Fluctuating hydrodynamics confronts rapidity dependence of p_T-correlations

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Outlines

- Motivation
- Viscosity and Hydrodynamics of Fluctuations
- Diffusion of p_t correlations
- Results and STAR measurements
- Summary

Motivation

- Modification of transverse momentum fluctuations by viscosity

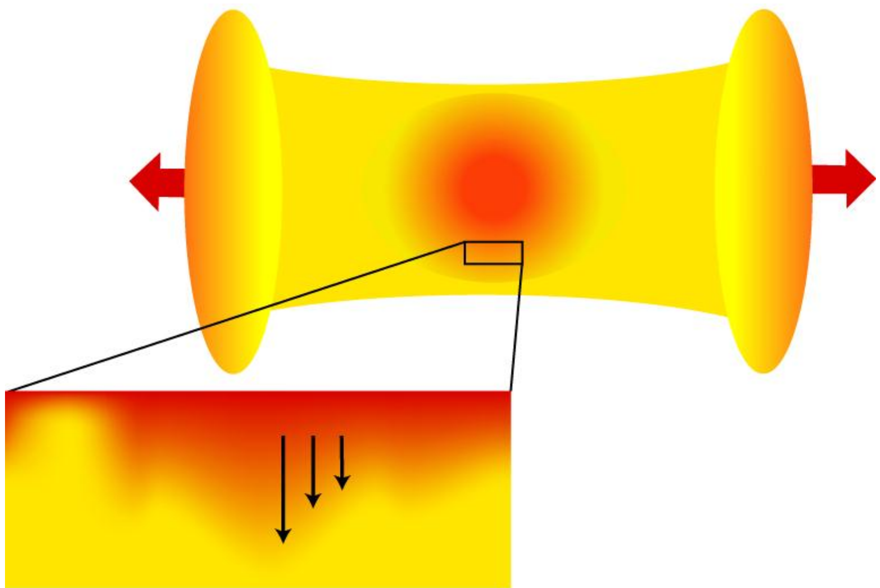
- Transverse momentum fluctuations have been used as an alternative measure of viscosity

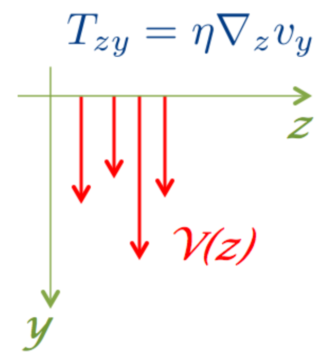
Sean Gavin & Mohamed Abdel-Aziz, Phys. Rev. Lett. 97 (2006) 162302

STAR: H. Agakishiev et al, Phys.Lett. B704 (2011) 467

- Estimate the impact of viscosity on fluctuations using best information on EOS, transport coefficients, and fluctuating hydrodynamics

Fluctuations of transverse flow





- Small variations of initial transverse flow in each event
- Viscosity arises as the fluid elements shear past each other
- Shear viscosity drives the flow toward the average
- damping of radial flow fluctuations ←→ viscosity

Sean Gavin & Mohamed Abdel-Aziz, Phys. Rev. Lett. 97 (2006) 162302

Transverse momentum fluctuations

Momentum density current $g_i = T_{0i} - < T_{0i} > \approx (\epsilon + p)\delta u_i$

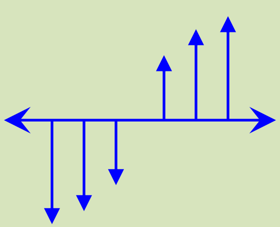
First, from ***non-relativistic*** hydro:

Linearized Navier-Stokes → $\partial_t g_i + \nabla_i p = \frac{\eta/3 + \zeta}{Ts}\nabla_i(\vec{\nabla}.\vec{g}) + \frac{\eta}{sT}\nabla^2 g_i$

Helmholtz decomposition: $\vec{g} = \vec{g}_l + \vec{g}_t$

Transverse modes:

$\vec{\nabla} \cdot \vec{g}_t = 0$

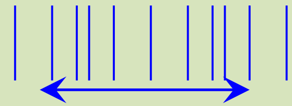


$\partial_t \vec{g}_t = \frac{\eta}{Ts}\nabla^2 \vec{g}_t$

viscous diffusion

Longitudinal modes: $\vec{\nabla} \times \vec{g}_l = 0$

$\partial_t \vec{g}_l + \vec{\nabla} p = \frac{\frac{4}{3}\eta + \zeta}{sT}\vec{\nabla}(\vec{\nabla} \cdot \vec{g}_l)$



sound waves (damped by viscosity)

It is the transverse modes that we are interested in.

Dissipative relativistic hydrodynamics

Conservation of energy-momentum: $\partial_\mu T^{\mu\nu} = 0$

$$T^{\mu\nu} = \underbrace{\epsilon u^\mu u^\nu - p\Delta^{\mu\nu}}_{\text{ideal}} + \underbrace{\Pi^{\mu\nu}}_{\text{dissipative}}$$

→ Equations of relativistic viscous hydrodynamics

$$D\epsilon + (\epsilon + p)\partial_\mu u^\mu - \Pi^{\mu\nu}\nabla_{(\mu}u_{\nu)} = 0$$
$$(\epsilon + p)Du^\lambda - \nabla^\lambda p + \Delta^\lambda_\nu \partial_\mu \Pi^{\mu\nu} = 0.$$

First order (Navier-Stokes) hydro: $\pi_{\mu\nu} = \eta \nabla_{<\mu}u_{\nu>}$ $\quad \Pi = \zeta \nabla_\alpha u^\alpha$

$$\Pi^{\mu\nu} = \eta(\nabla^\mu u^\nu + \nabla^\nu u^\mu) + (\zeta - \frac{2}{3}\eta)\Delta^{\mu\nu}\nabla_\alpha u^\alpha$$

Second order (Israel-Stewart) hydro:

$$\pi^{\mu\nu} = \eta\left[\nabla^{<\mu}u^{\nu>} - \pi^{\mu\nu}TD(\frac{\beta_2}{T}) - 2\beta_2 D\pi^{\mu\nu} - \beta_2\pi^{\mu\nu}\partial_\alpha u^\beta\right] \qquad \beta_2 = \frac{\tau_\pi}{2\eta}$$

A. Muronga, Phys.Rev. C69, 034903 (2004)

Linearized hydro and diffusion of flow fluctuations

Linearized Navier-Stokes for transverse flow fluctuations:

$$(\epsilon_0 + p_0)\partial_t \delta u^y + \partial_z \delta \Pi^{zy} = 0 \qquad \delta \Pi^{zy} = -\eta_0 \partial_z \delta u^y$$

$$\rightarrow \quad \frac{\partial}{\partial t}\delta u^y = \nu \frac{\partial^2}{\partial z^2}\delta u^y$$

$$\frac{\partial g}{\partial t} = \nu \nabla_z^2 g \qquad \nu = \frac{\eta}{\epsilon + p} = \frac{\eta}{s}\frac{1}{T} \qquad g \approx (\epsilon + p)\delta u^y$$

g transverse component

First order diffusion violates causality!

