Initial Conditions with Flow from MV model with Transverse Dynamics

GUANGYAO CHEN
RAINER J. FRIES
TEXAS A&M UNIVERSITY

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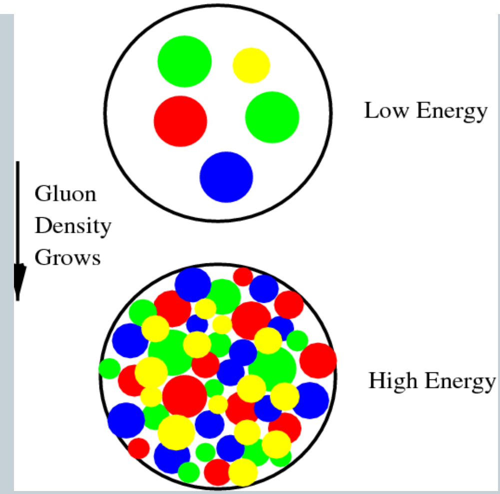
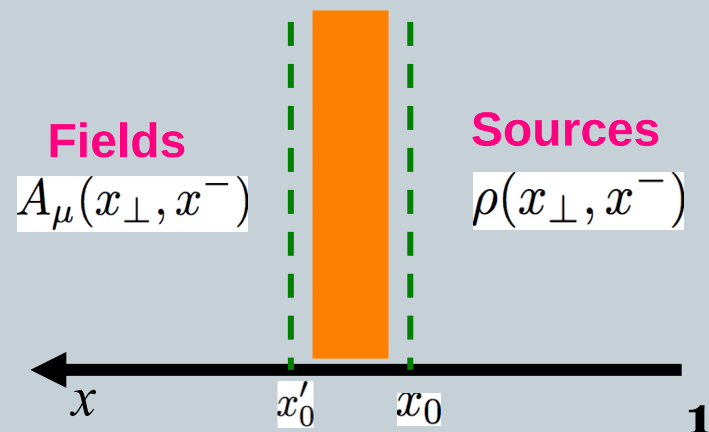
COPAMARINA, PUERTO RICO

Color Glass Condensate

- **Ultrarelativistic Nuclei**
  - Gluon Saturation → scale $Q_s >> \Lambda_{QCD}$
  - Coherent quarks & gluons
  - Large occupation numbers → quasi-classical fields.



- **CGC: Classical Effective Theory of QCD**
  - Static color sources–large x partons
  - Dynamic gluons as Fields—small x partons
  - Gluon Field dynamics governed by Yang-Mills equation $\left[D_\mu, F^{\mu\nu}\right] = J^\nu$
  - RGE(JIMWLK/BK) describe how QCD dynamics changes with energy
  - MV model: Random Color Sources with Weight W[ρ], $\langle O \rangle_\rho = \int [d\rho]\, O(\rho) W(\rho)$



Colliding Nuclei as a Color Capacitor

Yang-Mills equations(ρ_1, ρ_2):

- Currents on light cone J_1, J_2 (from ρ_1, ρ_2), YM equations for gluon field $A^\mu(\rho_1, \rho_2)$.
- Forward light cone: free YM equations

$$A^i = A^i_\perp(\tau, x_\perp)$$

$$A^\pm = \pm x^\pm A(\tau, x_\perp)$$

- Boundary Condition

$$A^i_\perp(\tau=0, x_\perp) = A^i_1(x_\perp) + A^i_2(x_\perp)$$

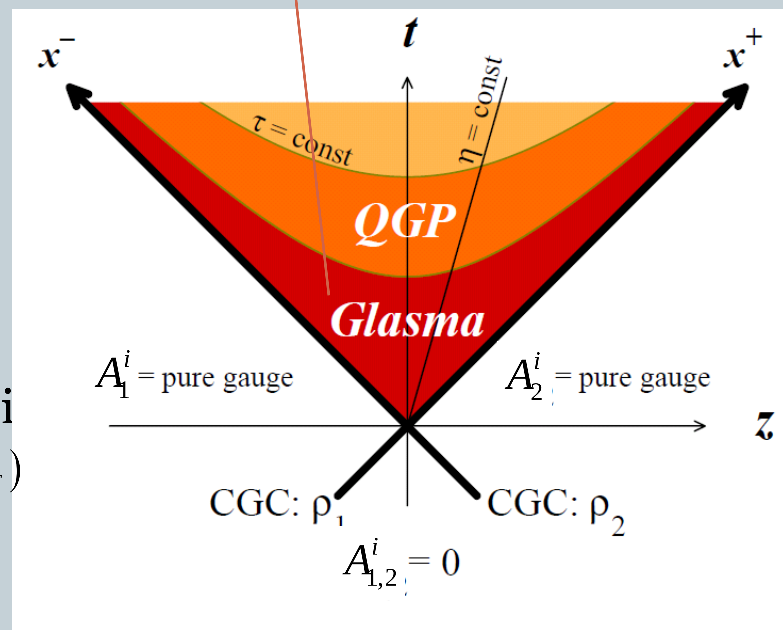
$$A(\tau=0, x_\perp) = -\frac{ig}{2}\left[A^i_1(x_\perp), A^i_2(x_\perp)\right]$$

$$\frac{1}{\tau^3}\partial_\tau \tau^3 \partial_\tau A - \left[D^i,\left[D^i, A\right]\right] = 0$$

$$\frac{1}{\tau}\left[D^i, \partial_\tau A^i_\perp\right] - ig\tau\left[A, \partial_\tau A\right] = 0$$

$$\frac{1}{\tau}\partial_\tau \tau \partial_\tau A^i_\perp - ig\tau^2\left[A,\left[D^i, A\right]\right] - \left[D^j, F^{ji}\right] = 0$$

