

# Two-nucleon transfer reactions

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## Possible Analogy between the Excitation Spectra of Nuclei and Those of the Superconducting Metallic State

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(Received January 7, 1958)

The evidence for an energy gap in the intrinsic excitation spectrum of nuclei is reviewed. A possible analogy between this effect and the energy gap observed in the electronic excitation of a superconducting metal is suggested.

THE nuclear structure exhibits many similarities with the electron structure of metals. In both cases, we are dealing with systems of fermions which may be characterized in first approximation in terms of independent particle motion. For instance, the statistical level density, at not too low excitation energies, is expected to resemble that of a Fermi gas. Still, in both systems, important correlations in the particle motion arise from the action of the forces between the particles and, in the metallic case, from the interaction with the lattice vibrations. These correlations decisively influence various specific properties of the system. We here wish to suggest a possible analogy between the correlation effects responsible for the energy gaps found in the excitation spectra of certain types of nuclei and those responsible for the observed energy gaps in superconducting metals.

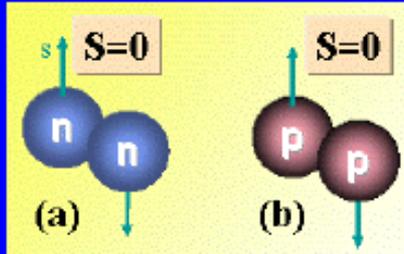
proximately<sup>1</sup>

$$\delta \approx 50A^{-1} \text{ Mev}, \quad (1)$$

where  $A$  is the number of particles in the nucleus.

If the intrinsic structure could be adequately described in terms of independent particle motion, we would expect, for even-even nuclei, the first intrinsic excitation to have on the average an energy  $\frac{1}{2}\delta$ , when we take into account the possibility of exciting neutrons as well as protons. Empirically, however, the first intrinsic excitation in heavy nuclei of the even-even type is usually observed at an energy of about 1 Mev (see Fig. 1). The only known examples of intrinsic excitations with appreciably smaller energy are the  $K=0-$  bands which occur in special regions of nuclei, and which may possibly represent collective octupole vibrations.<sup>2</sup>

# Neutron and proton pairing in nuclei



Pairing of even numbers of neutrons or protons outside closed shells

\*David Pines to Niels Bohr's Institute in Copenhagen, Summer 1957, just as BCS was being finished in Urbana.

\*Aage Bohr, Ben Mottelson and Pines (57) suggest BCS pairing in nuclei to explain energy gap in single particle spectrum

– odd-even mass differences

\*Rehovat Conference, Sept. 1957

\*Pairing gaps deduced from odd-even mass differences:

$$\Delta \simeq 12 A^{-1/2} \text{ MeV for both protons and neutrons}$$

# Pairing correlations enhance two-nucleon transfer cross sections

2.G

*Nuclear Physics* **33** (1962) 685—692; © North-Holland Publishing Co., Amsterdam

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## NOTE ON THE TWO-NUCLEON STRIPPING REACTION

SHIRO YOSHIDA †

*Radiation Laboratory, University of Pittsburgh, Pittsburgh, Pennsylvania* ††

Received 9 February 1962

**Abstract:** The magnitude of the two-nucleon stripping reactions is calculated using the pairing interaction model. The calculation also is applied to final states of collective type. For some types of reaction a collective enhancement of the reaction cross section is predicted.

## Form factor for the transfer of a $J=0$ pair

PHYSICAL REVIEW

VOLUME 156, NUMBER 4

20 APRIL 1967

## Relative-Angular-Momentum-Zero Part of Two-Nucleon Wave Functions\*

B. F. BAYMAN AND A. KALLIO†

*School of Physics, University of Minnesota, Minneapolis, Minnesota*

(Received 28 November 1966)

A method is given for finding the relative-angular-momentum-zero part of the wave function of two particles moving in a finite single-particle potential. The results are applied to form factors for two-nucleon transfer reactions and to two-nucleon interaction matrix elements.

# Review of two-neutron transfer experiments; pairing rotations and pairing vibrations

Reprinted from:  
ADVANCES IN NUCLEAR PHYSICS  
VOLUME 6  
Edited by Michael Baranger and Erich Vogt  
(Plenum Press, 1973)

## Chapter 3

### TWO-NEUTRON TRANSFER REACTIONS AND THE PAIRING MODEL

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and*

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Karl Marx Stadt, D.D.R.*

# 2<sup>nd</sup> order DWBA: Sequential and simultaneous contributions

PHYSICAL REVIEW C

VOLUME 26, NUMBER 4

OCTOBER 1982

## One-step and two-step contributions to two-nucleon transfer reactions

B. F. Bayman and Jongsheng Chen\*

*School of Physics and Astronomy, University of Minnesota,  
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(Received 26 March 1982)

PHYSICS REPORTS (Review Section of Physics Letters) 199, No. 1 (1991) 1-72. North-Holland

## TWO-NUCLEON TRANSFER REACTION MECHANISMS

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Editor: G.E. Brown

Received April 1990

Coupled channel effects

# Treatise on Heavy-Ion Science

Volume 1  
Elastic and Quasi-Elastic Phenomena

EDITED BY

D. ALLAN BROMLEY

*Henry Ford II Professor of Physics  
Yale University  
New Haven, Connecticut*

One- and Two-Nucleon Transfer  
Reactions Induced by Heavy Ions—  
Interplay of Nuclear Structure and  
Reaction Mechanisms

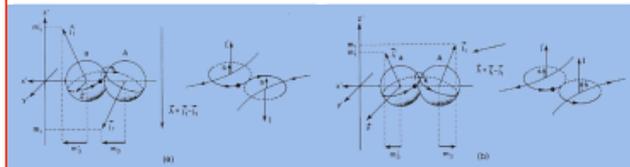
ROBERT J. ASCUITTO AND ERNEST A. SEGLIE

Semiclassical theory

FRONTIERS IN PHYSICS

# HEAVY ION REACTIONS

THE ELEMENTARY PROCESSES,  
PARTS I & II



ABP

Ricardo A. Broglia  
Aage Winther

## **Pairing correlations of nucleons and multi-nucleon transfer between heavy nuclei**

**W von Oertzen<sup>1,2</sup> and A Vitturi<sup>3</sup>**

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<sup>2</sup> Fachbereich Physik, Freie Universität Berlin, Germany

<sup>3</sup> Dipartimento di Fisica, Università di Padova, and INFN, Padova, Italy

A. Vitturi, Kyoto, DCEN 2011

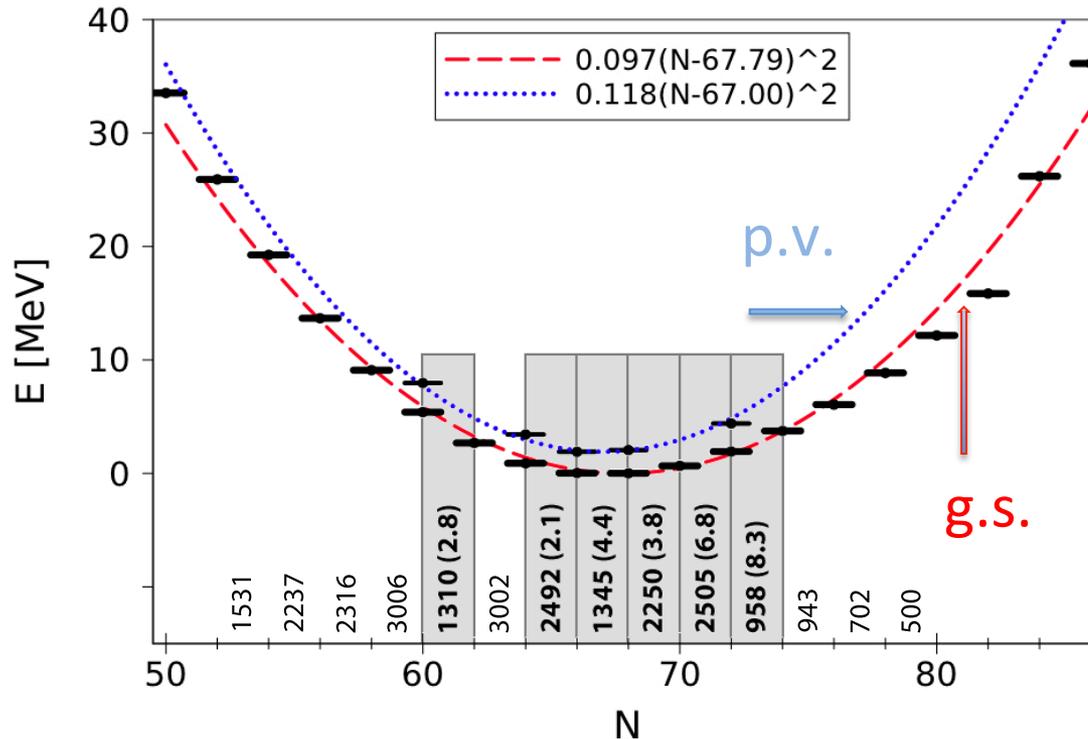
<http://www2.yukawa.kyoto-u.ac.jp/~ykis2011/dcen/slide/workshop4/vitturi.pdf>

50 years of nuclear BCS ,eds. R.A. Broglia and V. Zelevinsky,  
World Scientific, to be published