Linearized Israel-Stewart for transverse flow fluctuations:

$$\tau_\pi \partial_t \delta \pi^{zy} + \delta \pi^{zy} = -\eta_0 \partial_z \delta u^y$$

$$\rightarrow \quad \tau_\pi \frac{\partial^2 \delta u^y}{\partial t^2} + \frac{\partial}{\partial t}\delta u^y = \nu \frac{\partial^2}{\partial z^2}\delta u^y$$

$$\tau_\pi \frac{\partial^2 g}{\partial t^2} + \frac{\partial g}{\partial t} = \nu \nabla_z^2 g$$

This saves causality!

Two-particle transverse momentum correlations and first order diffusion

Two-particle momentum correlation

$$r = \langle g_1 g_2 \rangle - \langle g_1 \rangle \langle g_2 \rangle \qquad g_1 \equiv g(\mathbf{x_1}) \qquad g_2 \equiv g(\mathbf{x_2})$$

g satisfies the diffusion equation → r satisfies

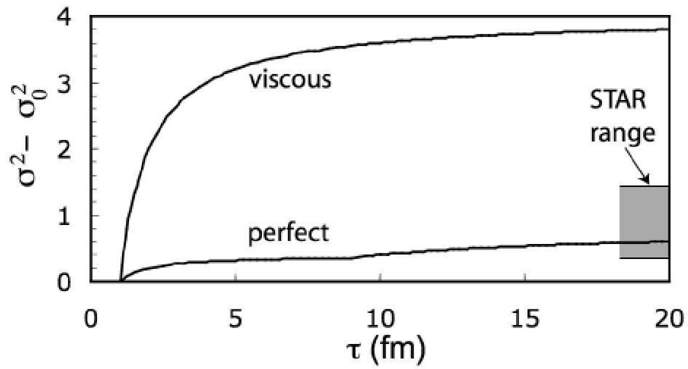
$$\frac{\partial r}{\partial t} = \nu(\nabla_1^2 + \nabla_2^2) r + Noise \rightarrow \qquad \frac{\partial \Delta r}{\partial t} = \nu(\nabla_1^2 + \nabla_2^2)\Delta r$$

$$\Delta r = r - r_{eql}$$

→ $$\frac{\partial \Delta r}{\partial \tau} = \frac{\nu}{\tau^2}(\partial^2/\partial\eta_1^2 + \partial^2/\partial\eta_2^2)\Delta r$$ first order

$$\sigma^2 - \sigma_0^2 = 4\nu(1/\tau_0 - 1/\tau)$$

Sean Gavin & Mohamed Abdel-Aziz,
Phys. Rev. Lett. 97 (2006) 162302

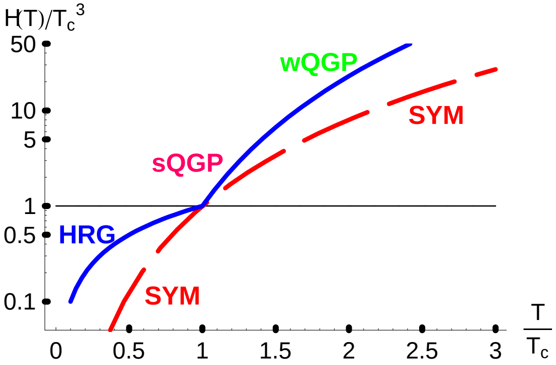


Second order diffusion of transverse momentum correlations

$$\tau_\pi \frac{\partial^2 \Delta r}{\partial \tau^2} + \frac{\partial \Delta r}{\partial \tau} = \frac{\nu}{\tau^2}(\partial^2/\partial\eta_1^2 + \partial^2/\partial\eta_2^2)\Delta r$$

$$\nu = \frac{\eta}{s}\,\frac{1}{T}$$

$\frac{\eta}{s}$ is temp. dependent



T. Hirano and M. Gyulassy, Nucl. Phys. A769, 71(2006), nucl-th/0506049.

Entropy density *s(T)* depends on equation of state (EOS)

EOS I -> Lattice s95p-v1

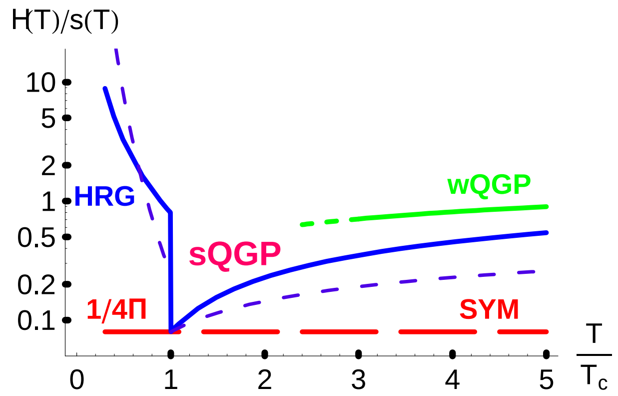
P. Huovinen and P. Petreczky, Nucl.Phys. A837, 26(2010), 0912.2541

EOS II -> Hirano & Gyulassy

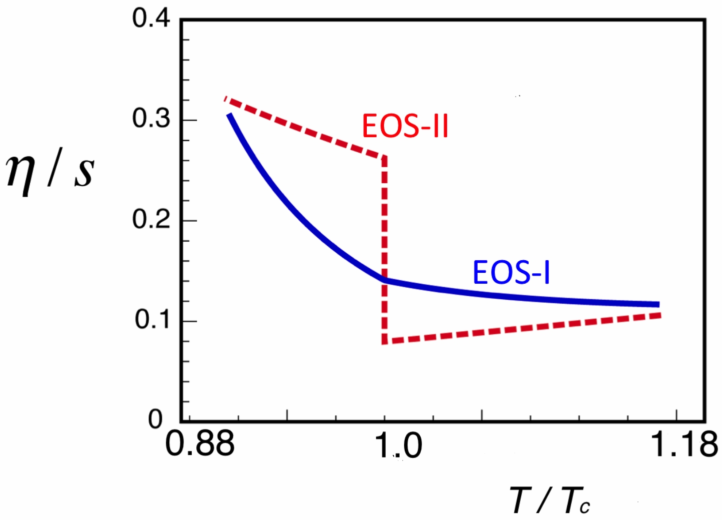
T. Hirano and M. Gyulassy, Nucl. Phys. A769, 71(2006), nucl-th/0506049.

