Testing pairing interaction in multi-nucleon transfer reactions
Direct reactions are the traditional tool to get structure information from nuclear dynamics.

**What are direct reactions? (from Wikipedia)**

An intermediate energy projectile transfers energy or picks up or loses nucleons to the nucleus in a single quick ($10^{-21}$ second) event.

Energy and momentum transfer are relatively small.

These are particularly useful in experimental nuclear physics, because the reaction mechanisms are often simple enough to calculate with sufficient accuracy to probe the structure of the target nucleus.
Use of structure information is maximized by the introduction of microscopic potentials and formfactors into the reaction mechanism, fully exploiting the knowledge of the nuclear many-body functions

BUT

“In order to make theoretical predictions which may be compared with the experimental data on direct reactions, it is necessary to introduce a number of simplifying assumptions to reduce the many-body problem to a tractable form” (from Daphne Jackson)
Coulomb excitation, inelastic to collective states and single-particle transfer reactions are to large extent well described by first-order one-step mechanism. But two-nucleon transfer reactions are already complicated processes (and one can imagine now for multi-pair transfer .....)

It is widely accepted that pairing correlations strongly effect (and enhance) two-particle transfer reactions. But the quantitative connection is not obvious. Will cross section scale with the square of the two-particle transfer matrix elements? Or the radial dependence of the two-particle transition densities contain more information? And how this information enter into the reaction mechanism?
Example of structure investigation of pairing correlations: can we discriminate among different forms of pairing interactions? Example: can we distinguish pure surface pairing interaction from mixed (volume plus surface) used in HFB Calculations? (from Grasso, Lacroix, AV)

How the different behavior in the tail enter in the reaction mechanism? Is only the integrated value pair strength $T_0$ relevant?
BASIC PROBLEM: The reaction mechanism (hopefully reasonably physically correct but on the same time sufficiently simple)

Large variety of models on the market (and this is not a good signal)

The fully microscopic approach is based on sequential two-step process (each step transfers one particle)

Microscopy: Pairing enhancement comes from the coherent interference of the different paths through the different intermediate states in (a-1) and (A+1) nuclei, due to the correlations in initial and final wave functions

Building blocks: single-particle formfactors and many-body wf’s

Problems: quantal calculations rather complex (taking into account full recoil), semiclassical more feasible (but approximate treatment of recoil)
All microscopy and nuclear structure information are contained in the two-particle transfer amplitudes (from correlated initial and final wave functions, so provided by structure models), which give the weight of each two-step path, and in the single particle transfer formfactors, which need single particle wavefunctions in target and projectile.

Obs: Basic idea: dominance of mean field, which provides the framework for defining the single-particle content of the correlated wave functions.

Cf talk by Enrico Vigezzi
But moving from the stability line .......
Two-particle transfer will proceed mainly by constructive interference of successive transfers through the (unbound) continuum intermediate states.

The integration over the continuum intermediate states can become feasible by continuum discretization: but how many paths should we include? Thousands or few, for example only the resonant (Gamow) states?

But continuum is a Pandora vase ......
But, aside from continuum, we have other serious problems, including

- Q-value effects
- Population of multipole states other than $0^+$ states (which are supposed to be those displaying pairing correlations) with the risk of completely masking the monopole response
Q-value effect

Keeping fixed any other parameter, the probability for populating a definite final channel depends on the Q-value of the reaction. The dependence (in first approximation a gaussian distribution centered in the optimum Q-value) is very strong in the case of heavy-ion induced reactions, weaker in the case of light ions.

The optimum Q-value depends on the angular momentum transfer and on the charge of the transferred particles. In the specific case of L=0 two-neutron transfer, the optimal Q-value is close to zero.

But the actual Q-value for two-particle transfer to the ground states may be different from zero .....
Experimental evidence

Negligible transitions to GS due to Q-value effects. What information on pairing correlations?

Example

$^{96}\text{Zr} + ^{40}\text{Ca}$

Selecting final $^{42}\text{Ca}$ mass partition
Playing with different combinations of projectile/target (having different $Q_{gg}$-value) one can favour different energy windows.

