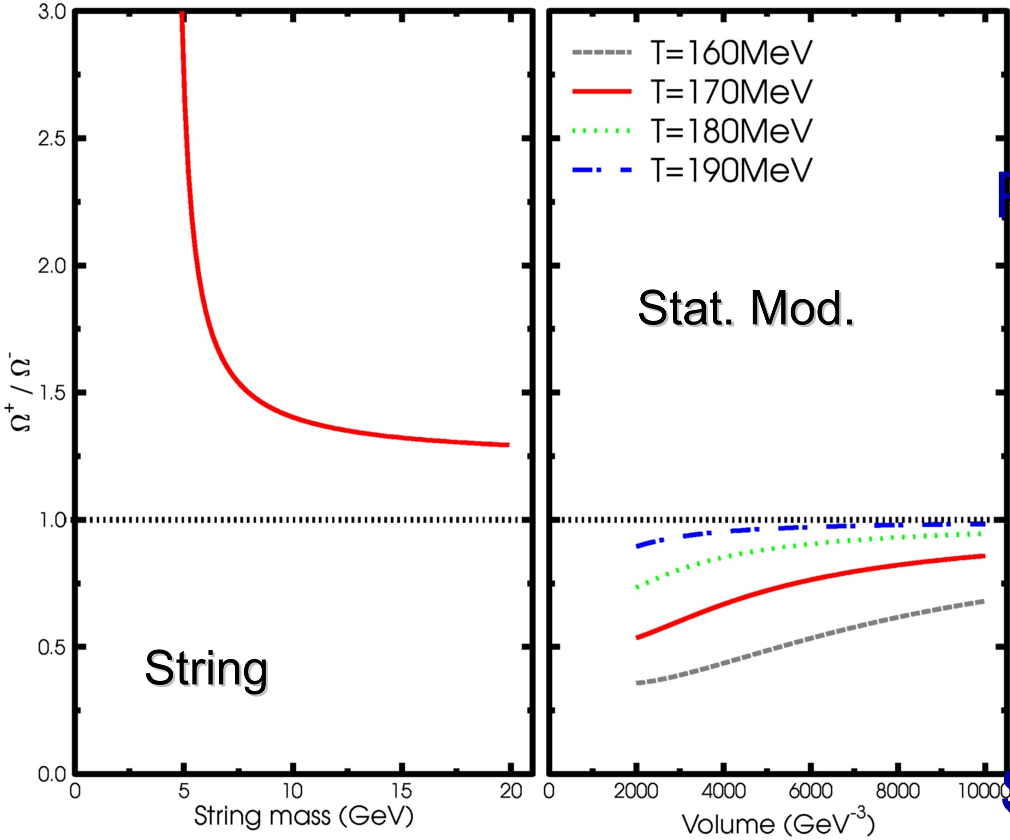


pp-like cluster, *M/V=0.4* GeV/fm^3



Proposed test: $\bar{\Omega}/\Omega$ in pp collisions

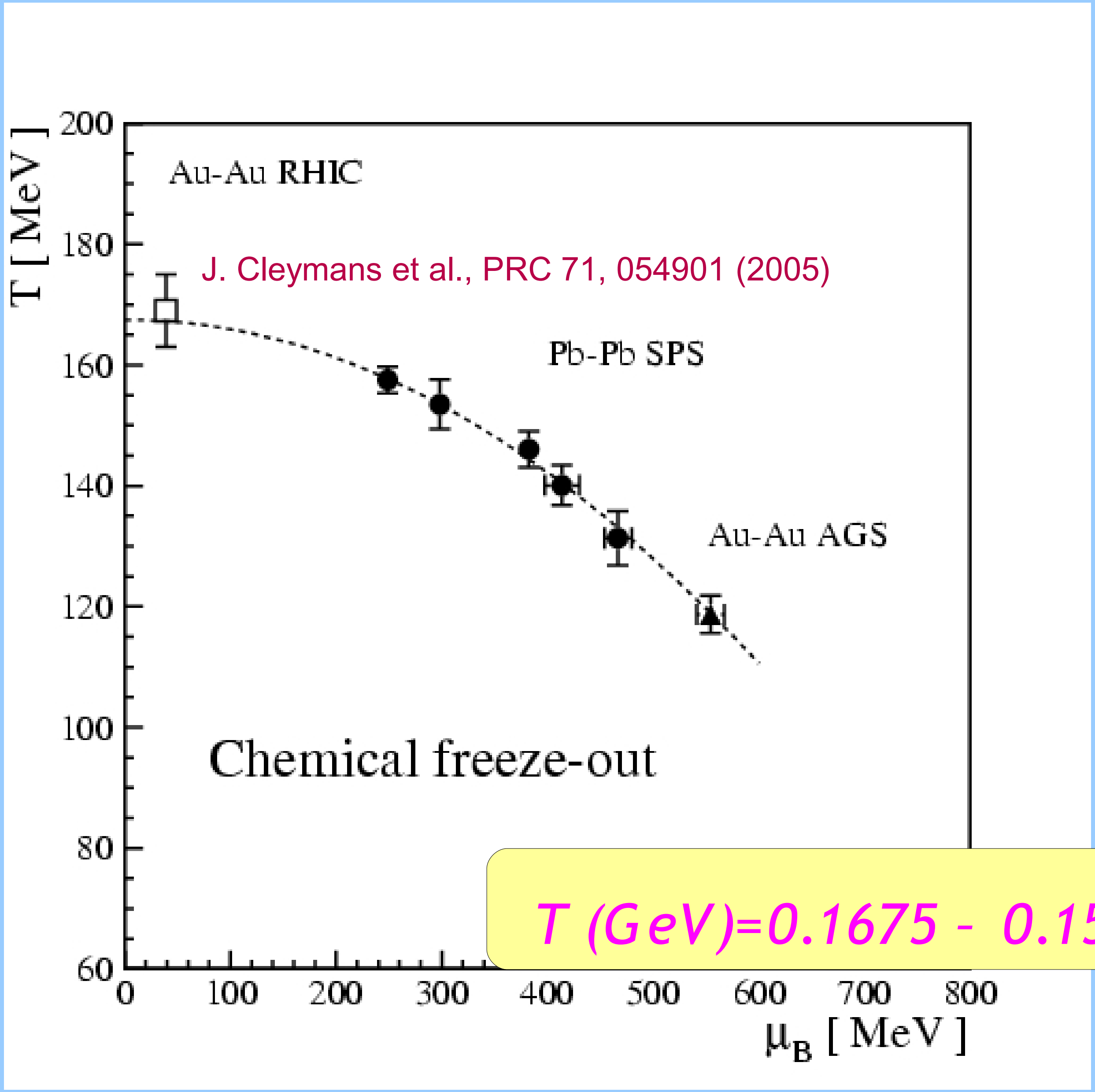
M. Bleicher et al., Phys. Rev. Lett. 88 (2002) 202501



Preliminary measurement by NA49 in pp @ 17.2 GeV (SQM03):