[A. Kovner, L. McLerran, H. Weigert, 95']



Average Over Color Charge μ^2_i

- $\left\langle \rho_i^a(x_1)\rho_j^b(x_2)\right\rangle_\rho = \frac{g^2}{N_c^2-1}\delta_{ij}\delta^{ab}\lambda\left(x_1^\mp\right)\delta\left(x_1^\mp - x_2^\mp\right)\delta^2\left(\mathbf{x}_{1T} - \mathbf{x}_{2T}\right)$
- $\mu_i^2(\mathbf{x}_T) = \int dx^\mp \lambda\left(x^\mp, \mathbf{x}_T\right)$

A Recursive Solution

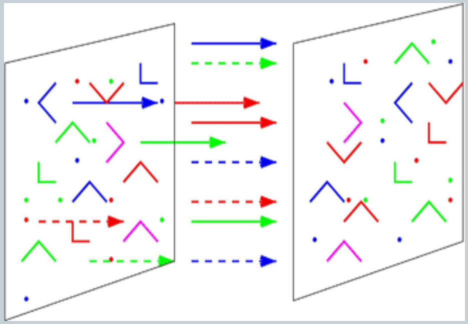
- Series expansion $A(\tau, x_\perp) = \sum_{n=0}^{\infty} \tau^n A_{(n)}(x_\perp)$ with $A^i_{\perp(0)}(x_\perp) = A^i_1(x_\perp) + A^i_2(x_\perp)$

$$A^i_\perp(\tau, x_\perp) = \sum_{n=0}^{\infty} \tau^n A^i_{\perp(n)}(x_\perp) \qquad A_{(0)}(x_\perp) = -\frac{ig}{2}\left[A^i_1(x_\perp), A^i_2(x_\perp)\right]$$

- Recursive Solution

$$A_{(n)} = \frac{1}{n(n+2)} \sum_{k+l+m=n-2} \left[D^i_{(k)}, \left[D^i_{(l)}, A_{(m)}\right]\right]$$

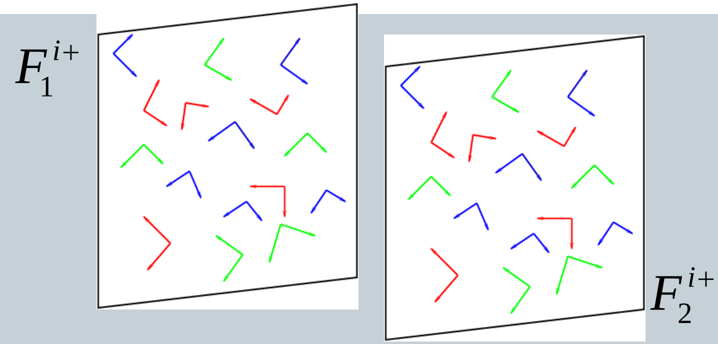
$$A^i_{\perp(n)} = \frac{1}{n^2}\left(\sum_{k+l=n-2}\left[D^j_{(k)}, F^{ji}_{(l)}\right] + ig \sum_{k+l+m=n-4}\left[A_{(k)}, \left[D^i_{(l)}, A_{(m)}\right]\right]\right)$$



- Advantages
  - Accurate for early time (small τ) physics quantities
  - Dependence on space-time coordinates
  - Analytically! (as much as possible)

[RJF, J. Kapusta, Y. Li, 06']

Chromo E&M Fields

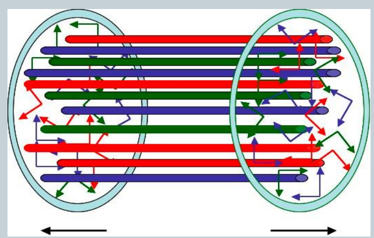


Before the Collision

- $\vec{E} \perp \vec{B} \perp \vec{z}$ Initial fields!
- $F_1^{i+} = \delta\left(x^-\right) A_1^i \quad , \quad F_2^{i+} = \delta\left(x^-\right) A_2^i$

Immediately after collision

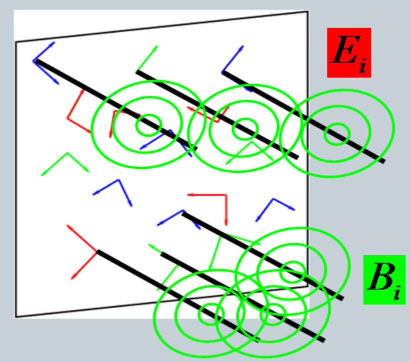
- $E_0 = F_{(0)}^{+-} = ig\left[A_1^i, A_2^i\right]$
- $B_0 = F_{(0)}^{21} = ig\epsilon^{ij}\left[A_1^i, A_2^j\right]$
- Non-abelian effects!



