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Simulations of QGP instabilities in A+A collisions

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HELMHOLTZ GEMEINSCHAFT

The second international workshop of the
Virtual Institute on Strongly Interacting Matter

Heavy Ion Physics Perspectives

Bad Liebenzell, 13. September 2007

Outline

How to numerically solve equations of motion for particles coupled to fields.
Two approaches:

- **Simulations in the hard loop (HL) limit**
  - Instability growth in anisotropic systems
  - Non-Abelian effects
  - Cascade to the UV
  - Numerical issues

- **Wong-Yang-Mills simulation**
  - Instability growth in anisotropic systems
  - Avalanche
  - Isotropization
  - Inclusion of hard collisions

What all simulations do…

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Solve the Vlasov equation for hard particles coupled to soft fields

$$V^\mu[D_\mu(X), f(P,X)] + gV^\mu F_{\mu\nu}\partial^\nu_{(p)} f(P,X) = \mathcal{C}$$

Drift term

Vlasov term
('Lorentz' force)

Collisions

$f(P,X)$ is the phase space distribution of hard particles

$F_{\mu\nu}$ is the field tensor and $D_\mu = \partial_\mu + ig[A_\mu, \cdot]$

The Vlasov equation describes the evolution of a system of particles under the effects of self-consistent (color-) electro-magnetic fields.

All quite similar to electrodynamics, but we have color degrees of freedom:

$f = \sum_a f^a t^a$, with the generators of the group t^a

$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu]$ → Non-Abelian interactions

Approach I: HL approximation

In the hard loop (HL) approximation, the back reaction of the fields on the particles is neglected. In practice it can be obtained by linearizing the Vlasov equation

$$f(P,X) \rightarrow \underset{\substack{\uparrow\\ \text{background}}}{f(\mathbf{p})} + \underset{\substack{\uparrow\\ \text{fluctuations}}}{\delta f(P,X)}$$

No collisions ↓

$$V^{\mu}[D_{\mu}(X), \color{red}{\delta f(P,X)}\color{black}] + gV^{\mu}\color{green}{F_{\mu\nu}}\color{black}\partial^{\nu}_{(p)} f(\mathbf{p}) = 0$$

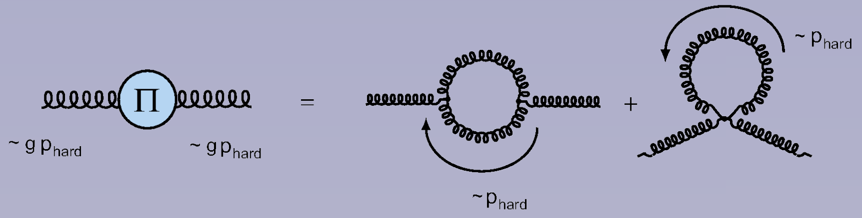
The Vlasov equation is coupled to the Yang-Mills equation:

$$D_{\mu}\color{green}{F^{\mu\nu}}\color{black} = J^{\nu} = g\int_{\mathbf{p}} V^{\nu}\color{red}{\delta f(X,P)}$$

(particle currents generate fields)

Why the name ‘hard loop‘ approximation?

Calculating the gluon polarization tensor from the current above leads to the same result as in the diagrammatic calculation using a hard loop resummation:

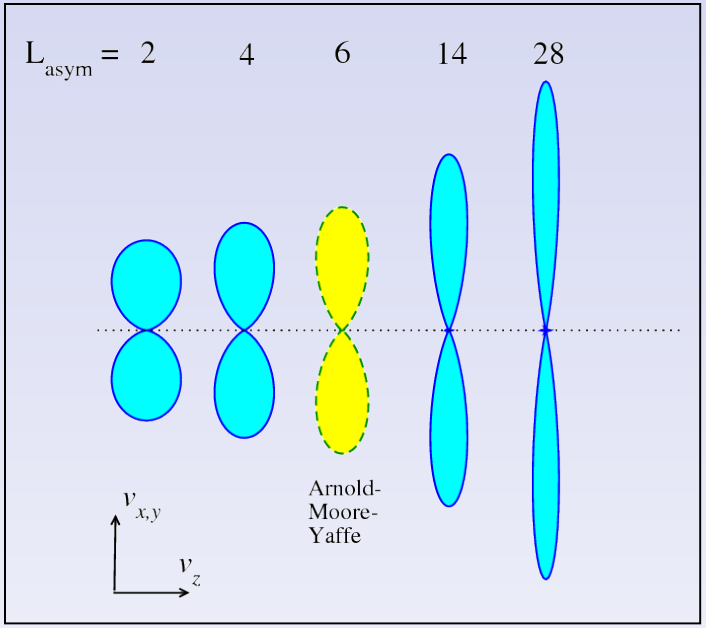


with $p_{\text{hard}} \gg g p_{\text{hard}}$

HL-Calculation: Discretization

Create an anisotropic distribution by expanding $f(|\mathbf{p}|)$ in spherical harmonics:

$$f(\mathbf{p}) = \sum_{l=0}^{L_{\text{asym}}} f_l(|\mathbf{p}|) Y_{l,0}(\mathbf{v})$$



D. Bödeker, K. Rummukainen, e-Print: arXiv:0705.0180

also expand the fluctuations on f in spherical harmonics.

Arnold, Moore, Yaffe, Bödeker, Rummukainen

OR

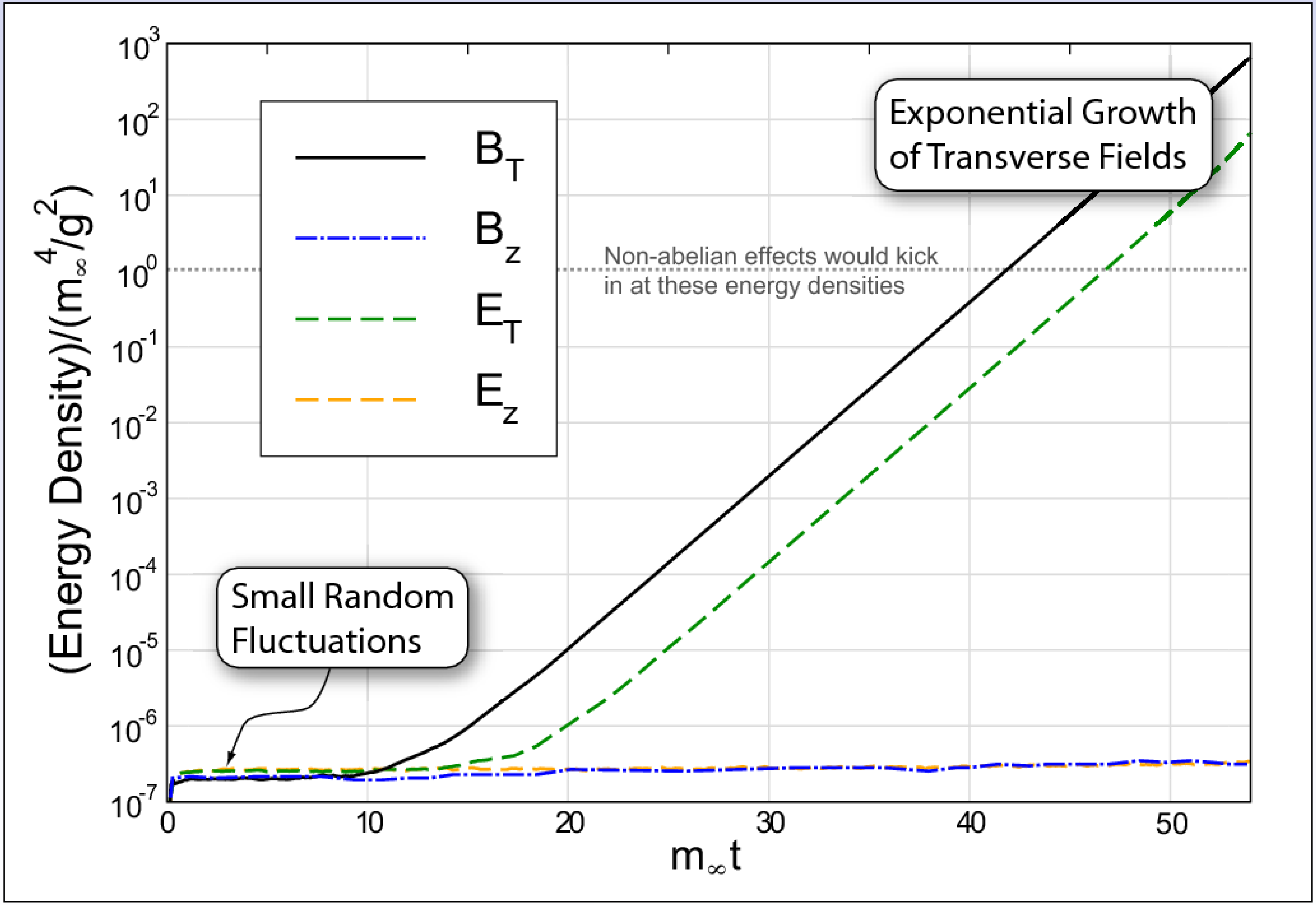
discretize the velocity directions $\mathbf{v}$

The anisotropy can then be parametrized e.g. like

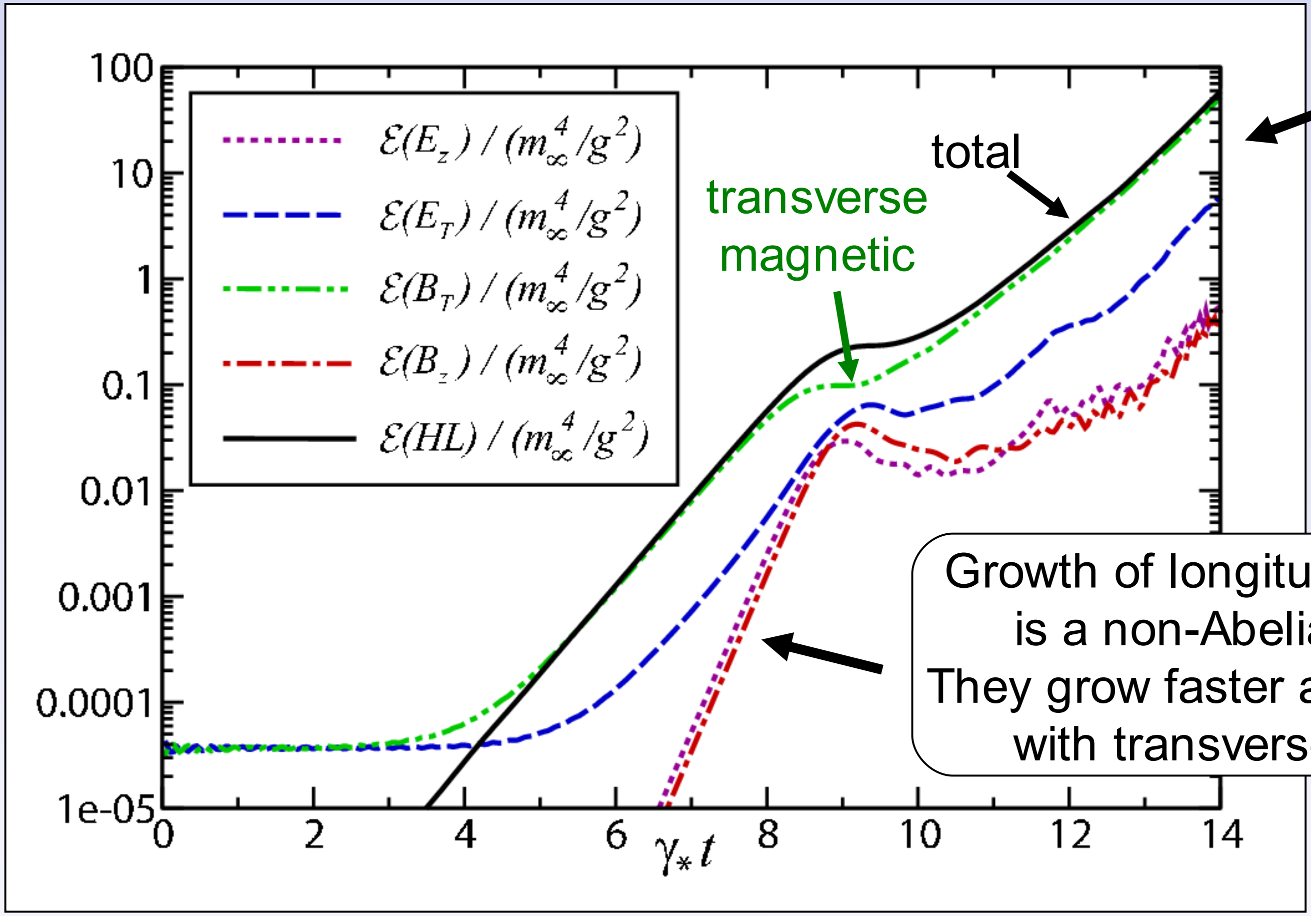
$$f(\mathbf{p}) = f_{\text{iso}}(\mathbf{p}^2 + \xi p_z^2)$$

