LANCASTER UNIVERSITY

The Cockcroft Institute
of Accelerator Science and Technology

High Luminosity LHC

Modelling LLRF performance for Crab Cavities

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EuCARD

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Plan

1. RF parameters and assumptions for RF modelling
2. Power requirement based on assumptions
3. Simplistic RF layout appropriate for IQ control
4. RF model
5. Parameters for cavity and RF system used for modelling
6. Simulation of de-tuning, re-tuning, RF on and RF off
7. Performance in the absence of measurement errors
8. Simulation of cavity quench
9. Spectral coefficients with measurement errors
10. Simulation of LLRF set point error
11. Spectral noise and relative kicks on particles

Parameters assumed for model

Parameter	Unit	Nominal	Scenario 1	Scenario 2
Beam energy (E_o)	TeV	7	7	7
Particles in bunch (N)		1.15×10^{11}	2.0×10^{11}	3.3×10^{11}
Bunch charge (q)	C	1.84×10^{-8}	3.2×10^{-8}	5.28×10^{-8}
Bunches (n)		2808	2808	1404
Bunch repetition frequency (f_{rep})	MHz	40	40	20
Bunch separation	ns	25	25	50
Crossing angle (θ_c)	μ rad	300	420	520
β^*	m	0.55	0.2	0.2
β at crab cavity	m		4167	4167
R12 (crab cavity to IP)	m		28.9	28.9
ε_s	eV	2.5	2.5	2.5
ε_n	m	3.75×10^{-6}	2.5×10^{-6}	3.0×10^{-6}
Bunch length (σ_z)	m	0.0755	0.0755	0.0755
Bunch width at IP (σ_x)	m	16.6×10^{-6}	8.2×10^{-6}	9.0×10^{-6}
Piwinski parameter		0.68	1.94	2.2
Peak Luminosity		1.3×10^{34}	8.5×10^{34}	8.7×10^{34}
Crab cavity Frequency (f)	MHz	N/A	400	400
Required Crab Transverse Kick	MV	N/A	6.07	7.52
Max. bunch offset at cavity (Δx)	μm	N/A	236	259
Beamloading at max. offset	kW	N/A	15.4	17.3

Power Requirement

Peak power requirement P depends on the maximum offset Δx
Beam to beam interactions at the IP reduce beam lifetime if the bunches not accurately aligned.
Maximum offset at the crab cavity is related to the maximum offset at the IP as

$$\sigma_y(\text{crab})/\sigma_y(\text{ip}) = \sqrt{\beta(\text{crab})/\beta(\text{ip})} = \sqrt{4167/0.2} = 144$$

Assume $$\Delta y = 0.2\,\sigma_y$$

Maximum offset at crab is 236 μm for scenario 1 and 259 μm for scenario 2.

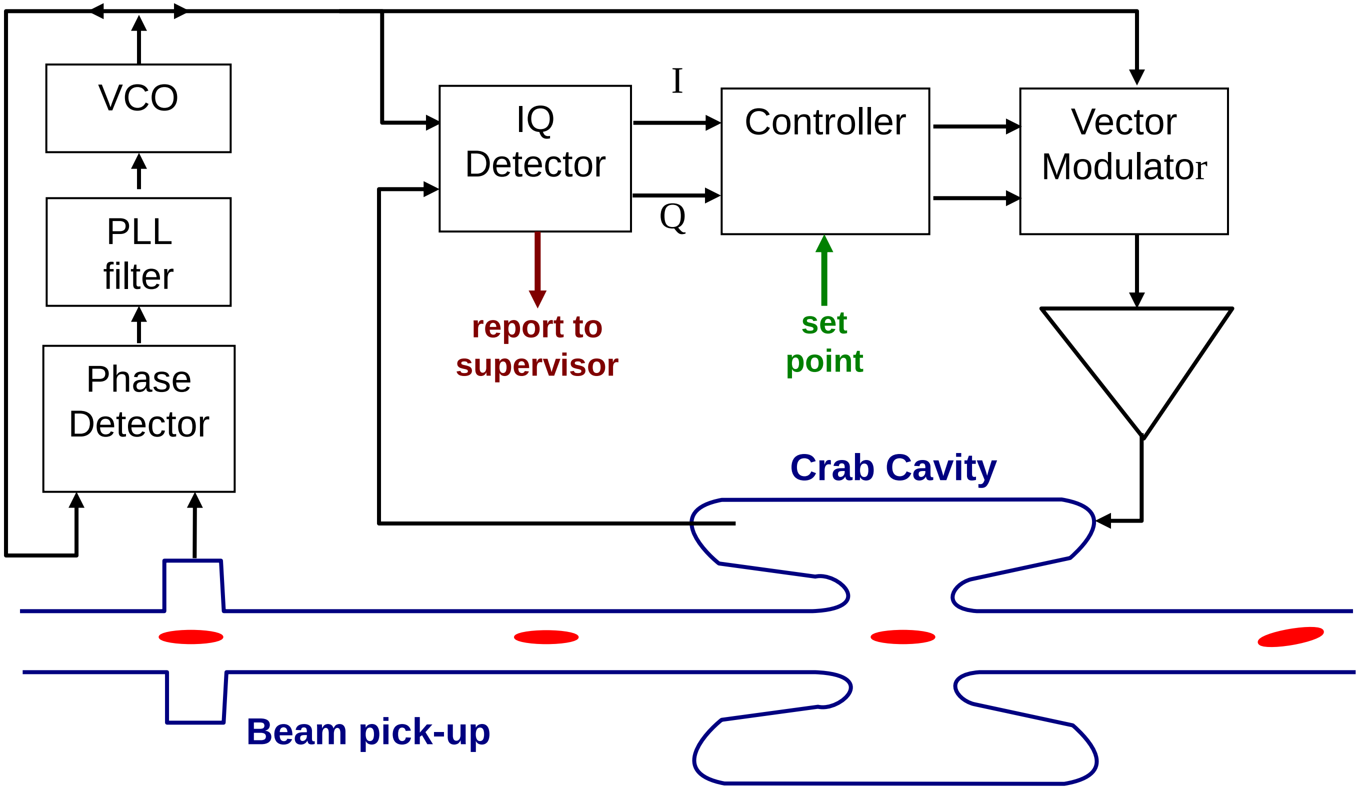
Assume active steering through the crab cavities to achieve these values
Applying Panofsky Wenzel gives

$$P = q\, f_{\text{rep}} V_z = q\, f_{\text{rep}} \Delta x \frac{\omega}{c} V_\perp \sim 3.2\times10^{-8}\times4\times10^{7}\times2.36\times10^{-4}\times0.119\times6.07\times10^{6} \quad = \quad 15.4\,\text{kW}$$

Compact crab cavity designs are expected to deliver a maximum kick between 3MV and 5MV.
Layout as planned has 3 cavities to kick and 3 three to un-kick

Power requirement per cavity ~ 6 kW hence the RF power source is likely to be solid state.

RF Layout assumed for LLRF model



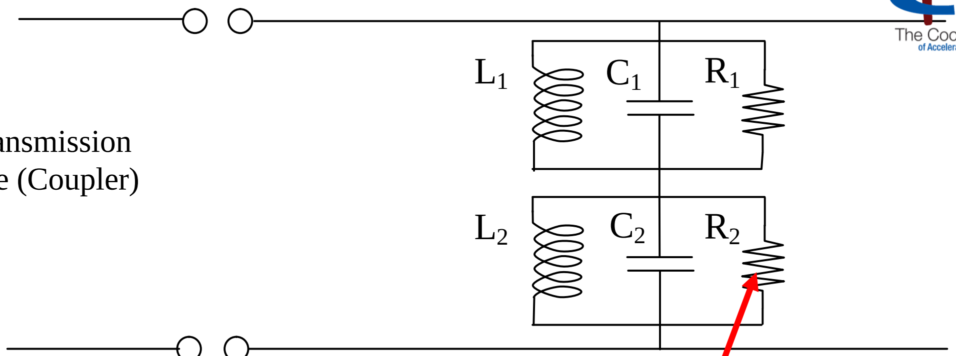
Lancaster experience is with IQ controllers hence stick to this.

If the LLRF can get its own reference from the beam then it only needs to know the set point.