[L. McLerran, T. Lappi, 06']
[RJF, J.I. Kapusta, Y. Li, 06']

As time goes by

- $F_{(1)}^{i\pm} = -\frac{e^{\pm\eta}}{2\sqrt{2}}\left(\varepsilon^{ij}\left[D_{(0)}^j, B_0\right] \pm \left[D_{(0)}^i, E_0\right]\right)$
- Transverse E i and B i
- $E_{(2)}^3, B_{(2)}^3, F_{(3)}^{i\pm}$



Higher Accuracy

- $E_{(4)}^3 = \frac{1}{64}[D^i,[D^i,[D^j,[D^j,E_0]]]] + \frac{1}{16} ig\epsilon^{ij}[[D^i,E_0],[D^j,B_0]]$,... [GC,RJF]

Energy Momentum Tensor

General Structure

$$\vec{S} = \vec{E} \times \vec{B}$$

$$\frac{1}{2}\left(E^2 + B^2\right)$$

$$T_{\mathrm{f}}^{\mu\nu} = \begin{pmatrix} \varepsilon_0 + O\left(\tau^2\right) & \alpha^1 \cosh\eta + \beta^1 \sinh\eta & \alpha^2 \cosh\eta + \beta^2 \sinh\eta & O\left(\tau^2\right) \\ \alpha^1 \cosh\eta + \beta^1 \sinh\eta & \varepsilon_0 + O\left(\tau^2\right) & O\left(\tau^2\right) & \alpha^1 \sinh\eta + \beta^1 \cosh\eta \\ \alpha^2 \cosh\eta + \beta^2 \sinh\eta & O\left(\tau^2\right) & \varepsilon_0 + O\left(\tau^2\right) & \alpha^2 \sinh\eta + \beta^2 \cosh\eta \\ O\left(\tau^2\right) & \alpha^1 \sinh\eta + \beta^1 \cosh\eta & \alpha^2 \sinh\eta + \beta^2 \cosh\eta & -\varepsilon_0 + O\left(\tau^2\right) \end{pmatrix}$$

Interpretation of Poynting Vector

$$\alpha^i = -\frac{\tau}{2}\nabla^i \varepsilon_0$$

Hydro-Like flow, ; even in rapidity.

$$\beta^i = \frac{\tau}{2}\varepsilon^{ij}\left(\left[D^j, B_0\right]E_0 - \left[D^j, E_0\right]B_0\right)$$

Odd in rapidity, Suprise???!! Will be back soon!!

[RJF, J.I. Kapusta, Y. Li, (2006)]

MV model with Transverse Dynamics

- Energy Density — Hydro-Like Flow
  - $\varepsilon_0 \sim \mu_1^2 \mu_2^2 \ln^2 \frac{Q_0^2}{m^2}$ [Lappi, 09'] [RJF, Kapusta, Li, 06'] [Fujii, Fukushima, Hidaka, 09'] $\alpha^i \sim \nabla^i \left(\mu_1^2 \mu_2^2 \right)$
- MV Model
  - Infinite Large Nuclei— μ^2_i = const.
  - All transverse flow vanish.
- MVTD
  - Realistic Nuclei profile— μ^2_i $(\mathbf{x})$.
  - Non-vanish flow comes in from shape of nuclei.
  - Infrared Safe if $\mu^2(\mathbf{x}_T) >> m^{-1} \left| \nabla^i \mu^2(\mathbf{x}_T) \right| >> m^{-2} \left| \nabla^i \nabla^j \mu^2(\mathbf{x}_T) \right|$, where m is an infrared regulator (gluon mass) which $m << Q_s$.
  - μ^2_i almost constant over length scale 1/ Q_s .

[GC,RJF]

MV model with Transverse Dynamics

- MV model — MVTD
  - Charge Density correlation

$$\left\langle \rho^a(x_1)\rho^b(x_2)\right\rangle \sim \delta^{ab}\lambda\left(x_1^-\right)\delta\left(x_1^- - x_2^-\right)\delta^2\left(\mathbf{x}_{1T}-\mathbf{x}_{2T}\right)$$

$$\left\langle \rho^a(x_1)\rho^b(x_2)\right\rangle \sim \delta^{ab}\lambda\left(x_1^-,\mathbf{x}_T\right)\delta\left(x_1^- - x_2^-\right)\delta^2\left(\mathbf{x}_{1T}-\mathbf{x}_{2T}\right)$$

  - Infrared Safety

$$\Gamma\left(\mathbf{x}_T,\mathbf{y}_T\right)=\mu^2\frac{r^2}{8\pi}\ln m^2r^2$$

$$\Gamma\left(\mathbf{x}_T,\mathbf{y}_T\right)=\mu^2(\mathbf{R})\frac{r^2}{8\pi}\ln m^2r^2+\frac{\nabla^i\nabla^j\mu^2(\mathbf{R})}{m^2}\frac{r^2}{48\pi}\left(-\delta^{ij}+\frac{r^ir^j}{r^2}\frac{13}{2}\right)$$

$$\mathbf{R}=\mathbf{x}_T+\mathbf{y}_T \qquad \mathbf{r}=\mathbf{x}_T-\mathbf{y}_T$$