Rebhan, Romatschke, Strickland

Anisotropic **U(1)** plasma – Weibel instability (1959)



1+1d SU(2) HL-simulation



Continued exponential growth in 1+1d

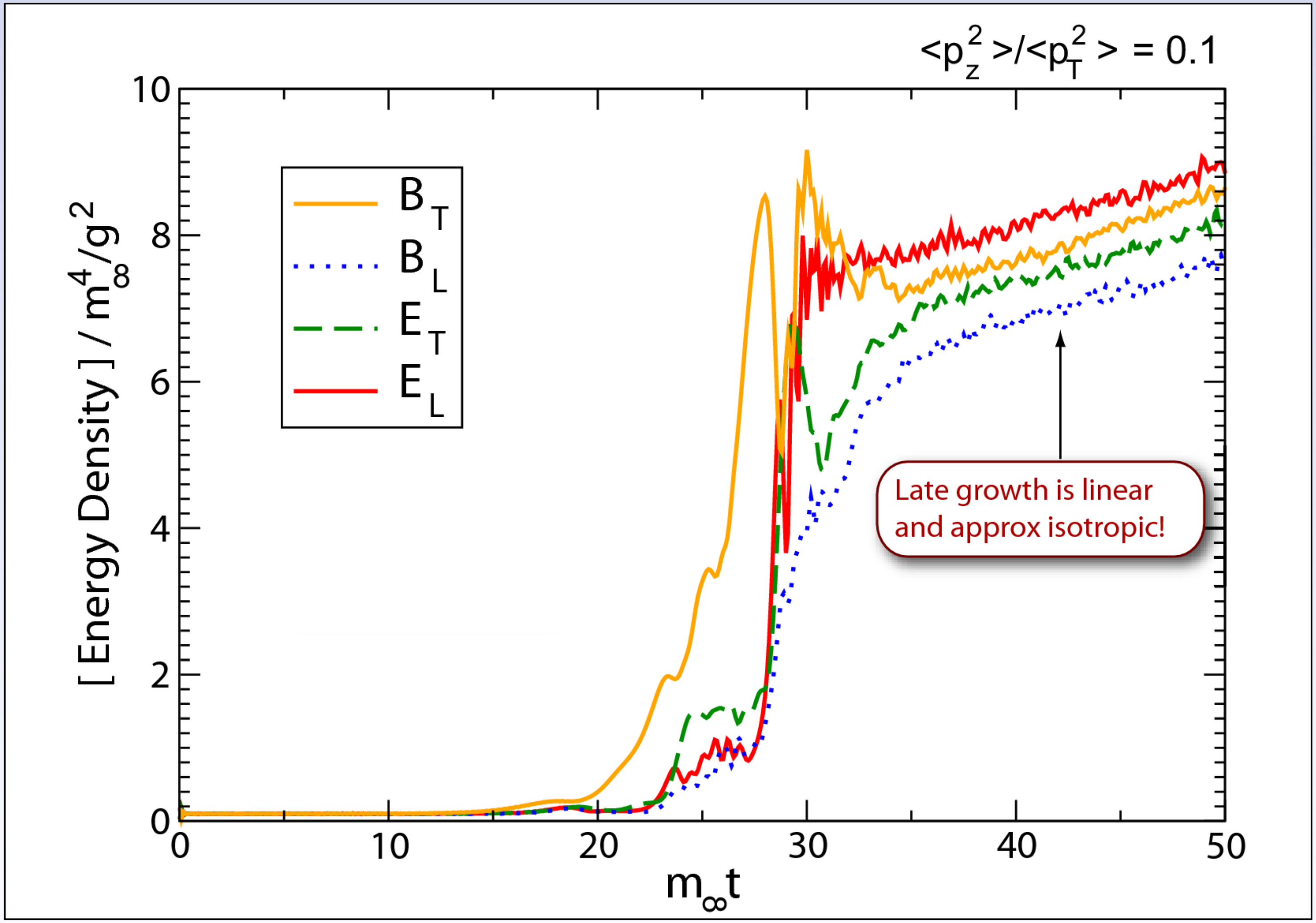
Growth of longitudinal modes is a non-Abelian effect. They grow faster and catch up with transverse modes.

takes into account only the z-direction (in space): $A_a^\mu = A_a^\mu(t,z)$

γ_* is the growth rate of the maximally unstable mode

A. Rebhan, P. Romatschke, M. Strickland, *Phys.Rev.Lett.* **94**:102303,2005.

3+1 d SU(2) HL-simulation

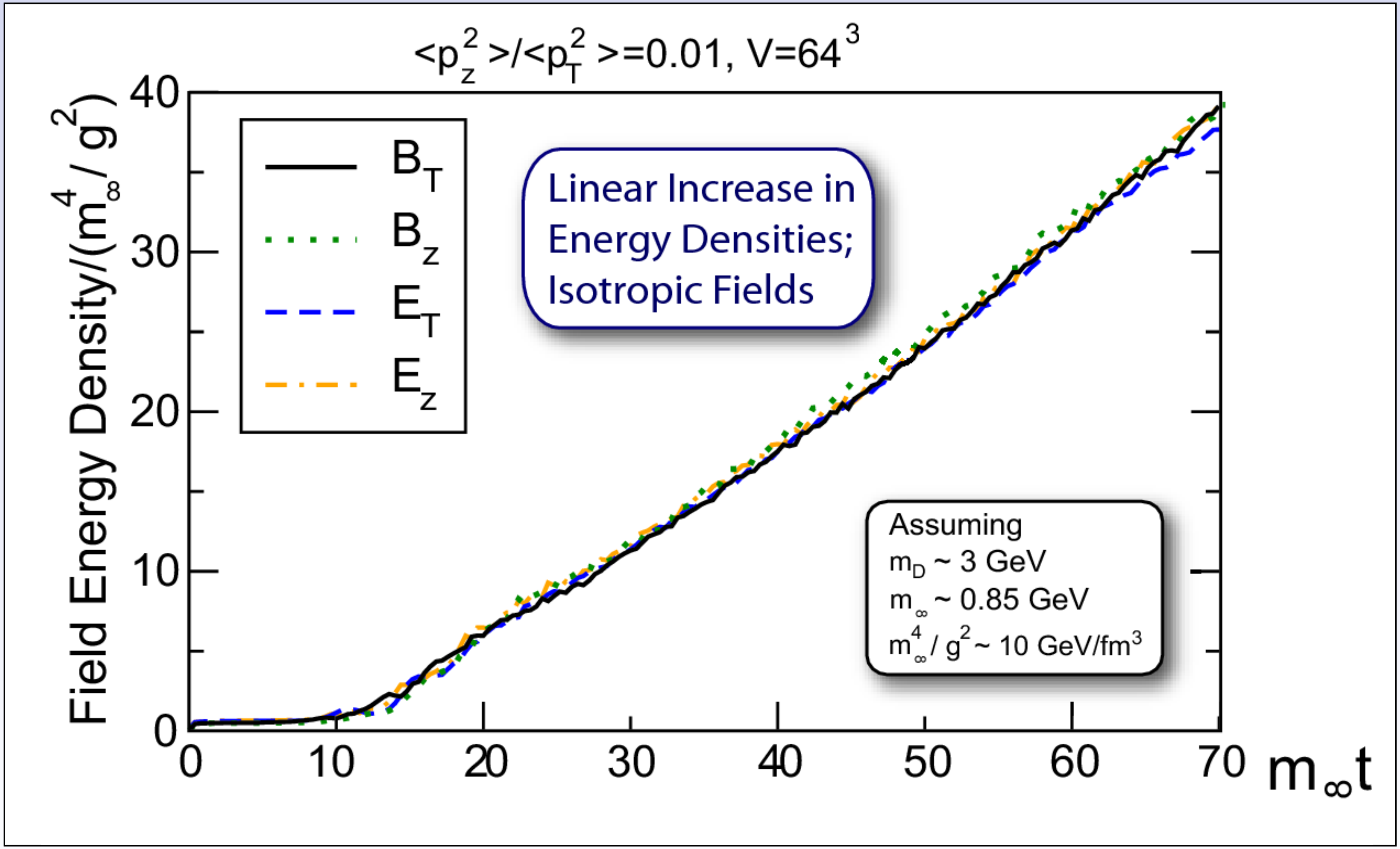


Late growth is linear and approx isotropic!

A. Rebhan, P. Romatschke, M. Strickland, *Phys.Rev.Lett.***94**:102303,2005.
P. Arnold, G. Moore, L. Yaffe, *Phys.Rev.***D72**:054003,2005

3+1 d SU(2) HL-simulation

Stronger initial fields



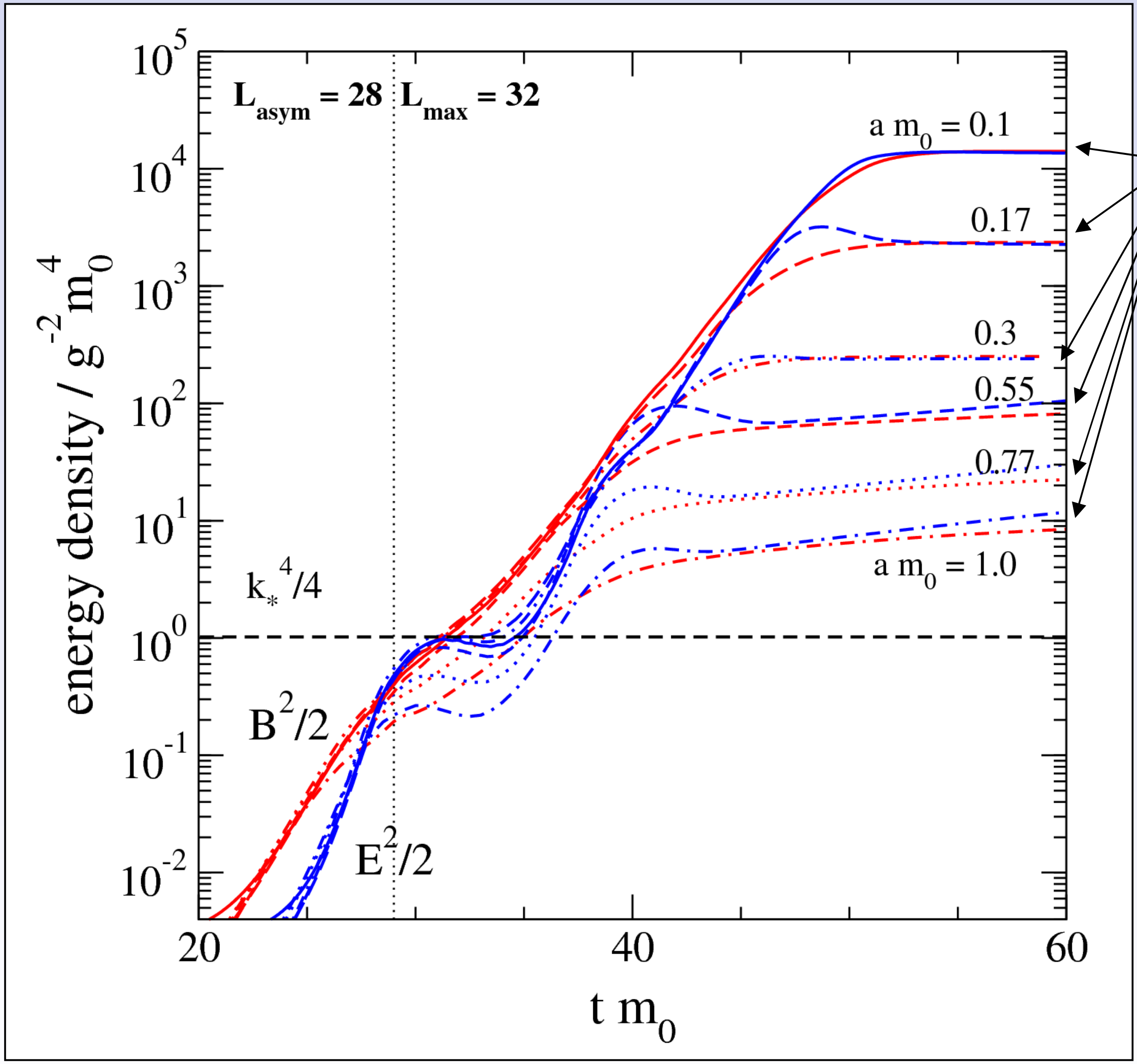
Linear Increase in Energy Densities; Isotropic Fields