Cavity Model

Input coupler

Transmission line (Coupler)



Mode damping included in mode circuit resistance

$$\frac{d^2V_i}{dt^2}+\frac{\omega_i}{Q_{Li}}\frac{dV_i}{dt}+\frac{1}{Q_{ei}}\omega_i\sum_{\substack{j=1\\ j\neq i}}^{N}\frac{dV_j}{dt}+\omega_i^2V=\frac{2\omega_o}{Q_e}\frac{d}{dt}\{\mathcal{F}\exp(-j\omega t)\}$$

$$V_m(t)=\{A_{mr}(t)+jA_{mi}(t)\}\exp\{-j\omega t\}$$

$$\dot{A}_{mr}=-\frac{\omega_m}{4Q_{om}}\left(\frac{\omega_m^2}{\omega^2}+1\right)A_{mr}-\frac{\omega_m}{4Q_{em}}\sum_{j=1}^{N}\left(\frac{\omega_j^2}{\omega^2}+1\right)A_{jr}+\left(\omega_m^2-\omega^2\right)\frac{A_{mi}}{2\omega}-\frac{\omega_m}{\omega Q_{em}}\left(\dot{\mathcal{F}}_i-\omega\mathcal{F}_r\right)$$

$$\dot{A}_{mi}=-\frac{\omega_m}{4Q_{om}}\left(\frac{\omega_m^2}{\omega^2}+1\right)A_{mi}-\frac{\omega_m}{4Q_{em}}\sum_{j=1}^{N}\left(\frac{\omega_j^2}{\omega^2}+1\right)A_{ji}-\left(\omega_m^2-\omega^2\right)\frac{A_{mr}}{2\omega}+\frac{\omega_m}{\omega Q_{em}}\left(\dot{\mathcal{F}}_r+\omega\mathcal{F}_i\right)$$

Solve in time domain with 4th order Runge Kutta one time step per RF cycle

Model History

2005 – 2007	**FORTRAN code written to understand how measurement errors, adjacent modes and time delays in the LLRF system limit stability limits and the level of phase control achievable for the ILC crab cavities**
2010	**Modified for Beta beam feasibility studies so that bunches do not have to be at a sub harmonic frequency of the RF.** **Switching of RF frequency enabled.**
2012	**Added FFT analysis of waveforms** **Added LHC bunch structure**

full	empty	full	empty	full	empty	full	empty
72	8	72	8	72	38		
72	8	72	8	72	38		
72	8	72	8	72	8	72	39
72	8	72	8	72	38		
72	8	72	8	72	38		
72	8	72	8	72	8	72	39
72	8	72	8	72	38		
72	8	72	8	72	38		
72	8	72	8	72	8	72	39
72	8	72	8	72	38		
72	8	72	8	72	38		
72	8	72	8	72	119		

Beamloading

Beamloading is included by letting in-phase and quadrature cavity voltages jump in proportion to the image charge deposited in the cavity after the passage of the bunch.

For a dipole mode the jump is determined as

$$A_r(\text{final}) = A_r(\text{initial}) + \frac{r_b \omega}{c} \omega \left(\frac{R}{Q}\right)_F q \cos\alpha$$

$$A_i(\text{final}) = A_i(\text{initial}) + \frac{r_b \omega}{c} \omega \left(\frac{R}{Q}\right)_F q \sin\alpha$$

α is the phase angle between the bunch passing the centre of the cavity and the initial RF phase of the cavity,
q is the bunch charge,
r_b is the offset of the bunch **(have also used Δx elsewhere).**

Here the R/Q is defined as

$$\left(\frac{R}{Q}\right)_F = \frac{|V_L(r)|^2}{2\omega U \left(\frac{r\omega}{c}\right)^2}$$

where the cavity voltage V_L includes transit time effects.

Cavity Parameters assumed

Number of cavity modes : 3
Operating mode order (1-dipole) : 1
Operating mode centre frequency (MHz) : 400.8
Operating mode Q factor : 2.000d+09
Operating mode external Q factor : 5.700d+05
Operating mode R over Q (Ohms per cell) : 912.6

Mode 2 order : 0
Mode 2 centre frequency (MHz) : 375.2
Mode 2 intrinsic Q factor (with damping) : 10000
Mode 2 external Q factor : 5.000d+06
Mode 3 relative pickup coupling : 0.1
Mode 2 relative beam coupling : 0.136d00

Mode 3 order : 1
Mode 3 centre frequency (MHz) : 436.6
Mode 3 intrinsic Q factor (with damping) : 10000
Mode 3 external Q factor : 5.000d+06
Mode 3 relative pickup coupling : 0.1
Mode 3 relative beam coupling : 0.0153d00

LLRF parameters

Master oscillator frequency (MHz) : 400.8
Bucket frequency (MHz) : 40.08
Energy set point (Joules per cell) : 1.89
Maximum Amp Power (Watts per cell) : 8500.0
Maximum beam offset (mm) : -0.250
1 for random offset 0 for periodic : 0
Offset fluctuation frequency (Hz) : various (200 -5000)
Initial Bunch phase retard (degrees) : 0.0
Bunch phase jitter (degrees) : 0.0
1 for random charge fluc 0 for periodic : 0
Phase jitter frequency (Hz) : 5000
Bunch charge fluctuation (fraction) : 0.005
1 for random charge fluc 0 for periodic : 1
Charge fluctuation frequency (Hz) : 5000
Bunch charge (Coulombs) : 3.2e-8
Bunch train length (seconds) : 86.90166e-6
Bunch train gap length (seconds) : 2.02096e-6
RF advance time (seconds) : 200.0e-6
Cavity freq. shift from microphonics Hz : 40.0
Vibration frequency of cavity (Hz) : various (200 -2000)
Initial vibration phase (degrees, sin) : 0
Measurement phase error in degrees : various (0.0, 0.005,…)
Measurement amplitude error as fraction : various (0.0, 0.0001. …)
Delay for control system in seconds : 0.5e-6
Control update interval in seconds : 0.5e-6
Initial gain constant for controller : various (5-10 optimum)
Amplifier Bandwidth : 5.0e6
Measurement filter bandwidth : 10.0e6
Feed forward sum jump (~1.2e6) : 0.0e6

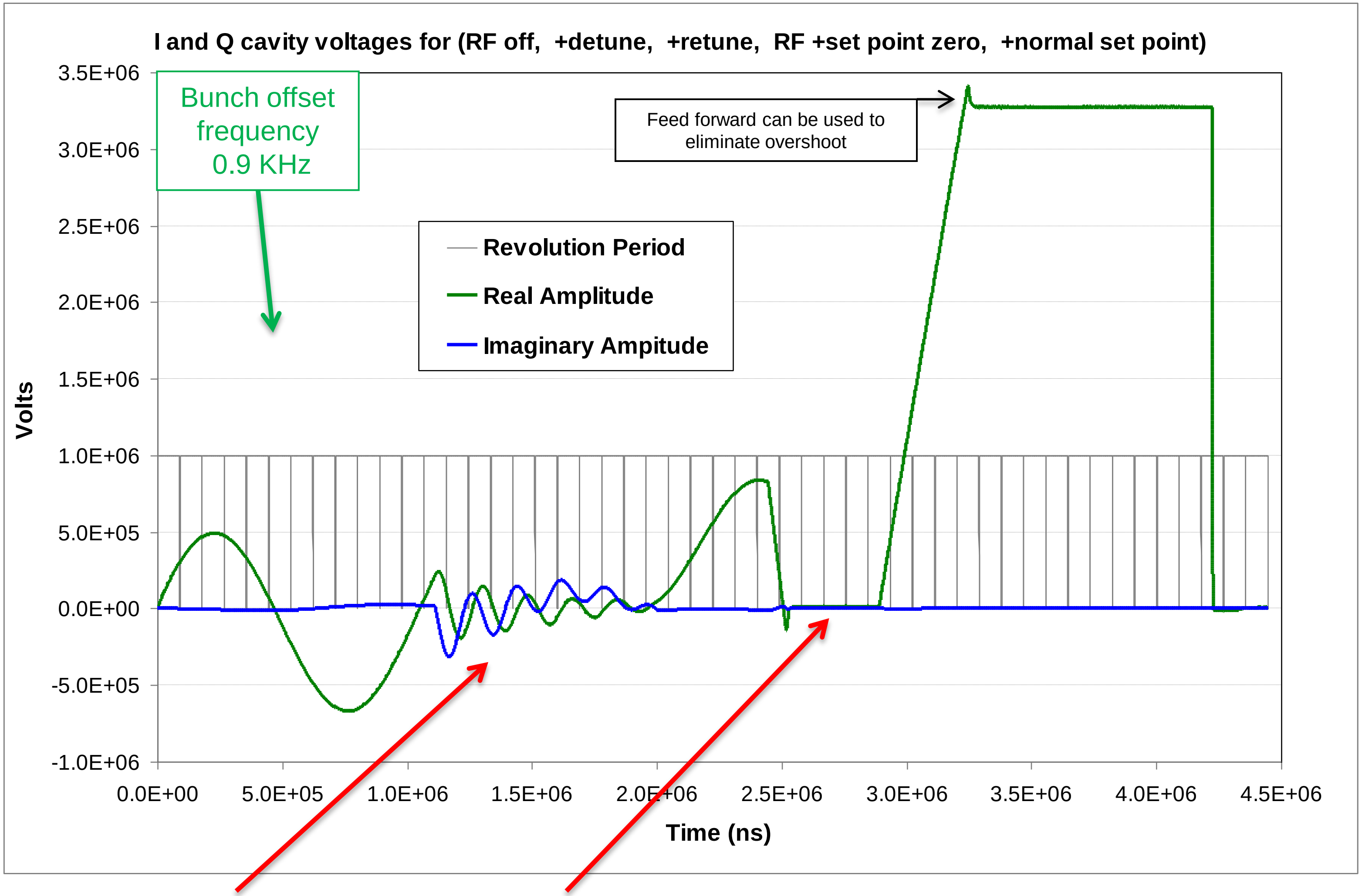
Bunch structure was hard coded

Largest cavity perturbation is the bunch offset

A stiff cavity should minimise microphonics

Simulation input and output

Can vary set points and cavity tune during simulation.
Primary outputs are I and Q voltages of each mode from which others are derived


I and Q cavity voltages for (RF off, +detune, +retune, RF +set point zero, +normal set point)