Entropy density and EOS

EOS I and EOS II



T. Hirano and M. Gyulassy, Nucl. Phys. A769, 71(2006), nucl-th/0506049



Lattice: P. Huovinen and P. Petreczky, Nucl.Phys. A837, 26(2010), 0912.2541

Entropy production

ideal: $$\frac{ds}{d\tau} + \frac{s}{\tau} = 0$$

first order: $$\frac{ds}{d\tau} + \frac{s}{\tau} = \frac{\Phi}{T\tau}$$

second order: $$\tau_\pi \frac{d\Phi}{d\tau} + \left(1 + \frac{\tau_\pi}{2\tau} + \frac{1}{2}\eta T \frac{d}{d\tau}\left(\frac{\tau_\pi}{\eta T}\right)\right)\Phi = \frac{4\eta}{3\tau}$$

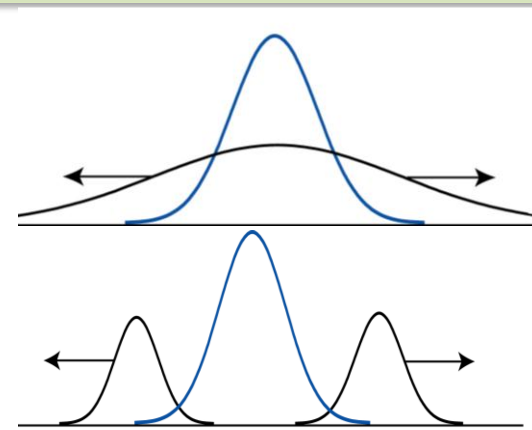
A. Muronga, Phys.Rev. C69, 034903 (2004)

We have used used ***both*** in our numerical solutions.

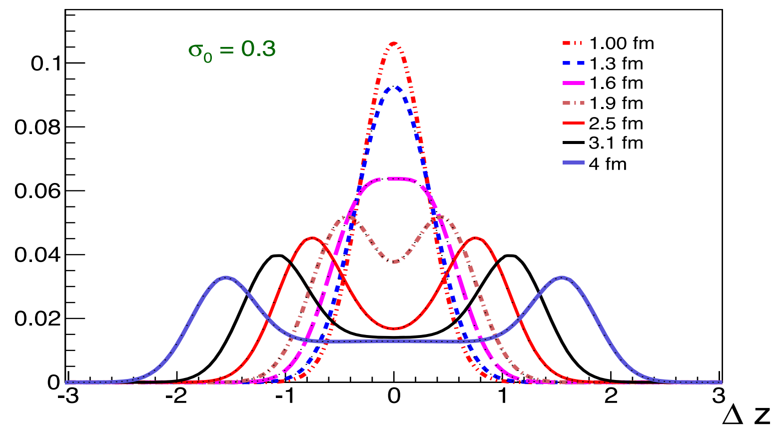
Diffusion vs Waves

$$\tau_\pi \frac{\partial^2 \Delta r}{\partial t^2} + \frac{\partial \Delta r}{\partial t} = \nu(\nabla_{z1}^2 + \nabla_{z2}^2)\Delta r$$

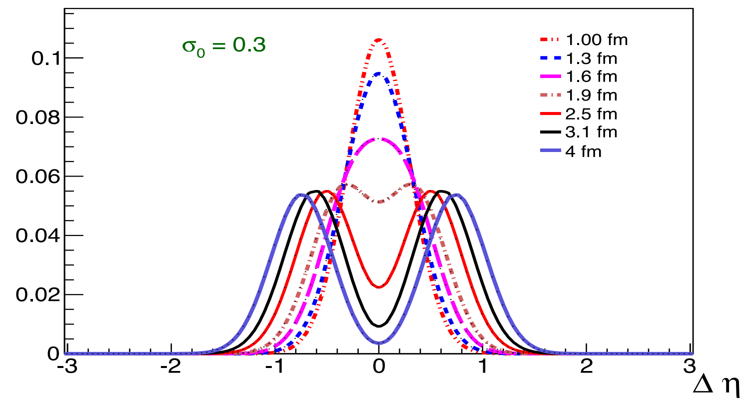
$$\tau_\pi \frac{\partial^2 \Delta r}{\partial \tau^2} + \frac{\partial \Delta r}{\partial \tau} = \frac{\nu}{\tau^2}(\partial^2/\partial\eta_1^2 + \partial^2/\partial\eta_2^2)\Delta r$$



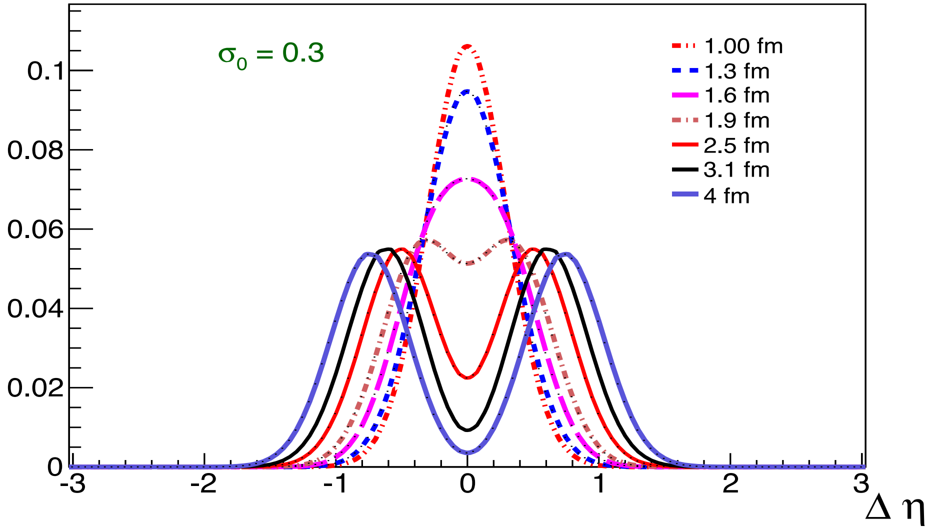
Diffusion fills the gap in between the propagating Wave