# Pairing rotational band in superfluid tin isotopes

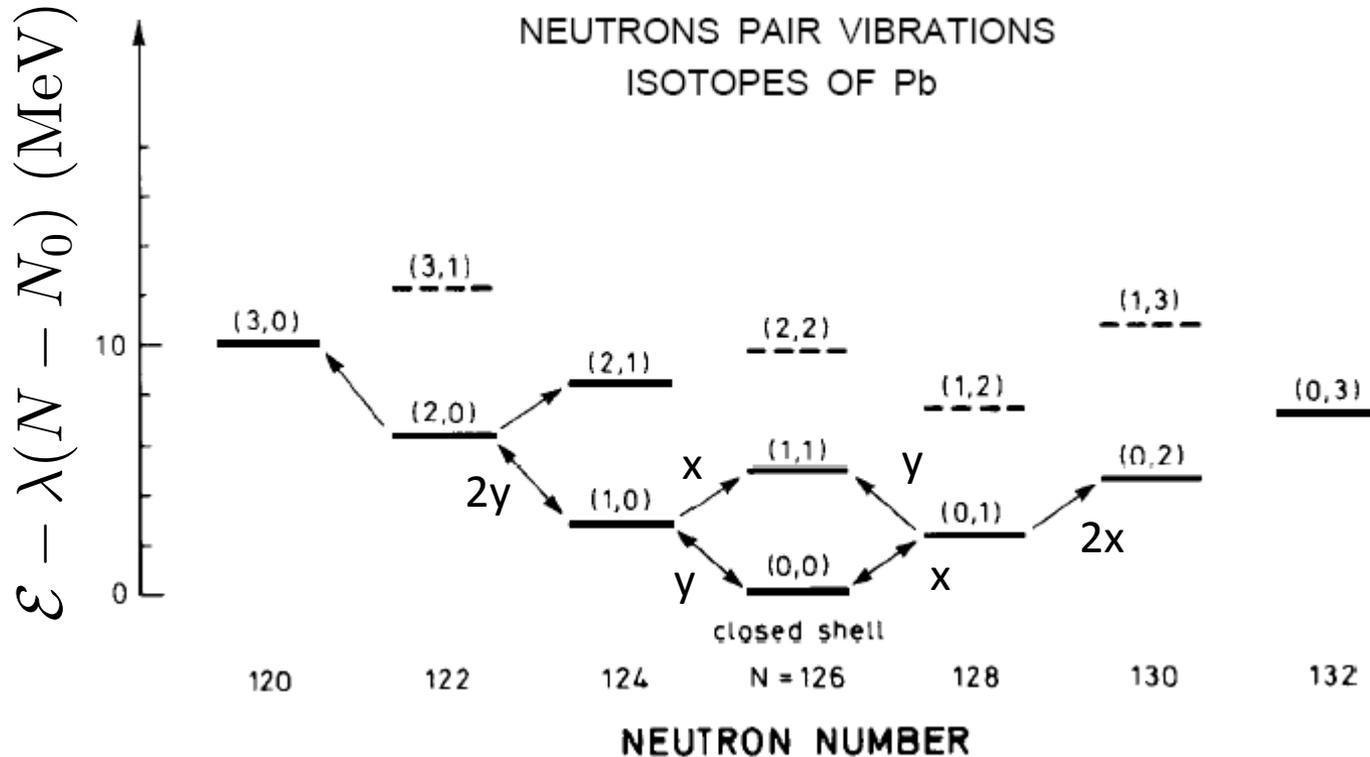
- Static deformation of the pair field
- Rotational-like spectrum formed by a sequence of ground states of even- $N$  systems

$$E_N = \frac{1}{2\mathcal{J}}(N - N_0)^2$$



# Pairing Vibrations

(Nobel Lecture, Ben R. Mottelson, 1975)



- Near closed shell nuclei (like  $^{208}\text{Pb}$ ) no static deformation of pair field
- Vibrational-like excitation spectrum.
- Enhanced pair-addition and pair-removal cross-sections seen in (t,p) and (p,t) reactions (indicated by arrows).

Two-neutron transfer is the specific probe  
of pairing correlations

In superfluid nuclei,

$$\alpha_0 = \langle BCS | P^+ | BCS \rangle = \sum_{\nu > 0} U_\nu V_\nu \sim \Delta / G$$

$$d\sigma / d\Omega(A, g.s. \rightarrow A + 2, g.s) \sim \alpha_0^2$$

In normal nuclei,  $\alpha_0 = 0$

$$d\sigma / d\Omega \sim \langle (\alpha - \alpha_0)^2 \rangle = [\langle 0 | P^+ P | 0 \rangle + \langle 0 | P P^+ | 0 \rangle] / 2$$

A wise opinion (John Schiffer)

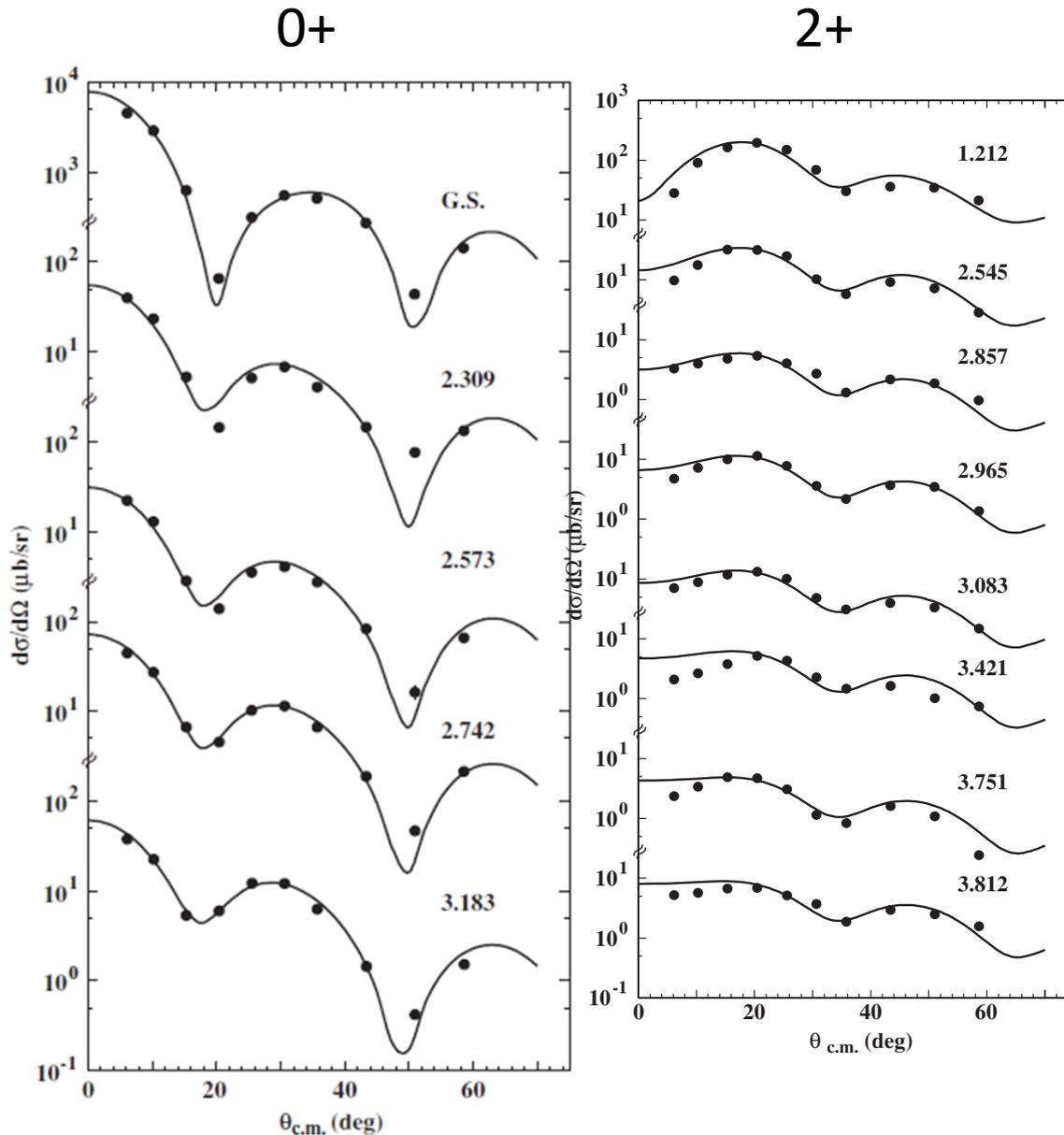
**Simple ways of treating data** can allow us to extract essential structural information, even if reaction theorists are not completely happy.

One hopes that a new generation will be able to start from what was Learned, and build on it, **and not get bogged down with the fascinating sophistries of reaction theories**. We need not have to rediscover all the blind alleys (of two-step processes, coupled channels ... etc) that obscured simple underlying information

All experiments should be feasible, exciting and interpretable, but they most rarely enjoy more than two of the three properties, frequently failing the interpretability test (V. Telegdi)

## Some recent 2-nucleon transfer experiment

$t(^{30}\text{Mg}, ^{32}\text{Mg})p$	K. Wimmer et al. ,PRL 105 (2011) 252501
$p(^{11}\text{Li}, ^9\text{Li})t$	I. Tanihata et al., PRL 100 (2008) 192502
$p(^8\text{He}, ^6\text{He})t$	N. Keeley et al., PLB 646 (2007) 222
$^{121}\text{Sb}(p,t)^{119}\text{Sb}$	P. Guazzoni et al., J.Phys.G 34 (2007)2665
$^{120}\text{Sn}(p,t)^{118}\text{Sn}$	P. Guazzoni et al., PRC 78 (2008)064608
$^{134}\text{Ba}(p,t)^{132}\text{Ba}$	S. Pascu et al., PRC 81 (2010)014304
$^{65}\text{Cu}(^6\text{He}, ^4\text{He})^{67}\text{Cu}$	A. Chatterjee et al., PRL 101 (2008) 032701
$^9\text{Be}(^{18}\text{O}, ^{16}\text{O})^{11}\text{Be}$	M. Cavallaro et al., J. Phys G Conf. Ser. 312(2011)092020
$^{40}\text{Ca}(^{96}\text{Zr}, ^{94}\text{Zr})^{42}\text{Ca}$	L. Corradi et al. , PRC 84(2011)034603

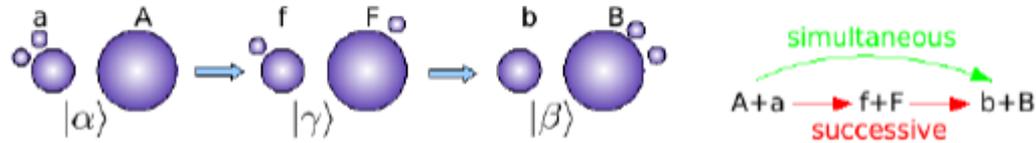


Systematic high-quality  $^{A+2}\text{Sn}(p,t)^A\text{Sn}$  angular distributions for  $112 < A < 124$ :  
 Guazzoni et al., PRC 60 054603 (1999);  
 74 054605 (2006); ecc.