Bunch offset frequency 0.9 KHz

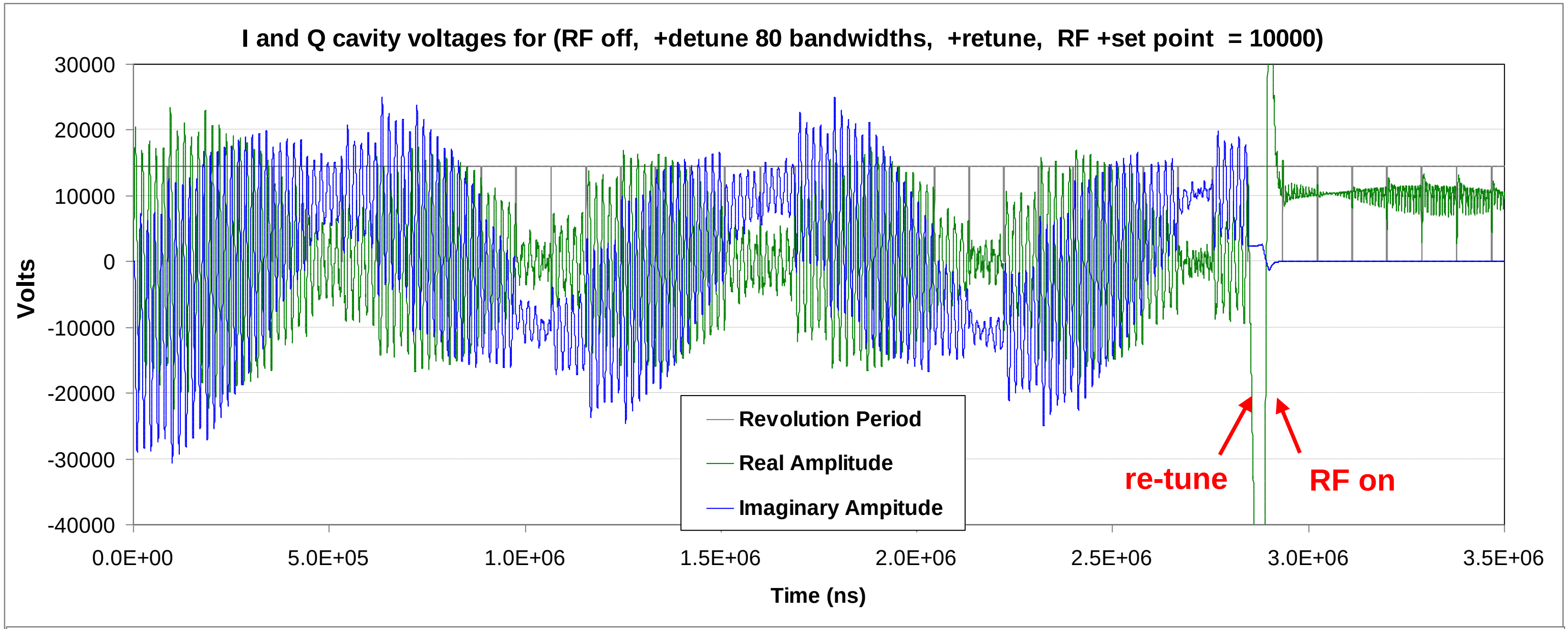
Feed forward can be used to eliminate overshoot

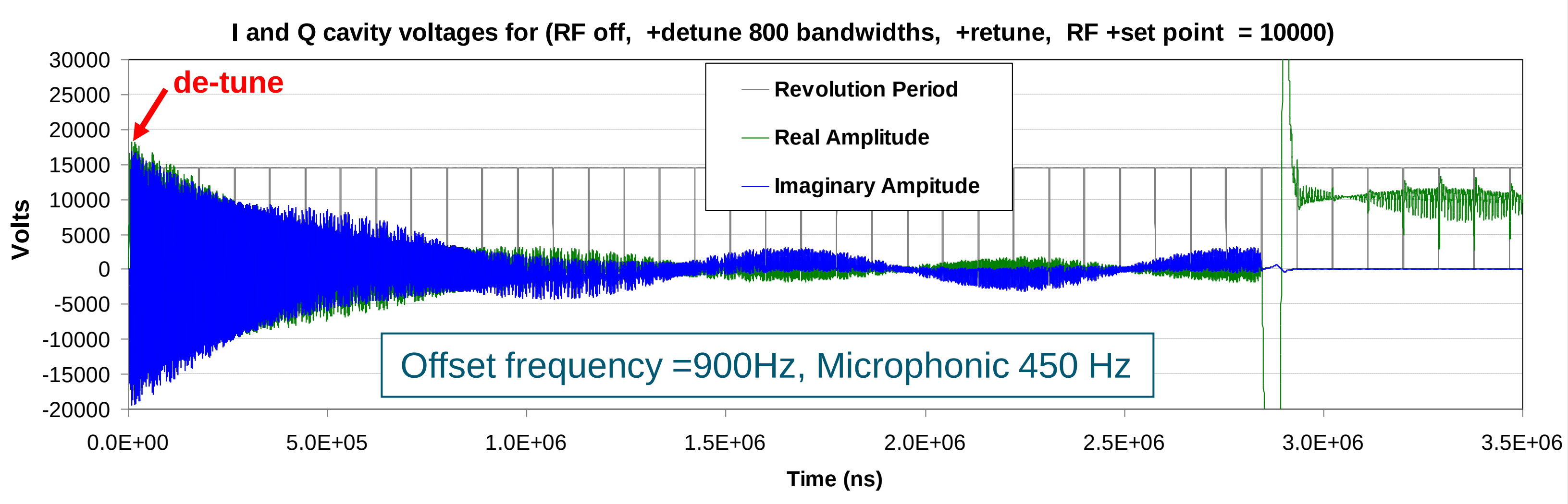
De-tune by 8 bandwidths

Set point zero

De-tune 80 and 800 Bandwidths

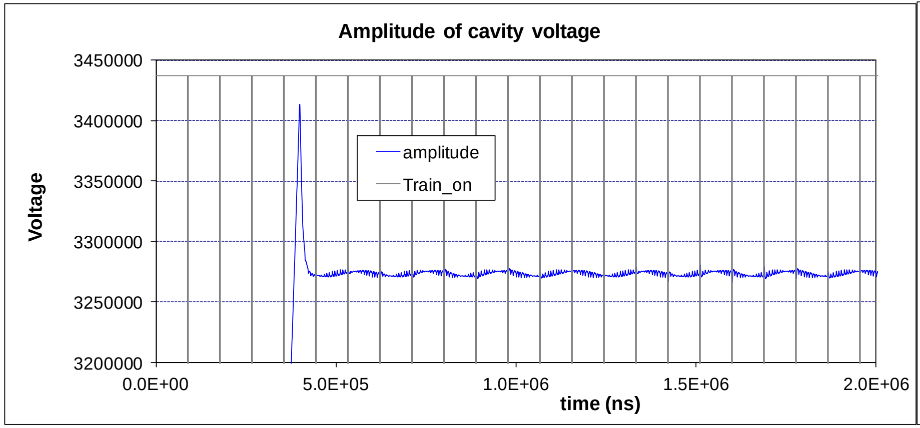
Keeping RF on with set point zero – better option than de-tuning?

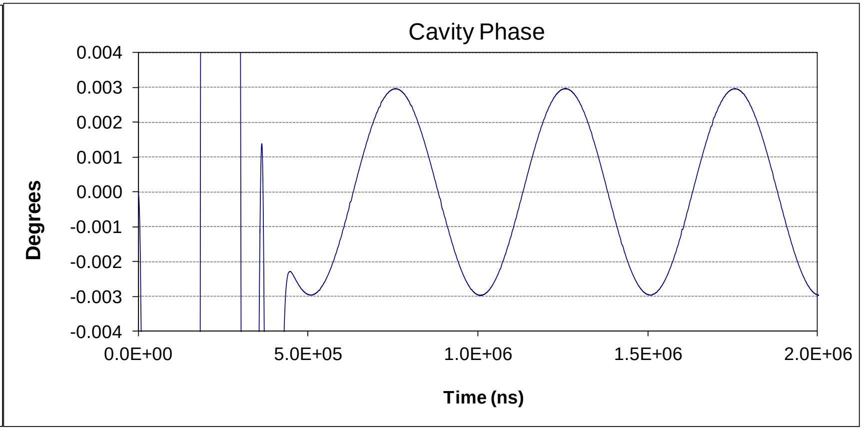


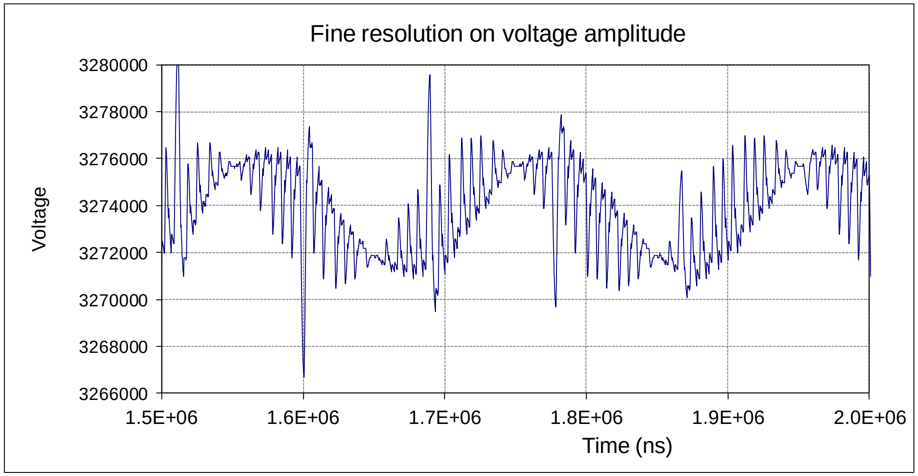


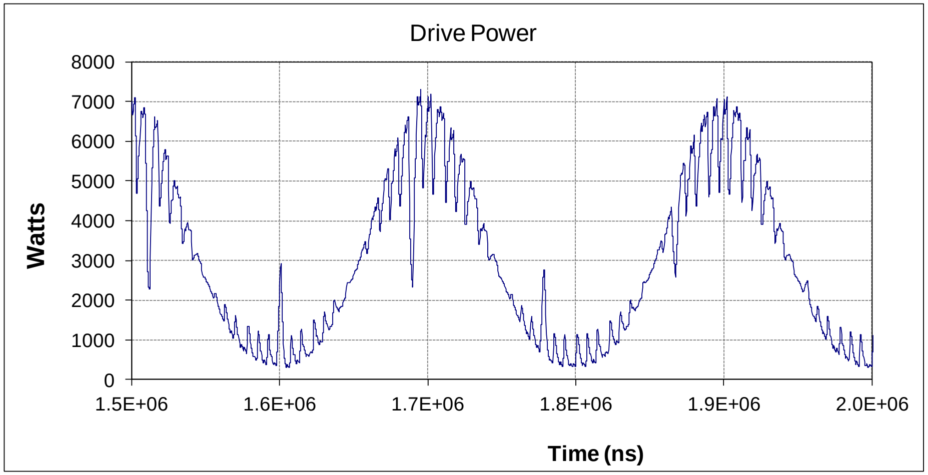
Offset frequency =900Hz, Microphonic 450 Hz