0.67+-0.62

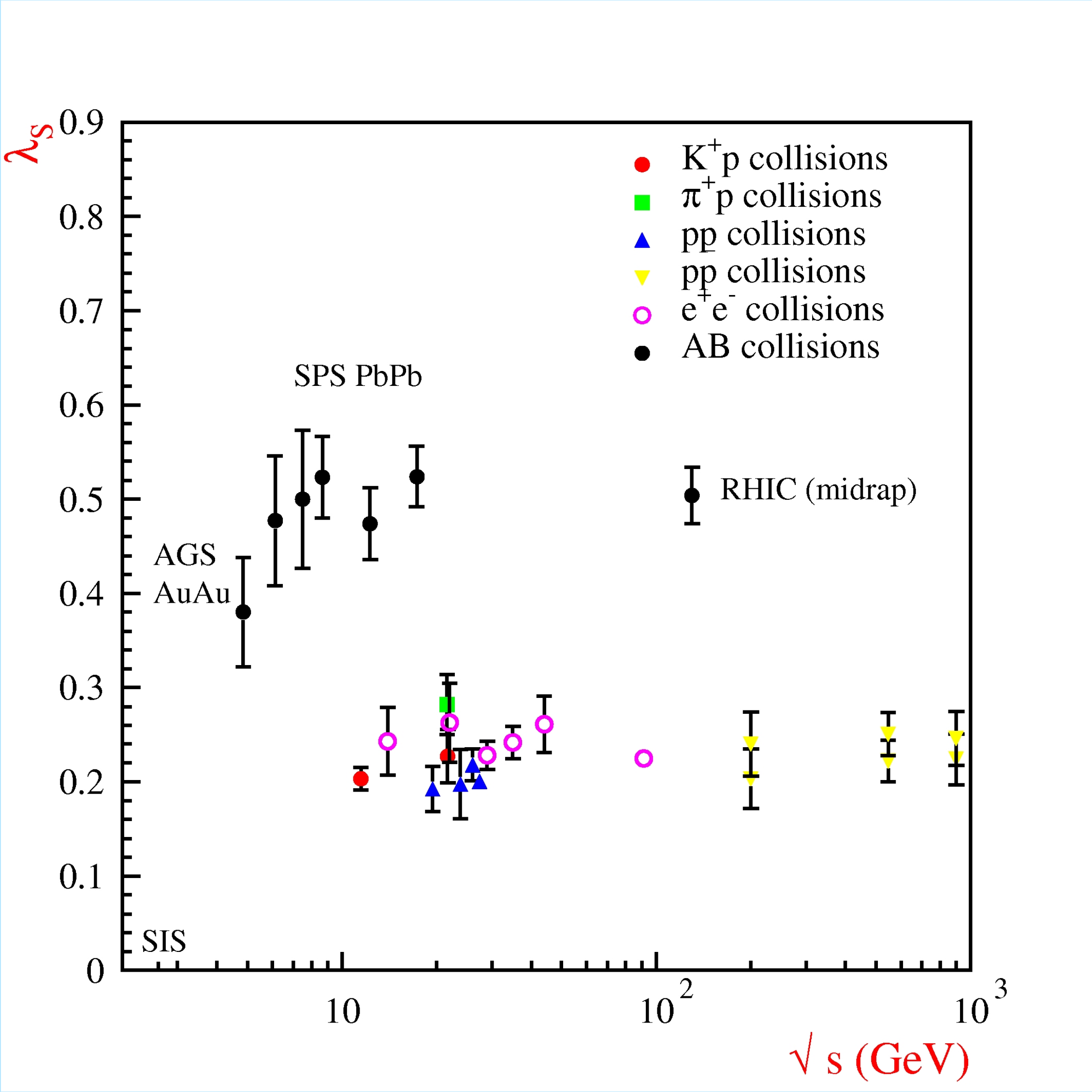
Statistics might be a problem



$$T\,(GeV) = 0.1675 - 0.1583\,\mu^2$$

F.B., J. Manninen, M. Gazdzicki, Phys. Rev. C 73 (2006) 044905

Strangeness enhancement



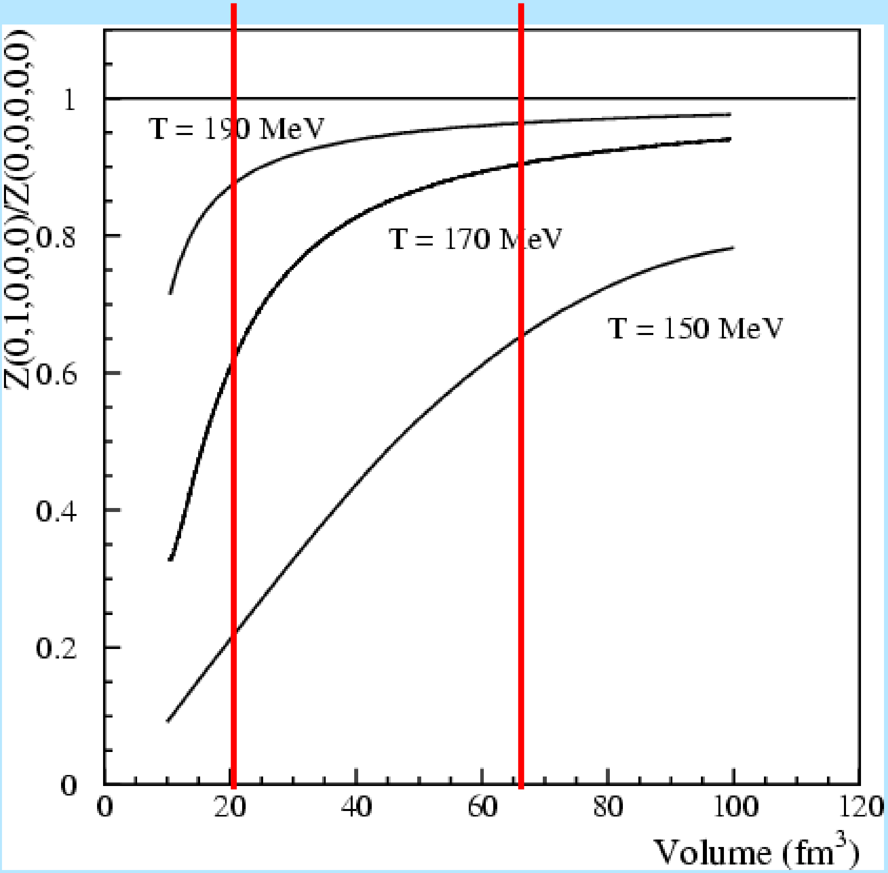
$$\lambda_s = \frac{2\langle s\bar{s}\rangle}{\langle d\bar{d}\rangle + \langle u\bar{u}\rangle}$$

By using multiplicities estimated in the model

HIC are not a linear Superposition of NN

Canonical ensemble does matter

Example: neutron chemical factor in a completely neutral cluster



In GC should be:

$$\langle n_j \rangle = \frac{V(2J_j+1)}{(2\pi)^3} \int d^3p \, \exp\left(-\sqrt{p^2+m_j^2}/T\right)$$

Whilst in C

$$\langle n_j \rangle = \frac{V(2J_j+1)}{(2\pi)^3} \int d^3p \, \exp\left(-\sqrt{p^2+m_j^2}/T\right) \frac{Z(-q_j)}{Z(0)}$$