Rapidity separation of the fronts saturates



Choosing different initial widths

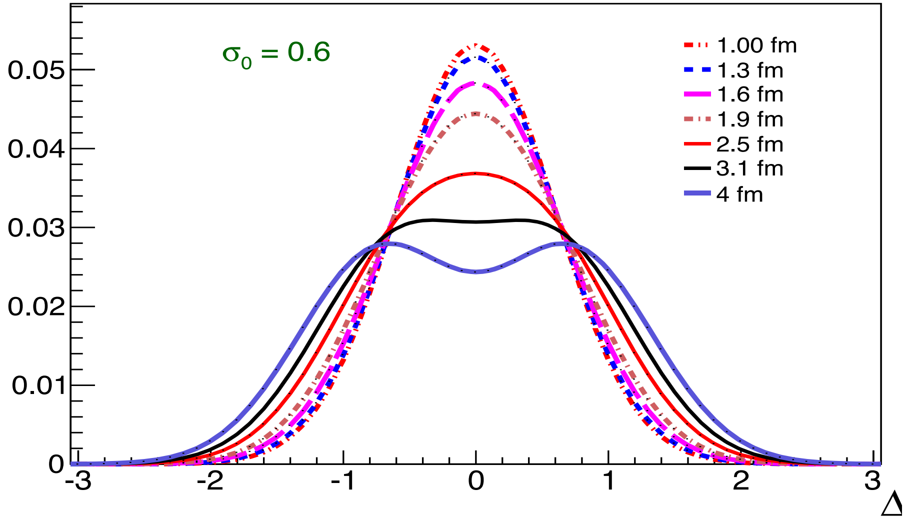


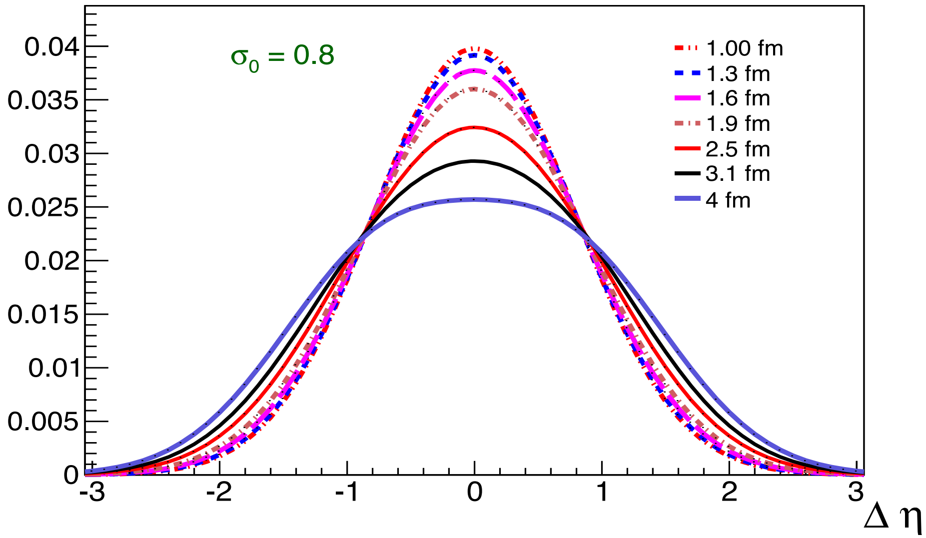
Initial correlation is a normalized Gaussian