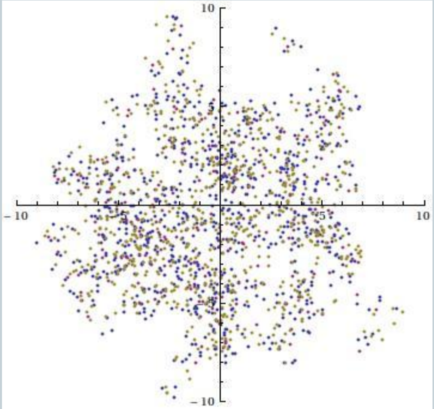
- Two Gluon Correlator in MVTD

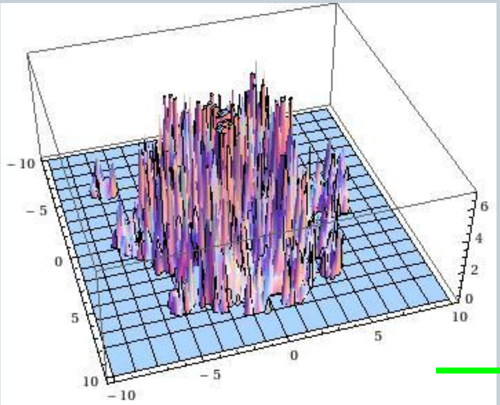
$$\left\langle A^{i,a}(\mathbf{x}_T)A^{j,b}(\mathbf{x}_T)\right\rangle=\delta^{ab}\frac{g^2\mu^2(\mathbf{x}_T)}{8\pi\left(N_c^2-1\right)}\left[\delta^{ij}\ln\frac{Q^2}{m^2}+\frac{\nabla^k\nabla^l\mu^2(\mathbf{x}_T)}{m^2\mu^2(\mathbf{x}_T)}\left(\frac{1}{6}\delta^{ij}\delta^{kl}-\frac{7}{12}\delta^{ik}\delta^{jl}\right)\right]$$

[GC,RJF]

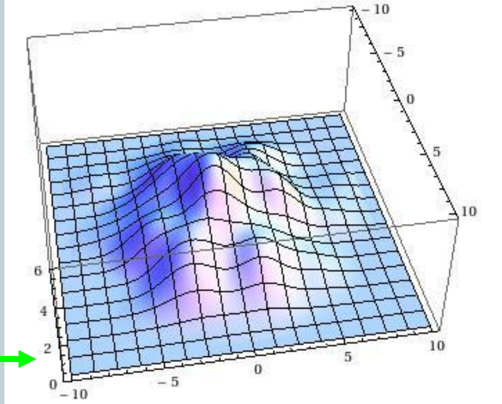
E-by-E Initial Condition

- Procedures:
  - Step 1: MC nucleon position based on Wood-Saxon profile.
  - Step 2: MC large x parton inside each nucleon.
  - Step 3: Assume charge density ~ Parton number density





Smear out



  - Step 4: Smear out charge density. Reason one: large number of sources not fulfilled for realistic nulei; reason two: charge should be almost constant over saturation scale.
  - Step 5: Calculate each component of $T^{\mu\nu}$
  - Step 6: Matching!

[GC,RJF, Salzillo]