Assuming
$m_D \sim 3$ GeV
$m_\infty \sim 0.85$ GeV
$m_\infty^4 / g^2 \sim 10$ GeV/fm^3

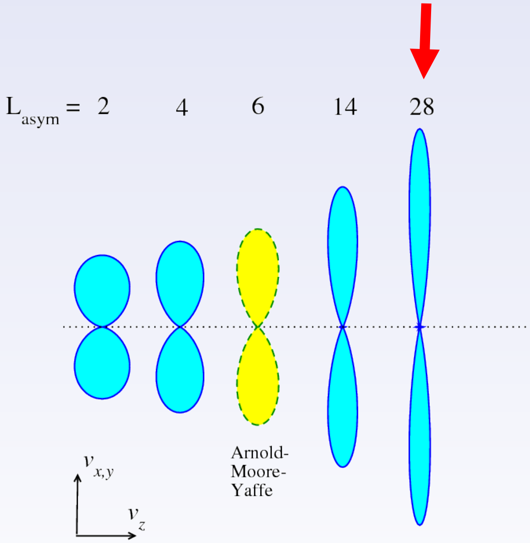
A. Rebhan, P. Romatschke, M. Strickland, *Phys.Rev.Lett.* **94**:102303,2005.

Strong anisotropies

Very weak initial fields

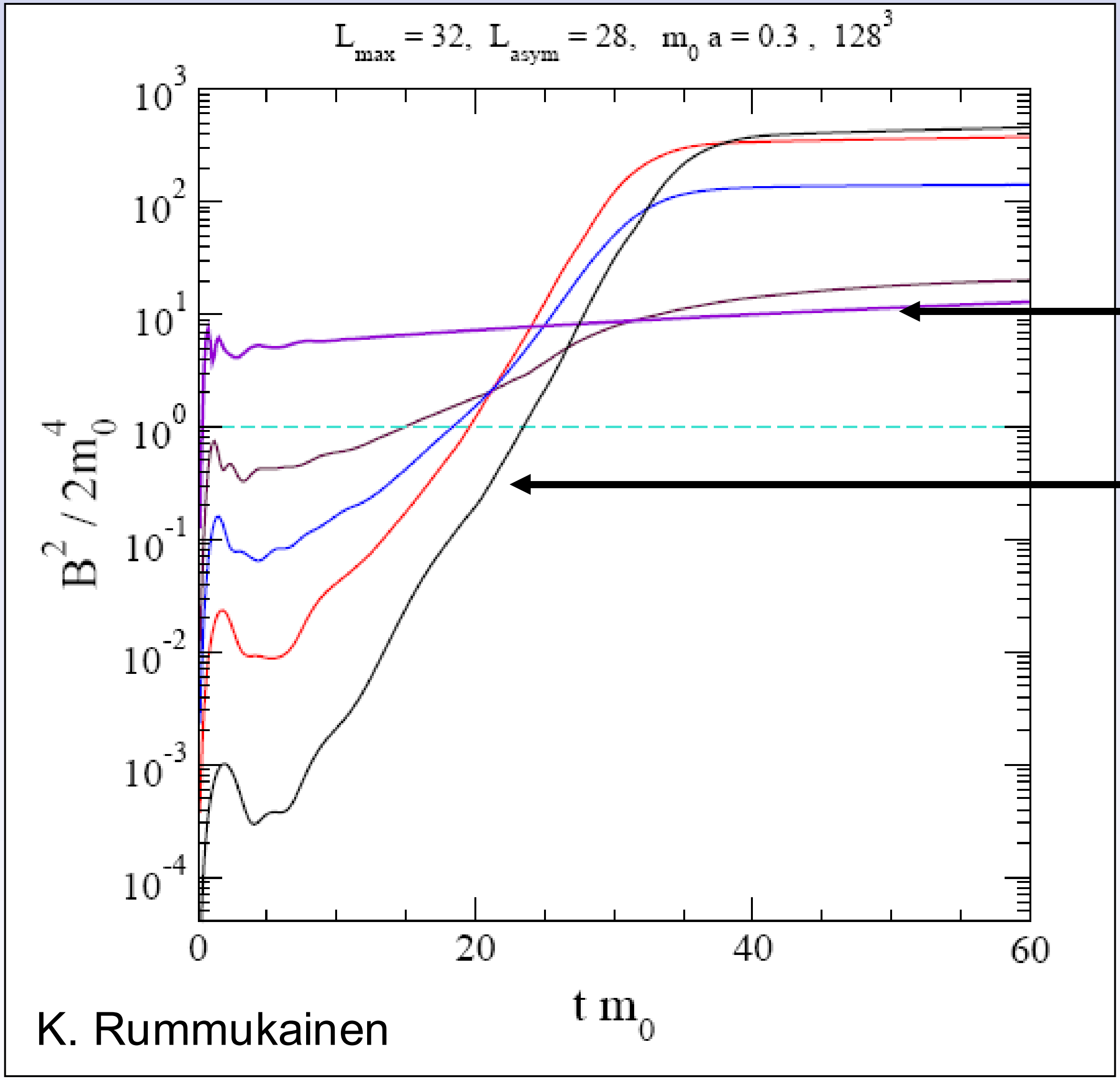


Saturation is a
finite lattice size
artifact –
regime of linear growth
is not reached
in these simulations



D. Bödeker, K. Rummukainen,
JHEP 0707:022, 2007

Initial field configuration matters!



K. Rummukainen

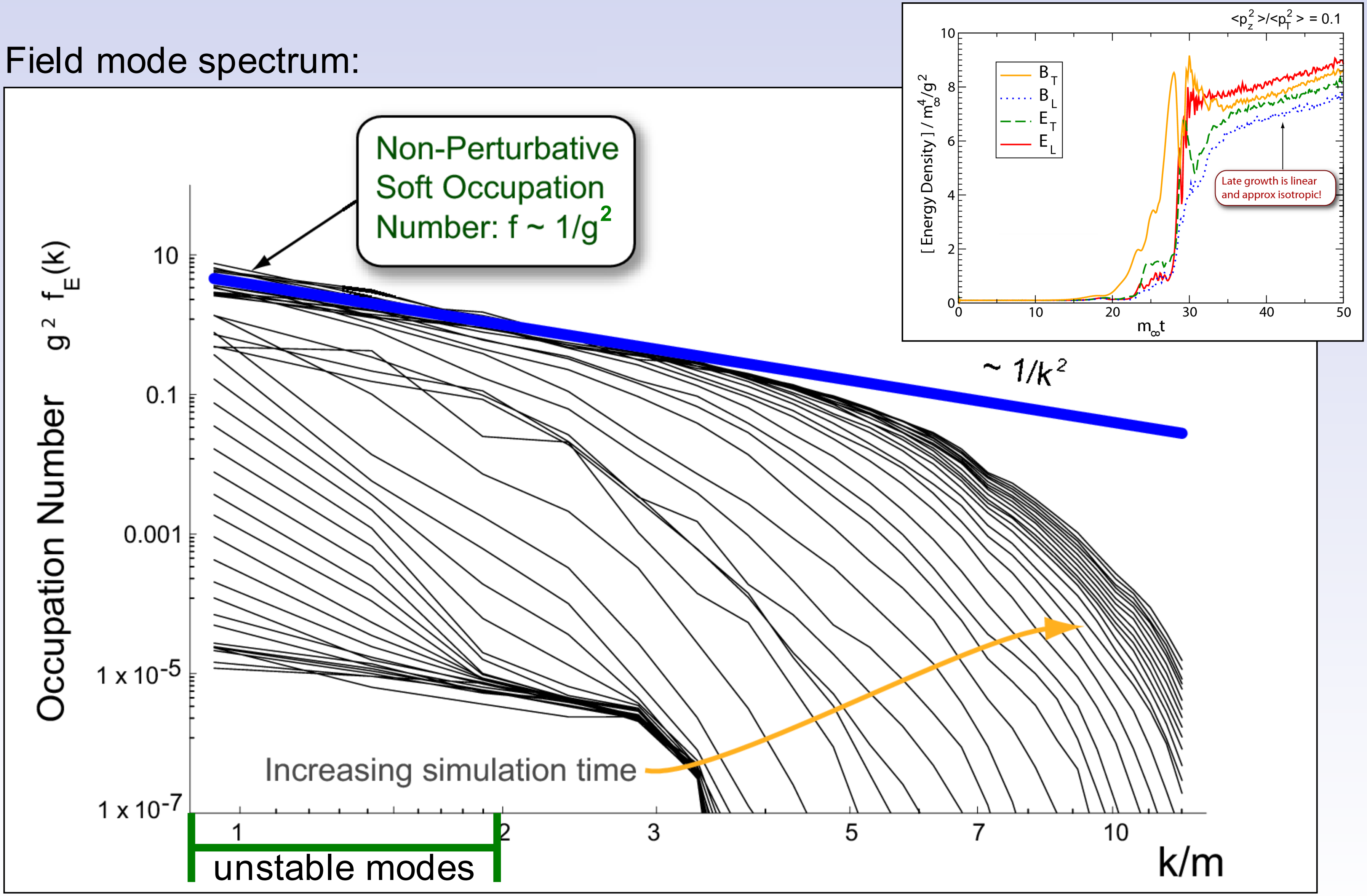
Growth depends on the initial field strength:

Strong initial fields:
linear growth

Weak initial fields:
exponential growth
and unphysical saturation

Kolmogorov cascade – turbulent fields

Field mode spectrum:



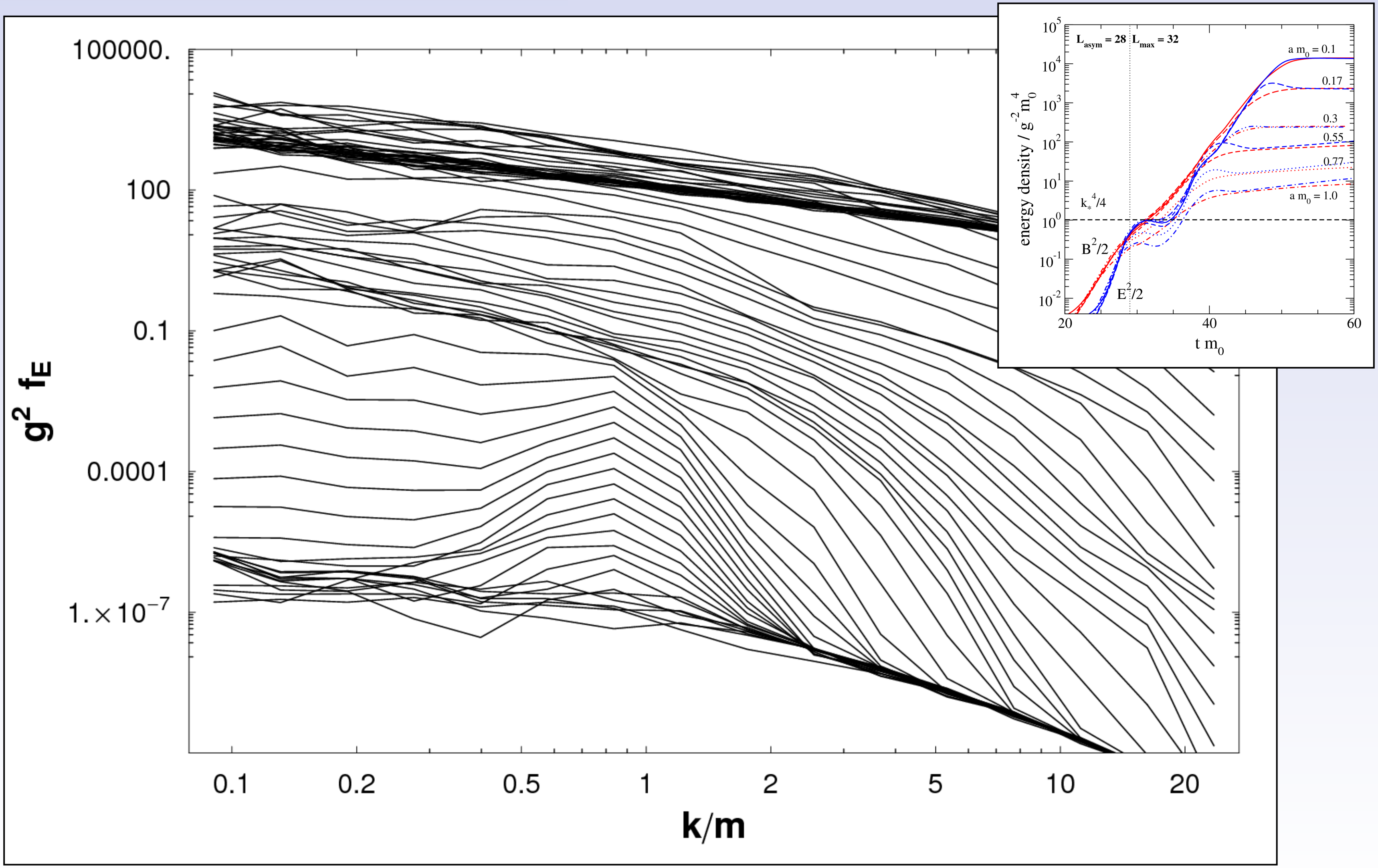
Non-Perturbative Soft Occupation Number: $f \sim 1/g^2$

Increasing simulation time

Late growth is linear and approx isotropic!

Lattice artifacts

When the finite lattice size causes saturation, the spectrum has a different shape



Lattice artifacts - continued

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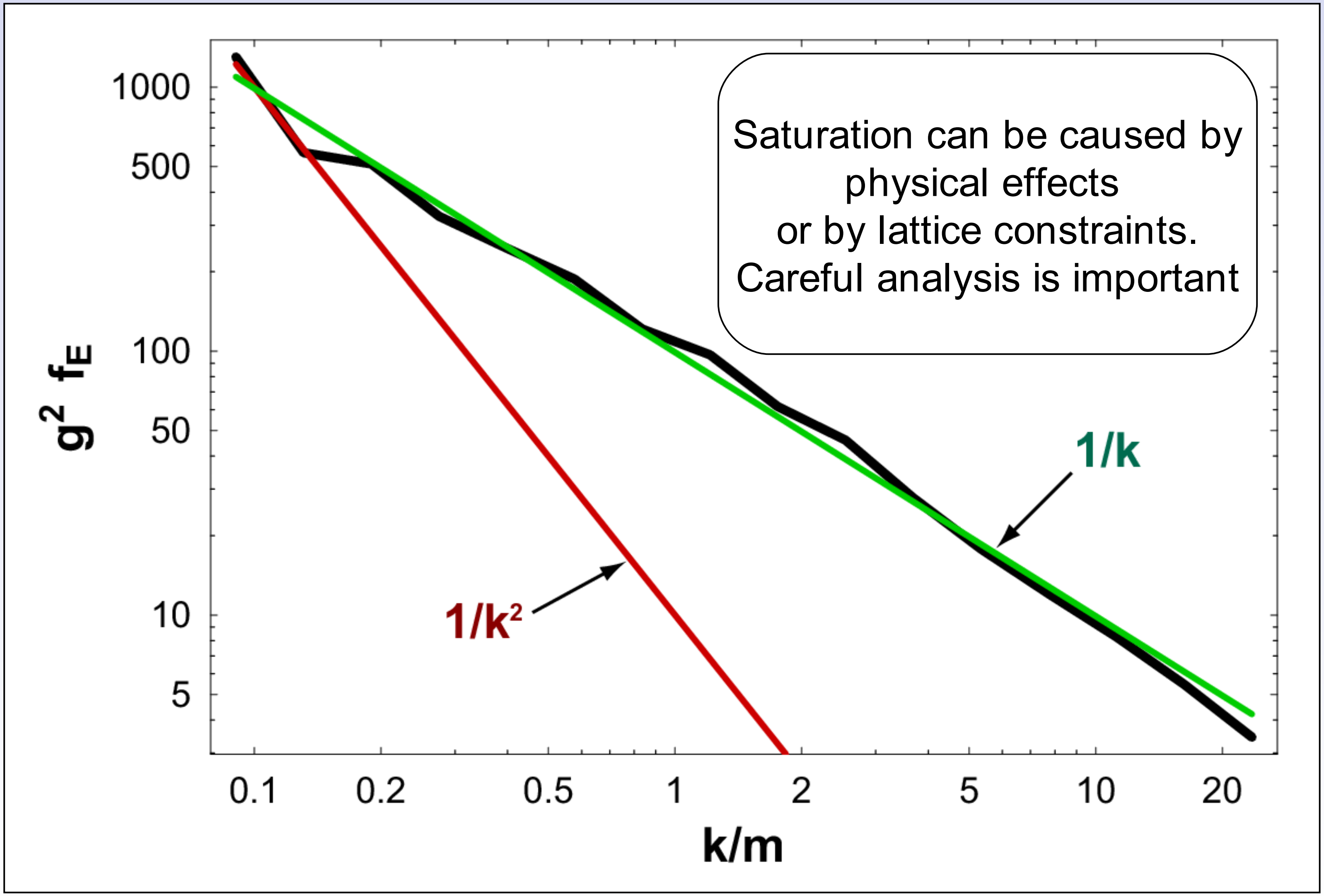
The final spectrum behaves as $\sim 1/k$



Saturation can be caused by physical effects
or by lattice constraints.
Careful analysis is important

Approach II: 3D Colored-Particle-In-Cell

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Solve the Vlasov equation coupled to the Yang Mills equation
without linearization:

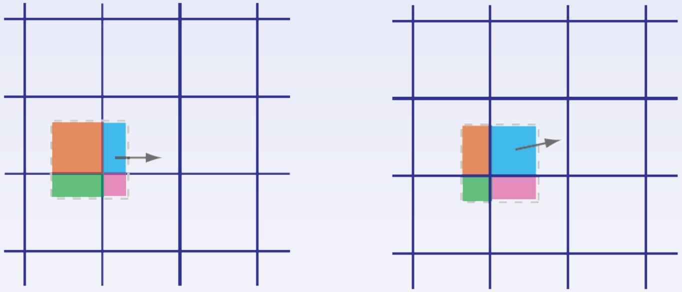
0 for now…

$$V^\mu[D_\mu(X), f(P,X)] + gV^\mu F_{\mu\nu}\partial^\nu_{(p)} f(P,X) = \mathcal{C}$$

using a test particle ansatz, leading to the Wong-Yang-Mills equations:

$$\begin{aligned}
\frac{dx_i}{dt} &= \mathbf{v}_i \\
\frac{dp_i}{dt} &= g\, q_i^a (\mathbf{E}^a + \mathbf{v}_i \times \mathbf{B}^a) \\
\frac{dq_i}{dt} &= ig\, v_i^\mu\, [A_\mu, \mathbf{q}_i] \\
J^{a\,\nu} &= \frac{g}{N_{\text{test}}} \sum_i q_i^a v^\nu \delta(\mathbf{x} - \mathbf{x}_i(t))
\end{aligned}$$

Solve using smeared colored particles

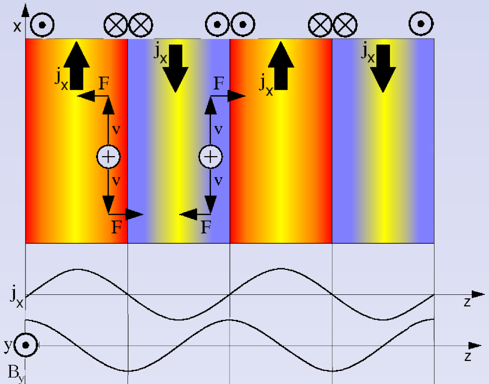


→ convergent results with less test particles

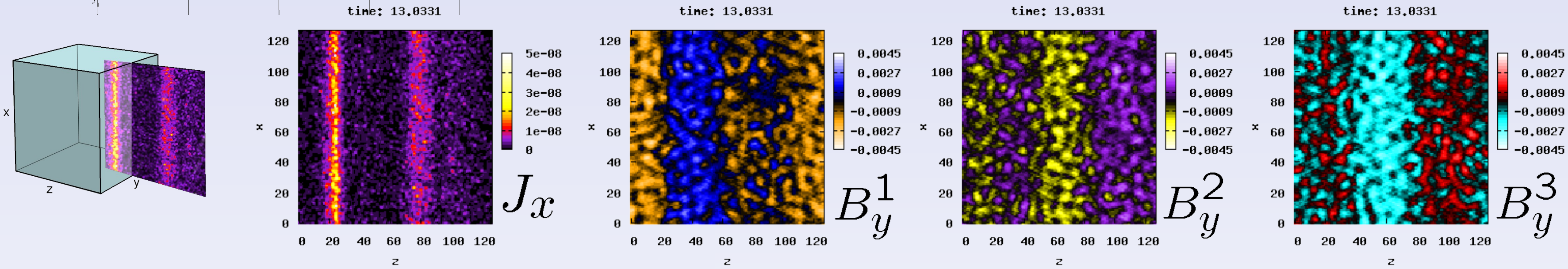
In this approach the back-reaction of the fields on the particles is included!