For $V \to \infty$ $\langle n \rangle_C = \langle n \rangle_{GC}$

$\langle n \rangle_C < \langle n \rangle_{GC}$

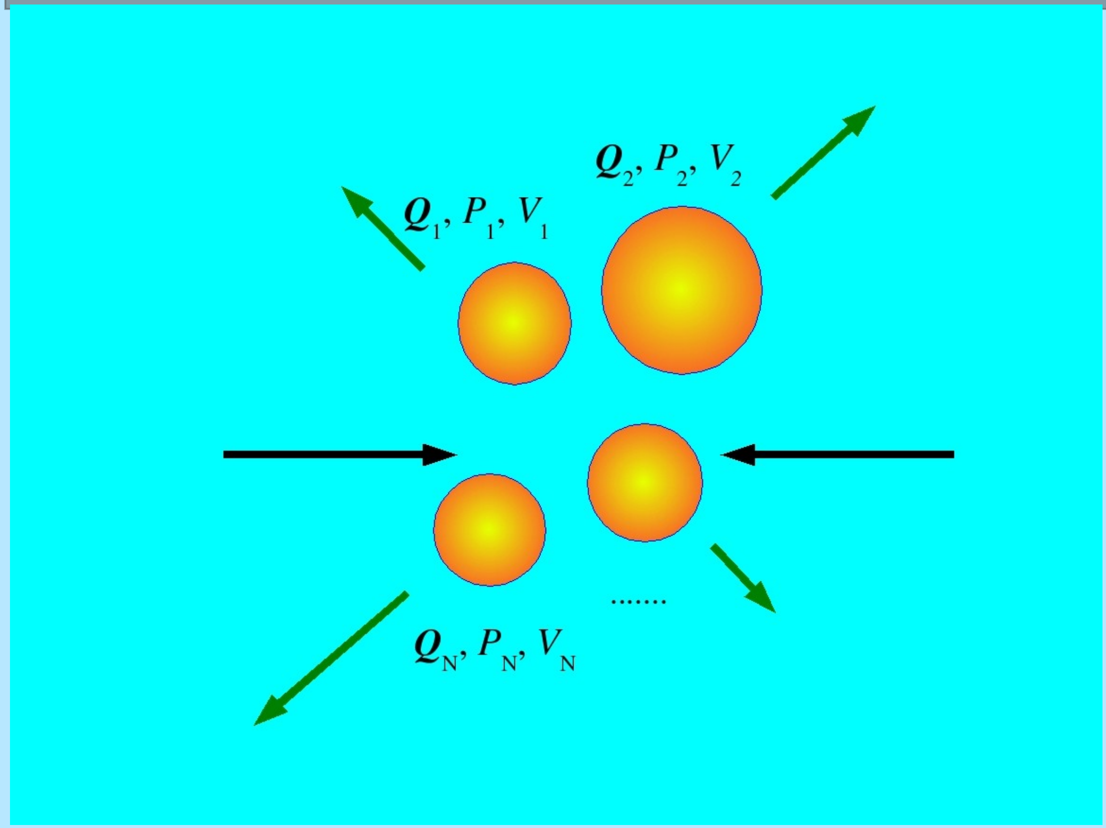
Canonical suppression

The statistical model in high energy collisions

- **QCD dynamics leads to colourless cluster formation (preconfinement)**

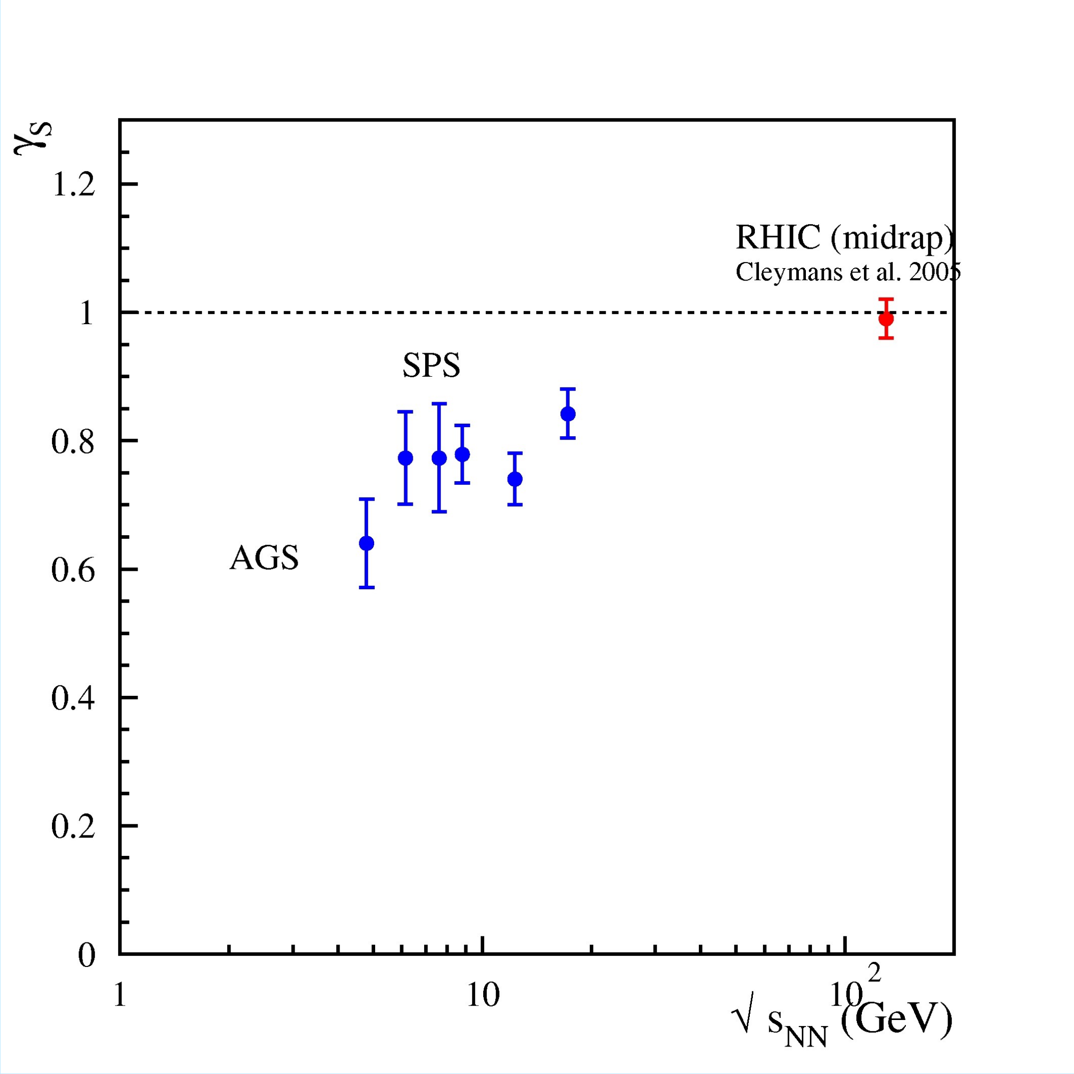
D. Amati, G. Veneziano, Phys. Lett. B 83 (1979) 87

- **Every multihadronic state localized within the cluster compatible with conservation laws is equally likely**



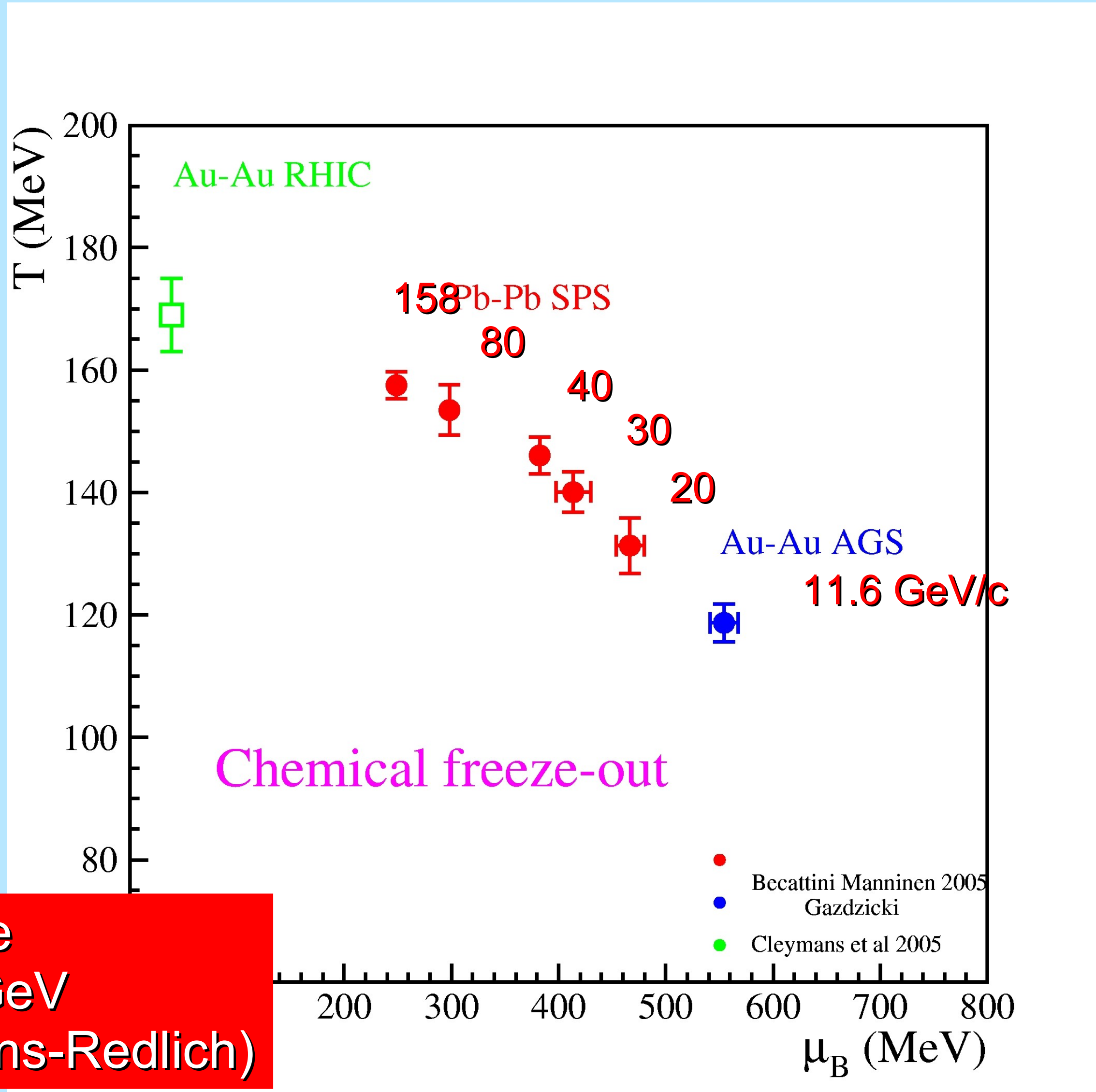
Key idea:
finite extension of clusters
(like MIT bags)