Simulation with no measurement errors


Amplitude of cavity voltage


Cavity Phase


Fine resolution on voltage amplitude


Drive Power

Power follows beam offset
Dips follow gaps in bunch structure
Amplitude correction depends on gain
Phase follows microphonics

Fourier Coefficients

no measurement errors

Beam offset frequency = 5.000 kHz
Cavity microphonics = 2.000 kHz
Revolution frequency = 11.245 kHz

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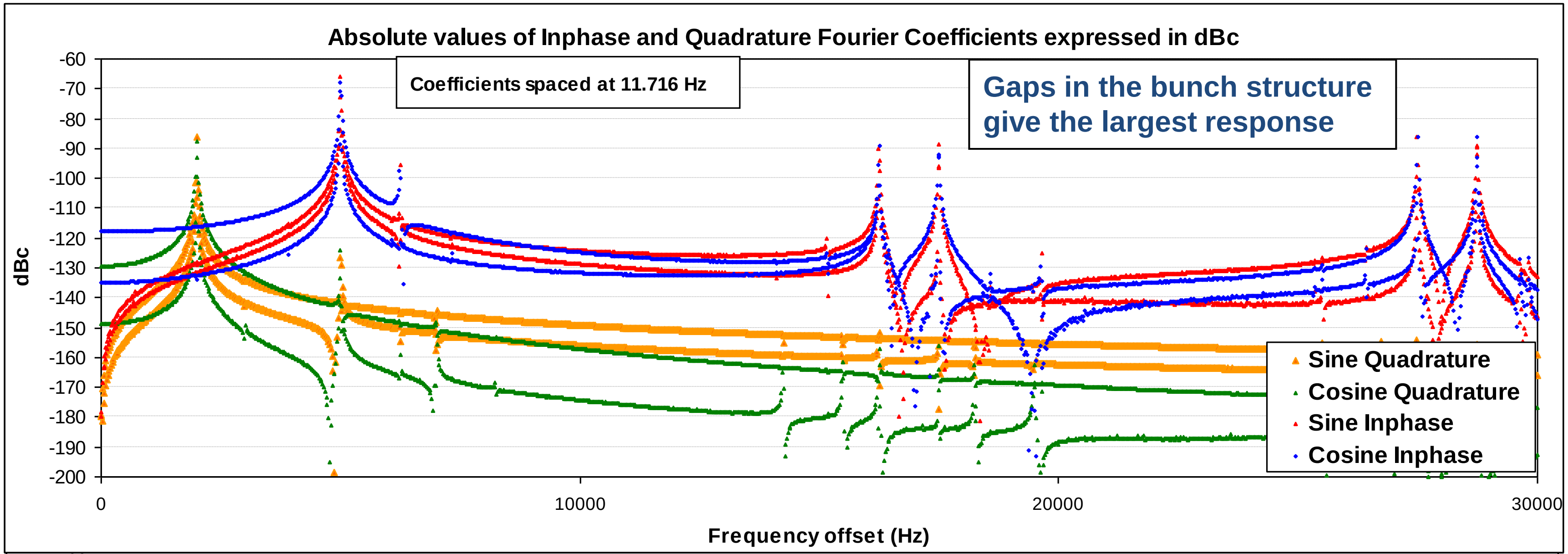
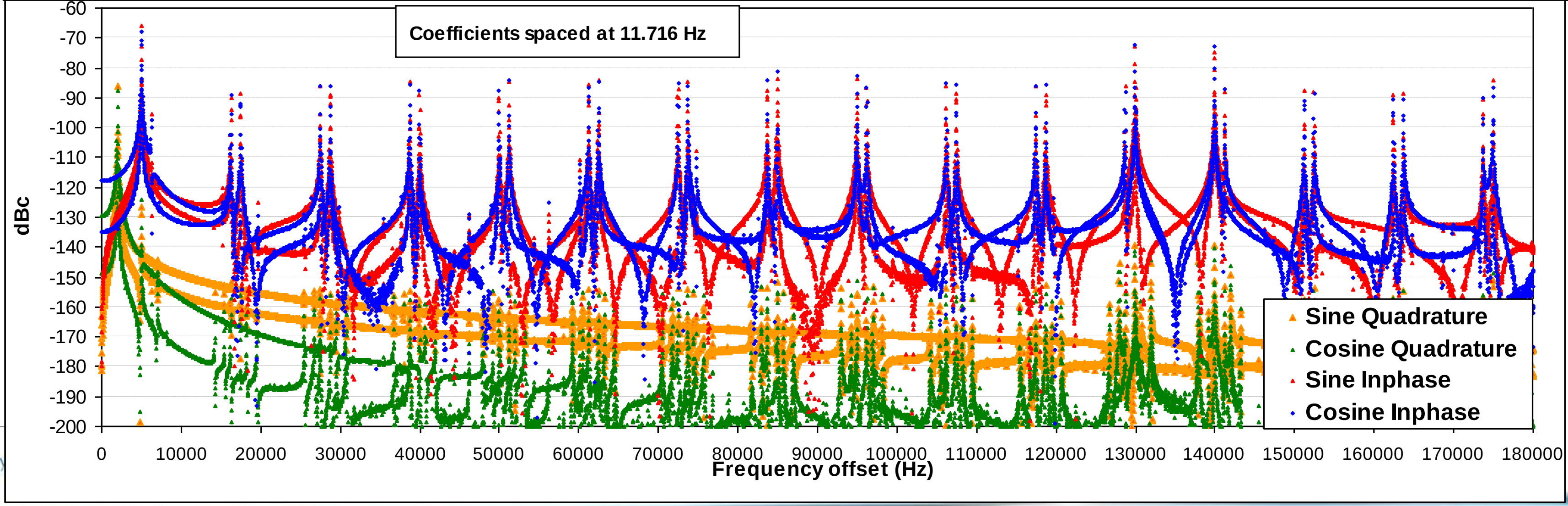

Absolute values of Inphase and Quadrature Fourier Coefficients expressed in dBc

Coefficients spaced at 11.716 Hz

Gaps in the bunch structure give the largest response


Coefficients spaced at 11.716 Hz

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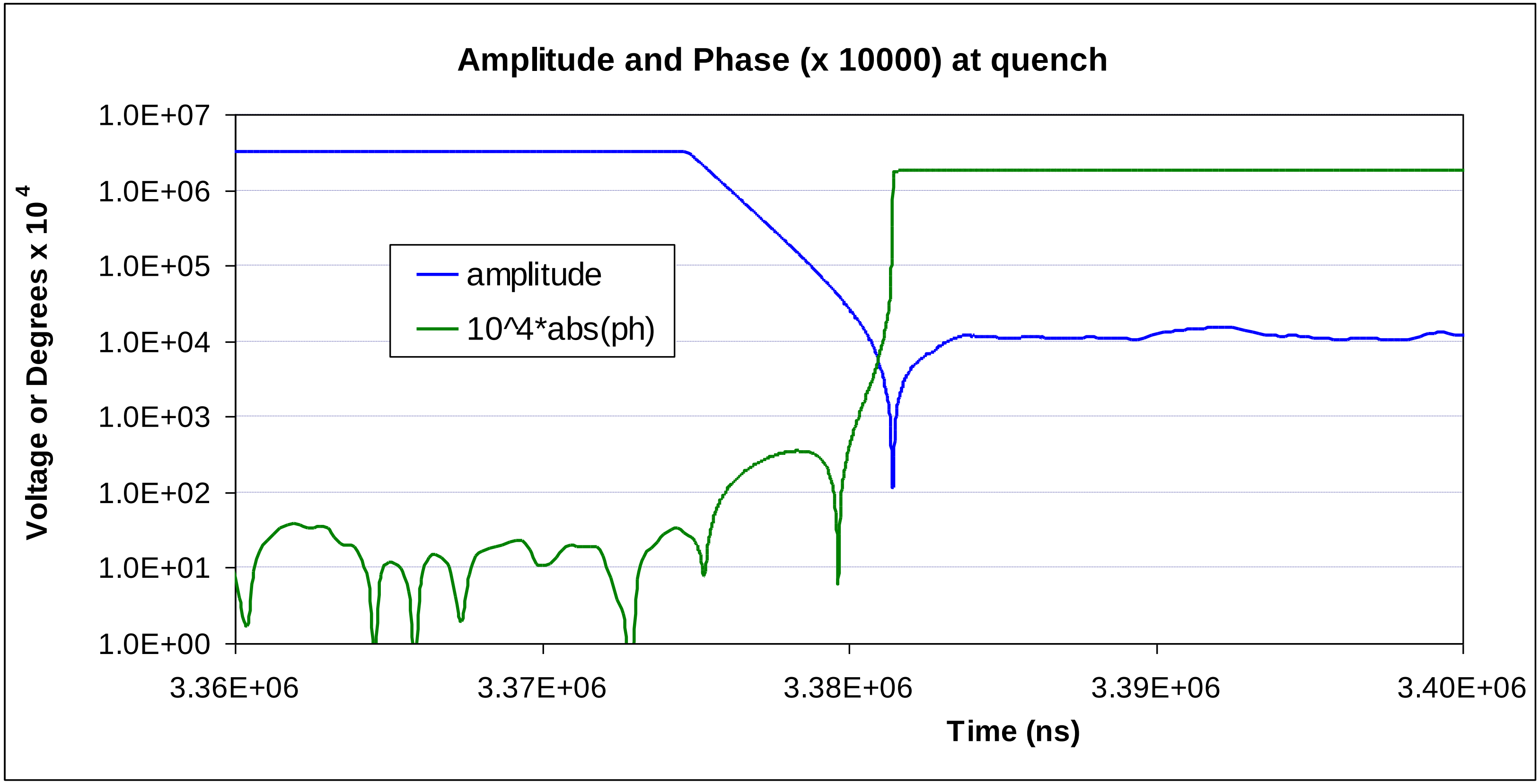
Quench