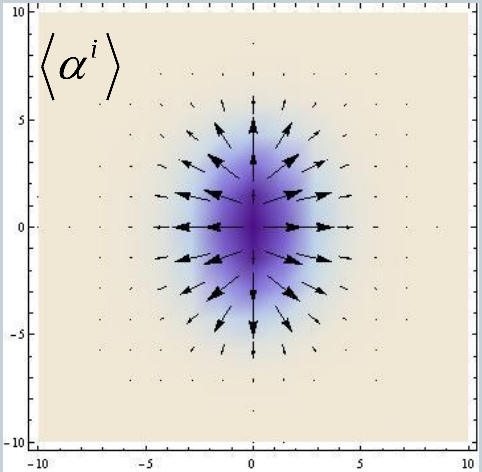
Poynting Vector

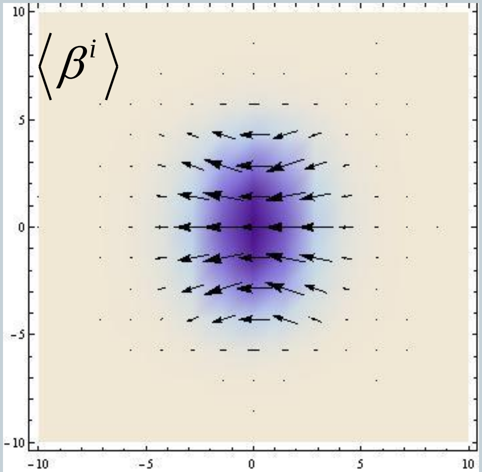
- Expectation Value in MVTD

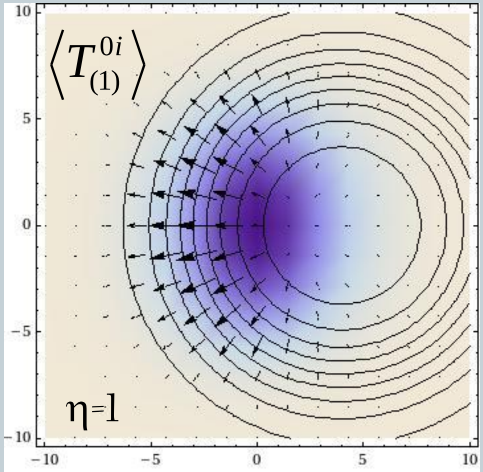
$$\langle \beta^i \rangle = -\tau \frac{g^6 N_c (N_c^2 - 1)}{64\pi^2} \left(\mu_2^2 \nabla^i \mu_1^2 - \mu_1^2 \nabla^i \mu_2^2 \right) \ln^2 \frac{Q^2}{m^2} \qquad \langle \alpha^i \rangle = \tau \frac{g^6 N_c (N_c^2 - 1)}{64\pi^2} \nabla^i \left(\mu_1^2 \mu_2^2 \right) \ln^2 \frac{Q^2}{m^2}$$

  - Does not vanish in MVTD with finite b, or asymmetry nuclei.
- $\langle \beta^i \rangle$ directed flow! $\langle \alpha^i \rangle$ Elliptic Flow!
  - $\langle T_{(1)}^{0i} \rangle = \frac{1}{2} \cosh \eta \langle \alpha^i \rangle + \frac{1}{2} \sinh \eta \langle \beta^i \rangle$

[GC,RJF]



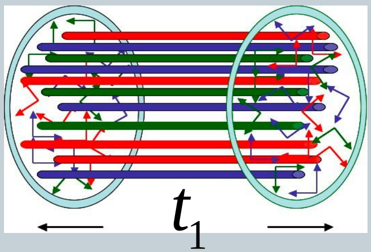




Directed Flow From Maxwell's Equations

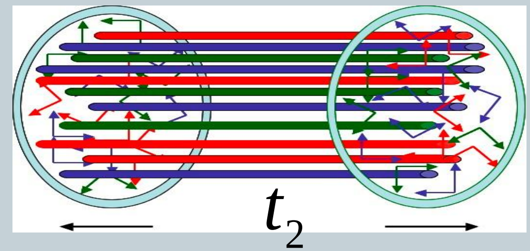
An Intuitive Interpretation

- Ampere(Faraday) 's Law gives rapidity-even transverse E&M fields



$$t_1 < t_2$$

$$E_z, B_z(t_1) > E_z, B_z(t_2)$$

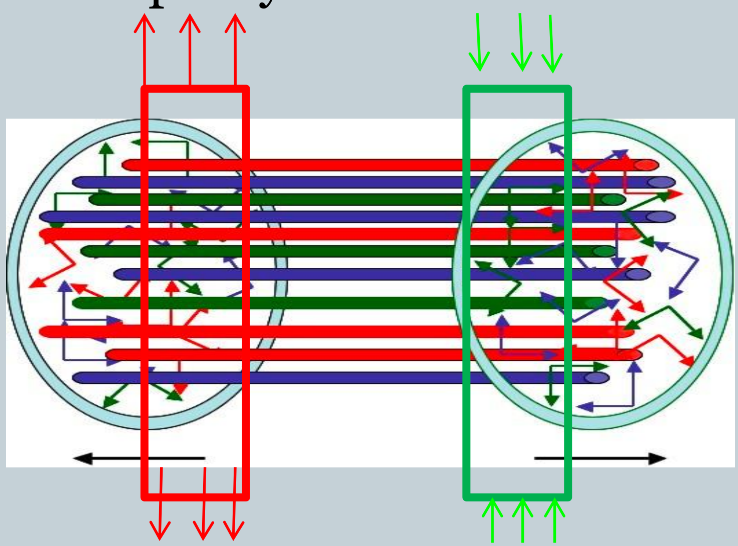


- Gauss 's Law gives rapidity-odd transverse E&M fields

$$\eta < 0$$

$$z_1 < z_2$$

$$E_z, B_z(z_1) > E_z, B_z(z_2)$$



$$\eta < 0$$

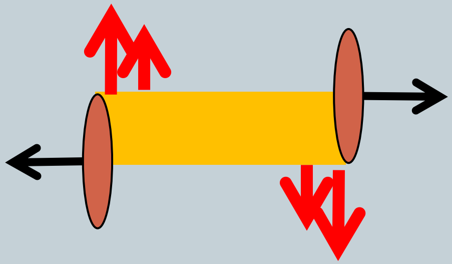
$$z_1 < z_2$$

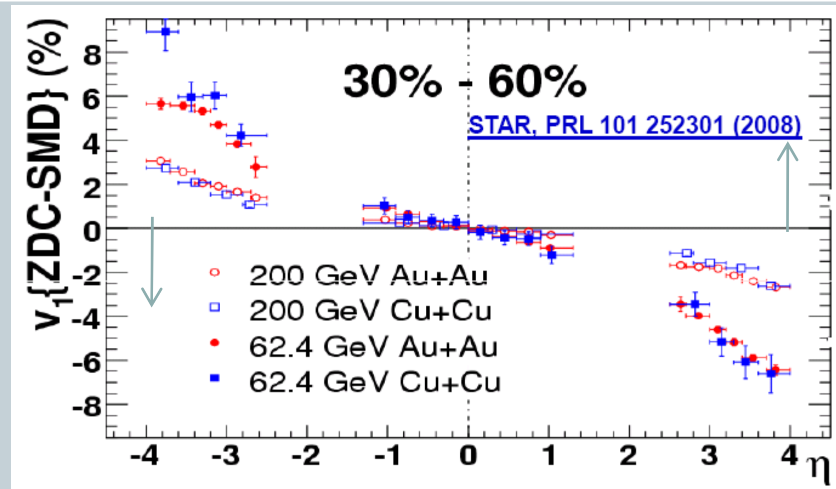
$$E_z, B_z(z_1) < E_z, B_z(z_2)$$

[GC,RJF]