Note change of scale of cross sections and of angular distributions between  $0+$  and  $2+$

Fits obtained using normalization factors and assuming dineutron transfer (one step); angular distributions are well described.

# simultaneous and successive contributions



2-body spect. ampl.

$U_{jf} V_{jf}$

$$T = \sum_{j_i j_f} B_{j_f}(B, A) B_{j_i}(a, b) T^{(1)}(j_f, j_i) + C_{j_f}(B, F) C_{j_f}(F, A) C_{j_i}(a, f) C_{j_i}(f, b) \times [T_{succ}^{(2)}(j_f, j_i) - T_{NO}^{(2)}(j_f, j_i)]$$

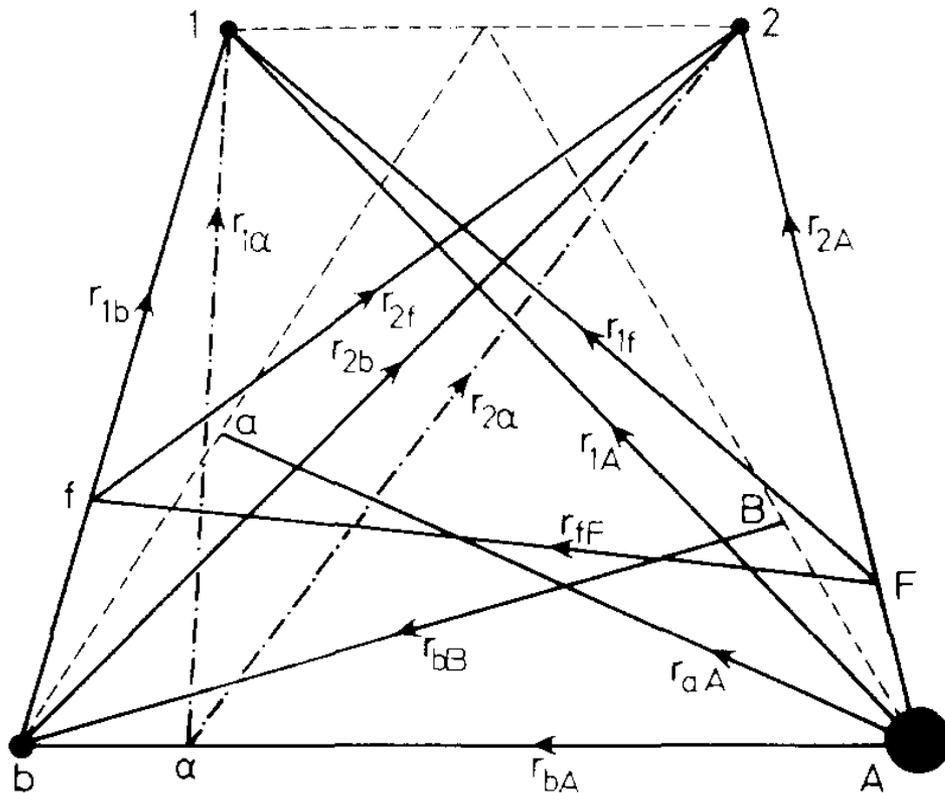
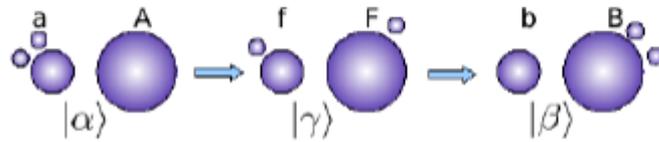
$U_{jf} \times V_{jf}$

1-body spect. ampl.

Both successive and simultaneous terms are coherent

# Calculation of absolute two-nucleon transfer cross section by finite-range DWBA calculation

simultaneous and successive contributions

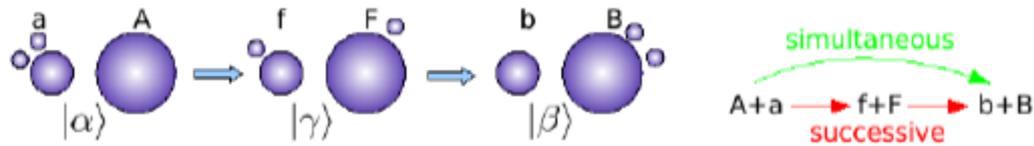


$$|\alpha\rangle = \phi_a(\xi_b, \mathbf{r}_1, \mathbf{r}_2) \times \phi_A(\xi_A) \chi_{aA}(\mathbf{r}_{aA})$$

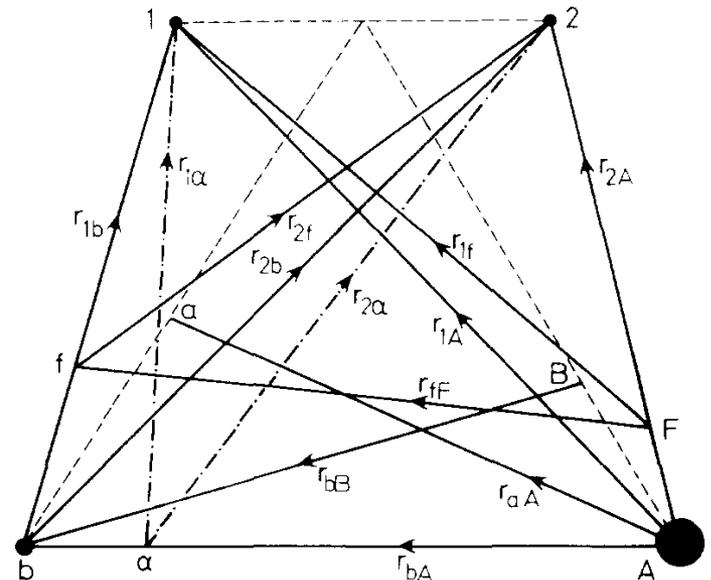
$$|\beta\rangle = \phi_b(\xi_b) \phi_B(\xi_A, \mathbf{r}_1, \mathbf{r}_2) \times \chi_{bB}(\mathbf{r}_{bB})$$

B.F. Bayman and J. Chen,  
Phys. Rev. C 26 (1982) 1509  
G. Potel et al., arXiv:0906.4298

# Simultaneous transfer

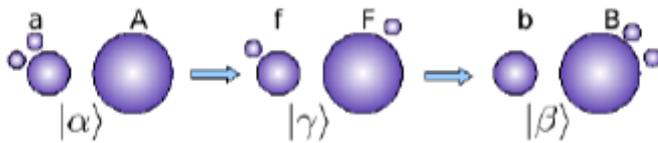


$$T^{(1)}(j_i, j_f) = 2 \sum_{\sigma_1 \sigma_2} \int d\mathbf{r}_{fF} d\mathbf{r}_{b1} d\mathbf{r}_{A2} [\Psi^{j_f}(\mathbf{r}_{A1}, \sigma_1) \Psi^{j_f}(\mathbf{r}_{A2}, \sigma_2)]_0^{0*} \chi_{bB}^{(-)*}(\mathbf{r}_{bB}) \\ \times v(\mathbf{r}_{b1}) [\Psi^{j_i}(\mathbf{r}_{b1}, \sigma_1) \Psi^{j_i}(\mathbf{r}_{b2}, \sigma_2)]_0^0 \chi_{aA}^{(+)}(\mathbf{r}_{aA})$$

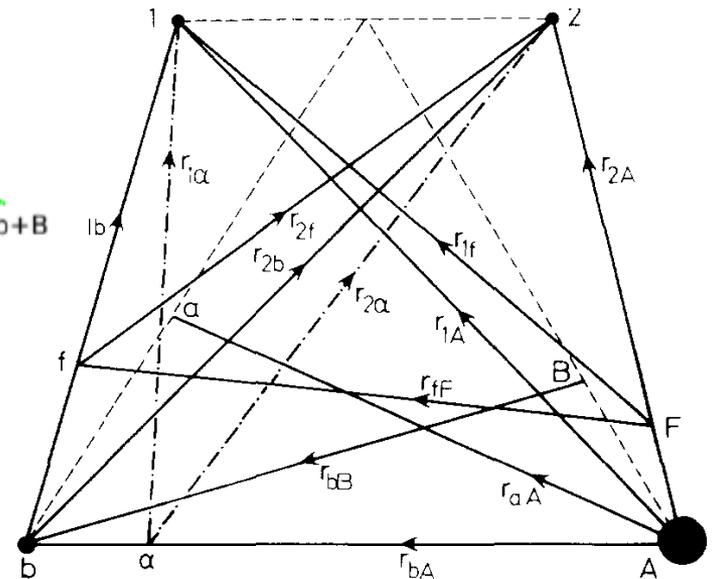


# Successive transfer

$$\begin{aligned}
 T_{succ}^{(2)}(j_i, j_f) &= 2 \sum_{K, M} \sum_{\substack{\sigma_1 \sigma_2 \\ \sigma'_1 \sigma'_2}} \int d\mathbf{r}_{fF} d\mathbf{r}_{b1} d\mathbf{r}_{A2} [\Psi^{j_f}(\mathbf{r}_{A1}, \sigma_1) \Psi^{j_f}(\mathbf{r}_{A2}, \sigma_2)]_0^{0*} \\
 &\times \chi_{bB}^{(-)*}(\mathbf{r}_{bB}) v(\mathbf{r}_{b1}) [\Psi^{j_f}(\mathbf{r}_{A2}, \sigma_2) \Psi^{j_i}(\mathbf{r}_{b1}, \sigma_1)]_M^K \\
 &\times \int d\mathbf{r}'_{fF} d\mathbf{r}'_{b1} d\mathbf{r}'_{A2} G(\mathbf{r}_{fF}, \mathbf{r}'_{fF}) [\Psi^{j_f}(\mathbf{r}'_{A2}, \sigma'_2) \Psi^{j_i}(\mathbf{r}'_{b1}, \sigma'_1)]_M^K \\
 &\times \frac{2\mu_{fF}}{\hbar^2} v(\mathbf{r}'_{f2}) [\Psi^{j_i}(\mathbf{r}'_{A2}, \sigma'_2) \Psi^{j_i}(\mathbf{r}'_{b1}, \sigma'_1)]_0^0 \chi_{aA}^{(+)}(\mathbf{r}'_{aA})
 \end{aligned}$$