Smaller initial widths resolves better

Constant diffusion coefficient used in these examples plots

R. Pokharel, S. Gavin, G. Moschelli in preparation

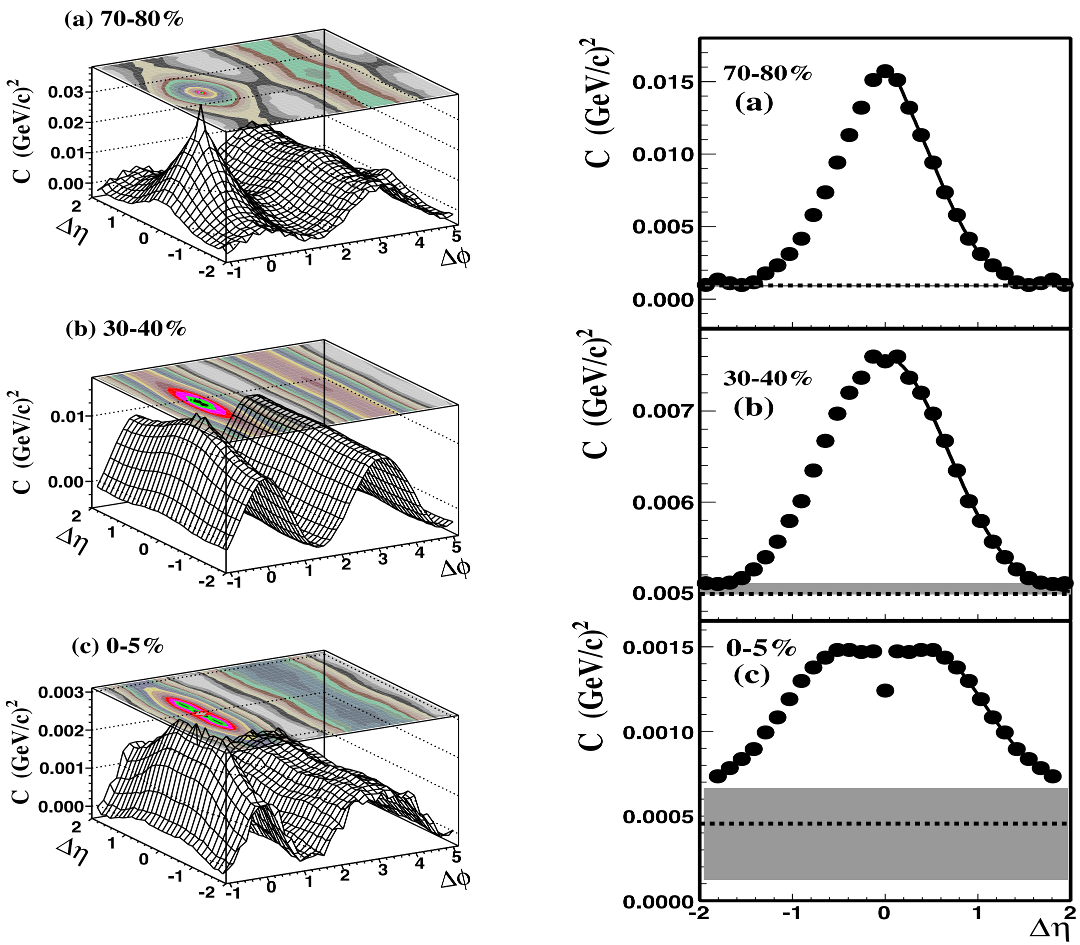




STAR measured these correlations

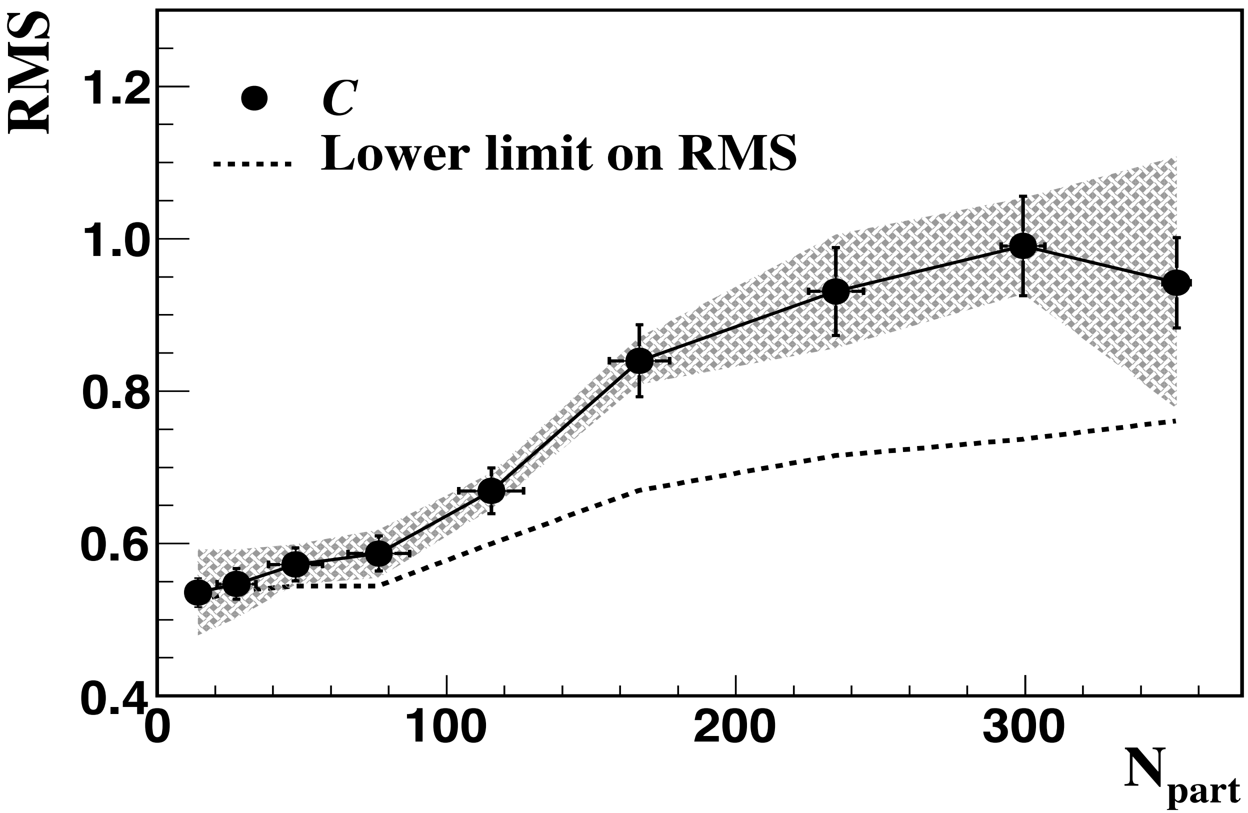
$$C = \frac{1}{< N >^2} \left\langle \sum_{pairs} p_{ti} p_{tj} \right\rangle - < p_t >^2 = \frac{1}{< N >^2} \int (r - r_{eq}) dx_1 dx_2$$

Sean Gavin & Mohamed Abdel-Aziz, Phys. Rev. Lett. 97 (2006) 162302



STAR: H. Agakishiev et al, Phys.Lett. B704 (2011) 467

STAR measured these correlations



STAR: H. Agakishiev et al, Phys.Lett. B704 (2011) 467

$$\sigma_{central} = 1.0 \pm 0.2$$
$$\sigma_{peripheral} = 0.54 \pm 0.02$$

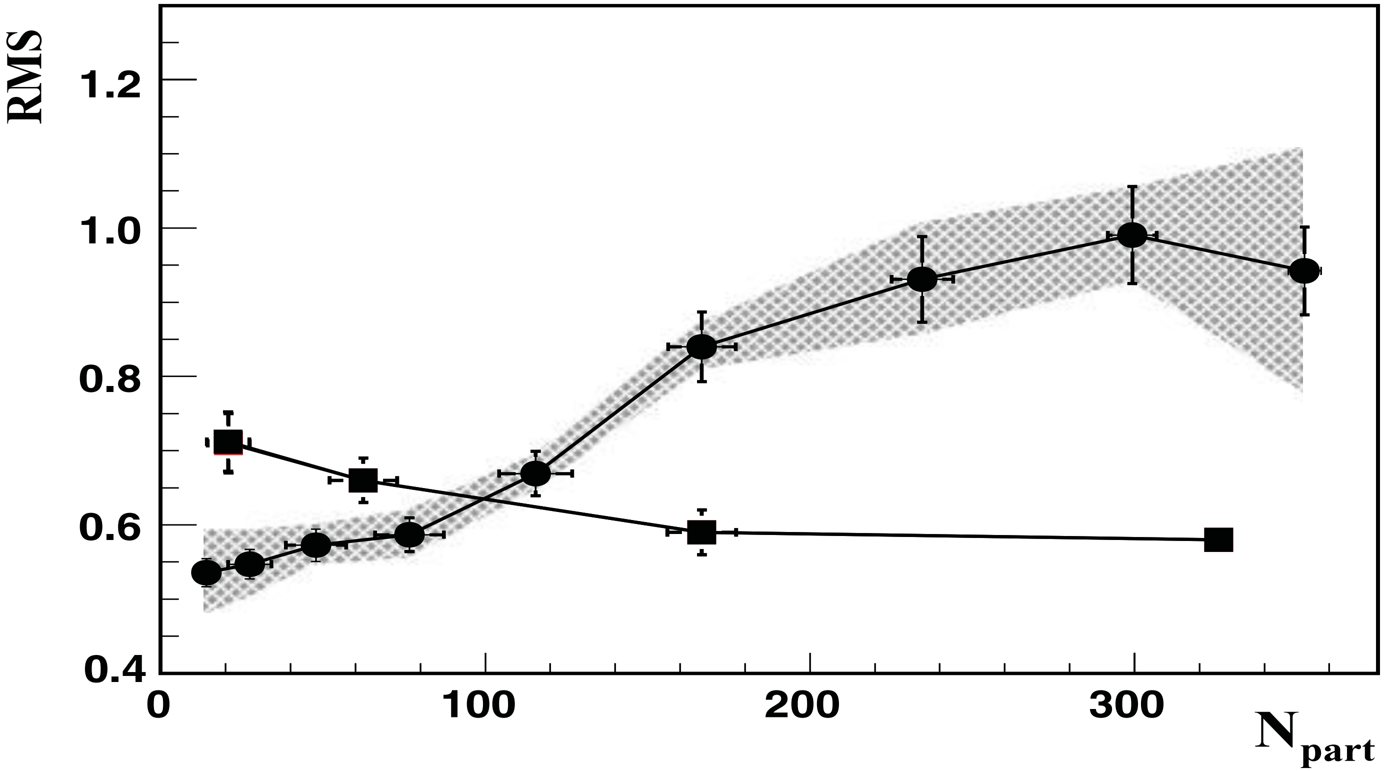
Central vs. peripheral increase consistent with