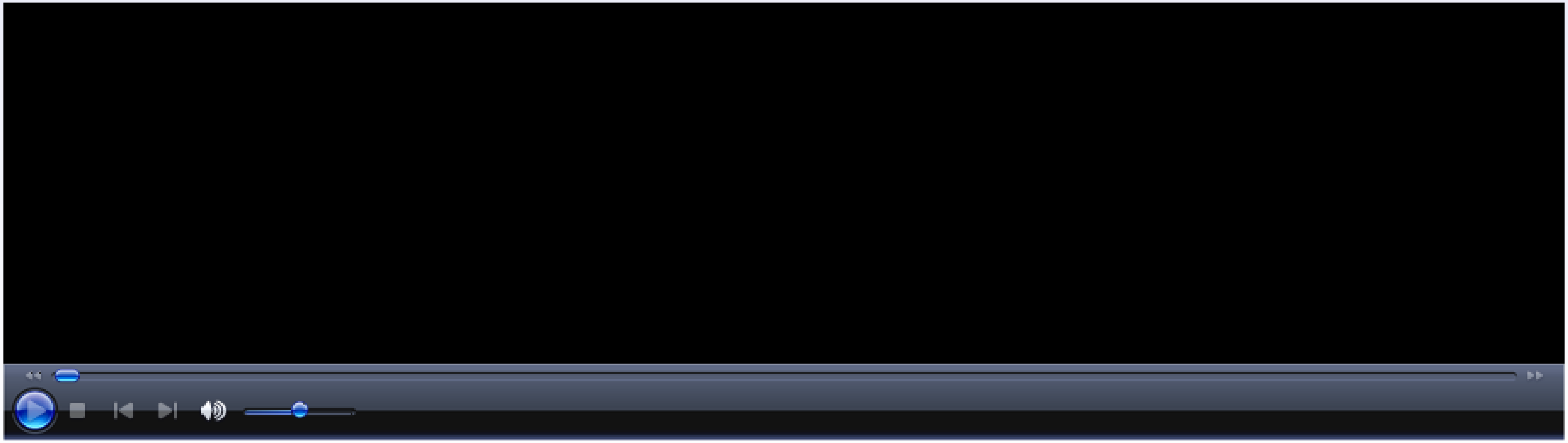
A. Dumitru, Y. Nara, M. Strickland, *Phys.Rev.**D75**:025016*, 2007

Visualization of filamentation



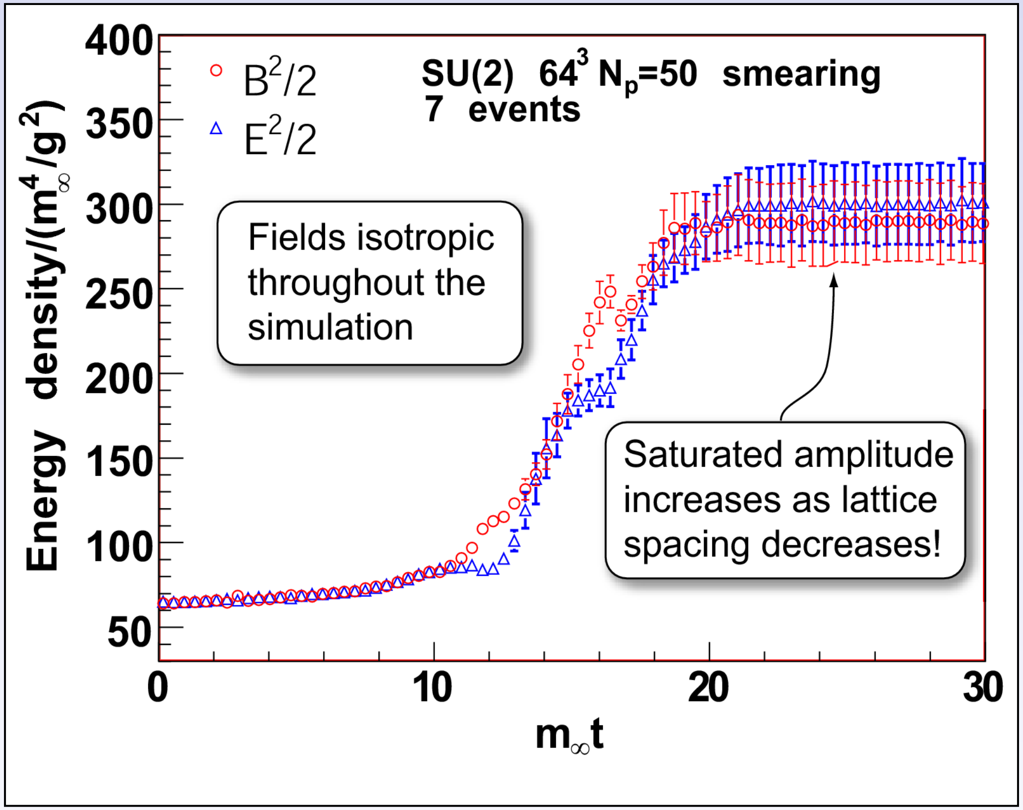
The filamentation is directly visible in the code. Shown are the current in the x-direction and the three color-components of the magnetic field in the y-direction.





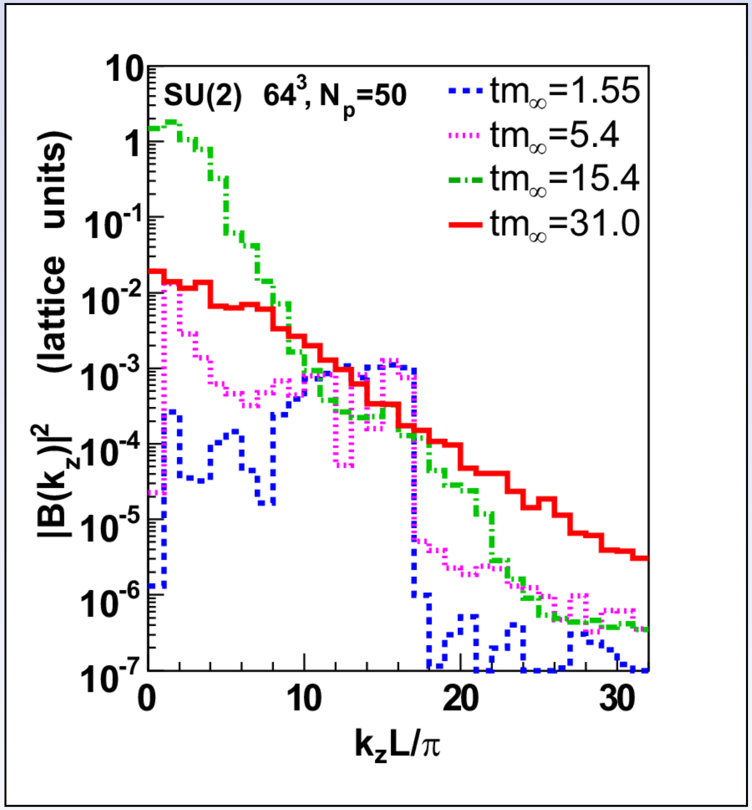
Ultraviolet Avalanche

Start with extremely anisotropic particle distribution $f(\mathrm{p}) = n_g \frac{2\pi}{p_h} \delta(p_z) e^{-p_t/p_h}$



Fields isotropic throughout the simulation

Saturated amplitude increases as lattice spacing decreases!



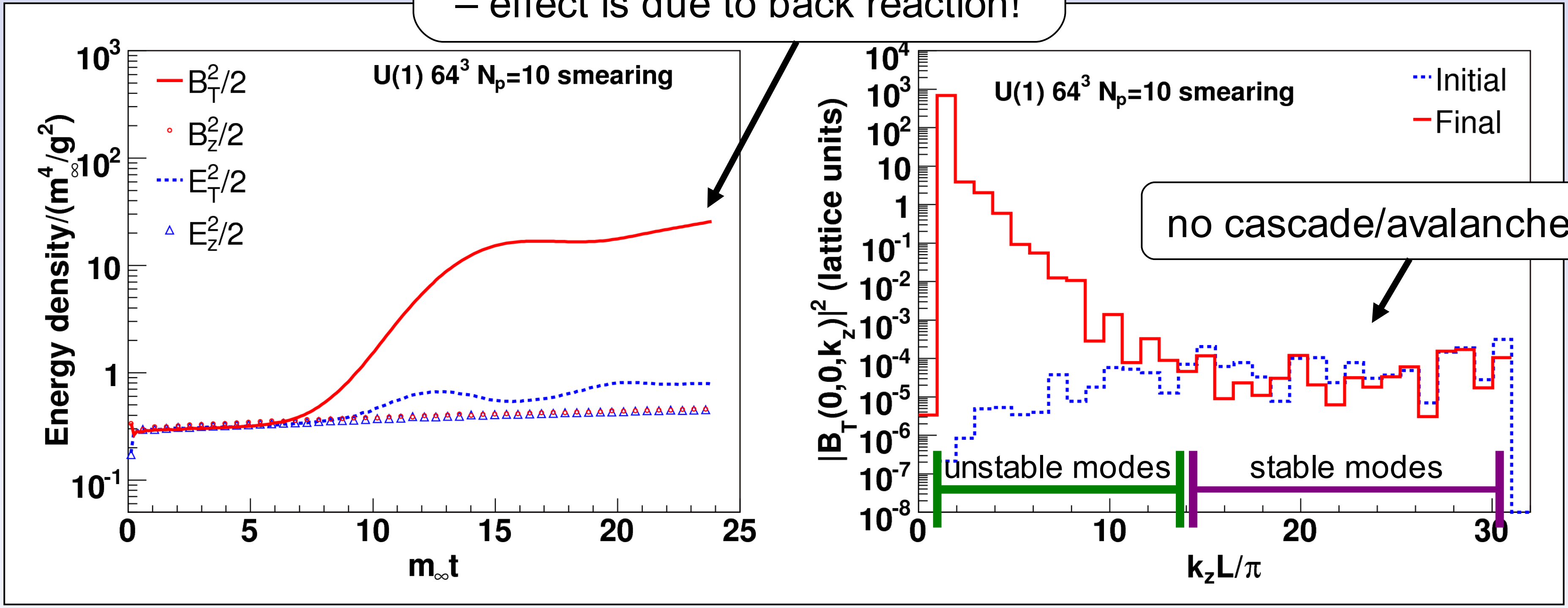
$L = 5\mathrm{fm}, p_{\mathrm{hard}} = 16\mathrm{GeV},$
$g^2 n_g = 10/\mathrm{fm}^3, m_\infty = 0.12\mathrm{GeV}$

Coulomb gauge-fixed color-magnetic field spectrum at four different times

A. Dumitru, Y. Nara, M. Strickland, *Phys.Rev.**D75**:025016*, 2007

U(1) control

Saturation is lattice size independent
– effect is due to back reaction!



no cascade/avalanche

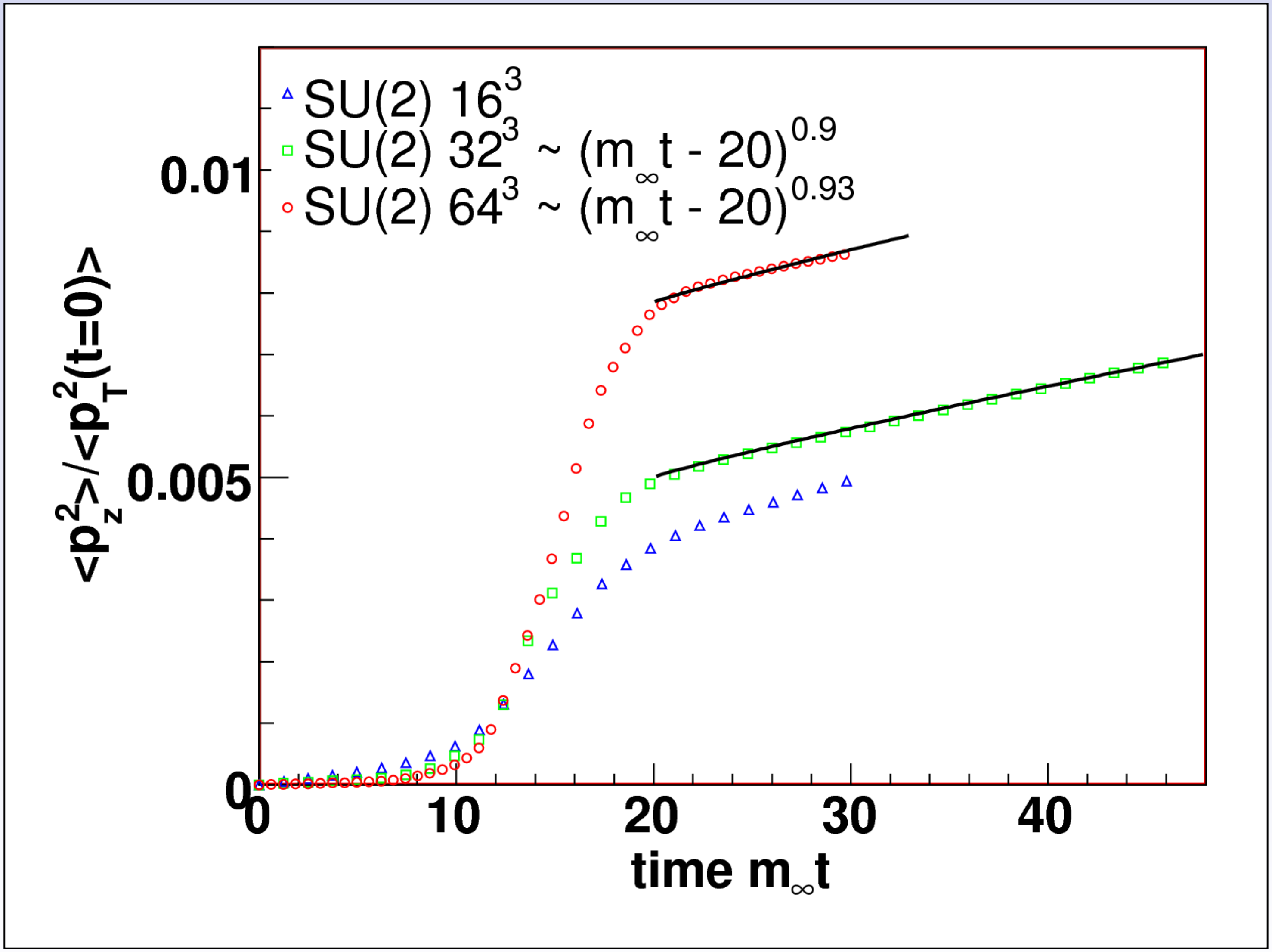
$$L = 5\text{fm}, p_\text{hard} = 8\text{GeV},$$
$$g^2 n_g = 100/\text{fm}^3, m_\infty = 0.3\text{GeV}$$

Magnetic field spectrum
at different times

Back-reaction is important for the dynamics: growth saturates; not all modes unstable.