Example: Target $^{208}_{\text{Pb}}$ Final $^{210}_{\text{Pb}}$ (at bombarding energy $E_{cm} = 1.2 \times E_{\text{barrier}}$)
The width of the Q-value window increases with the bombarding energy.
The pairing strength is therefore modulated by the Q-value cut-off to yield the final two-particle cross section.
$^{208}\text{Pb} (^6\text{He}, ^4\text{He})^{210}\text{Pb} (0^+ \text{ states})$

- **RPA**
- **TDA**
- **sp**

**Graphs**:
- **gs** and **gpv**
- **cross section** and **strength**
- **E (MeV)**

- **RPA**
- **TDA**
- **sp**
As a result, the correlated states may be populated in a much weaker way than uncorrelated states.

Example: $^{96}{\text{Zr}}^+{^{40}}{\text{Ca}}$, leading to $^{42}{\text{Ca}}$. In this case is favored the excitation of an “uncorrelated” $0^+$ state at about 6 MeV.

Corradi, Pollarolo et al, LNL
Multipole selection

“standard” pairing interaction is associated with $\lambda=0$ states and $\lambda=0$ two-particle creation (or removal) response.

To investigate pairing interaction one needs therefore to select the population of $\lambda=0$ states (trivial issue for theorists, not for experimentalists).

Possible clue: shape of angular distributions

* note, however, the role played by “quadrupole” pairing in deformed nuclei
Angular distribution

With light ions at forward angles one excites selectively $0^+$ states
The excited states in $^{114}$Sn are of proton character at $Z=50$ closed shell
Pair transfer intensities

PV

removal phonon

A-2

addition phonon

A+2

A
Vibrational pairing spectrum around closed shell: neutron case around $^{208}\text{Pb}$
Proton pairing vibration at $Z=50$ closed shell $\text{He}^3,n$ reactions
Lowest 0+, 2+, 4+ states

Guazzoni et al.

Obs: Cross section to 0+ state orders of magnitude larger at 0° degrees
Angular distribution

Situation different for heavy-ions induced pair transfer processes: angular distributions are always peaked around the grazing angle, independently of the multipolarity

$^{48}\text{Ca}(^{18}\text{O},^{16}\text{O}_\text{gs})^{50}\text{Ca}(\lambda)$
Angular distribution
Situation different for heavy-ions induced pair transfer processes:
angular distributions are always peaked around the grazing angle, independently of the multipolarity

$^{48}$Ca$(^{18}$O, $^{16}$O$_{gs}$)$^{50}$Ca($\lambda$)

Note however forward angles
Higher multipolarities

Far from the very forward angles the pairing vibrational states are overwhelmed by states with other multipolarities

Example:
predicted total cross sections in $^{120}\text{Sn}(p,t)^{118}\text{Sn}^*$ reaction
Searching the Giant Pairing Vibration 

\[ \frac{d\sigma}{dE} \]

\[ ^{120}_{\text{Sn}}(p,t)^{118}_{\text{Sn}} \]

\[ E_p = 42 \text{ MeV} \]

Bortignon and AV
Bump at 10 MeV does not come from GPV, but from incoherent sum of different multipolarities.
Multi-nucleon transfer reactions

The situation is becoming orders of magnitude more complex in the case of multi-particle (or multi-pair) transfers. By definition it cannot be treated as a “genuine” direct process. When restricted to the population of the 0+ ground states it is a key case as test of pairing modes in the “vibrational multiphonon-like” and in the “rotor-like” pairing cases. But in fact one is progressively populating also the excited states, and the whole process is highly coupled, involving pairing, single particle, collective excitations, non-collective excitations, etc. The whole process is fundamental in describing the transition from grazing reactions to more central deep-inelastic collisions.