Strangeness suppression



F.B., J. Manninen, M. Gazdzicki, in preparation

Chemical freeze-out in HIC



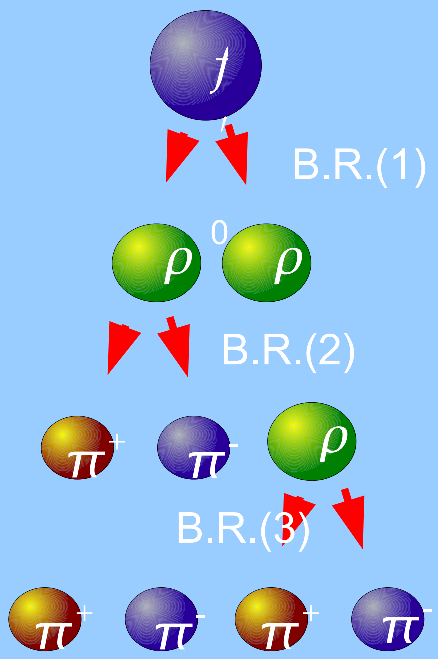
Points lie on the
$\langle E\rangle/\langle N\rangle \sim 1$ GeV
curve (Cleymans-Redlich)

F.B., J. Manninen, M. Gazdzicki, Phys. Rev. C 73 044905 (2006)

microcanonical ensemble(2)

(Interacting relativistic hadron gas)

from primary to



$$\Omega^{final}(\pi^+\pi^-\pi^+\pi^-)=\Omega(\pi^+\pi^-\pi^+\pi^-)+B.R.(3)\,\Omega(\pi^+\pi^-\rho)+$$

$$+B.R.(3)\,B.R.(2)\,\Omega(\rho\,\rho)+$$

$$+B.R.(3)\,B.R.(2)\,B.R.(1)\,\Omega(f'_0)+\ldots \textit{all possible decay trees}$$

single-particle's channels weight:

$$\Omega_h(M,V)\propto V\frac{m_h\Gamma_h}{(M^2-m_h^2)^2+m_h^2\Gamma_h^2}$$

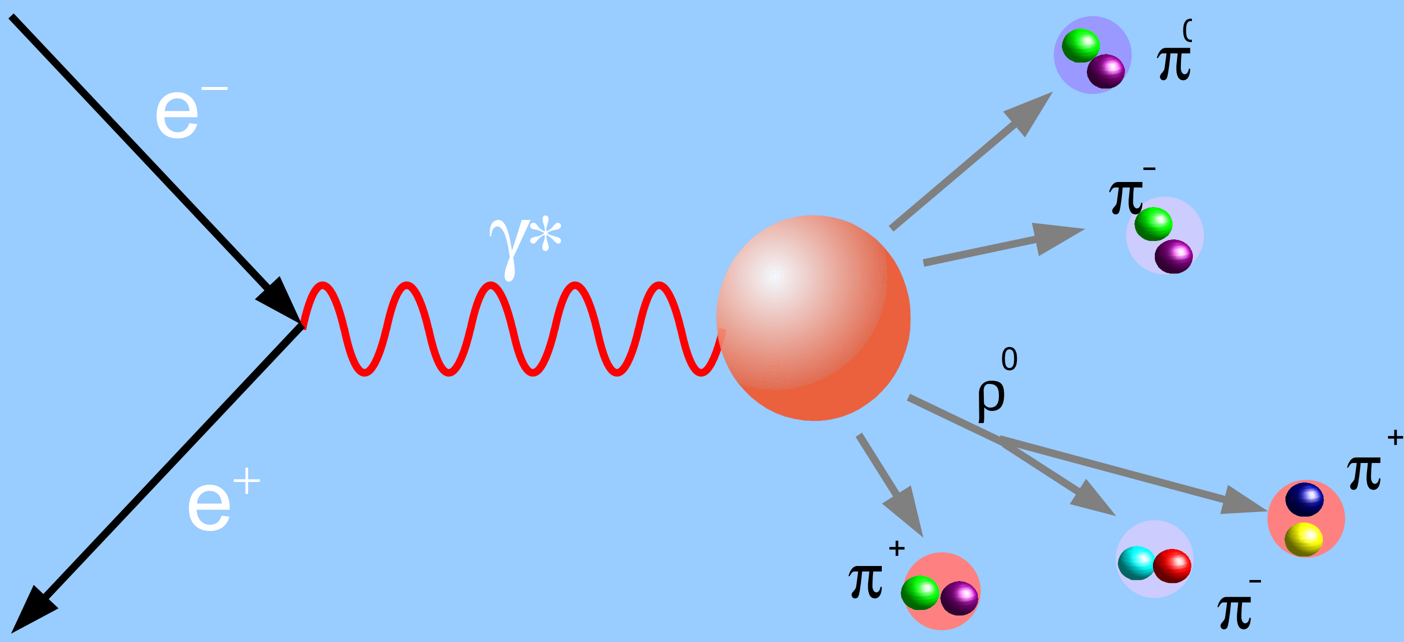
Safe energy range: **2 GeV ≤ √s ≤ 3 GeV**

Preliminary comparison with data

Model assumptions:

Minimize χ^2

$$\Omega^{final}(channel) = A \cdot \sigma(channel)$$



- *Isospin Mixing $f_1(I=1) + (1-f_1)(I=0)$*
- SU(3)→SU(2)⊗U(1)

$\sqrt{s}$= 2.1 GeV e^+e^- collisions

channel	σ (nb) (model)				σ (nb) (exp.)
	$\boldsymbol{\rho = 0.1}$ **GeV/fm**^{3} $f_1 = 0.9$ $\gamma_S = 1.0$	$\rho = 0.5$ GeV/fm^3 $f_1 = 0.9$ $\gamma_S = 1.2$	$\rho = 1.0$ GeV/fm^3 $f_1 = 0.9$ $\gamma_S = 1.2$	$\rho = 1.5$ GeV/fm^3 $f_1 = 0.9$ $\gamma_S = 1.2$	
$\pi^+\pi^-$	$0.25 \pm 0.004 \pm 0.02$	$0.49 \pm 0.004 \pm 0.06$	$0.6 \pm 0.004 \pm 0.1$	$0.7 \pm 0.004 \pm 0.1$	0.35 ± 0.17
$\pi^+\pi^-\pi^0$	$0.07 \pm 0.005 \pm 0.01$	$0.09 \pm 0.0005 \pm 0.02$	$0.1 \pm 0.0004 \pm 0.02$	$0.1 \pm 0.0003 \pm 0.02$	0.398 ± 0.099
$\pi^+\pi^-\pi^0\pi^0$	$6.6 \pm 0.1 \pm 0.5$	$3.8 \pm 0.02 \pm 0.4$	$3.4 \pm 0.01 \pm 0.4$	$3.2 \pm 0.01 \pm 0.3$	15.8 ± 3.4
$\pi^+\pi^-\pi^+\pi^-$	$5.4 \pm 0.1 \pm 0.5$	$3.3 \pm 0.02 \pm 0.4$	$2.9 \pm 0.01 \pm 0.4$	$2.7 \pm 0.006 \pm 0.4$	5.142 ± 0.263
$\pi^+\pi^-\pi^+\pi^-\pi^0$	$1.46 \pm 0.02 \pm 0.07$	$0.92 \pm 0.004 \pm 0.07$	$0.81 \pm 0.003 \pm 0.06$	$0.73 \pm 0.002 \pm 0.05$	2.5 ± 2.4
$\mathrm{p\overline{p}}$	$0.468 \pm 0.002 \pm 0$	$0.662 \pm 0.002 \pm 0$	$0.744 \pm 0.002 \pm 0$	$0.763 \pm 0.002 \pm 0$	0.571 ± 0.068
$\mathrm{n\overline{n}}$	$0.473 \pm 0.002 \pm 0$	$0.662 \pm 0.002 \pm 0$	$0.745 \pm 0.002 \pm 0$	$0.764 \pm 0.002 \pm 0$	0.95 ± 0.23
$K^+K^-\pi^+\pi^-\pi^+\pi^-$	$0.0381 \pm 0.0003 \pm 0.0002$	$0.00113\,(\pm 0.7 \pm 1)\,10^{-5}$	$0.000195\,(\pm 1 \pm 3)\,10^{-6}$	$6.8\,10^{-5}\,(\pm 0.7 \pm 2)\,10^{-6}$	0.015 ± 0.015
$K^+K^-\pi^+\pi^-$	$3.4 \pm 0.04 \pm 0.1$	$3.39 \pm 0.01 \pm 0.07$	$2.84 \pm 0.01 \pm 0.05$	$2.53 \pm 0.006 \pm 0.05$	3.5 ± 0.27
$\eta\pi^+\pi^-$	$1.2 \pm 0.03 \pm 0.2$	$1.5 \pm 0.01 \pm 0.2$	$1.5 \pm 0.01 \pm 0.2$	$1.4 \pm 0.01 \pm 0.1$	0.22 ± 0.16
$\omega\pi^+\pi^-$	$0.285 \pm 0.004 \pm 0.001$	$0.234 \pm 0.001 \pm 0.002$	$0.225 \pm 0.001 \pm 0.004$	$0.216 \pm 0.0006 \pm 0.005$	0.27 ± 0.12