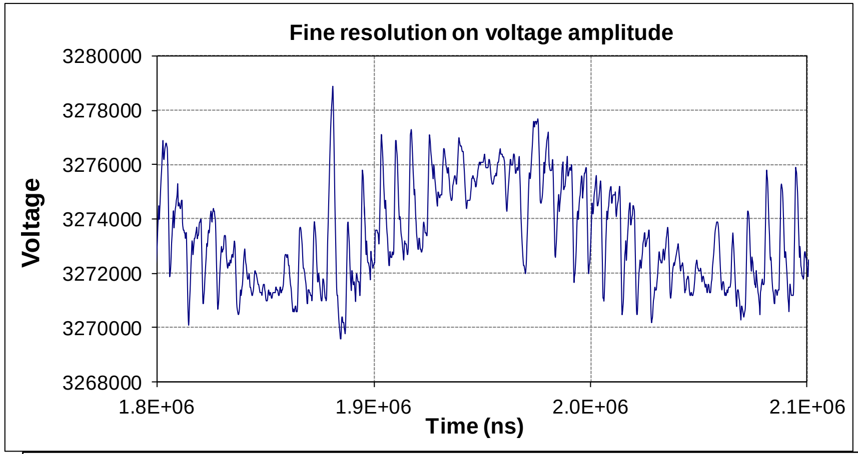
Quench is simulated by letting intrinsic Q fall to 1500
As expected the amplitude falls to a small level before the phase shift is significant

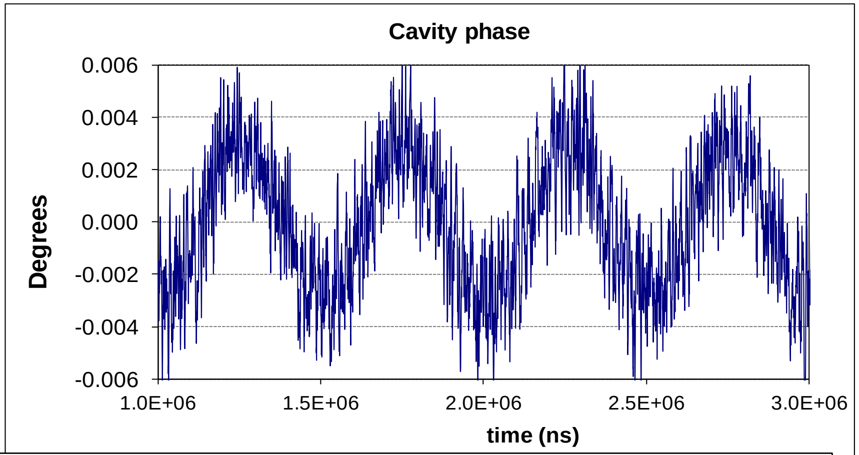


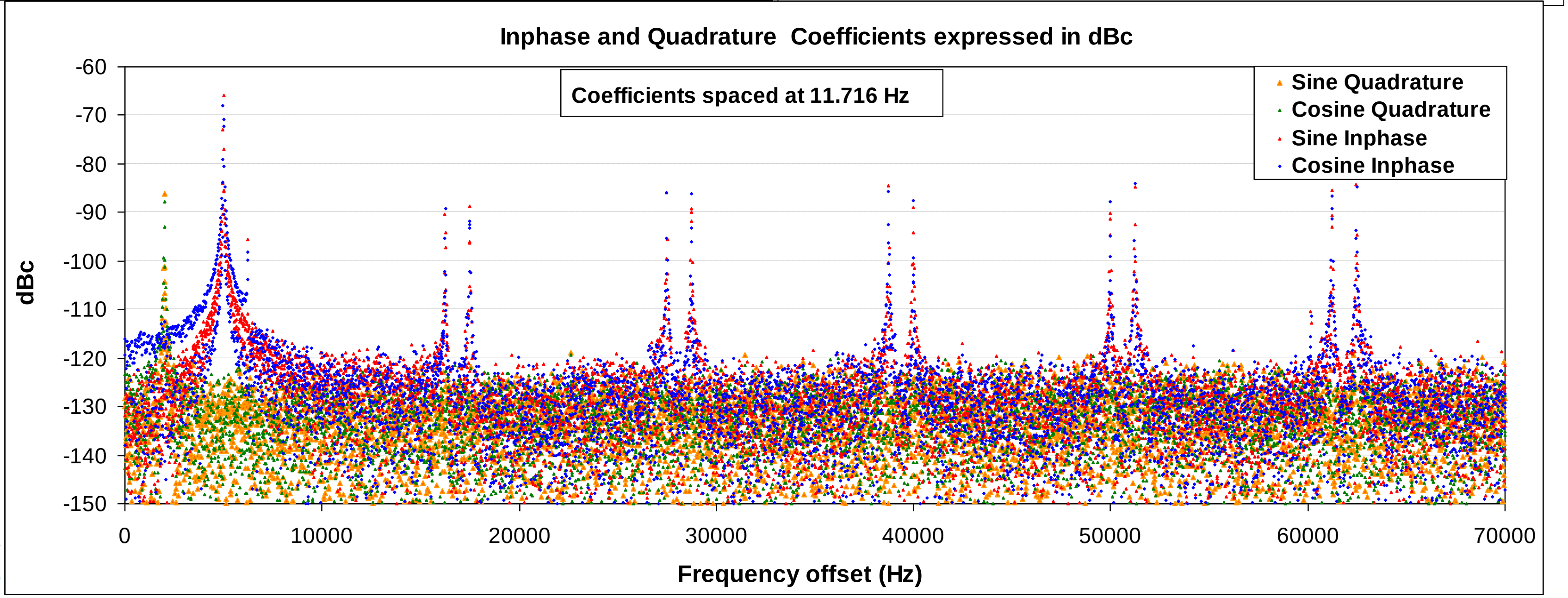
After quench and even if the beam is steered through cavity then the beam could still deposit 6 kW in cavity which is way too much for the cryo system. There is still a requirement to dump the beam after a crab cavity quench.

With errors

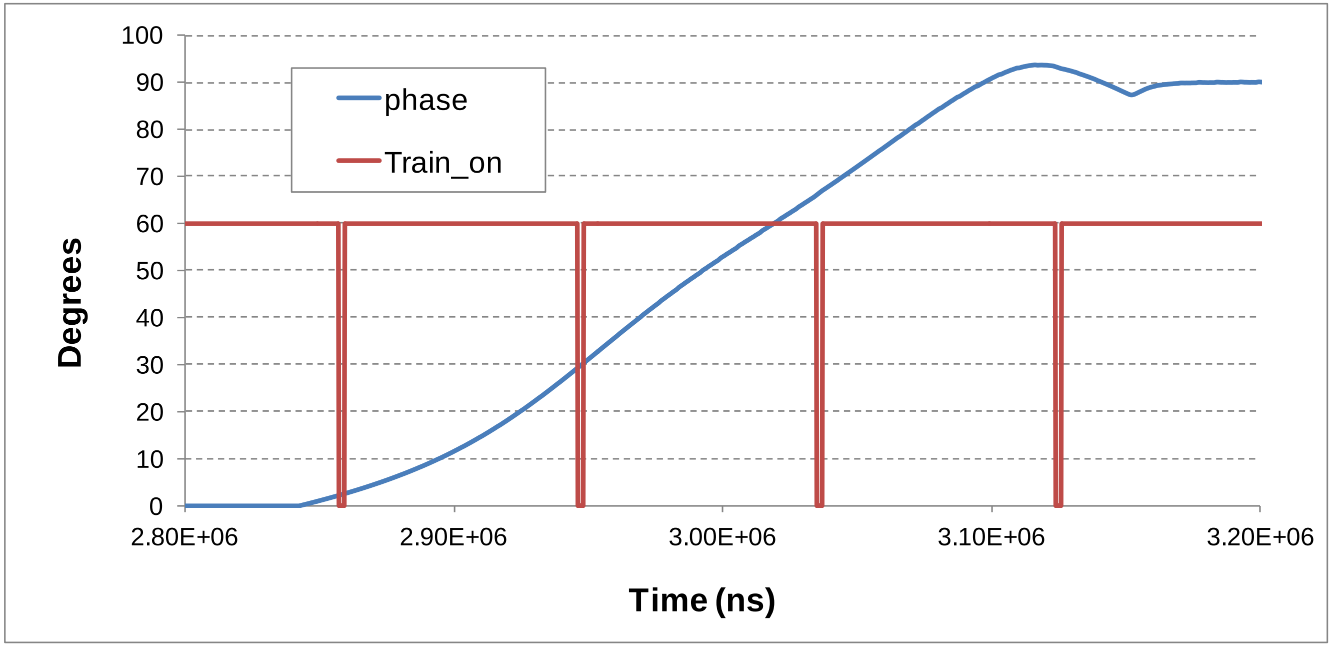
Amplitude measurement error = 0.01%
Phase measurement error = 0.005 degrees
Time delay = 0.5 μs







Detected RF Failure –input requests 90º error



Worst case scenario – LLRF input error requesting 90º cavity phase error. Response depends on the maximum RF power and external Q factor.