Explain the Experiments

- The direction of flow (√)

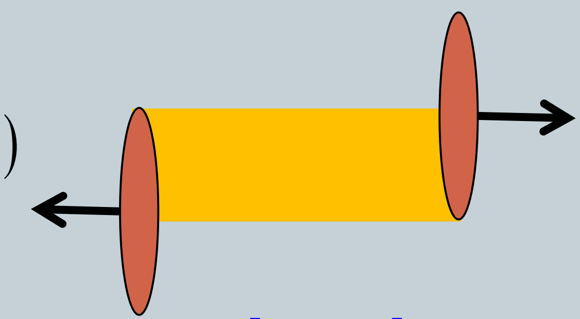




- Independent of the system-size(Cu-Cu or Au-Au) (√)

$$\langle \beta^i \rangle \sim \left(\mu_2^2 \nabla^i \mu_1^2 - \mu_1^2 \nabla^i \mu_2^2 \right)$$

$$\mu_{Au}^2 \approx \sqrt[3]{3} \mu_{Cu}^2$$

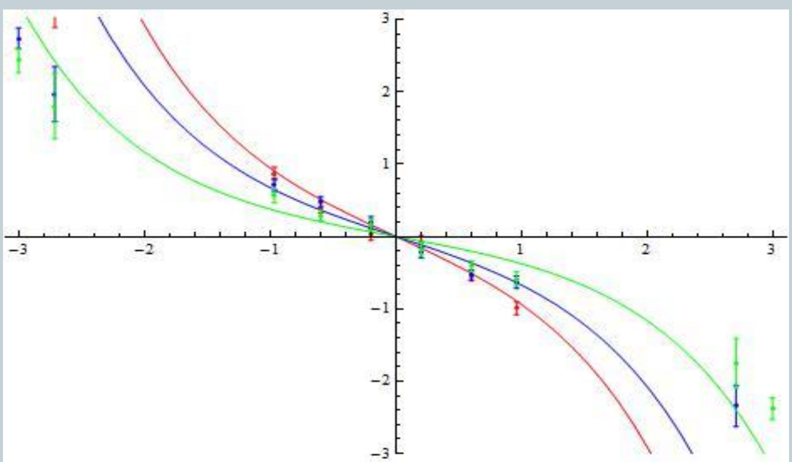


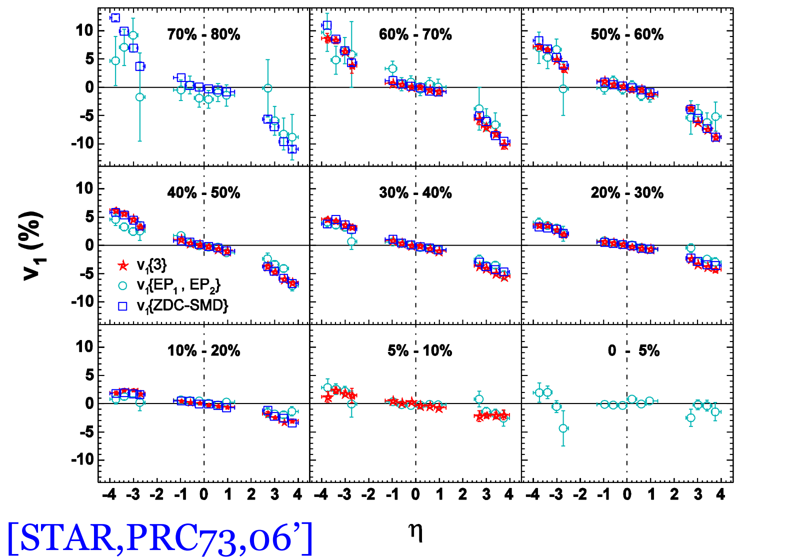
[GC,RJF]

- Energy dependence (Maybe!)
  - $\langle v^1 \rangle \sim \langle p^x \rangle / \langle p^t \rangle$, $\langle p^t \rangle$ increase fast with $\sqrt{s}$, $\langle p^x \rangle$ probably not change.

Explain the Experiments

- Centrality Dependence (√)
  - Directed flow from different centrality is fix up to one constant.
  - Normalization from 30-40% data(red), comparison of 20-30%(Blue) and 10-20%(Green) data.





[STAR,PRC73,06']

  - Conclusion: Fit within Uncertainties!

Note: Such approach may break down for high centrality data!

[GC,RJF]

From Fields to Plasma

Fields to Plasma at τ = τ₀ .