A. Dumitru, Y. Nara, M. Strickland, *Phys.Rev.**D75**:025016*, 2007

Isotropization



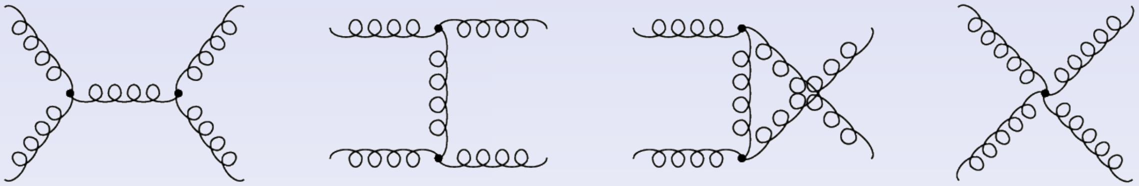
A. Dumitru, Y. Nara, M. Strickland, *Phys.Rev.**D75**:025016*, 2007

Including hard collisions

Now solve the Boltzmann-Vlasov equation coupled to the Yang Mills equation

$$V^{\mu}[D_{\mu}(X), f(P,X)] + gV^{\mu}F_{\mu\nu}\partial^{\nu}_{(p)} f(P,X) = \mathcal{C}$$

including elastic binary collisions with hard momentum exchange:



Separation of scales:

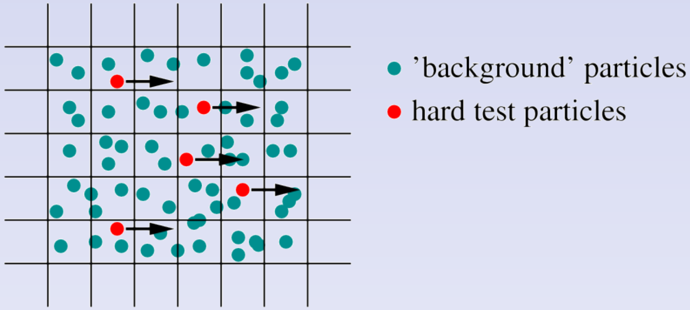
Include hard momentum transfers above highest field momentum (on the lattice)

$$\sigma_{\mathrm{tot}} = \int_{\kappa^2}^{s/2} \frac{d\sigma}{dq^2} dq^2$$

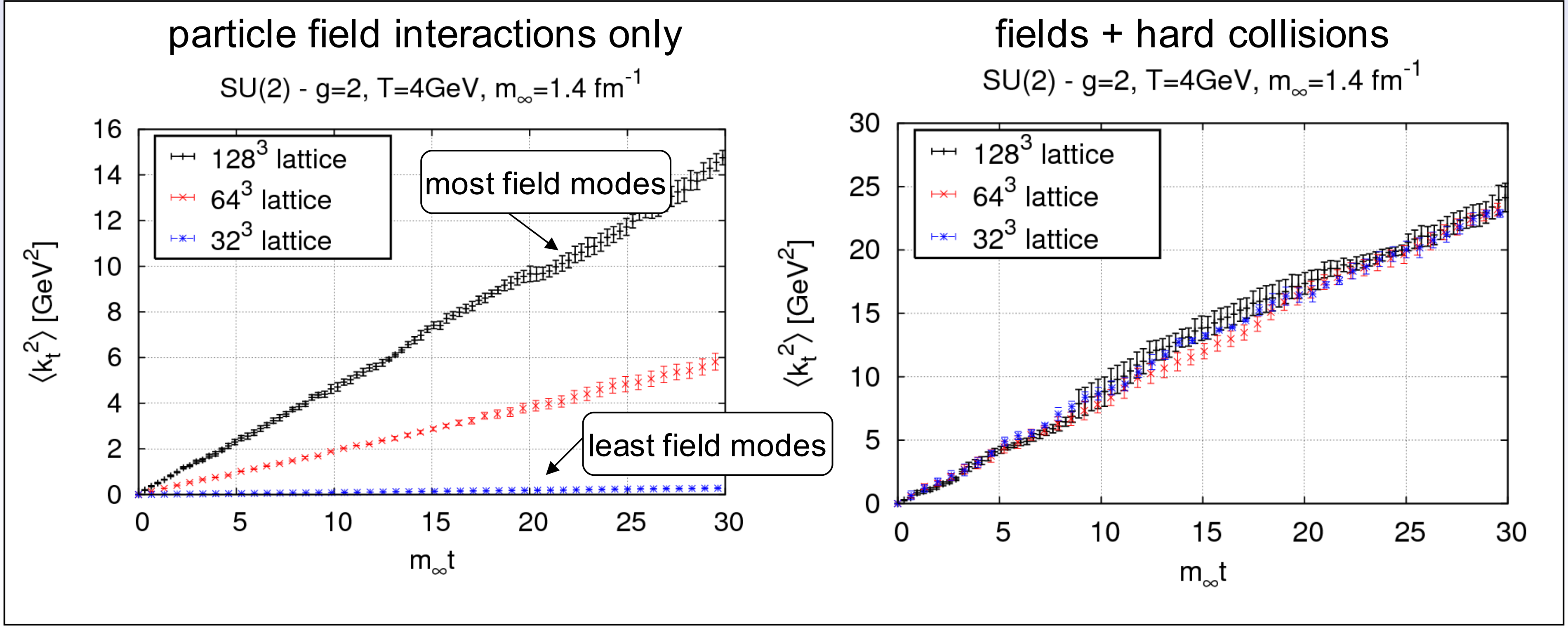
This way we avoid double counting of scatterings that are already described by interactions via backgound fields

Momentum space diffusion

CPIC with collisions is the first parton-cascade without a (large) cutoff



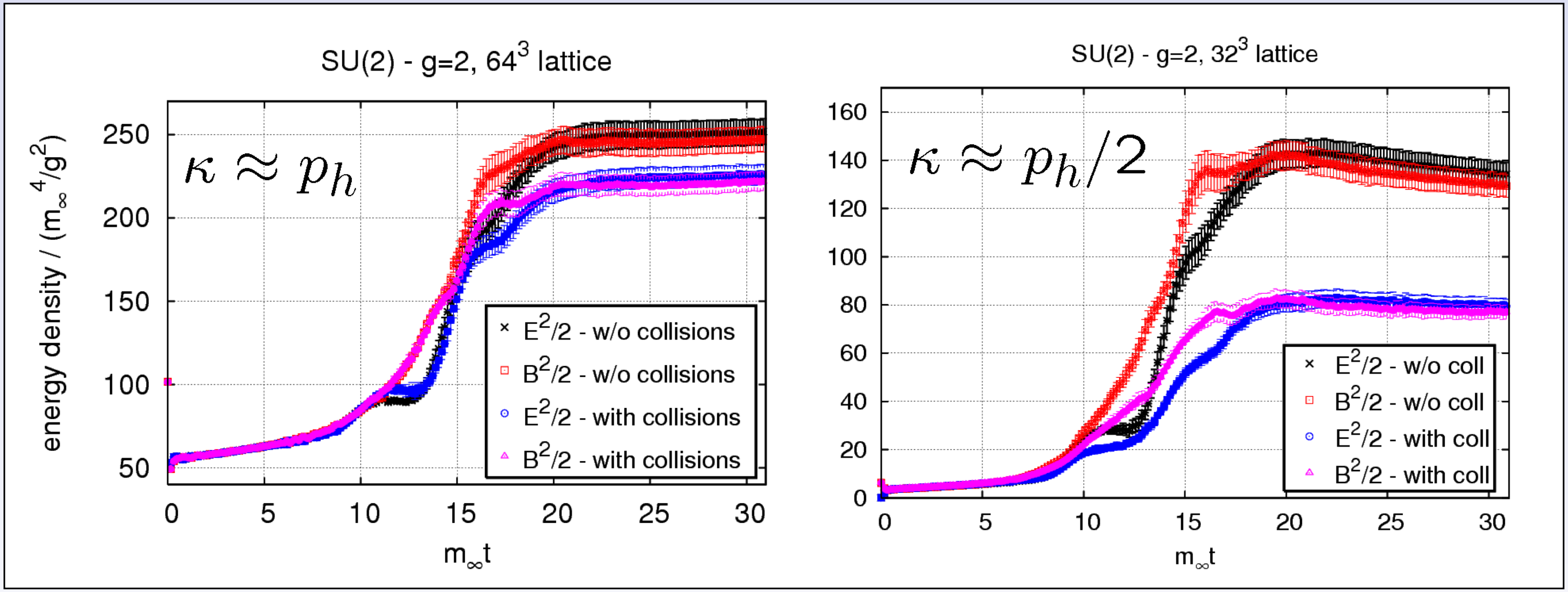
Can produce lattice independent results in an isotropic situation when matching the energy density of the fields to that of the particles



Including hard collisions

Collisions reduce instability growth!