Phase error ~30º after 1 revolution – unacceptable deflection.

Once deviation detected transverse damping system could start correction hence some mitigation.

Cutting power does not solve problem.

Steering beam through cavity will provide protection.

RF Spectral Noise and Beam Effects

Sum voltage acting on a particle starting displacement at $x(t_o)$ with phase $\phi(t_o)$ is given as

$$V_x\{x(t_o),\phi(t_o)\} = \mathrm{Re}\left[-j\sum_m \{A_r(t)+jA_i(t)\}\exp\{\,j\omega_{RF}\,\phi(t)/c\,\}\,\delta(t-mT_o)\right]$$

$$V_z\{x(t_o),\phi(t_o)\} = \mathrm{Re}\left[\sum_m \{A_r(t)+jA_i(t)\}\frac{x(t)\omega}{c}\exp\{j\omega_{RF}\,\phi(t)/c\}\,\delta(t-mT_o)\right]$$

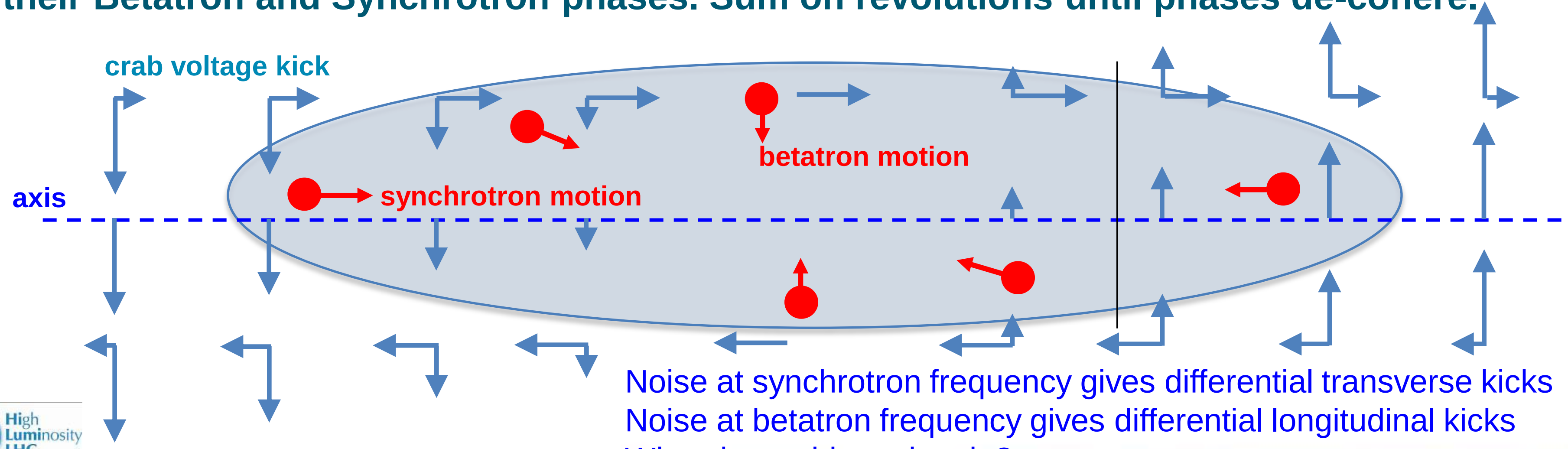
Both the phase ϕ and the offset x are oscillatory

$$x(s) = a\sqrt{\beta(s)}\cos\left(\int_0^s \frac{ds'}{\beta(s)} - \theta\right)$$

at crab cavity

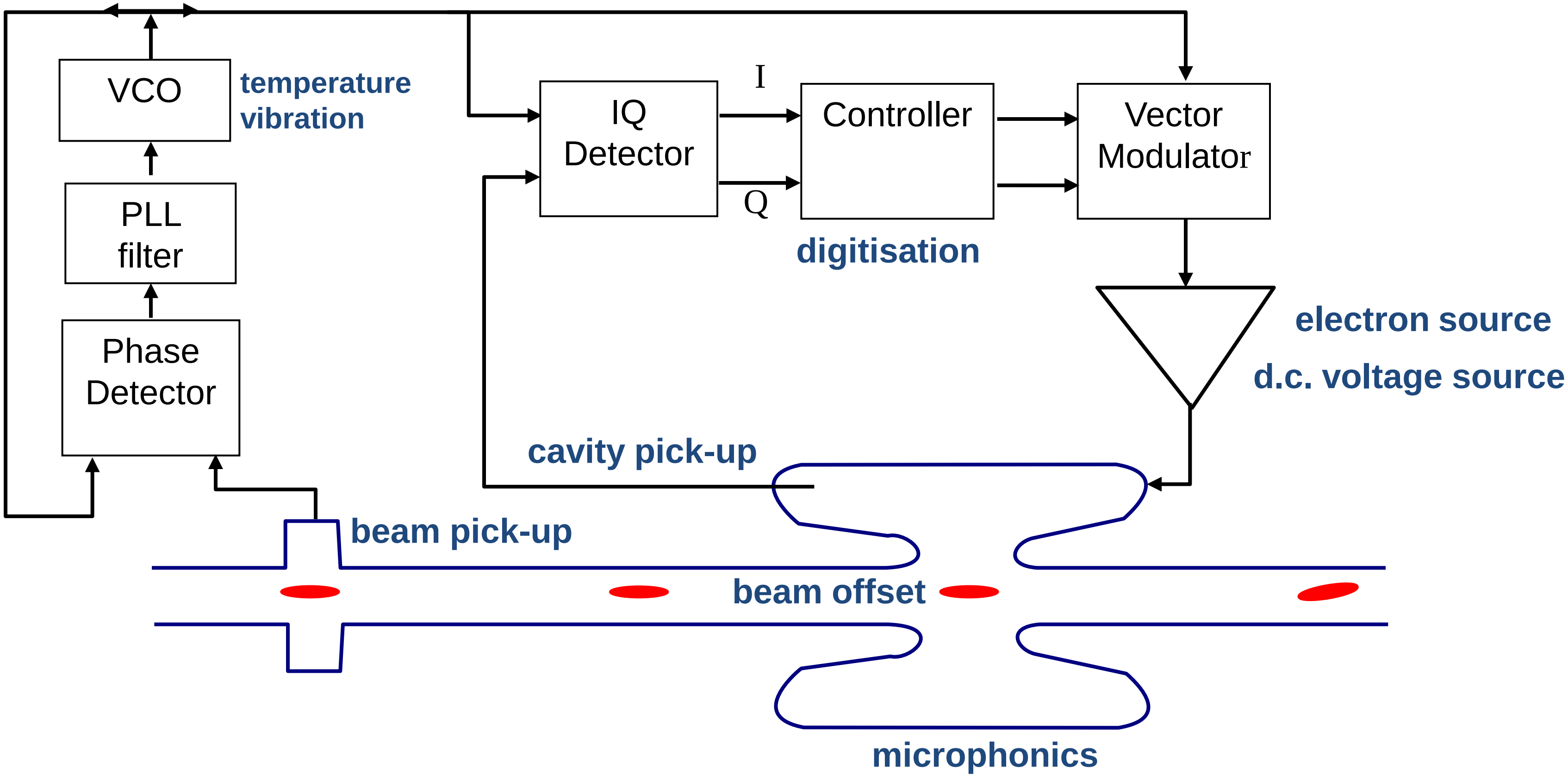
$$x_m = a\sqrt{\beta(s_c)}\cos(2\pi m\nu - \theta)$$

Can do explicit summations to get effective voltage acting on particles as a function of their Betatron and Synchrotron phases. Sum on revolutions until phases de-cohere.



Noise at synchrotron frequency gives differential transverse kicks
Noise at betatron frequency gives differential longitudinal kicks
What does white noise do?