$$h/s = 0.17 \pm 0.08$$

- Measured: rapidity width of near side peak – the ridge
- Fit peak + constant offset
- Offset is ridge, i.e., long range rapidity correlations
- Report rms width of the peak

NeXSPheRIO

Fluctuating ideal hydro, NeXSPheRIO vs Ideal hydro



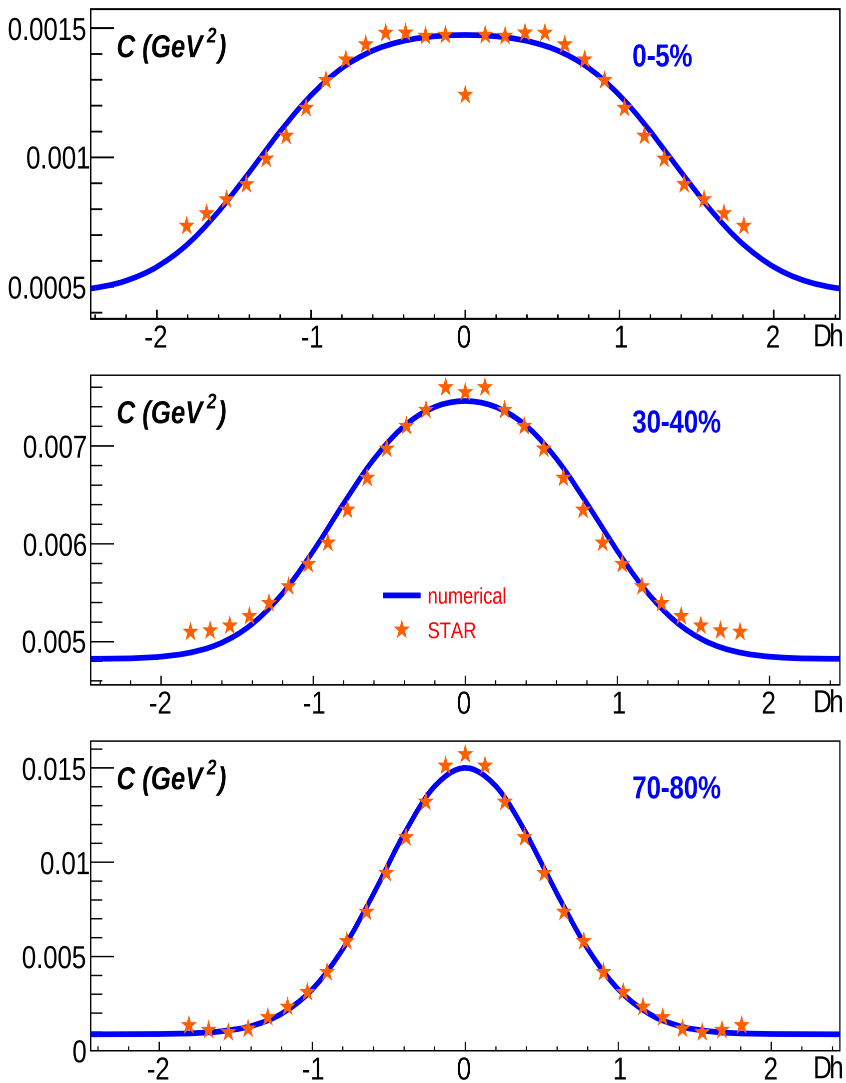
NeXSPheRIO calculations of width fails match the STAR data

NeXSPheRIO: Sharma et al., Phys.Rev. C84 (2011) 054915

STAR: H. Agakishiev et al, Phys.Lett. B704 (2011) 467

Results

Numerical results vs STAR



$$\tau_\pi = \beta \frac{\eta}{Ts}, \quad \beta = 6$$
$$\tau_0 = 1fm, \quad \sigma_0 = 0.54$$
$$T_F = 150MeV, \quad , \tau_{Fc} = 9.0fm$$
$$\tau - \tau_0 \propto (R - R_0)^2$$

Relaxation time:

τ_π = 5-6, AMY, Phys. Rev. D79, 054011 (2009), 0811.0729

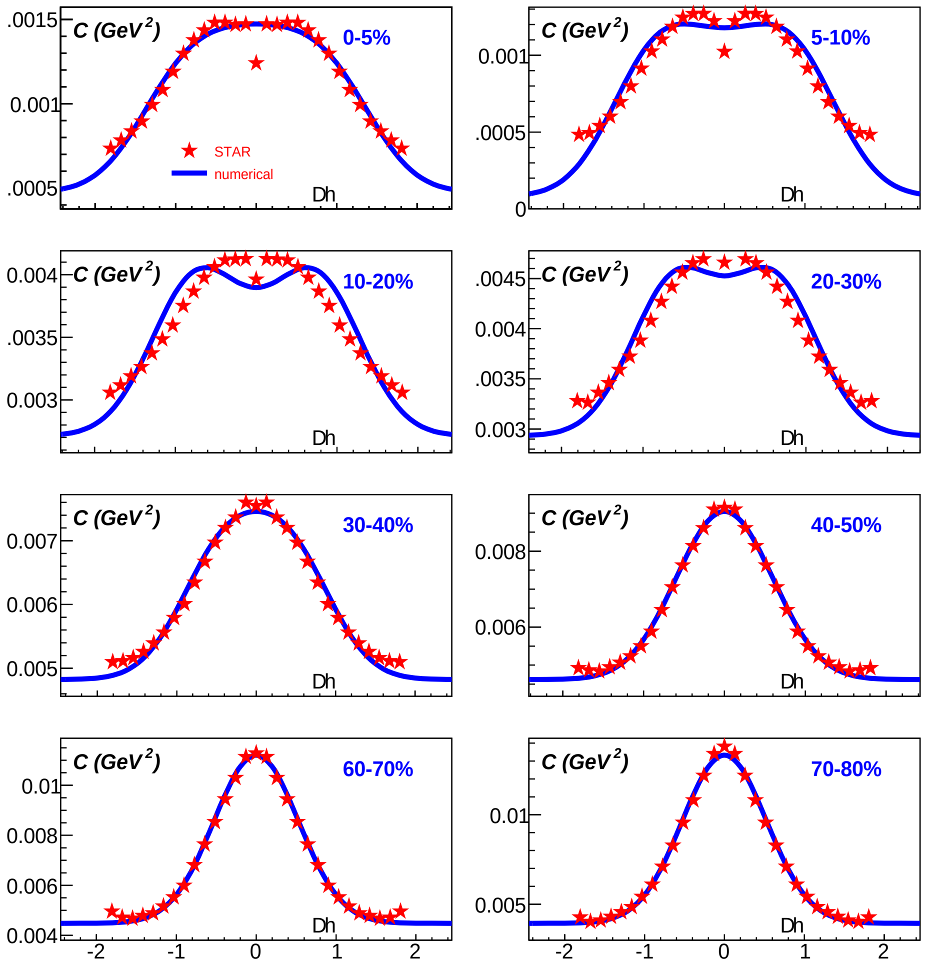
τ_π = 6.3, J. Hong, D. Teaney, and P. M. Chesler (2011), 1110.5292

R. Pokharel, S. Gavin, G. Moschelli in preparation

STAR: H. Agakishiev et al, Phys.Lett. B704 (2011) 467

Results

C : second order vs STAR



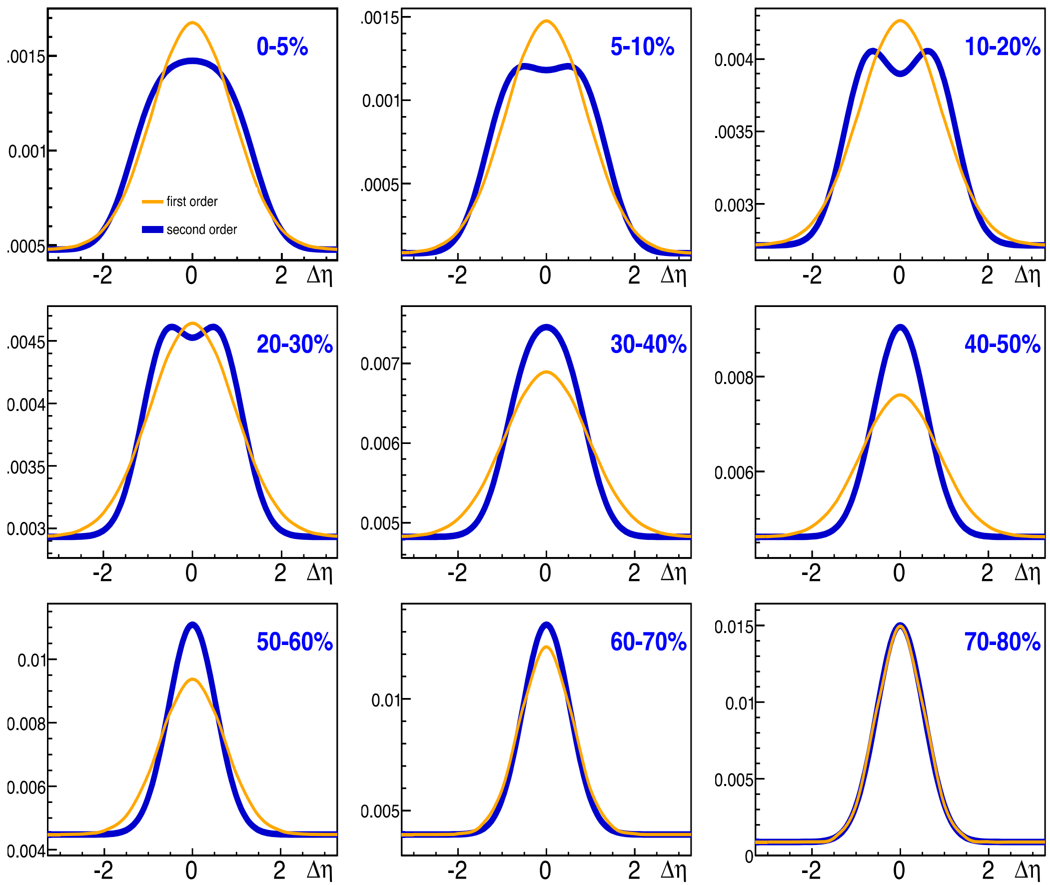
- Bumps in central to mid-central case in data and **second order diffusion** calculations
- No such bumps in the first order case means bumps are second order diffusion phenomena
- First and second order **entropy** production equations give virtually the same results (plots not shown here).

R. Pokharel, S. Gavin, G. Moschelli in preparation

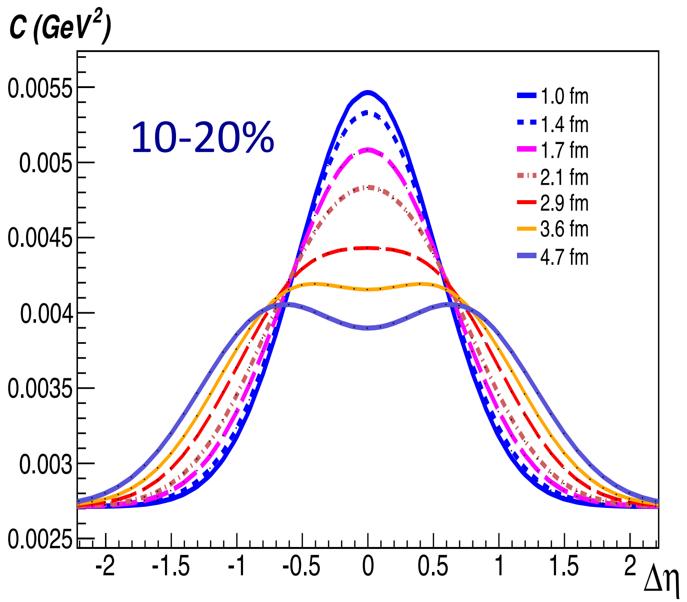
STAR: H. Agakishiev et al, Phys.Lett. B704 (2011) 467
STAR unpublished data: M. Sharma and C. Pruneau, private communications.