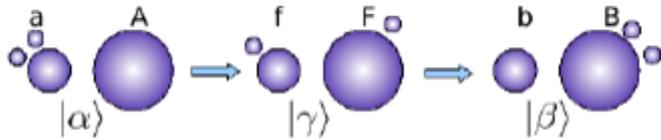


simultaneous  
 $A+a \rightarrow f+F \rightarrow b+B$   
 successive

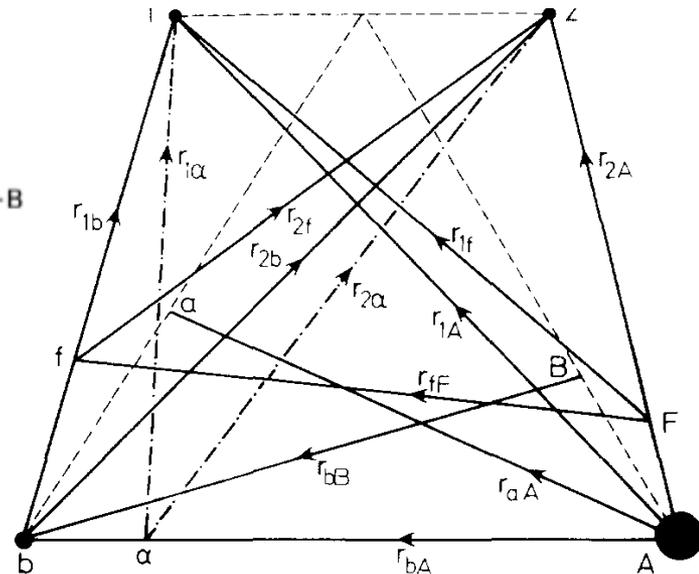


## Non orthogonal contribution

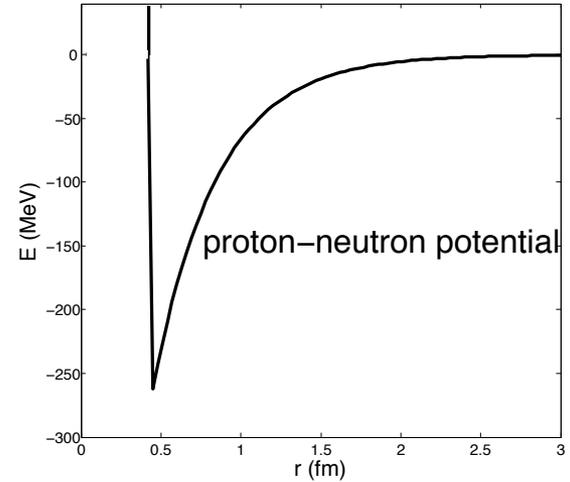
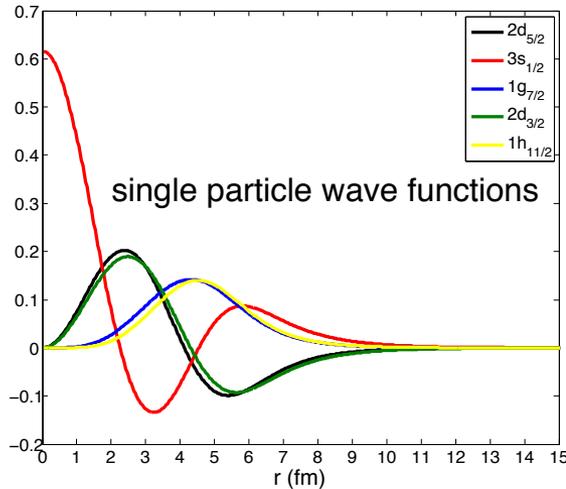
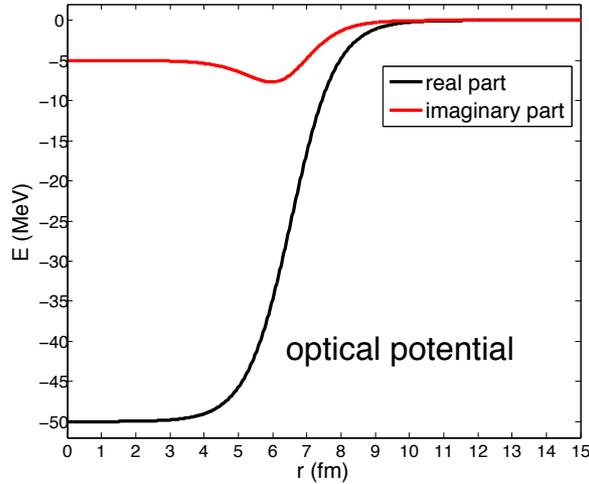
$$\begin{aligned}
 T_{NO}^{(2)}(j_i, j_f) &= 2 \sum_{K, M} \sum_{\substack{\sigma_1 \sigma_2 \\ \sigma'_1 \sigma'_2}} \int d\mathbf{r}_{fF} d\mathbf{r}_{b1} d\mathbf{r}_{A2} [\Psi^{j_f}(\mathbf{r}_{A1}, \sigma_1) \Psi^{j_f}(\mathbf{r}_{A2}, \sigma_2)]_0^{0*} \\
 &\times \chi_{bB}^{(-)*}(\mathbf{r}_{bB}) v(\mathbf{r}_{b1}) [\Psi^{j_f}(\mathbf{r}_{A2}, \sigma_2) \Psi^{j_i}(\mathbf{r}_{b1}, \sigma_1)]_M^K \\
 &\times \int d\mathbf{r}'_{b1} d\mathbf{r}'_{A2} [\Psi^{j_f}(\mathbf{r}'_{A2}, \sigma'_2) \Psi^{j_i}(\mathbf{r}'_{b1}, \sigma'_1)]_M^K \\
 &\times [\Psi^{j_i}(\mathbf{r}'_{A2}, \sigma'_2) \Psi^{j_i}(\mathbf{r}'_{b1}, \sigma'_1)]_0^0 \chi_{aA}^{(+)}(\mathbf{r}'_{aA})
 \end{aligned}$$



simultaneous  
 $A+a \rightarrow f+F \rightarrow b+B$   
successive



# Elements of the calculation for the $^{112}\text{Sn}(p,t)^{110}\text{Sn}$ reaction



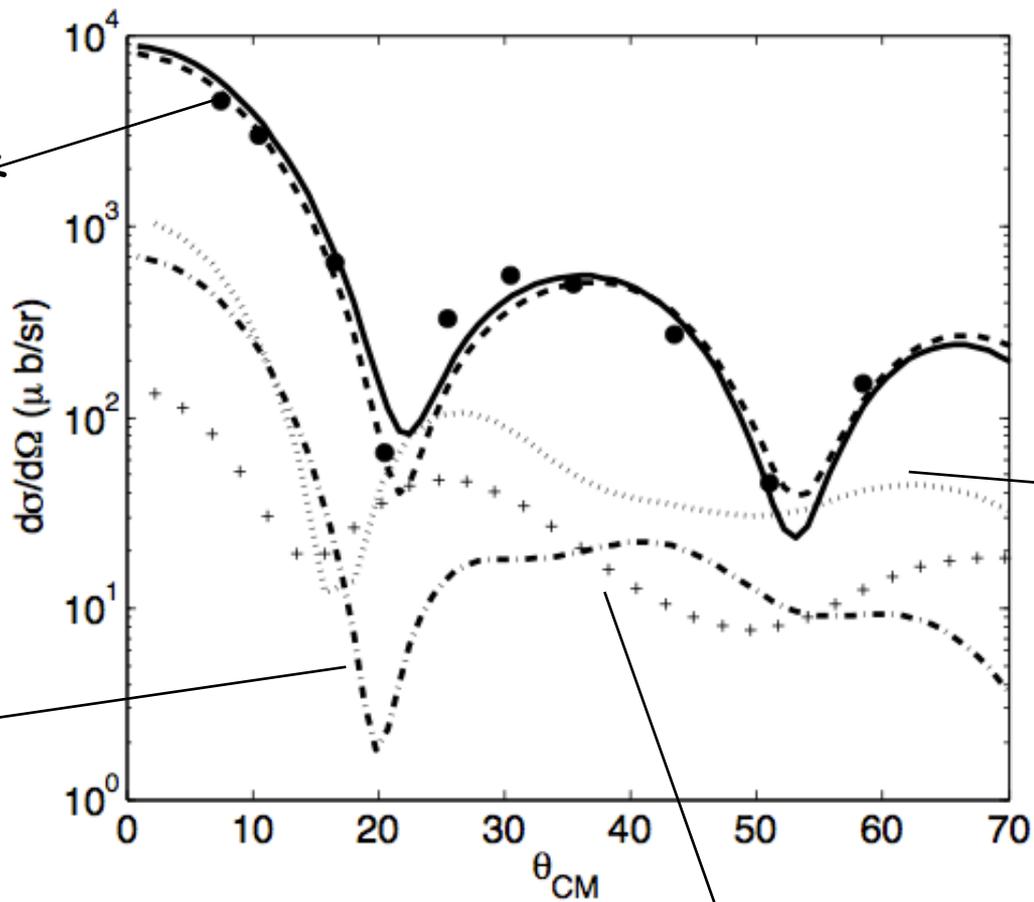
$$T = \sum_j B_j \left[ T^{(1)}(j) + T_{succ}^{(2)}(j) + T_{NO}^{(2)}(j) \right]$$