Noise sources



getting 16 bit accuracy at 100 MBPS is difficult
main interest here is measurement errors between cavity pick-up and controller
beam offset probably known but use arbitrary values here
cavity can be modified to change microphonic spectrum but arbitrary values used

Kick contribution after m revolutions

For one disturbance frequency f_n the previous formulation for small bunches yields

$$\tilde{V}_x = \sum_{m=1}^{m=Revs}\left[\tilde{A}_{nr}\frac{\omega_{RF}}{2c}\Delta\phi\left\{\cos\left(\frac{f_n-f_s}{f_o}2\pi m-\theta_s\right)+\cos\left(\frac{f_n+f_s}{f_o}2\pi m+\theta_s\right)\right\}+\tilde{A}_{ni}\cos\left(\frac{f_n}{f_o}2\pi m\right)\right]$$

Transverse damping can only remove this term

$$\begin{aligned}\tilde{V}_z = \sum_{m=1}^{m=Revs}\Bigg[&\tilde{A}_{nr}\frac{\omega_{RF}}{2c}\Delta x\left\{\cos\left(\frac{f_n-f_b}{f_o}2\pi m-\theta_b\right)+\cos\left(\frac{f_n+f_b}{f_o}2\pi m+\theta_b\right)\right\}\\ &-\tilde{A}_{ni}\frac{1}{2}\left(\frac{\omega_{RF}}{c}\right)^2\Delta\phi\,\Delta x\left\{\cos\left(\frac{f_n-f_b}{f_o}2\pi m+\theta_b\right)+\cos\left(\frac{f_n+f_b}{f_o}2\pi m+\theta_b\right)\right\}\Bigg]\end{aligned}$$

From before Fourier coefficients can be determined for known disturbances

Terms containing the phase are different for differing particles in the same bunch

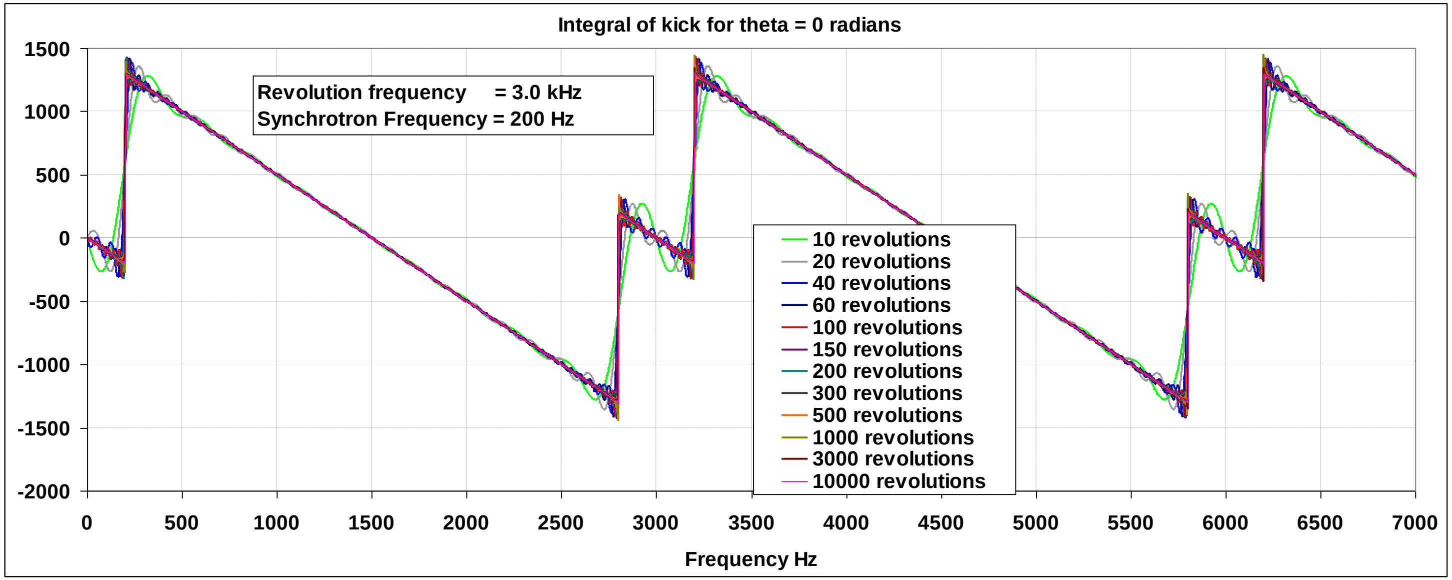
For all frequencies we are interested in summations of the form

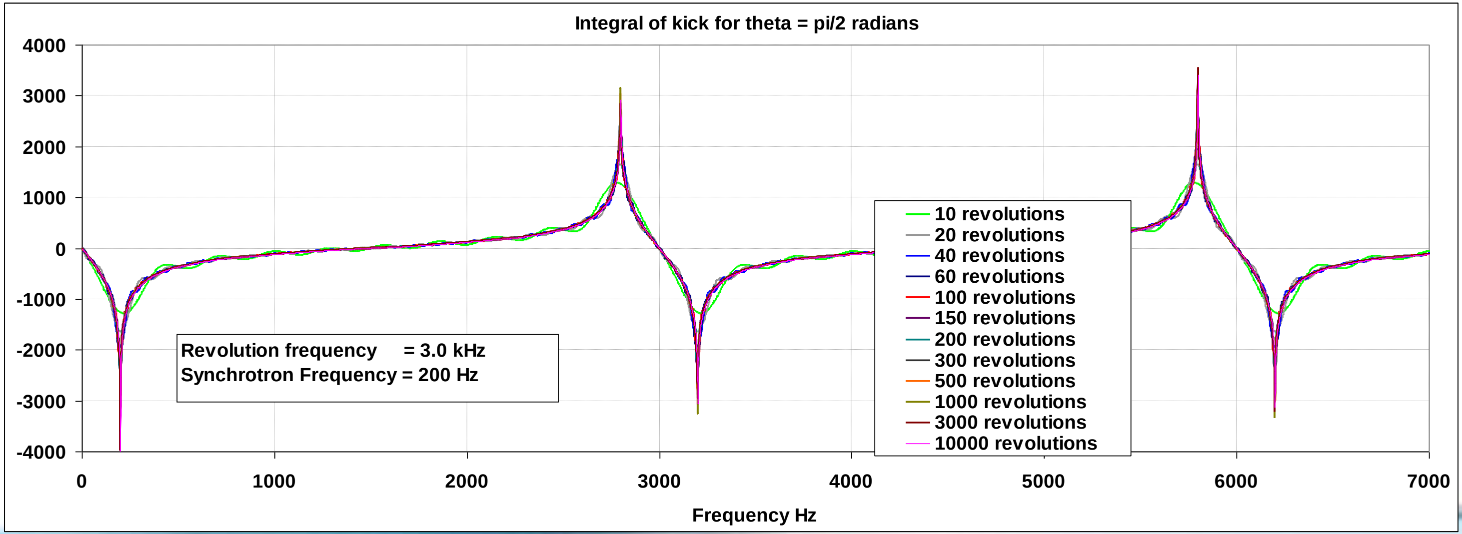
$$R = \sum_{m=1}^{m=Revs}\sum_{n=1}^{n=N}\tilde{A}_r(f_n)\left\{\cos\left(\frac{f_n-f_p}{f_o}2\pi m-\theta_p\right)+\cos\left(\frac{f_n+f_p}{f_o}2\pi m+\theta_p\right)\right\}$$

$$S = \sum_{m=1}^{m=Revs}\sum_{n=1}^{n=N}\tilde{B}_r(f_n)\left\{\sin\left(\frac{f_n-f_p}{f_o}2\pi m-\theta_p\right)+\sin\left(\frac{f_n+f_p}{f_o}2\pi m+\theta_p\right)\right\}$$

White noise

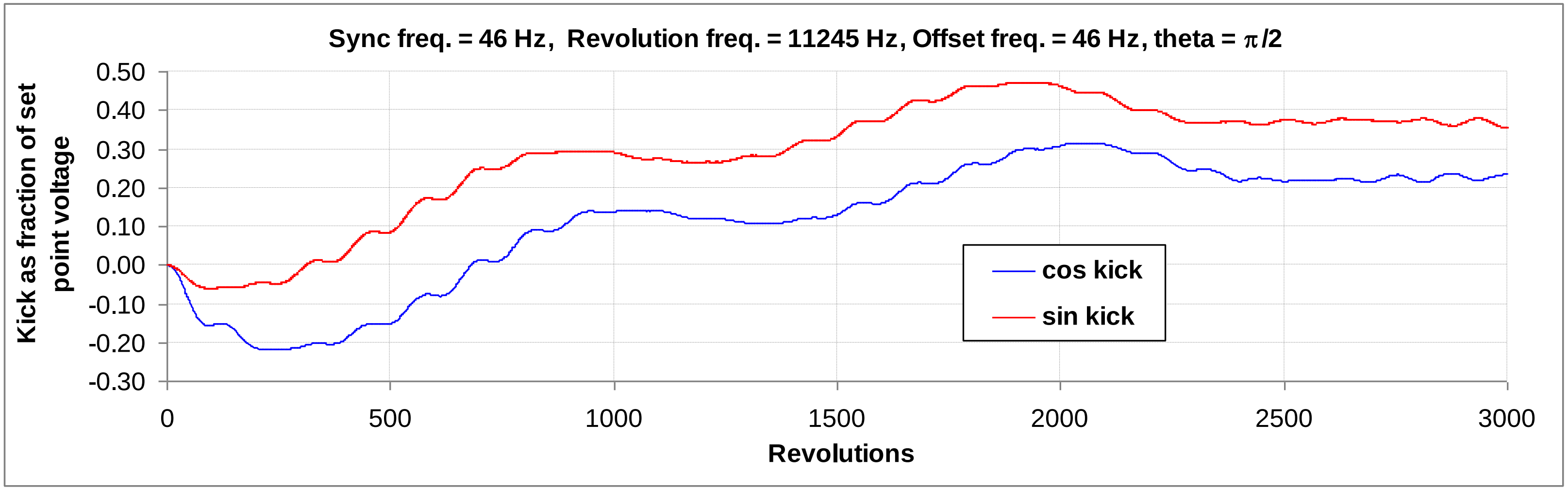
Big kicks clearly come from when frequencies are equal to revolution frequencies +/- the synchrotron or betratron frequency but for white noise there is perfect cancellation