OBS: instrumental for structure studies with γ-spectroscopy for systems far from stability, but this is another story .....
Example of multi-nucleon transfers at Legnaro (cf the talk of Lorenzo Corradi this afternoon)

\[ ^{40}\text{Ca}+^{208}\text{Pb} \]

Obs: transfer of particles on both directions
Transition from direct to deep inelastic (cf Q distributions)

Example: Neutron transfer channels
(odd-even transfer effect? Structure effect?)
A way to define a pairing “enhancement” factor, by plotting transfer probabilities not as function of the scattering angle, but as function of the distance of closest approach of the corresponding classical trajectory.

distance of closest approach
General problem: how separate the contribution of $0^+$ states? Q-distributions?
Proton transfer

\[ ^{208}\text{Pb} + ^{144}\text{Sm} \]

Transfer function \( P_{tr} \)

- \(-1p\)-Transfer
- \(-2p\)
- \(-3p\)
- \(-4p\)
- \(-5p\)
- \(-6p\)

\( d_0 \) \( \text{fm} \)

\( (P_{1p})^2 \)

\( (P_{2p})^3 \)
Basic and most popular approach: “Grazing model” (Nanni Pollarolo and Aage Winther)

- semiclassical description of trajectory
- single-particle transfers
- two-particle transfers (double counting?)
- collective inelastic excitations
- sufficient phase-space for multi-transfer?
- “bare” ion-ion potential?
- structure information?
- excellent for “average” behavior.
  Specific cases?
  Weakly-bound systems and treatment of continuum?
- collective vs non-collective transfer (and non L=0 pairs)

$^{40}\text{Ca} + ^{124}\text{Sn} \quad \text{E}_{\text{lab}} = 170 \text{ MeV}$

![Graphs showing neutron number distribution for different scenarios with experimental data and Grazing model predictions.](image)
Other approach: TDHF with particle number projection technique (Cedric Simenel), with reduction of computational time by a factor 100 (still in fieri and under discussion the range of validity: maximum number of transferred particles? Residual correlations? In particular pairing correlations? Reaction mechanism beyond mean field?)

Example: \(^{40}\text{Ca}^{124}\text{Sn} \ E_{\text{lab}}=174\text{ MeV}\)

Sekizawa and Yabana
SLy5

OBS: Overall agreement is good when transferred proton number is small
Simple models (for the multi-pair transfer to the ground state sequence)

Simple models can be developed exploiting the formal analogy between “macroscopic” models for surface vibrations (and rotations) and “macroscopic” models for pairing, In the latter case, it is the mass number that can vibrate in a two-dimensional “gauge” space (regime of pairing vibrations) or lead to a stable deformation (regime of pairing rotation) characterized by a “pairing” deformation parameter $\beta_P$ (proportional to the pairing gap $\Delta$), analogous to the “standard” deformation parameter $\beta$. 
All ground states share the same intrinsic "deformed" states.

\[ |A_0 + \Delta N\rangle = \exp\{i \Delta N \Phi\} |\psi^{A_0}_{\text{intr}}\rangle \]

\[ \rho_0 \rightarrow \delta \rho_2 \rightarrow \delta \rho_4 \rightarrow \delta \rho_6 \]

- Normal phase: closed-shell nuclei
- Superfluid phase: open-shell nuclei

Bohr & Mottelson

Brogia, et al.
Excitation of a normal rotor (classical description)

One has to average over the different possible initial orientations. Classically there is a maximum angular momentum transfer $J_{\text{max}}$ (for $\phi = 45$ degr), that depends on the deformation $\beta$, bombarding energy and charges. In the quantal treatment, interference among different orientation gives rise to oscillation in the classically allowed range.
In this case one has to average over the different possible initial orientations in gauge space.

"Classically" there is a maximum number of transferred pairs $\Delta N_{\text{max}}$ (for $\phi_{\text{gauge}} = 45$ degr), that depends on the pairing deformation $\beta_p$, bombarding energy and mass of projectile.

In the quantal treatment, interference among different orientation gives rise to oscillations in the classically allowed range

OBS: Realistic value of $\beta_p$
In the pairing case one cannot play with the strength of the Coulomb interaction (e.g. increasing the charge of the projectile) and the classical allowed maximum number of transferred pairs may be small, so that we fall practically always in the classically forbidden region (so having an always decreasing probability for increasing numbers of pairs).

Lotti, Dasso, Vitturi
Multi-pair transfer for deformed nuclei:
Simultaneous excitation of “normal” rotor
(angular momentum transfer) and “pairing” rotor
(mass transfer)

Oscillatory behavior in the spin excitation

Purely decaying behavior in the multiple mass transfer

Lotti, Dasso, Vitturi
Different cuts

Fixed final mass

Fixed final spin

Obs: clear spin vs mass transfer correlations