Two neutron spectroscopic amplitude:

$$B_j = \langle \Psi^{(A-2)} | P_j | \Psi^A \rangle$$

**BCS**  $\longrightarrow$   $B_j = (j + 1/2)^{1/2} U_j^{(A-2)} V_j^{(A)}$

$^{122}\text{Sn}(p,t)^{120}\text{Sn}$   $E_{\text{lab}} = 26 \text{ MeV}$



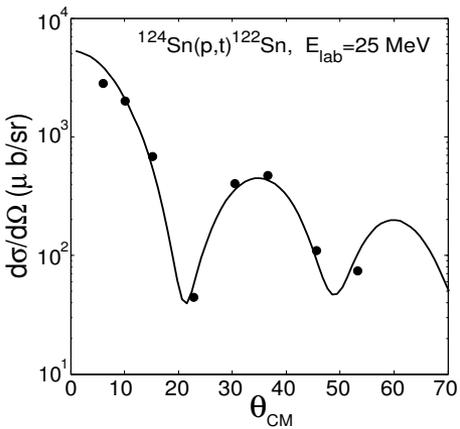
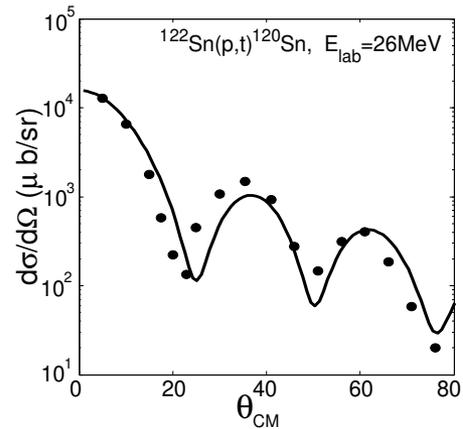
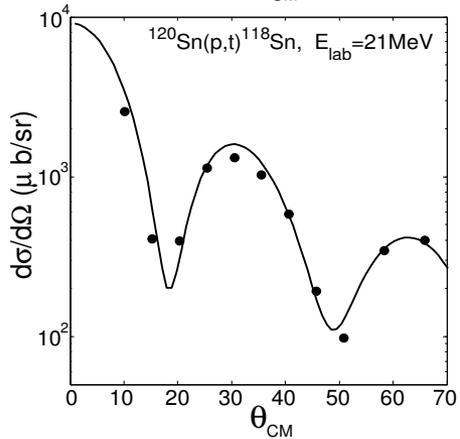
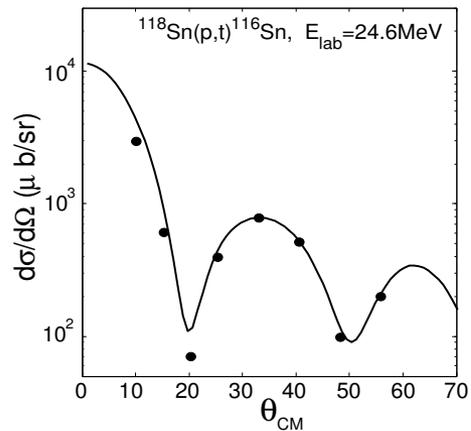
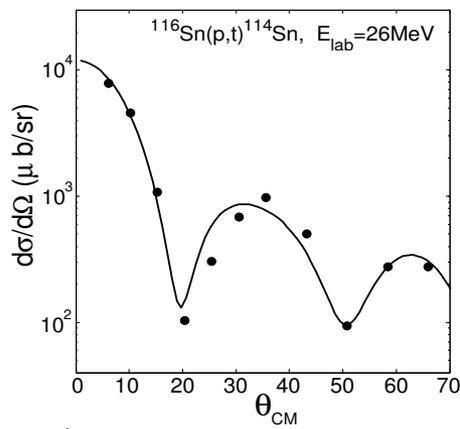
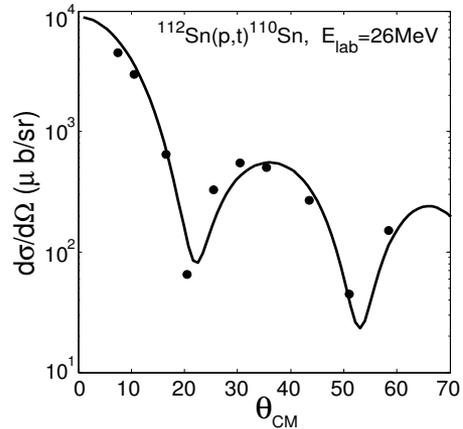
Successive

Simultaneous

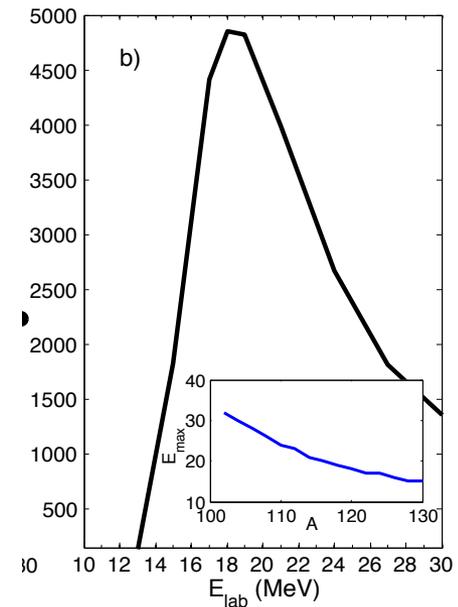
Non orthogonal

Simult.+Non orth.

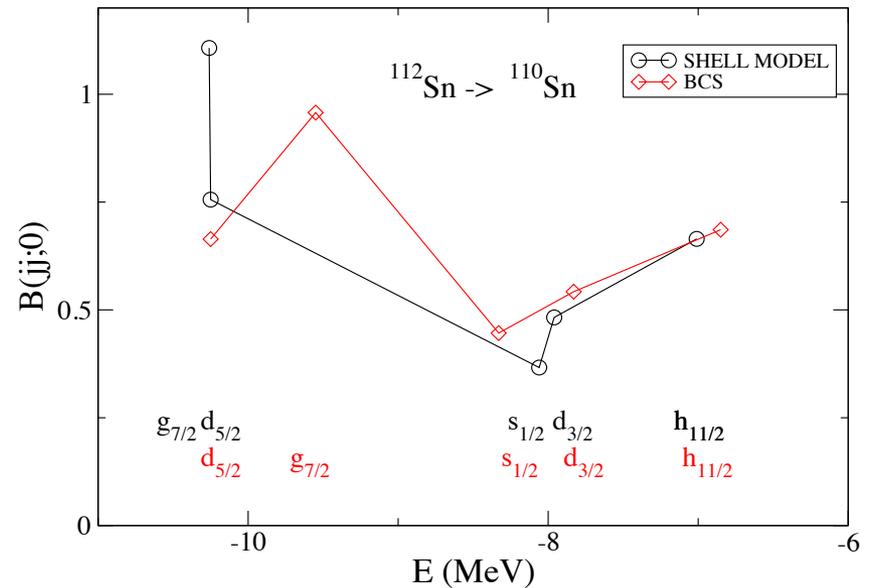
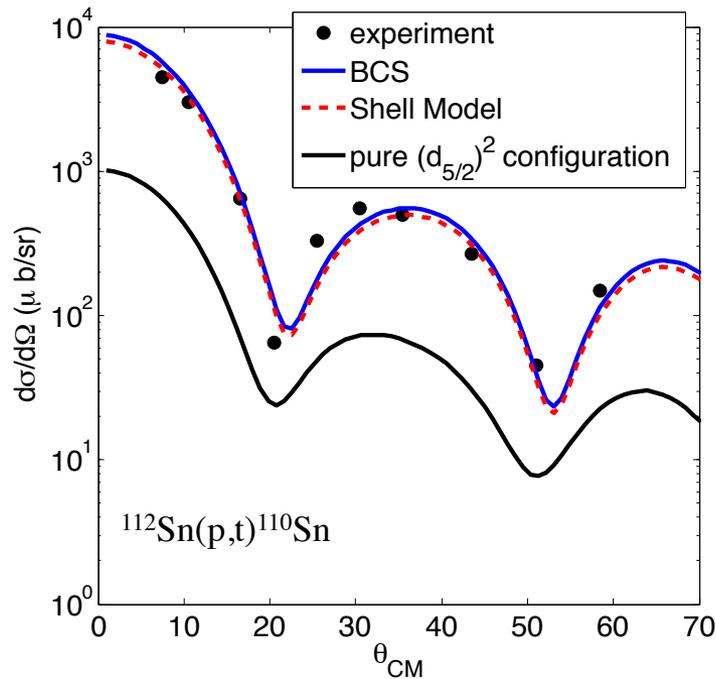
# $A\text{Sn}(p,t)^{A-2}\text{Sn}$ , results



- BCS wavefunctions reproducing experimental pairing gaps
- Tang-Herndon wavefunctions for the triton
- Optical potential fitted by Guazzoni et al



The ground state is a coherent state well described by BCS calculations constrained to reproduce the phenomenological pairing gap



Shell model : P. Guazzoni et al.,  
PRC 74 054605 (2006)

Some challenges:

Check pair vibrational scheme around new magic nuclei

Transfer from heavy, weakly bound nuclei

Effects of core excitation

Transfer from halo nuclei

Continuum and coupled channel effects

Pair transfer as probe of shape coexistence

# Pair vibrations around $^{100}\text{Sn}$ and $^{132}\text{Sn}$ in the harmonic approximation

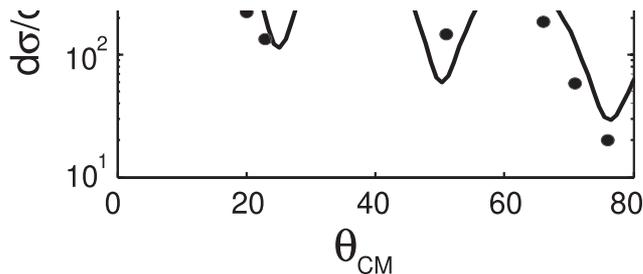


FIG. 2. Absolute calculated cross section comparison with the experimental results for  $^{100}\text{Sn}$  and  $^{132}\text{Sn}$ .