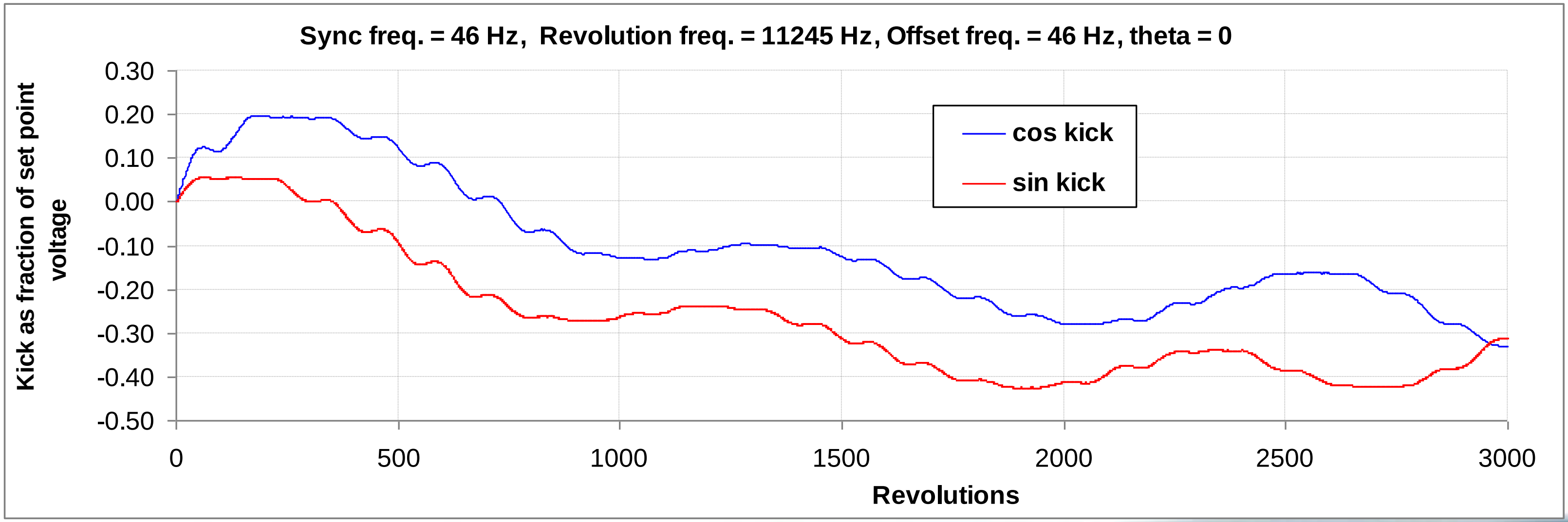




Sum kick for a disturbance at synchrotron freq.

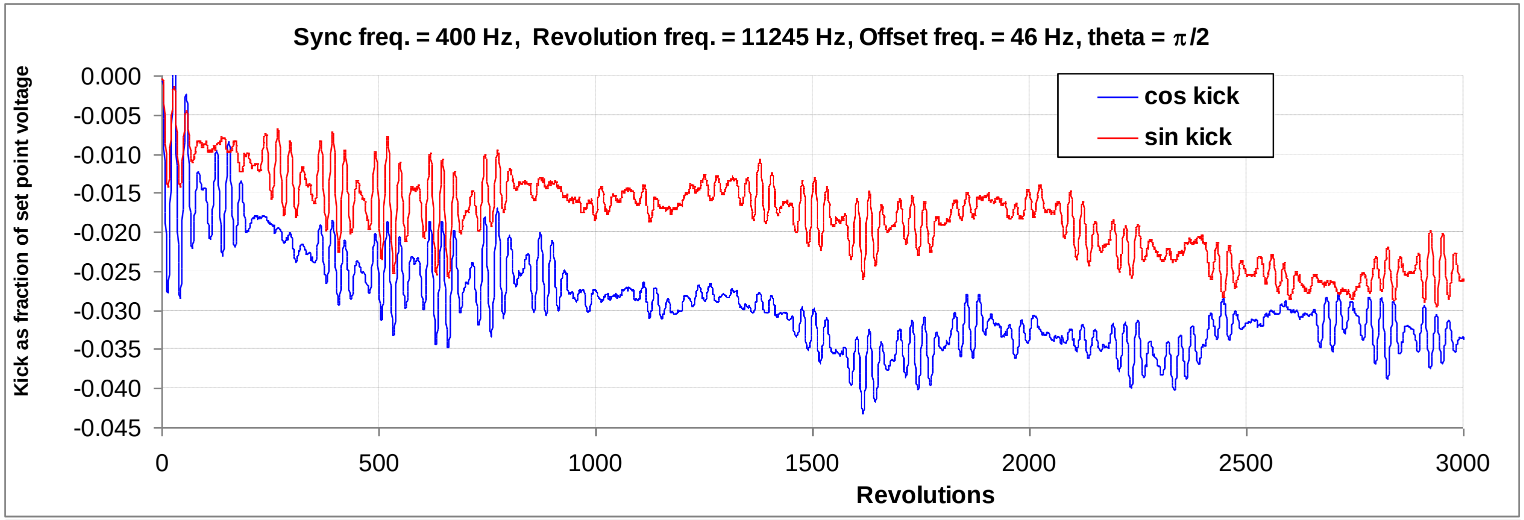
Graphs give voltage kicks acting on particles phased apart by 90 degrees in the same bunch. The kick becomes unacceptable after a few tens of turns.



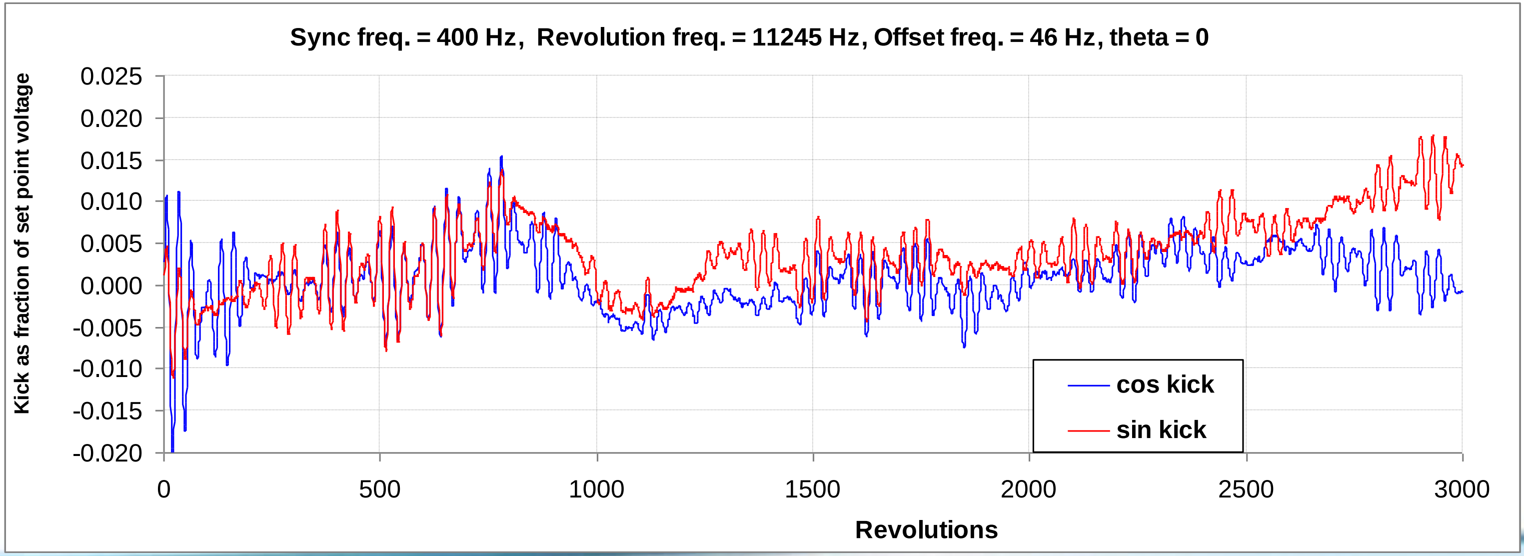


Kick as summation

For convenience have moved synchrotron frequency away from disturbance frequency
Kicks are worryingly large.



Sync freq. = 400 Hz, Revolution freq. = 11245 Hz, Offset freq. = 46 Hz, theta = $\pi/2$



Sync freq. = 400 Hz, Revolution freq. = 11245 Hz, Offset freq. = 46 Hz, theta = 0

Next Steps

1. **Obtain better time domain estimates of bunch offsets**
2. **Determine predictability of bunch offsets as this allows feed forward**
3. **Obtain likely microphonic spectra**
4. **Evaluate kick summations for a wide range of disturbance parameters**
5. **Repeat analysis for a crab cavity in the SPS**
6. **Determine key investigations that can validate expected bunch lifetimes**

Observations

1. **Steering beams through cavities is probably necessary for machine protection.**
2. **The largest contribution to RF noise (determining beam lifetime) is likely to be gaps in the bunch train. This can be mitigated by increasing the accuracy to which beams are steered through the crab cavities.**

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