Anisotropic system with low field amplitudes – no lattice independence:
Sensible choice for κ : between $p_h/2$ and p_h



$\kappa \approx p_h$ more field modes
less hard collisions

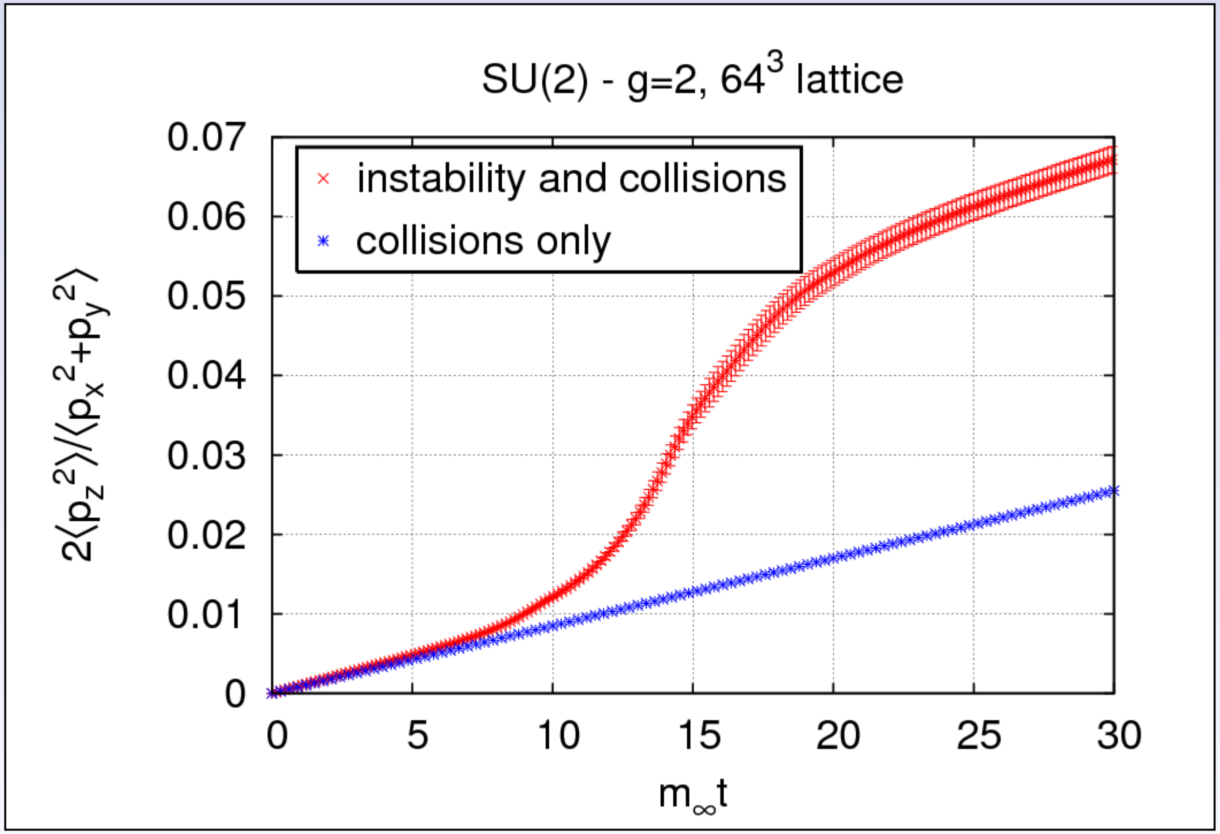
$\kappa \approx p_h/2$ less field modes
more hard collisions

Also seen in model calculation with BGK collision term:

B. Schenke, M. Strickland, C. Greiner, M.H. Thoma, *Phys.Rev.**D73**:125004,2006*

Isotropization: Collisions vs. Instability

For weak initial fields and the used density
the instability still dominates isotropization!



more isotropic

$$L = 5\text{fm}, p_{\text{hard}} = 16\text{GeV},$$

$$g^2 n_g = 10/\text{fm}^3, m_\infty = 0.12\text{GeV}$$

Summary

- **Presented numerical methods for solving coupled particle-field dynamics:**
    - **Hard loop (HL) approach**
    - **Wong-Yang-Mills simulation**

- Instability growth for systems anisotropic in momentum space

- Exponential growth - non-Abelian effects lead to linear growth

- Cascade or avalanche to the ultra violet

- Dimensionality of the simulation is crucial!

- Dependence on the initial conditions

- CPIC including collisions is the first parton cascade without a large cutoff

- Collisions reduce instability growth

- **Isotropization is sped up by instabilities** – enough to explain fast isotropization in HICs? Depends on initial conditions…

Outlook

- Expanding geometry – first HL results available (see Mike‘s talk)

- Conversion from fields to particles and vice versa in the CPIC simulation

- Expansion in CPIC simulation

The new generation of hard expanding loop (HEL) and CPIC simulations will allow for improved simulation of non-equilibrium plasmas and for predictions for non-equilibrium observables (see Mike‘s talk…)

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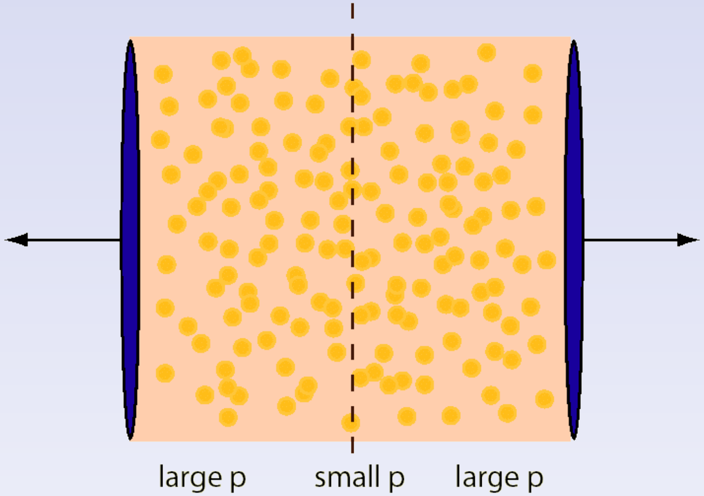
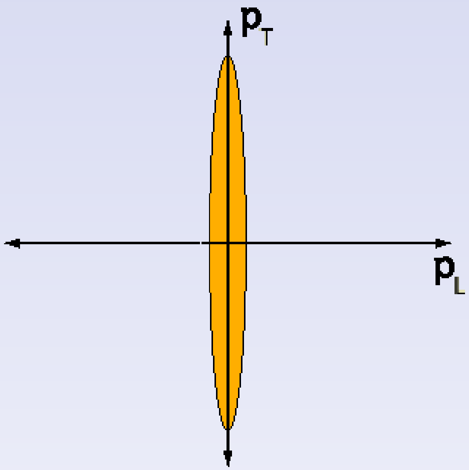
Backup Slides

Anisotropy in HICs



$$\langle p_T \rangle \sim Q_s$$
$$\langle p_L \rangle \sim 1/\tau$$



- Due to expansion the parton distribution functions become locally anisotropic in momentum space for times $\tau > \langle p_T \rangle^{-1}$.

HL-calculation: anisotropy (1)

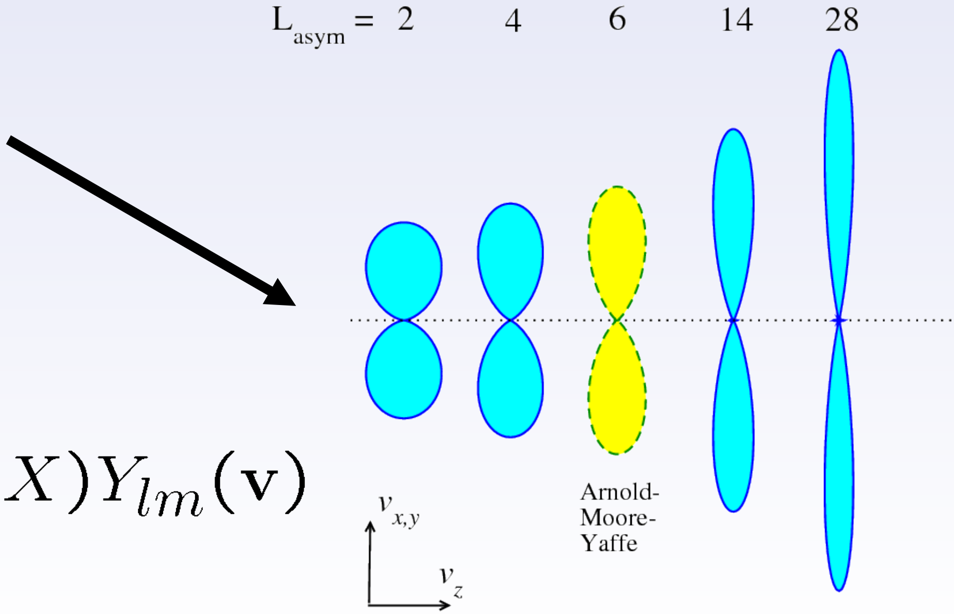
Integrate out irrelevant $|\mathbf{p}|$ dependence of $\delta f^a(P, X)$:

$$W^a(X, \mathbf{v}) := 4\pi g \int_0^\infty \frac{dpp^2}{(2\pi)^3} \delta f^a(X, p\mathbf{v})$$

and determine the equations of motion in terms of W^a...

Create an anisotropic distribution by expanding $f(|\mathbf{p}|)$ in spherical harmonics:

$$\boxed{f(\mathbf{p}) = \sum_{l=0}^{L_{\text{asym}}} f_l(|\mathbf{p}|) Y_{l,0}(\mathbf{v})}$$



Also expand W in spherical harmonics

$$W(X, \mathbf{v}) := \sum_{l=0}^{L_{\text{max}}} \sum_{m=-l}^{l} W_{lm}(X) Y_{lm}(\mathbf{v})$$

HL-calculation: anisotropy (2)

Introduce auxiliary fields $\mathcal{W}^\mu(\mathbf{v}, X)$

$$\delta f(\mathbf{p}, X) = g\frac{\partial f(\mathbf{p})}{\partial P^\mu}\mathcal{W}^\mu(\mathbf{v}, X)$$

And express the current as $j^\mu = -g^2 \int_{\mathbf{p}} V^\mu \frac{\partial f(\mathbf{p})}{\partial P^\beta}\mathcal{W}^\beta(\mathbf{v}, X)$

With the eqm for $\mathcal{W}^\mu(\mathbf{v}, X)$

$$[V \cdot D(A)]\mathcal{W}_\beta(\mathbf{v}, X) = F_{\beta\gamma}(A)V^\gamma$$

This is then solved together with the Yang-Mills equation $D_\mu(A)F^{\mu\nu} = j^\nu$

by discretizing the integral over the directions $\mathbf{v}$ in j^μ above. ➘

The anisotropy can then be parametrized e.g. like

$$f(\mathbf{p}) = f_{\text{iso}}(\mathbf{p}^2 + \xi p_z^2)$$

Colored-Particle-In-Cell (CPIC-) simulation

- Aim: Find solutions to the Vlasov equation

$$p^\mu \left[\partial_\mu - g\, q^a F^a_{\mu\nu} \partial^\nu_p - g\, f_{abc} A^b_\mu q^c \partial_{q^a}\right] f(x,p,q) = 0$$

which is coupled self-consistently to the

Yang-Mills equation for the soft gluon fields

$$D_\mu F^{\mu\nu} = j^\nu = g \int \frac{d^3p}{(2\pi)^3} dq\, q v^\nu f(t,\mathbf{x},\mathbf{p},q)$$