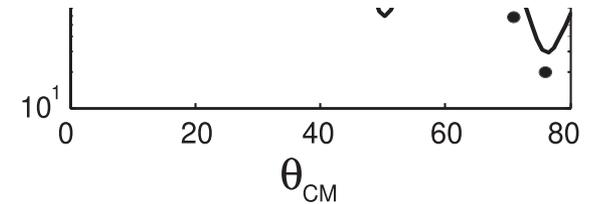
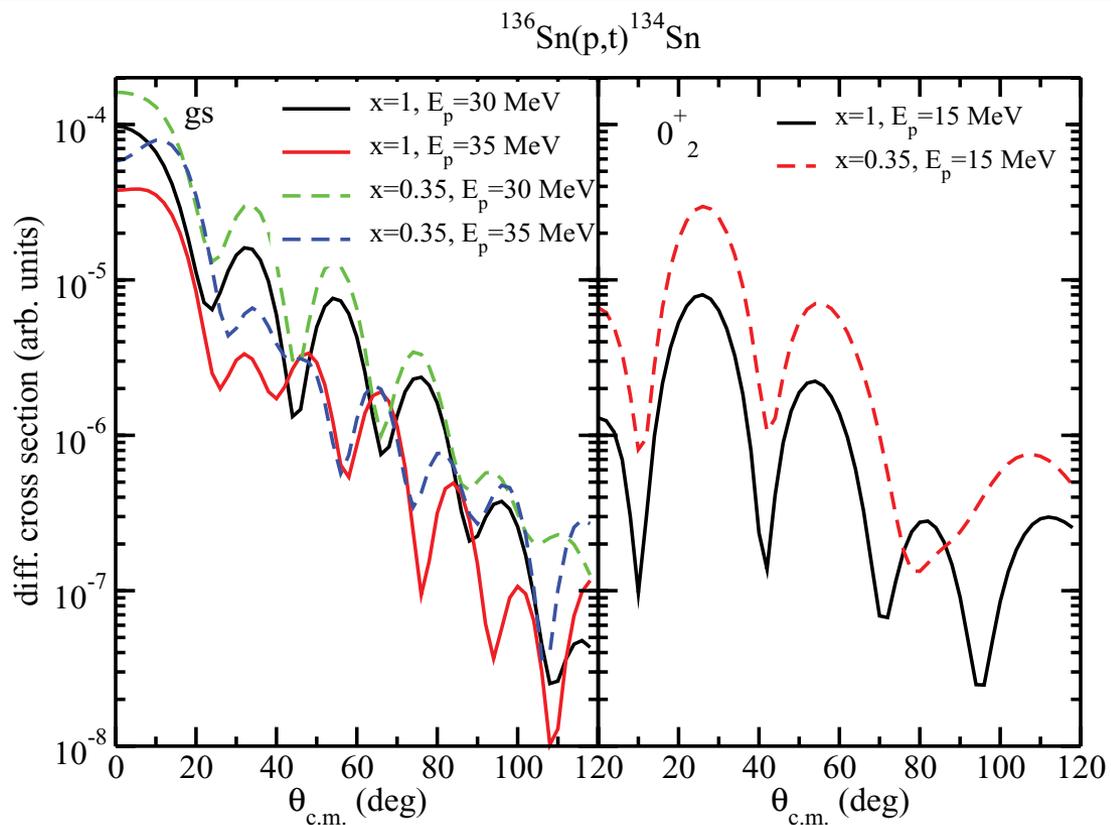


FIG. 2. Absolute calculated cross section comparison with the experimental results for  $^{100}\text{Sn}$  and  $^{132}\text{Sn}$ .

Two –neutron transfer cross sections for neutron-rich Sn isotopes  
(zero-range approximation)

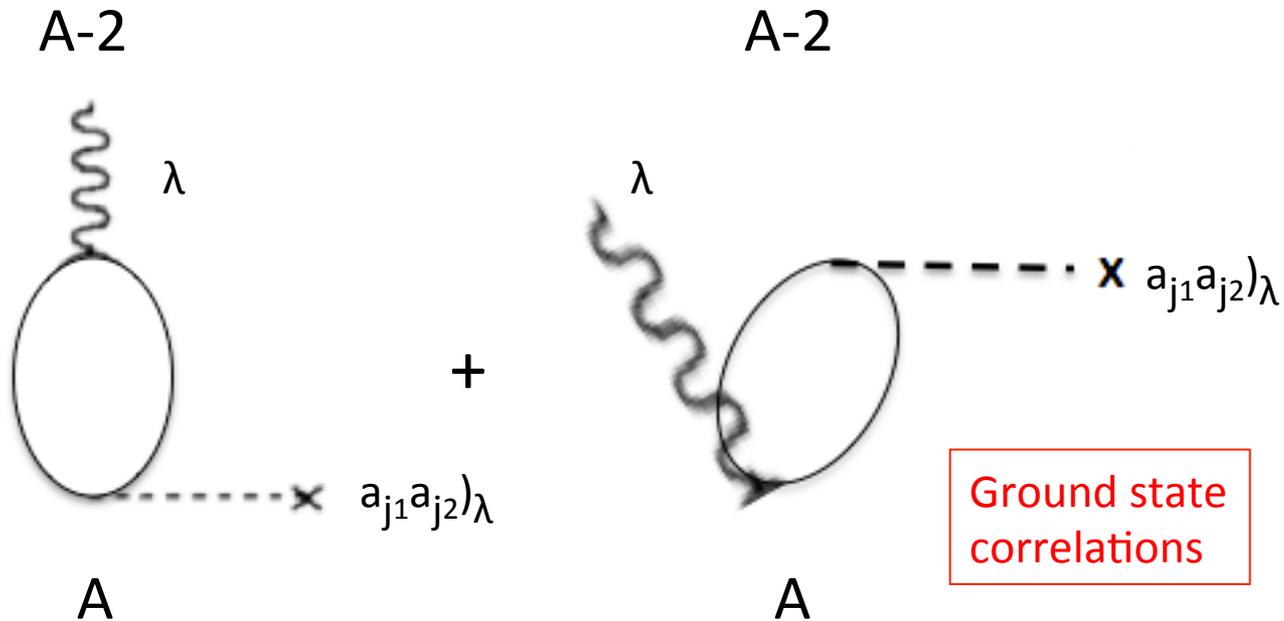


Pllumbi et al., PRC 83 (2010) 034613

BCS wavefunctions are enough to calculate  $2n$  transfer  
between ground states of superfluid nuclei.

**BUT**

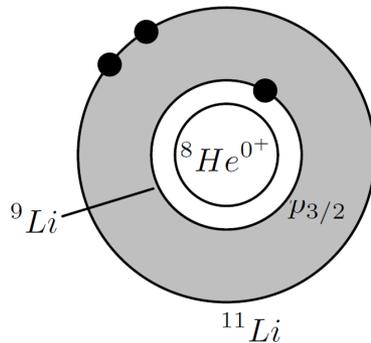
$2n$  transfer reactions to collective surface vibrational states  
may reveal the existence of shape fluctuations in the condensate



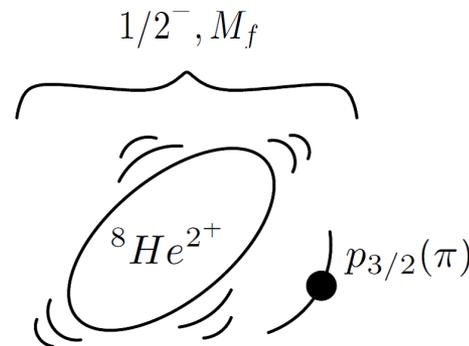
R.A. Broglia, C. Riedel,  
T. Udagawa,  
NPA169 (1971) 225

# Probing $^{11}\text{Li}$ halo-neutrons correlations via (p,t) reaction

We will try to draw information about the halo structure of  $^{11}\text{Li}$  from the reactions  $^1\text{H}(^{11}\text{Li}, ^9\text{Li})^3\text{H}$  and  $^1\text{H}(^{11}\text{Li}, ^9\text{Li}^*(2.69 \text{ MeV}))^3\text{H}$  (I. Tanihata *et al.*, Phys. Rev. Lett. **100**, 192502 (2008))