Exclusive cross sections in e^+e^- collisions at $\sqrt{s}$=2.1 GeV. The first error on model calculations is statistical and the second is the uncertainty due to poorly known resonance branching ratios. The energy density value in best agreement with data is in boldface.

$\sqrt{s}$= 2.4 GeV e^+e^- collisions

channel	σ (nb) (model)				σ (nb) (exp.)
	$\rho = 0.1$ GeV/fm^3 $f_1 = 0.5$ $\gamma_S = 0.8$	$\rho = 0.5$ GeV/fm^3 $f_1 = 0.3$ $\gamma_S = 1.4$	$\rho = 1.0$ GeV/fm^3 $f_1 = 0.3$ $\gamma_S = 1.4$	$\boldsymbol{\rho = 1.5}$ **GeV/fm^3** $f_1 = 0.3$ $\gamma_S = 1.4$	
$\pi^+\pi^-\pi^0$	$0.12 \pm 0.004 \pm 0.02$	$0.17 \pm 0.001 \pm 0.04$	$0.2 \pm 0.001 \pm 0.05$	$0.21 \pm 0.001 \pm 0.05$	0.254 ± 0.071
$\pi^+\pi^-\pi^+\pi^-$	$1.6 \pm 0.02 \pm 0.2$	$0.9 \pm 0.004 \pm 0.3$	$0.9 \pm 0.002 \pm 0.3$	$0.8 \pm 0.002 \pm 0.3$	1.3 ± 0.38
$p\bar{p}$	$0.101 \pm 0.001 \pm 0$	$0.165 \pm 0.001 \pm 0$	$0.215 \pm 0.001 \pm 0$	$0.233 \pm 0.001 \pm 0$	0.31 ± 0.12
$n\bar{n}$	$0.102 \pm 0.001 \pm 0$	$0.165 \pm 0.001 \pm 0$	$0.213 \pm 0.001 \pm 0$	$0.234 \pm 0.001 \pm 0$	0.69 ± 0.29
$K^+K^-\pi^+\pi^-\pi^+\pi^-$	$0.208 \pm 0.002 \pm 0.002$	$0.288 \pm 0.001 \pm 0.003$	$0.256 \pm 0.001 \pm 0.002$	$0.245 \pm 0.001 \pm 0.002$	0.295 ± 0.075
$\eta\pi^+\pi^-$	$0.17 \pm 0.008 \pm 0.04$	$0.28 \pm 0.004 \pm 0.065$	$0.3 \pm 0.002 \pm 0.07$	$0.301 \pm 0.002 \pm 0.07$	0.22 ± 0.11
$\omega\pi^+\pi^-$	$0.62 \pm 0.008 \pm 0.04$	$1.5 \pm 0.006 \pm 0.5$	$1.6 \pm 0.004 \pm 0.6$	$1.6 \pm 0.004 \pm 0.6$	0.24 ± 0.19

Exclusive cross sections in e^+e^- collisions at $\sqrt{s}$=2.4 GeV. The first error on model calculations is statistical and the second is the uncertainty due to poorly known resonance branching ratios. The energy density value in best agreement with data is in boldface.

resonances branching ratios

(Resonances as hadronizing clusters)

R. Hagedorn

Normalizing constant

$$\Omega^{final}(channel) = B \cdot B.R.(channel)$$

$K_2^*(1430)$					
channel	B.R. (model)				B.R (exp.)
	$\rho = 0.02$ GeV/fm^3 $\gamma_S = 1.0$	$\rho = 0.5$ GeV/fm^3 $\gamma_S = 1.0$	$\boldsymbol{\rho = 0.6}$ **GeV/fm^3** $\gamma_S = 1.0$	$\rho = 1.5$ GeV/fm^3 $\gamma_S = 1.0$	
$K\pi$	$10.5 \pm 0.2 \pm 0$	$51.5 \pm 0.5 \pm 0$	$53.2 \pm 0.6 \pm 0$	$68.3 \pm 1.6 \pm 0$	49.9 ± 1.2
$K^*(892)\pi$	$11.5 \pm 0.2 \pm 0$	$21.1 \pm 0.6 \pm 0$	$20.4 \pm 0.6 \pm 0$	$16 \pm 1.7 \pm 0$	24.7 ± 1.5
$K^*(892)\pi\pi$	$51 \pm 1 \pm 0$	$5.4 \pm 0.1 \pm 0$	$4.75 \pm 0.08 \pm 0$	$3.2 \pm 0.05 \pm 0$	13.4 ± 2.2
$K\rho$	$12.3 \pm 0.2 \pm 0$	$12.3 \pm 0.6 \pm 0$	$12.1 \pm 0.7 \pm 0$	$7.2 \pm 1.6 \pm 0$	8.7 ± 0.8
$K\omega$	$14.3 \pm 0.2 \pm 0$	$9.4 \pm 0.7 \pm 0$	$9.1 \pm 0.8 \pm 0$	$4.8 \pm 1.7 \pm 0$	2.9 ± 0.8

$\Lambda(1520)$					
channel	B.R. (model)				B.R (exp.)
	$\rho = 0.02$ GeV/fm^3 $\gamma_S = 1.0$	$\boldsymbol{\rho = 0.1}$ **GeV/fm^3** $\gamma_S = 1.0$	$\rho = 0.5$ GeV/fm^3 $\gamma_S = 1.0$	$\rho = 1.5$ GeV/fm^3 $\gamma_S = 1.0$	
$N\overline{K}$	$42.3 \pm 0.4 \pm 0$	$40.1 \pm 1.1 \pm 0$	$20.2 \pm 5.7 \pm 0$	$5.5 \pm 12.1 \pm 0$	45 ± 1
$\Sigma\pi$	$29.4 \pm 0.3 \pm 0$	$36.5 \pm 0.7 \pm 0$	$28.5 \pm 3.9 \pm 0$	$7.3 \pm 8.8 \pm 0$	42 ± 1
$\Lambda\pi\pi$	$23.7 \pm 0.4 \pm 0$	$19.3 \pm 0.1 \pm 0$	$43.4 \pm 0.2 \pm 0$	$74.9 \pm 0.3 \pm 0$	10 ± 1
$\Sigma\pi\pi$	$2.47 \pm 0.01 \pm 0$	$2.065 \pm 0.007 \pm 0$	$5.7 \pm 0.02 \pm 0$	$10.14 \pm 0.04 \pm 0$	0.9 ± 0.1