Back-reaction of the fields on the hard particles are of course included

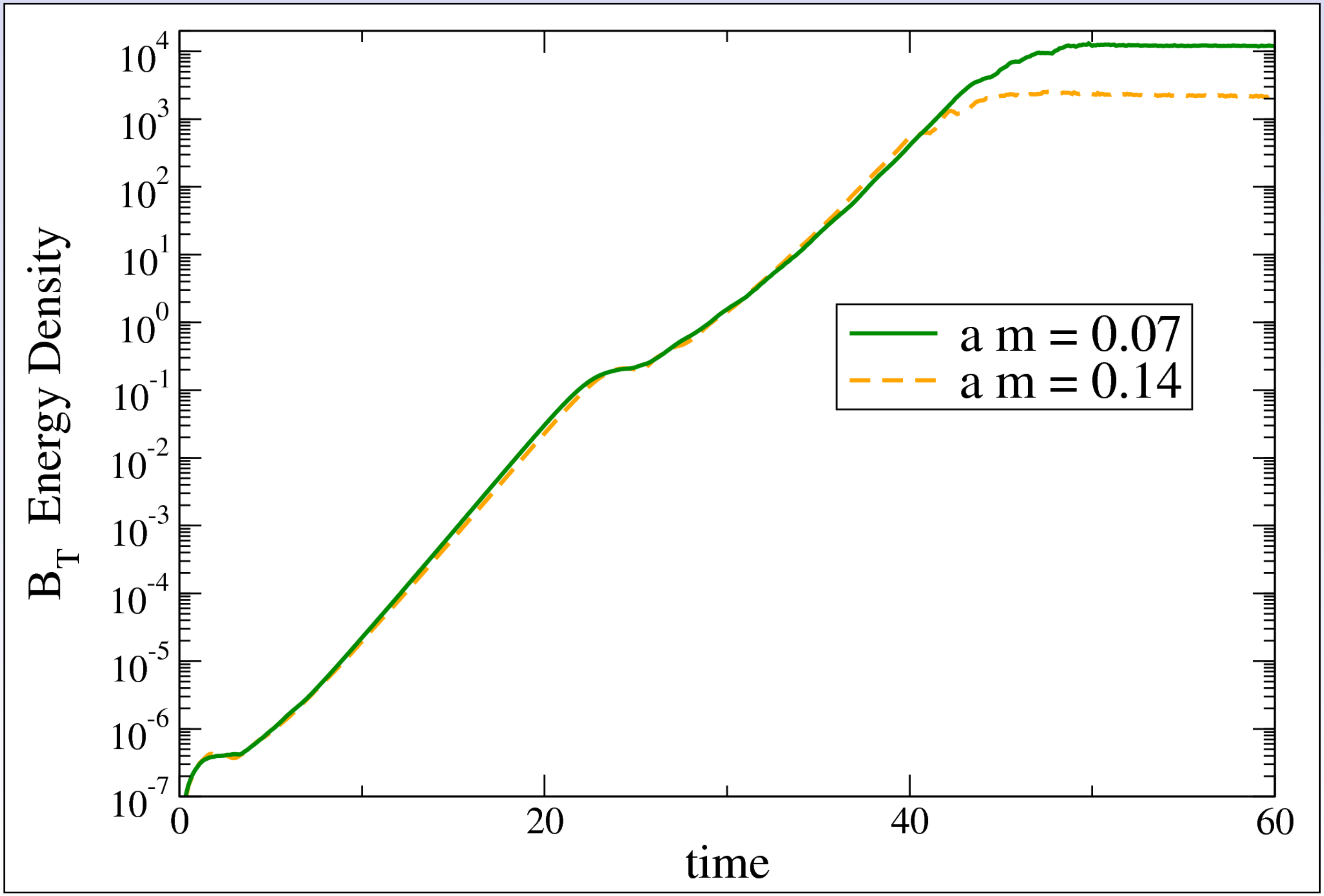
- Numerical solution by replacing the continuous single particle distribution by a large number of test particles:

$$f(x,p,q) = \frac{1}{N_{\mathrm{test}}} \sum_i \delta(\mathbf{x}-\mathbf{x}_i(t))(2\pi)^3 \delta(\mathbf{p}-\mathbf{p}_i(t))\delta(q^a - q^a_i(t))$$

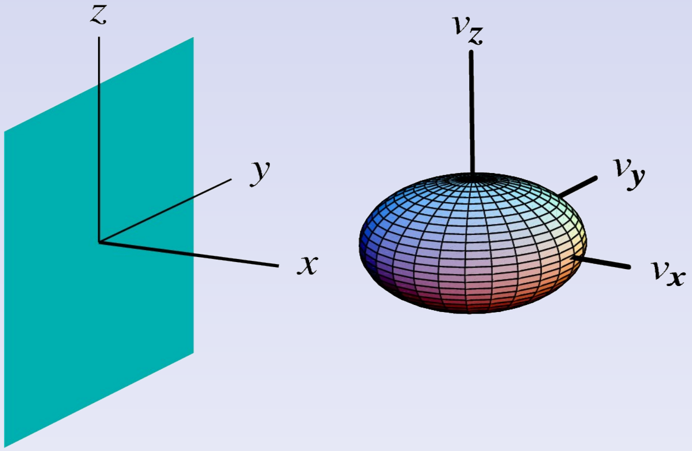
Leads to the Wong-Yang-Mills equations

Lattice artifacts

For very small initial fields, the exponential growth continues until it hits the compactness bound:

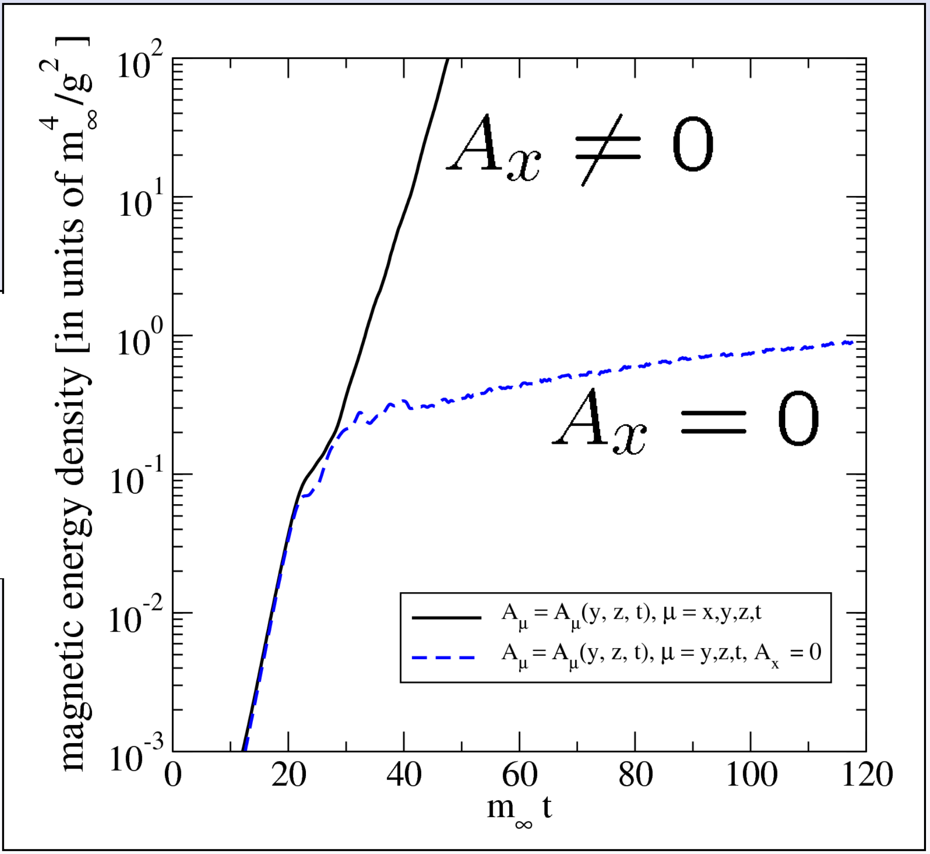


Difference between 1+1d and 3+1d: check 2+1 dimensional HL-simulation



All fields and distribution functions only depend on the y- and z- coordinates

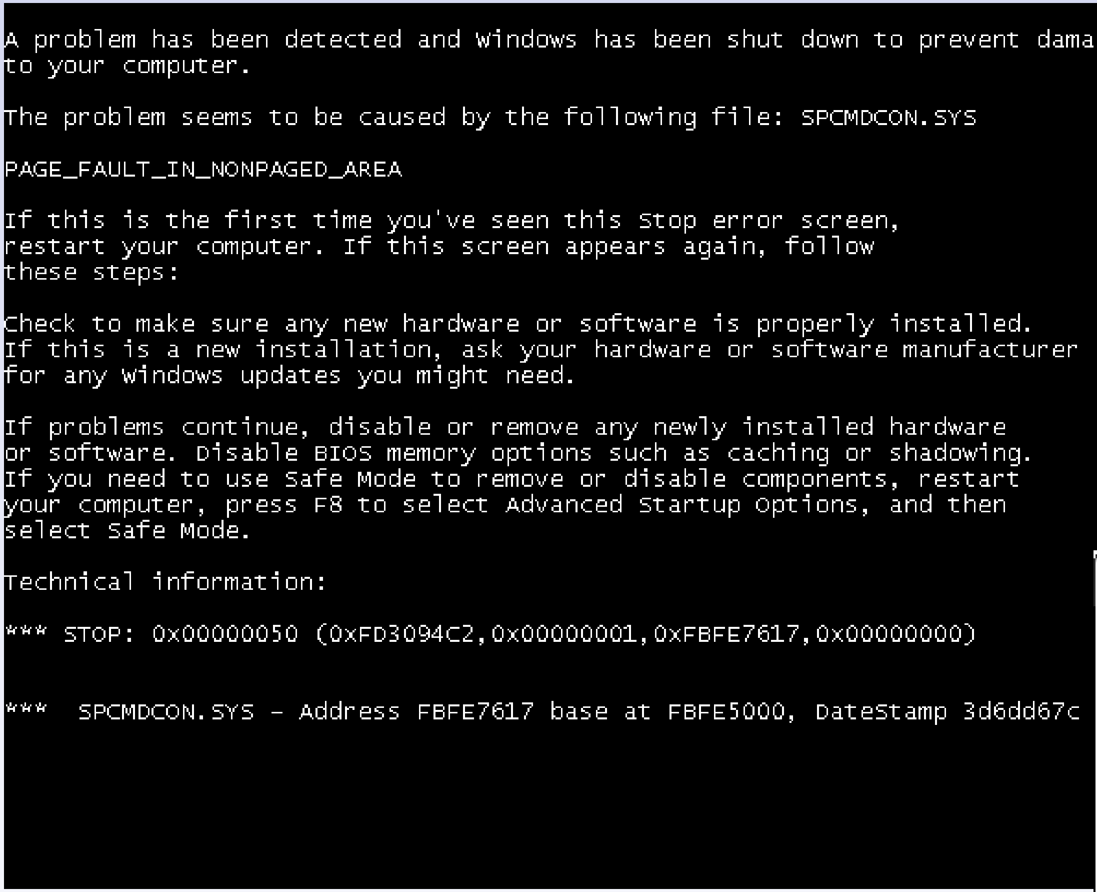
If $A_x = 0$ the results are similar to the 3+1d simulations, otherwise continued exponential growth as in the 1+1d case is found.



In the dimensionally reduced theory, scalar fields arise (A_x), which prevent non-Abelian effects from ending the field growth.

Peter Arnold, Po-Shan Leang, e-Print: arXiv:0704.3996 [hep-ph]

Another kind of instabilities …



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A problem has been detected and windows has been shut down to prevent damage
to your computer.

The problem seems to be caused by the following file: SPCMDCON.SYS

PAGE_FAULT_IN_NONPAGED_AREA

If this is the first time you've seen this Stop error screen,
restart your computer. If this screen appears again, follow
these steps:

Check to make sure any new hardware or software is properly installed.
If this is a new installation, ask your hardware or software manufacturer
for any windows updates you might need.

If problems continue, disable or remove any newly installed hardware
or software. Disable BIOS memory options such as caching or shadowing.
If you need to use Safe Mode to remove or disable components, restart
your computer, press F8 to select Advanced Startup Options, and then
select Safe Mode.

Technical information:

*** STOP: 0x00000050 (0xFD3094C2,0x00000001,0xFBFE7617,0x00000000)


***  SPCMDCON.SYS - Address FBFE7617 base at FBFE5000, DateStamp 3d6dd67c
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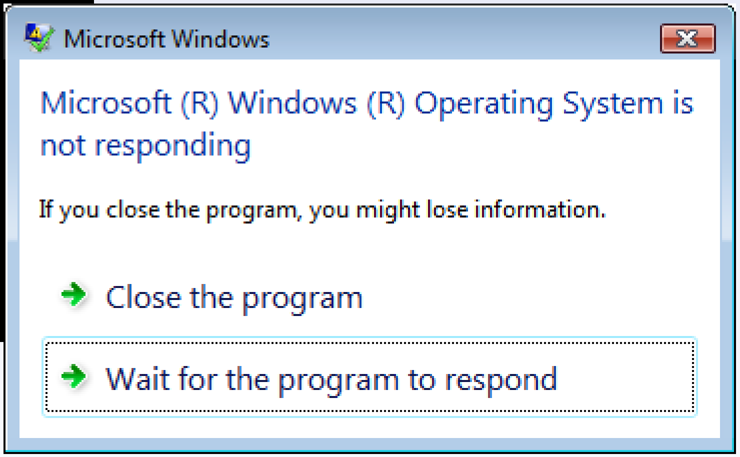

Microsoft (R) Windows (R) Operating System is not responding

If you close the program, you might lose information.