Schematic depiction of  $^{11}\text{Li}$

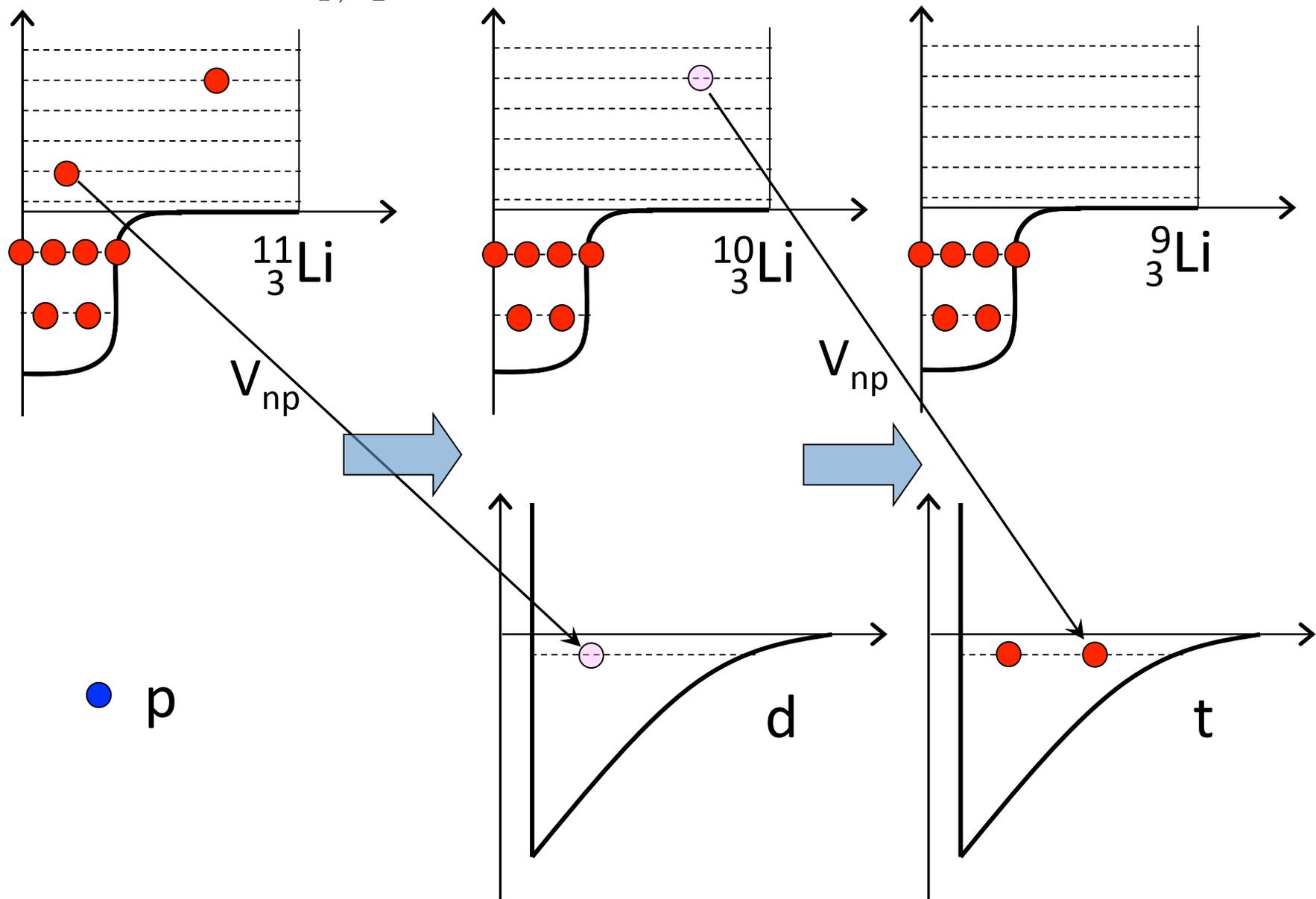


First excited state of  $^9\text{Li}$

$$|0\rangle = 0.45|s_{1/2}^2(0)\rangle + 0.55|p_{1/2}^2(0)\rangle + 0.04|d_{5/2}^2(0)\rangle$$

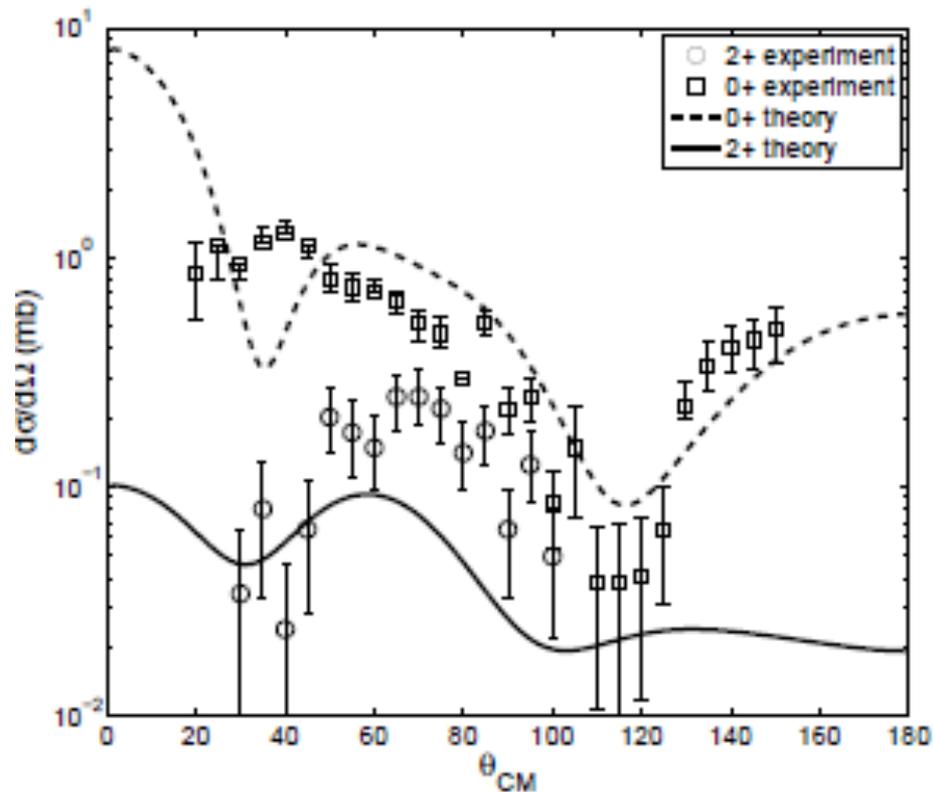
$$|\bar{0}\rangle = |0\rangle + 0.7|(ps)_{1^-} \otimes 1^-; 0\rangle + 0.1|(sd)_{2^+} \otimes 2^+; 0\rangle$$

$$\sum_{n_1, n_2} a_{n_1, n_2} [\psi_{n_1}(r_1) \psi_{n_2}(r_2)]_{00}$$



	$\sigma(^{11}\text{Li}(\text{gs}) \rightarrow ^9\text{Li}(\text{i}))$ (mb)		
i	$\Delta L$	Theory	Experiment
gs ( $3/2^-$ )	0	6.1	$5.7 \pm 0.9$
2.69 MeV ( $1/2^-$ )	2	0.5	$1.0 \pm 0.36$

## $^{11}\text{Li}(\text{p,t})^9\text{Li}$

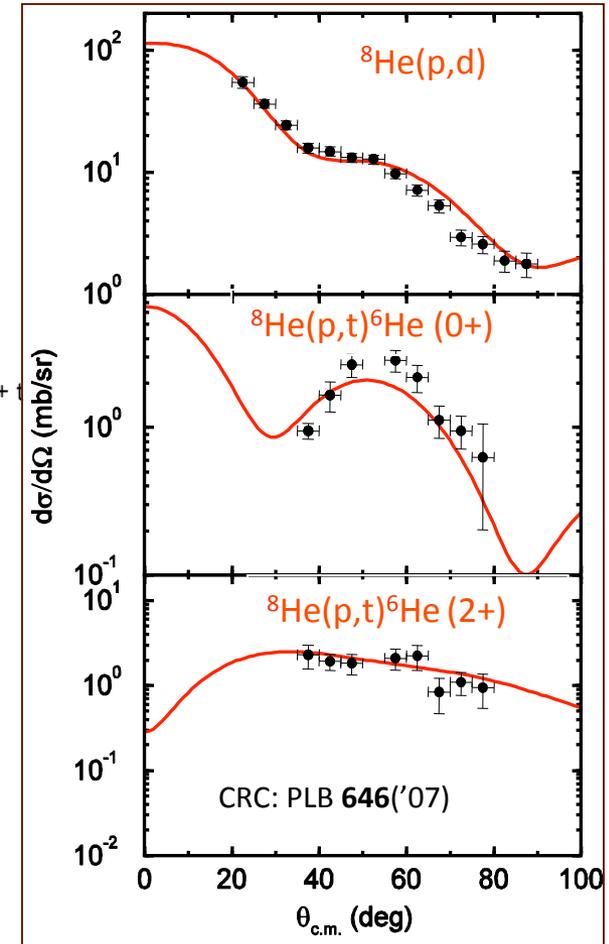
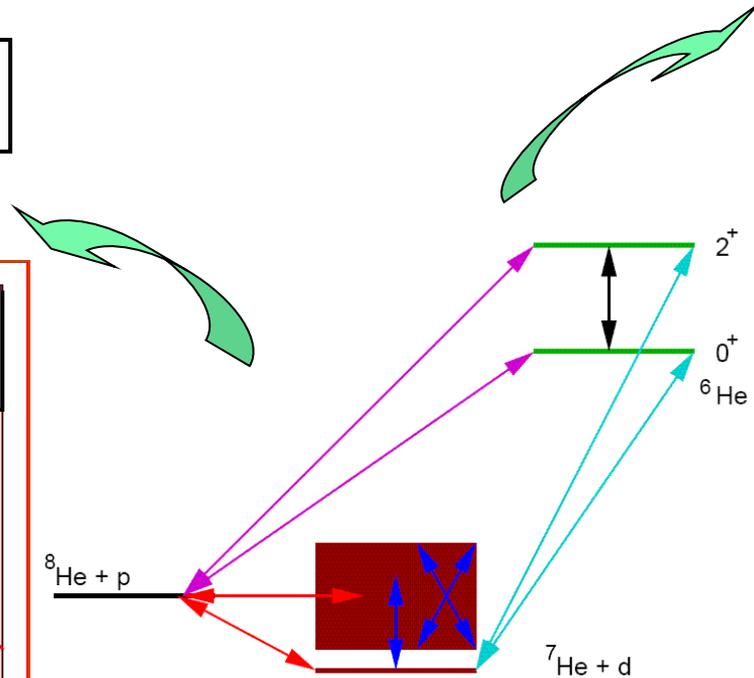
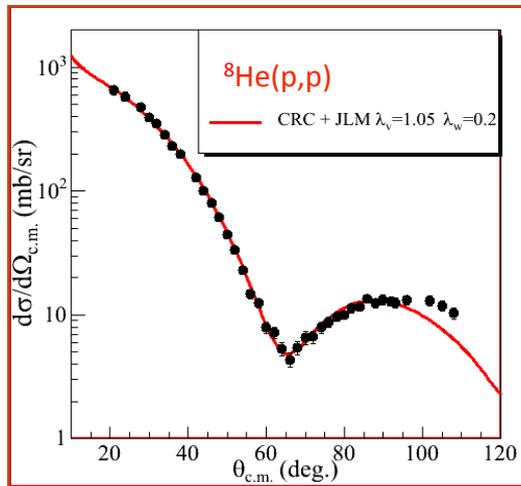


g.s.  $3/2^-$

First exc.  $1/2^-$

Coupled reaction channel (CRC) calculations needed:  
*Cf* $^8\text{He}+p$  Analysis  $\rightarrow$  N. Keeley, SPbN [now: univ of Warsaw]  
 F. Skaza *et al.*, PLB **619**, 82 ('05) ; PRC **73**, 044301 ('06)  
 N. Keeley *et al.*, PLB **646**, 222('07)

E405s –GANIL-MUST  
 $^8\text{He} + p$  @ 15.6 MeV/n



Spectroscopic factors  $C^2S$  from  
 $(d\sigma/d\Omega)_{\text{theo}} \% (d\sigma/d\Omega)_{\text{exp}}$