resonances branching ratios

$\pi_2(1670)$					
channel	B.R. (model)				B.R (exp.)
	$\rho = 0.02$ GeV/fm^3 $\gamma_S = 0.6$	$\rho = 0.1$ GeV/fm^3 $\gamma_S = 0.6$	$\rho = 0.5$ GeV/fm^3 $\gamma_S = 0.6$	$\boldsymbol{\rho = 1.5}$ **GeV/fm^3** $\gamma_S = 0.6$	
$f_2(1270)\pi$	$50 \pm 0.3 \pm 0$	$46.3 \pm 0.2 \pm 0$	$46.9 \pm 0.1 \pm 0$	$51 \pm 0.1 \pm 0$	56.2 ± 3.2
$\rho\pi$	$35.6 \pm 0.4 \pm 0$	$39.7 \pm 0.3 \pm 0$	$40.1 \pm 0.2 \pm 0$	$37.2 \pm 0.2 \pm 0$	31 ± 4
$K\overline{K}^*(892)$ + c.c	$5.73 \pm 0.05 \pm 0$	$5.37 \pm 0.03 \pm 0$	$4.31 \pm 0.03 \pm 0$	$3.18 \pm 0.02 \pm 0$	4.2 ± 1.4
$\omega\rho$ (EXCLUD.)	$76.3 \pm 0.7 \pm 0$	$62.9 \pm 0.5 \pm 0$	$43.8 \pm 0.3 \pm 0$	$29.7 \pm 0.2 \pm 0$	2.7 ± 1.1

$\rho_3(1690)$					
channel	B.R. (model)				B.R (exp.)
	$\rho = 0.02$ GeV/fm^3 $\gamma_S = 1.8$	$\boldsymbol{\rho = 0.3}$ **GeV/fm^3** $\gamma_S = 0.6$	$\rho = 0.5$ GeV/fm^3 $\gamma_S = 0.6$	$\rho = 1.5$ GeV/fm^3 $\gamma_S = 0.4$	
$\pi\pi\pi\pi$	$59.7 \pm 2.4 \pm 0.02$	$70.9 \pm 1.2 \pm 0.1$	$60.5 \pm 1.1 \pm 0.1$	$46 \pm 2.1 \pm 0.02$	71.1 ± 1.9
$\omega\pi$	$0.61 \pm 0.02 \pm 0$	$18.7 \pm 0.4 \pm 0$	$21 \pm 0.5 \pm 0$	$20.6 \pm 1.5 \pm 0$	16 ± 6
$K\overline{K}\pi$	$48.8 \pm 12.2 \pm 0.0002$	$2.4 \pm 0.1 \pm 0.02$	$2 \pm 0.1 \pm 0.0005$	$0.9 \pm 0.2 \pm 0.1$	3.8 ± 1.2
$K\overline{K}$	$3.18 \pm 0.06 \pm 0$	$1.25 \pm 0.02 \pm 0$	$1.5 \pm 0.03 \pm 0$	$0.33 \pm 0.02 \pm 0$	1.58 ± 0.26
$\pi\pi$	$3.74 \pm 0.08 \pm 0$	$22.8 \pm 0.3 \pm 0$	$31 \pm 0.5 \pm 0$	$48.2 \pm 1.4 \pm 0$	23.6 ± 1.3

resonances branching ratios

$K_3^*(1780)$					
channel	B.R. (model)				B.R (exp.)
	$\boldsymbol{\rho = 0.02}$ **GeV/fm^3** $\gamma_S = 1.2$	$\rho = 0.1$ GeV/fm^3 $\gamma_S = 1.4$	$\rho = 0.5$ GeV/fm^3 $\gamma_S = 1.6$	$\rho = 1.5$ GeV/fm^3 $\gamma_S = 1.8$	
$K\rho$	$25.4 \pm 0.6 \pm 0$	$19.4 \pm 0.4 \pm 0$	$11.9 \pm 0.4 \pm 0$	$6.4 \pm 0.7 \pm 0$	31 ± 9
$K^*(892)\pi$	$21.5 \pm 0.6 \pm 0$	$19.3 \pm 0.4 \pm 0$	$14 \pm 0.4 \pm 0$	$9.1 \pm 0.7 \pm 0$	20 ± 5
$K\pi$	$19.9 \pm 0.5 \pm 0$	$18.8 \pm 0.3 \pm 0$	$19.8 \pm 0.3 \pm 0$	$20.5 \pm 0.6 \pm 0$	18.8 ± 1
$K\eta$	$32.9 \pm 0.7 \pm 0$	$42.3 \pm 0.6 \pm 0$	$54.1 \pm 0.9 \pm 0$	$63.7 \pm 2.5 \pm 0$	30 ± 13

$K_4^*(2045)$					
channel	B.R. (model)				B.R (exp.)
	$\rho = 0.02$ GeV/fm^3 $\gamma_S = 1.2$	$\rho = 0.1$ GeV/fm^3 $\gamma_S = 1.0$	$\rho = 0.5$ GeV/fm^3 $\gamma_S = 1.0$	$\boldsymbol{\rho = 1.5}$ **GeV/fm^3** $\gamma_S = 0.8$	
$K\pi$	$0.055 \pm 0.002 \pm 0$	$0.429 \pm 0.006 \pm 0$	$1.99 \pm 0.03 \pm 0$	$3.3 \pm 0.1 \pm 0$	9.9 ± 1.2
$K^*(892)\pi\pi$	$4.7 \pm 0.1 \pm 0.03$	$8.3 \pm 0.1 \pm 0.1$	$8.3 \pm 0.1 \pm 0.2$	$7.6 \pm 0.3 \pm 0.2$	9 ± 5
$K^*(892)\pi\pi\pi$	$20.5 \pm 1.1 \pm 0.1$	$9.6 \pm 0.2 \pm 0.1$	$7 \pm 0.1 \pm 0.07$	$8.4 \pm 0.4 \pm 0.06$	7 ± 5
$K\rho\pi$	$7.3 \pm 1.4 \pm 0.02$	$12 \pm 0.5 \pm 0.06$	$10.9 \pm 0.4 \pm 0.08$	$9.6 \pm 0.5 \pm 0.09$	5.7 ± 3.2
$K\omega\pi$	$3.5 \pm 0.2 \pm 0.0002$	$6.6 \pm 0.2 \pm 0.001$	$7.7 \pm 0.1 \pm 0.001$	$9.6 \pm 0.4 \pm 0.0002$	5 ± 3
$K\phi\pi$	$4.1 \pm 0.4 \pm 0$	$2.67 \pm 0.09 \pm 0$	$2.6 \pm 0.09 \pm 0$	$1.2 \pm 0.1 \pm 0$	2.8 ± 1.4
$K^*(892)\phi$	$0.635 \pm 0.008 \pm 0$	$1.18 \pm 0.01 \pm 0$	$2.25 \pm 0.05 \pm 0$	$1.1 \pm 0.1 \pm 0$	1.4 ± 0.7

Conclusions & Outlook

- *We calculated the most general microcanonical ensemble of a relativistic hadron gas in a quantum field theory framework with full conservation of P,J,I,I_3,C,Π.*
- *We made a preliminary comparison with e^+e^- data at low energy and with branching ratios of some heavy resonance.*
- *Fair agreement with data... but the whole pattern has to be understood.*
- *Assess the effect of using the SU(3) flavour symmetry group and resonance interference terms.*
- *Definition of an equivalent probability assuming the "Phase space dominance" and comparison with SHM.*

J. Hormuzdiar, S. D. H. Hsu and G. Mahlon, Int. J. Mod. Phys. E 12, 649 (2003)

- *Further tests on other kinds of reactions.*

Motivations

We need deeper tests to understand the statistical behaviour

Tests on exclusive channels -better sensitivity-

Warning!