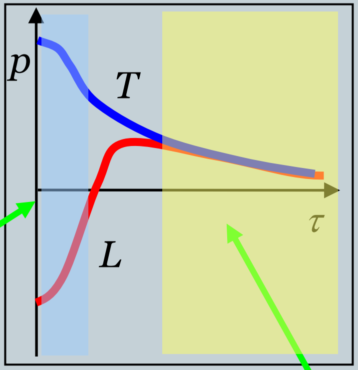
- Matching to Ideal Hydrodynamics from energy momentum conservation.

$$\vec{v}_\perp = \frac{1}{\cosh\eta}\frac{\vec{\alpha}}{\varepsilon_0+\varepsilon_0'+p},\quad v_L=\tanh\eta,\quad \epsilon+p=(\varepsilon_0+\varepsilon_0'+p)\left(1-\frac{\vec{\alpha}^2}{(\varepsilon_0+\varepsilon_0'+p)^2}\right)$$

[RJF, Kapusta, Li, 06']

- Matching to Viscous Hydrodynamics through

$$T^{\mu\nu}_{fields}(\tau_0,\vec{x},\eta)=T^{\mu\nu}_{viscoushydro}(\tau_0,\vec{x},\eta)$$



- 9 Independent components on each side, 9 equations!
- May only be solved numerically.

Longitudinal Fields dominate; CGC

(viscous) plasma

- In Progress....

[GC,RJF]

$$T^{\mu\nu}_{\mathrm{f}}=\begin{pmatrix}\varepsilon_{00}&\varepsilon_{01}&\varepsilon_{02}&\varepsilon_{03}\\\varepsilon_{10}&\varepsilon_{11}&\varepsilon_{12}&\varepsilon_{13}\\\varepsilon_{20}&\varepsilon_{21}&\varepsilon_{22}&\varepsilon_{23}\\\varepsilon_{30}&\varepsilon_{31}&\varepsilon_{32}&\varepsilon_{33}\end{pmatrix}\longrightarrow T^{\mu\nu}_{\mathrm{viscous}}(\tau=\tau_0,\vec{v},\varepsilon,\vec{\pi})$$

Summary

- Energy-Momentum Tensor from Recursive solution of Yang-Mills equation;
- MV model with Transverse dynamics;
- Directed flow emerges in MVTD;
- Can predict an initial condition for viscous-hydro;
- An E-by-E initial condition based on parton distribution.

Thank You!

Directed Flow in QCD

- Full QCD transverse Fields

$$E^i = -\frac{\tau}{2}\left(\sinh\eta\left[D^i, E_0\right] + \cosh\eta\ \varepsilon^{ij}\left[D^j, B_0\right]\right)$$

$$B^i = \frac{\tau}{2}\left(\cosh\eta\ \varepsilon^{ij}\left[D^j, E_0\right] - \sinh\eta\left[D^i, B_0\right]\right)$$

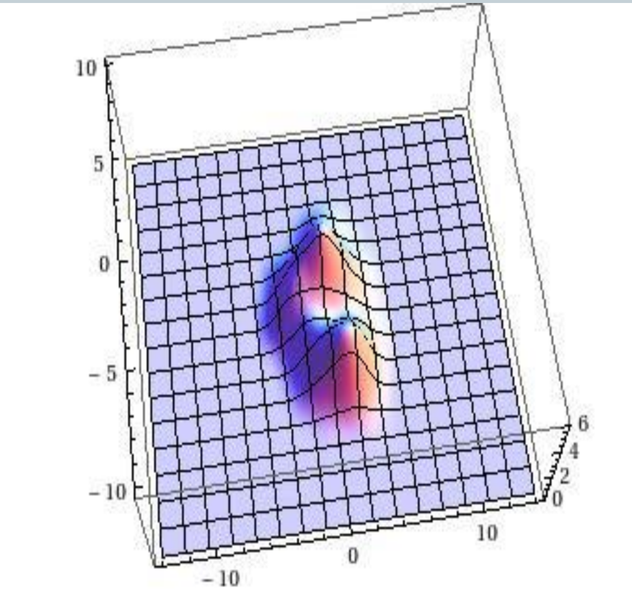
- Full QCD Poynting Vector

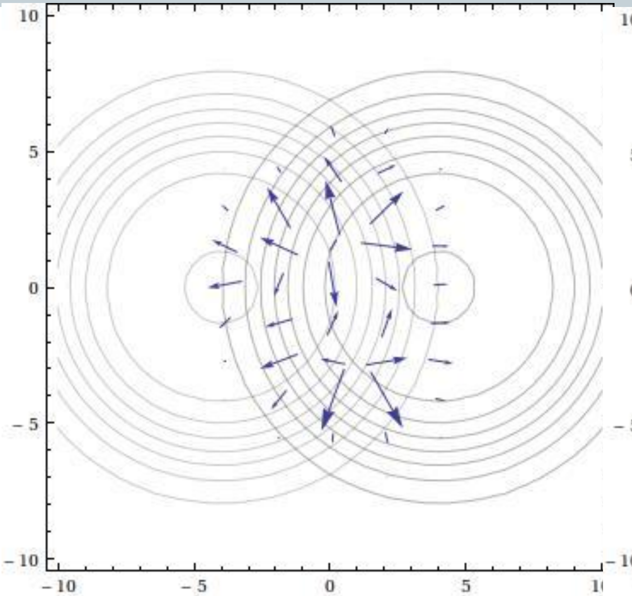
$$S^i_{\text{even}} = \frac{\tau}{2}\cosh\eta\left(E_0\left[D^i, E_0\right] + B_0\left[D^i, B_0\right]\right) = \alpha^i \cosh\eta$$