The transferred angular momentum  $L_t$  indicates  $J^\pi$

Challenge: radioactive target (TREX at ISOLDE)

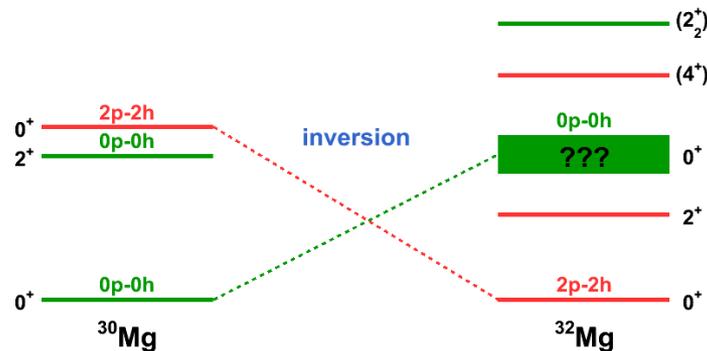
first experiment with a Tritium target and a radioactive heavy ion beam



## Two neutron transfer reaction to $^{32}\text{Mg}$

### coexistence of spherical and deformed states

- deformed  $2p-2h$  configuration becomes ground state in  $^{32}\text{Mg}$
- where is the excited  $0^+$  state?



predictions for the  $0_2^+$  state in  $^{32}\text{Mg}$ :  
between 1.5 and 3 MeV

E. Caurier et al., NPA **693** 374, T. Otsuka, EPJA **20** 69,  
R. Rodriguez-Guzmán et al., NPA **709** 201

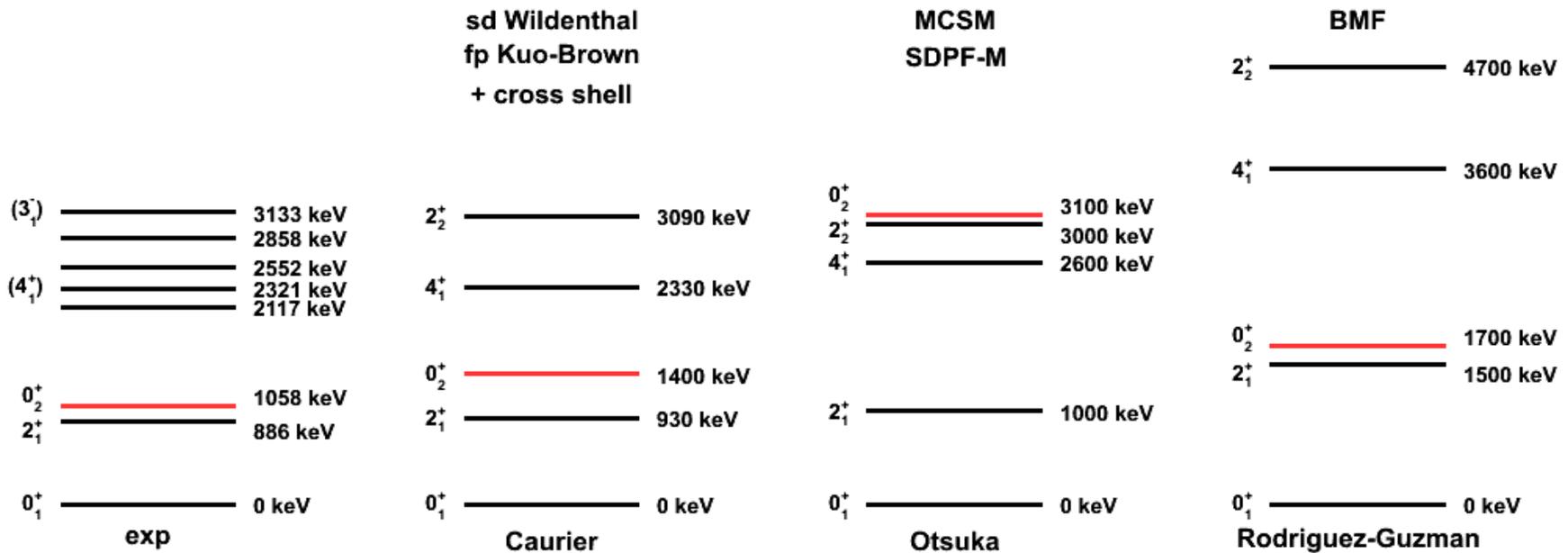
this state has not been observed so far

similar particle-hole structure:

populate the excited  $0^+$  state by a two neutron transfer reaction

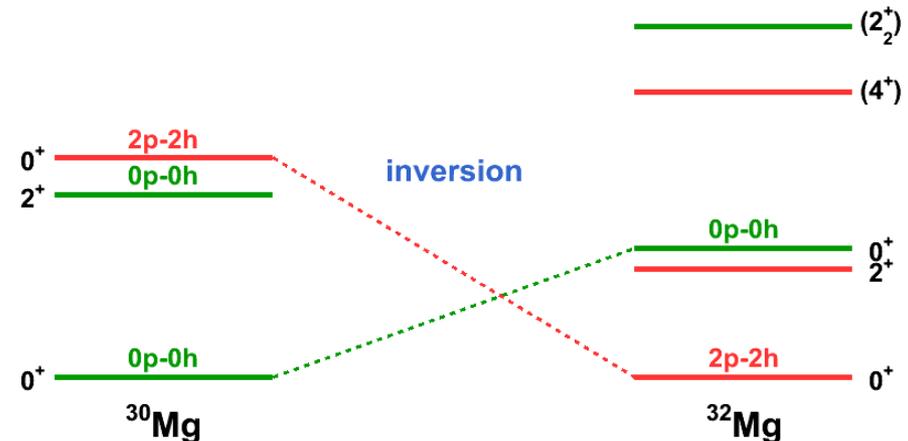
- large overlap of wavefunctions
- large spectroscopic factor for transfer





E. Caurier et al., NPA **693** 374, T. Otsuka, EPJA **20** 69, R. Rodriguez-Guzmán et al., NPA **709** 201

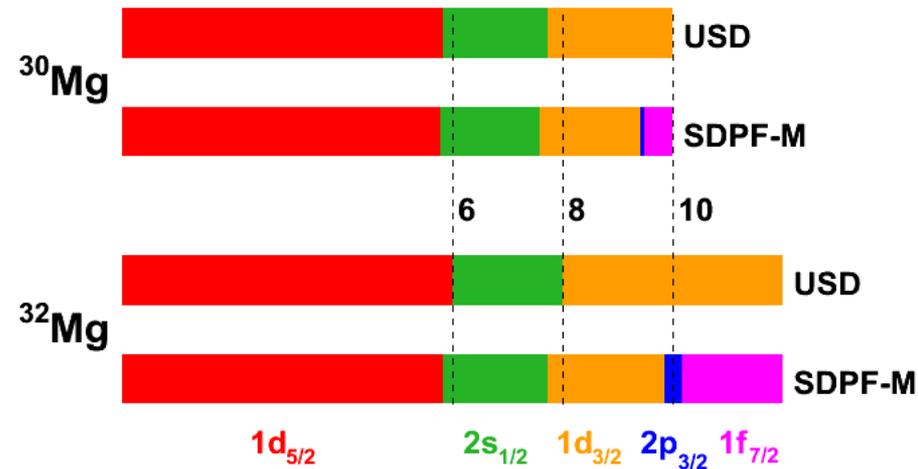
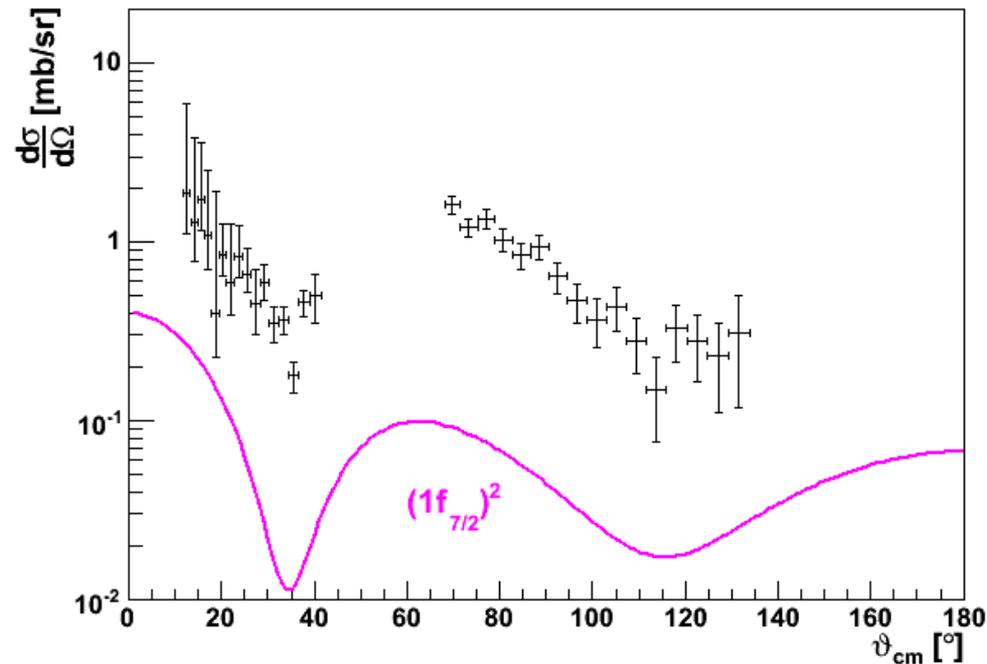
- $0_2^+$  state lower than predicted
- long lifetime several ns
- similar cross sections
- simple picture
  - ground state  $2p - 2h$
  - excited state  $0p - 0h$



Ground state:

- two particle - two hole configuration
- simple assumption: neutron pair in  $1f_{7/2}$
- Occupation numbers for the ground state within the Monte Carlo SM SDPF-M effective interaction

T. Otsuka et al., Prog. Part. Nucl. Phys. **47** 319



J. R. Terry et al., Phys. Rev. C **77** 014316

- simple assumption  $(1f_{7/2})^2$  amplitude can not reproduce the data