Data available only for low energy collisions

microcanonical ensemble

Microcanonical partition function:

$$\Omega = \sum_{states} \delta^4(P - P_{state})\,\delta_{Q-Q_{state}} = \sum_{h_V} \langle h_V | \delta^4(P - \hat{P})\,\delta_{Q-\hat{Q}} | h_V \rangle$$

Generalization:

projector onto initial state

$$\Omega = \sum_{h_V} \langle h_V | P_i | h_V \rangle = Tr(P_i P_V)$$

Probability to observe an asimptotic state f:

$$\Omega_f \propto \langle f | P_i P_V | f \rangle$$

$$\Omega = \sum_f \Omega_f$$

$$\langle O \rangle = \frac{\sum_f \Omega_f O(f)}{\Omega}$$

microcanonical ensemble(1)

(Non-interacting relativistic hadron gas)

$$\Omega_f \propto \langle f | \underline{P_i} P_V | f \rangle$$

Small systems

$$P_i = P_{PJ\lambda\pi} P_C P_{I,I_3} P_{\boldsymbol{Q}}$$

Extended Poincarè group IO(1,3)↑, Isospin SU(2), U(1) C-parity, Abelian charges

Cluster's rest frame

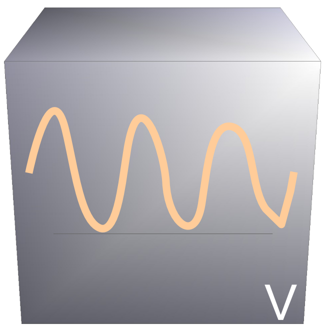
SU(2) measure

$$P_{PJ\lambda\pi} = \delta^4(P - \hat{\boldsymbol{P}})(2\mathrm{J}+1)\int d R\, D^J_{\lambda\lambda}(R^{-1})\,\hat{\boldsymbol{R}}\,\frac{\boldsymbol{I}+\pi\hat{\boldsymbol{\Pi}}}{2}$$

microcanonical ensemble(1)

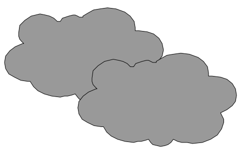
(Non-interacting relativistic hadron gas)

$$\Omega_f \propto \langle f | P_i \underline{P_V} | f \rangle$$



Non-relativistic quantum mechanics

$$P_V = \sum |\{N_j\}, k\rangle_V \langle \{N_j\}, k|_V$$



$$|1p\rangle_V = \alpha_1 |1p\rangle + \alpha_2 |2p\rangle + \alpha_3 |3p\rangle + \ldots$$

$$P_V = \int_V D\Psi |\Psi\rangle\langle\Psi|$$

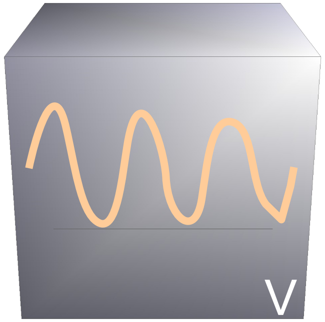
microcanonical field theory

microcanonical ensemble(1)

(Non-interacting relativistic hadron gas)

$$\Omega_f \propto \langle f | P_i \underline{P_V} | f \rangle$$

Non-relativistic quantum mechanics



$$P_V = \sum |\{N_j\}, k\rangle_V \langle \{N_j\}, k|_V$$

$$|1p\rangle_V = \alpha_1 |1p\rangle + \alpha_2 |2p\rangle + \alpha_3 |3p\rangle + \ldots$$

$$P_V = \int_V D\Psi |\Psi\rangle\langle\Psi|$$

microcanonical field theory

Insieme microcanonico (2)

(Gas adronico relativistico interagente)

Teorema di Dashen Ma Bernstein: $V \to \infty$ *Non Relativistico*

$$Tr\ \delta(E-\hat{H}) = Tr\ \delta(E-\hat{H}_0) + \frac{1}{4\pi i} Tr\left[S^{-1} \overleftrightarrow{\partial}_E S\right]$$

$V \to \infty$

Generalizzazione relativistica

$$Tr\ \delta^4(P-\hat{P}) = Tr\ \delta^4(P-\hat{P}_0) + \frac{1}{4\pi i} Tr\left[S^{-1} \overleftrightarrow{\partial}_E S\, \delta^3(\vec{P})\right]$$

$V \to \infty$

Insieme microcanonico completo

Risonanze

$$Tr\ P_i = Tr\ P_i^0 + \frac{1}{4\pi i} Tr\left[S^{-1} \overleftrightarrow{\partial}_E S\, P_i^0\right]$$

microcanonical ensemble(1)

(Non-interacting relativistic hadron gas)

$$\Omega_f \propto \langle f | P_i \underline{P_V} | f \rangle$$

$$P_V = \int_V D\Psi |\Psi\rangle\langle\Psi|$$

$$\Psi(\boldsymbol{x}) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int \frac{d^3 p}{\sqrt{2\epsilon}} D^S([p])\, a(p)\, e^{i\boldsymbol{p}\cdot\boldsymbol{x}} + D^S([p]C^{-1})\, b^\dagger(p)\, e^{-i\boldsymbol{p}\cdot\boldsymbol{x}}$$

$$\tilde{\Psi}(\boldsymbol{x}) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int \frac{d^3 p}{\sqrt{2\epsilon}} D^S([p]^{\dagger-1})\, a(p)\, e^{i\boldsymbol{p}\cdot\boldsymbol{x}} + D^S([p]^{\dagger-1} C)\, b^\dagger(p)\, e^{-i\boldsymbol{p}\cdot\boldsymbol{x}}$$

P. Moussa, R. Stora, *Angular Analysis of Elementary Particle Reactions*, in Proceedings of the 1966 International School on Elementary Particles, Hercegnovi (Gordon and Breach, New York, London, 1968).

microcanonical ensemble(1)

(Non-interacting relativistic hadron gas)

$$\Omega_f = \sum_{f'} \langle f' | P_V | f \rangle \langle f | P_i | f' \rangle$$

$$\langle f' | P_V | f \rangle = \prod_j^k \sum_{\eta_j} \chi(\eta_j)^{b_j} \prod_{n_j=1}^{N_j} F_V(\boldsymbol{p}_{\eta_j(n_j)}, \boldsymbol{p}_{n_j}') \sum_{\tau_{n_j}} D^{S_j}_{\lambda_{n_j}', \tau_{n_j}}([p_{n_j}']^{-1}) \, D^{S_j}_{\tau_{n_j}, \lambda_{\eta_j(n_j)}}([p_{\eta_j(n_j)}])$$

where $F_V(\boldsymbol{p}, \boldsymbol{p}') = \int_V \frac{d^3 x}{(2\pi)^3} e^{i(\boldsymbol{p}-\boldsymbol{p}')\cdot \boldsymbol{x}} = \frac{R^2}{2\pi^2} \frac{j_1(|\boldsymbol{p}-\boldsymbol{p}'|R)}{|\boldsymbol{p}-\boldsymbol{p}'|}$

(*For a sharp sphere*)

Provided contributions from outside the system can be subtracted away... *same result of non-relativistic quantum mechanics!*

microcanonical ensemble(1)

(Non-interacting relativistic hadron gas)

$$\Omega_f = \sum_{f'} \langle f' | P_V | f \rangle \langle f | P_i | f' \rangle$$

Generalization of Cerulus's formulation

F. Cerulus, N. Cim. 22 (1961) 958

$$P_{PJ\lambda} = \delta^4(P - \hat{P})(2J+1) \int dR\, D^J_{\lambda\lambda}(R^{-1}) \hat{\boldsymbol{R}}$$

$$\langle f' | P_{PJ\lambda} | f \rangle = \delta^4(P - \sum_n^N p_n)(2J+1) \int dR\, D^J_{\lambda\lambda}(R^{-1}) \prod_j^k \sum_{\rho_j} \chi(\rho_j)^{b_j} \times \prod_{n_j=1}^{N_j} D^{S_j}_{\lambda_{\rho_j(n_j)}, \lambda_{n_j}'}(\underline{[p_{\rho_j(n_j)}]^{-1} R [R p_{\rho_j(n_j)}]}) \delta^3(\boldsymbol{p}_{n_j}' - R\, \boldsymbol{p}_{\rho_j(n_j)})$$

Wigner rotations

Valid for any spin!

microcanonical ensemble(1)

(Non-interacting relativistic hadron gas)

Microcanonical channel weight (proportional to the probability)

$$\Omega_{\{N_j\}} = [\prod_{n=1}^{N} \sum_{\lambda_n} \int d^3 p_n] \Omega_f$$

$$\Omega_{\{N\}} = (2J+1) \int dR\, D^J_{\lambda\lambda}(R^{-1}) \left[\prod_{n=1}^{N} \int d^3 p_n \right] \delta^4 \left(P - \sum_{n=1}^{N} p_n \right)$$

$$\times \left[\prod_{n=1}^{N} tr[D^{S_n}(R)] F_V(\boldsymbol{p_n} - R^{-1} \boldsymbol{p_n}) \right] I^{\{N\}}(I, I_3)$$