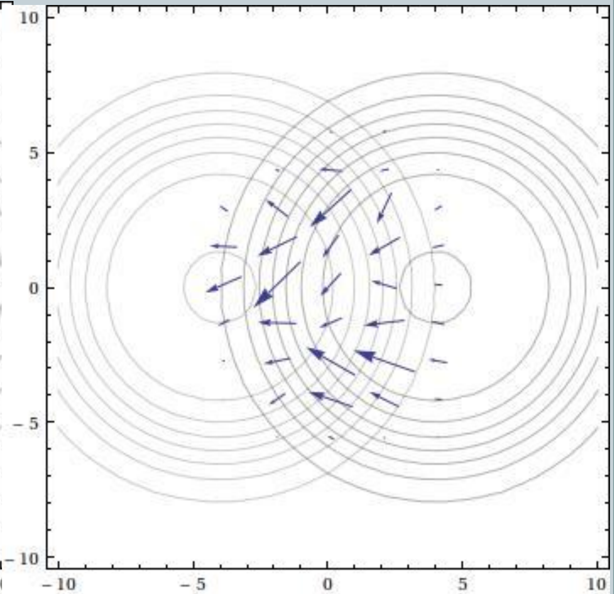
$$S^i_{\text{odd}} = \frac{\tau}{2}\sinh\eta\ \varepsilon^{ij}\left(E_0\left[D^j, B_0\right] - B_0\left[D^j, E_0\right]\right) = \beta^i \sinh\eta$$

A Typical E-by-E Simulation

- Energy density　　Radial Flow　　Directed flow

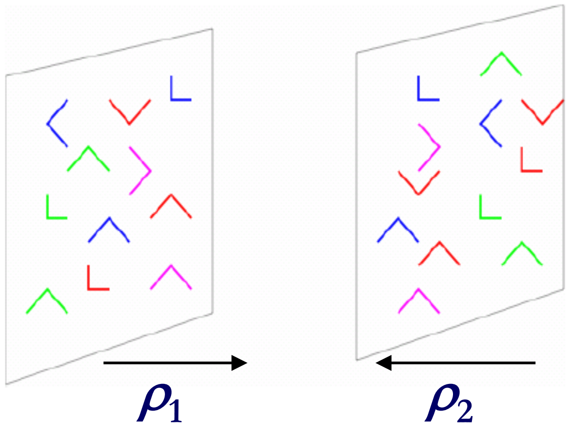




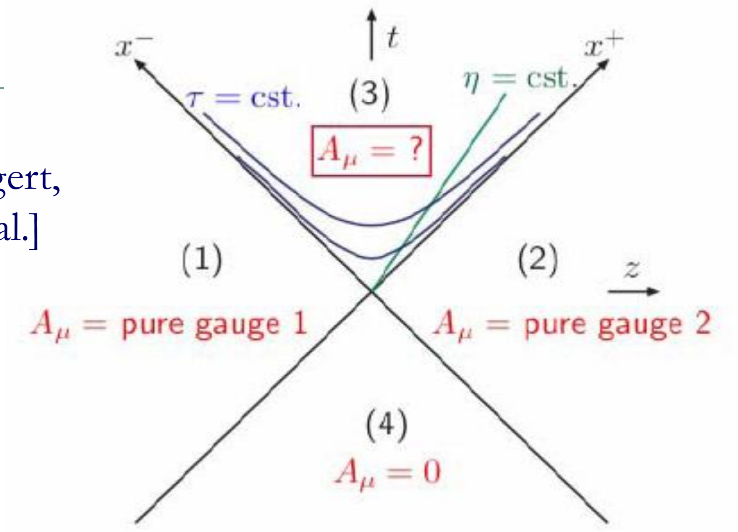


CGC as the initial condition for H.I.C.

HIC = Collision of two sheets



[Kovner, Weigert, McLerran, et al.]



Each source creates the gluon field for each nucleus.← Initial condition

$$J^\mu = \delta^{\mu +}\delta(x^-)\rho_1(\mathbf{x}_T) + \delta^{\mu -}\delta(x^+)\rho_2(\mathbf{x}_T)$$

$$-D_i\alpha^i_{(m)} = \rho_{(m)}(\mathbf{x}_\perp).$$ α_1 , α_2 : gluon fields of nuclei

In Region (3), and at $\tau = 0+$, the gauge field is determined by α_1 and α_2

$$A^\pm = \pm x^\pm \alpha(\tau, x_T)$$
$$A^i = \alpha^i_3(\tau, x_T).$$

$$\alpha^i_3 \,|_{\tau=0} = \alpha^i_1 + \alpha^i_2$$
$$\alpha \,|_{\tau=0} = \frac{ig}{2}\left[\alpha^i_1, \alpha^i_2\right]$$
$$\partial_\tau \alpha \,|_{\tau=0} = \partial_\tau \alpha^i_3 \,|_{\tau=0} = 0.$$

$$\left(x^+A^- + x^-A^+\right) = 0.$$