Results

Second order vs first order

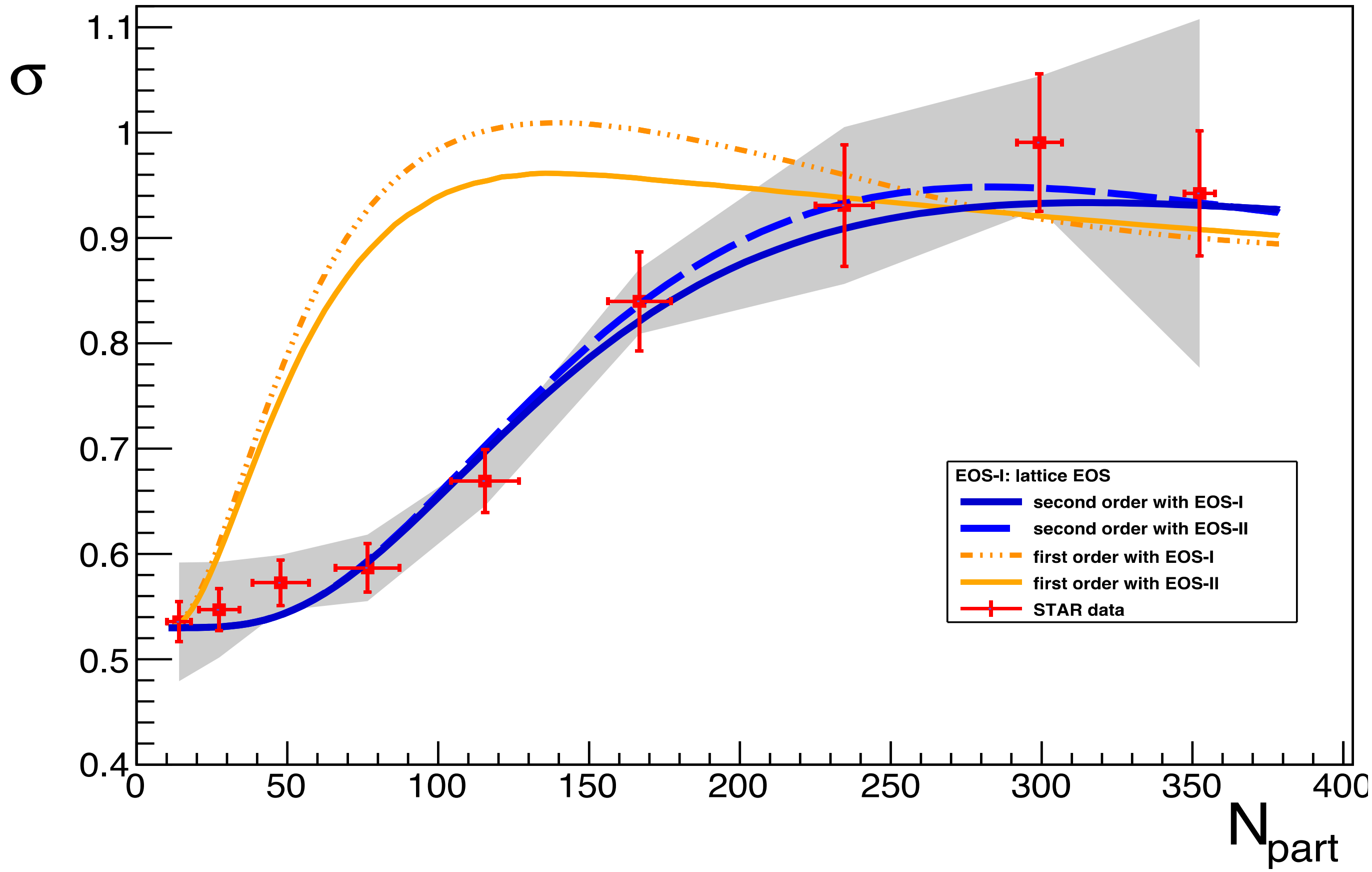


Evolution of C for 10-20%



R. Pokharel, S. Gavin, G. Moschelli in preparation

Results



σ
1.1
1
0.9
0.8
0.7
0.6
0.5
0.4
0
50
100
150
200
250
300
350
400
N_{part}
EOS-I: lattice EOS
second order with EOS-I
second order with EOS-II
first order with EOS-I
first order with EOS-II
STAR data

Summary

- First order hydro calculations do poor job in fitting the experimental (STAR) data.
- NeXSPheRIO calculations (ideal hydro + fluctuations) of width show opposite trend in the variation of width with centralities compared to the data.
- Widths given by second order diffusion agrees with STAR data.
- Second order diffusion calculations show bumps in C for central to mid-central cases and this is also indicated by data.
- Theory the bumps is clear: pronounced effect of wave part of the causal diffusion equation.

Thank You

Backups

Notations & conventions:

$$\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu \qquad g^{\mu\nu} = diag(1,-1,-1,-1)$$

$$\nabla^\lambda = \Delta^{\lambda\mu}\partial_\mu \qquad D = u^\mu \partial_\mu \qquad \Delta_{(\mu\nu)} = \frac{1}{2}(\nabla_\mu u_\nu + \nabla_\nu u_\mu)$$

$$\nabla_{<\mu}u_{\nu>} = (\nabla_\mu u_\nu + \nabla_\nu u_\mu) - \frac{2}{3}\Delta_{\mu\nu}\nabla_\alpha u^\alpha$$

$$\Pi^{\mu\nu} = \pi^{\mu\nu} + \Delta^{\mu\nu}\Pi$$

$$\eta = \frac{1}{2} ln\left(\frac{t+z}{t-z}\right) \qquad \tau = \sqrt{t^2 - z^2}$$

R. Baier, P. Romatschke, and U. A. Wiedemann, Phys.Rev. C73, 064903 (2006), U. W. Heinz, H. Song, and A. K. Chaudhuri, Phys.Rev. C73, 034904 (2006)

$$\tau_\pi D\pi^{\mu\nu} + \pi^{\mu\nu} = \eta\nabla^{<\mu}u^{\nu>}$$

$$\Delta r_g(y_r, y_a) \propto e^{-y_r^2/2\sigma^2 - y_a^2/2\Sigma^2}$$

$$\eta(T) = \begin{cases} [1 + w(T) ln(T/T_C)]^2 T^3 & \text{for } T > T_C \\ T_C^2 T & \text{for } T \leq T_C \end{cases}$$