Distinct particles
full conservation of Four momentum, angular momentum,Isospin
(Field theory framework)

Monte-Carlo integration

Importance Sampling Method:

$$I = \int_a^b f(x)\,dx \approx \frac{b-a}{M} \sum_{i=1}^{M} \frac{f(x_i)}{g(x_i)}$$

auxiliary distribution

We use a flat distribution for angles and rotation parameters, while for modulus of particles momenta we use the β function:

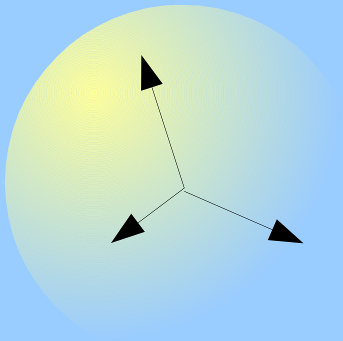
$$\beta\left(\frac{t}{t_{max}}\right) \propto \left(\frac{t}{t_{max}}\right)^{c_1 - 1} \left(1 - \frac{t}{t_{max}}\right)^{\frac{t_{max}}{T}}$$

$$c_1 - 1 = \frac{1}{2} + \frac{3}{2} e^{-2\mathrm{m}_i}$$

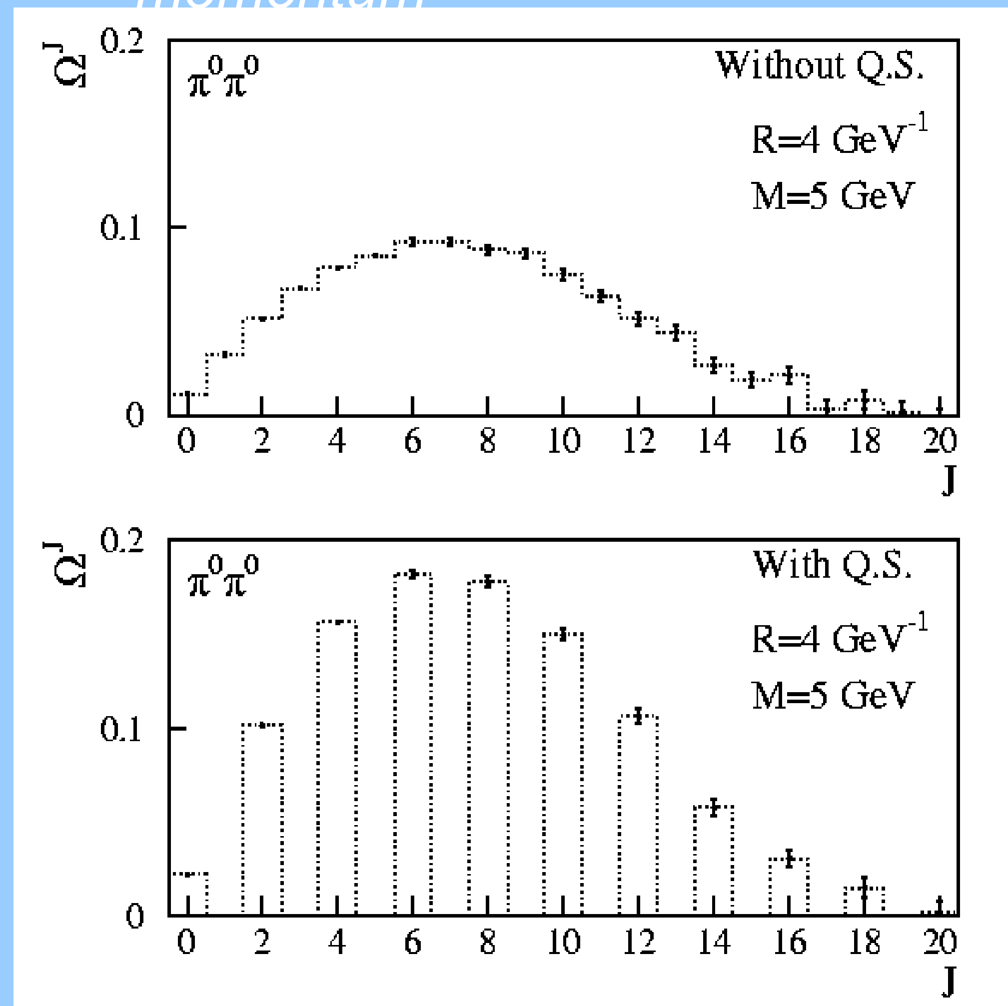
- Similar to kinetic energy distribution in the G.C.E.
- Easy to sample.

Effects of angular momentum conservation

(Non-interacting relativistic hadron gas)



Small V & low p= suppression of high angular momentum



Microcanonical weight of the channel $\pi^0\pi^0$ as a function of the total spin of the system with Boltzmann statistics (upper panel) and quantum statistics (lower panel).

Phase space dominance

$$\Omega_{\{N_j\}} = \frac{V^N}{(2\pi)^{3N}} \prod_j \frac{(2J_j+1)^{N_j}}{N_j!} \int d^3p_1 \ldots d^3p_N \, \delta^4\left(P - \sum_{i=1}^{N} p_i\right)$$

$$\Gamma_f \propto \frac{1}{(2\pi)^{3N}} \prod_j \frac{(2J_j+1)^{N_j}}{N_j!} \int \frac{d^3p_1}{2\varepsilon_1} \ldots \frac{d^3p_N}{2\varepsilon_N} \, |M_{if}|^2 \, \delta^4\left(P - \sum_{i=1}^{N} p_i\right)$$

VS

Discussed in detail in J. Hormuzdiar et al., Int. J. Mod. Phys. E (2003) 649, nucl-th 0001044 and F.B. hep-ph 0410403, J. Phys. Conf. Ser. 5 (2005) 175

If $|M_{if}|^2$ has a very weak dependence on kinematical independent variables, e.g. $p_i \cdot p_j$,we could somehow recover a pseudo-thermal shape of the multiplicity and p_T spectrum function

IF $|M_{if}|^2 = \alpha^N$ for large mults $\rightarrow$ $\langle n_j \rangle \simeq \frac{(2J_j+1)\alpha}{(2\pi)^3} \int \frac{d^3p}{2\varepsilon} \exp(-\beta\sqrt{p^2+m_j^2})$

Where β is such that:

$$M + \frac{\partial}{\partial\beta} \sum_j \frac{(2J_j+1)\alpha}{(2\pi)^3} \int \frac{d^3p}{2\varepsilon} \exp(-\beta\sqrt{p^2+m_j^2}) = 0$$

Conclusion: β is not a temperature and inclusive particle multiplicities are not sensitive enough to the different integration measure to distinguish between a genuine thermal behaviour and this pseudo-thermal function (phase space dominance)

Yet, there is a quantitative difference!

$$\langle n_j \rangle = \frac{(2J_j+1)V}{(2\pi)^3} \int d^3p \exp(-\beta\sqrt{p^2+m_j^2}) \qquad \langle n_j \rangle \simeq \frac{(2J_j+1)\alpha}{(2\pi)^3} \int \frac{d^3p}{2\varepsilon} \exp(-\beta\sqrt{p^2+m_j^2})$$

Average multiplicities (Boltzmann statistics limit)

- Grand-canonical ensemble

$$\langle n_j \rangle = \frac{V(2J_j+1)}{(2\pi)^3} \exp(\mu \cdot q_j/T) \int d^3p \, \exp\left(-\sqrt{p^2+m_j^2}/T\right)$$

$$Z(Q) \simeq Z_G \exp(-\mu \cdot Q/T)$$

- Canonical ensemble

$$\langle n_j \rangle = \frac{V(2J_j+1)}{(2\pi)^3} \int d^3p \, \exp\left(-\sqrt{p^2+m_j^2}/T\right) \frac{Z(Q-q_j)}{Z(Q)}$$

- Microcanonical ensemble

$$\Omega \simeq Z(Q) \exp(\beta \cdot P)$$

$$\langle n_j \rangle = \frac{V(2J_j+1)}{(2\pi)^3} \int d^3p \, \frac{\Omega(P-p_j, Q-q_j)}{\Omega(P,Q)}$$