Hydrodynamics and flow in heavy-ion collisions

A brief overview

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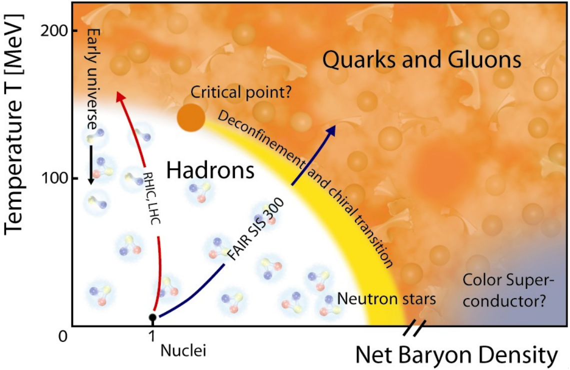
Hot Quarks
October 17, 2012

THEORETICAL MOTIVATION FOR HYDRODYNAMICS

- Fundamental theory of strong interactions: Quantum Chromodynamics (QCD)
- If sufficient separation of scales, system behaves as a fluid
- Hydrodynamic description is the correct description of QCD if system large enough / interactions strong enough



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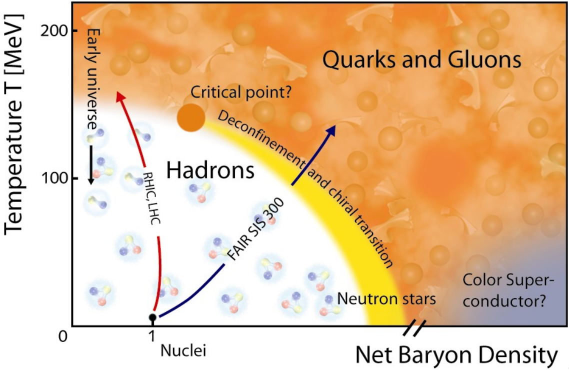

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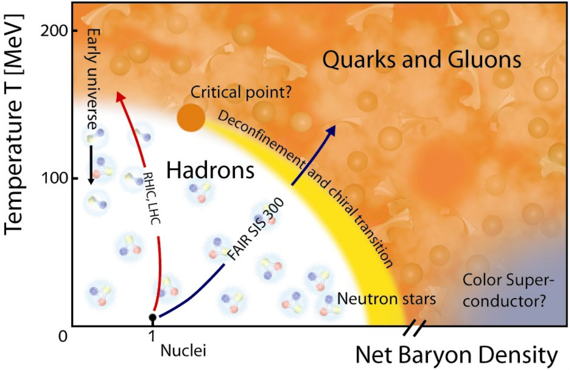

(RELATIVISTIC) HYDRODYNAMICS

- Hydrodynamic equations = conservation equations:

$$\partial_\mu T^{\mu\nu} = 0$$

- Define Local Rest Frame $\equiv$ zero momentum frame:

$$T^{0i}_{\text{rest}} \equiv 0$$

- Ideal hydro = isotropy in LRF:

$$T^{\mu\nu}_{\text{rest}} = T^{\mu\nu}_{0_{\text{rest}}} = \begin{pmatrix} \epsilon & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix}$$

- Viscosity/dissipation: gradient expansion

$$T^{\mu\nu} = T^{\mu\nu}_0 + \eta \nabla^{\langle\mu} u^{\nu\rangle} + \zeta \Delta^{\mu\nu} \nabla_\alpha u^\alpha + \dots$$

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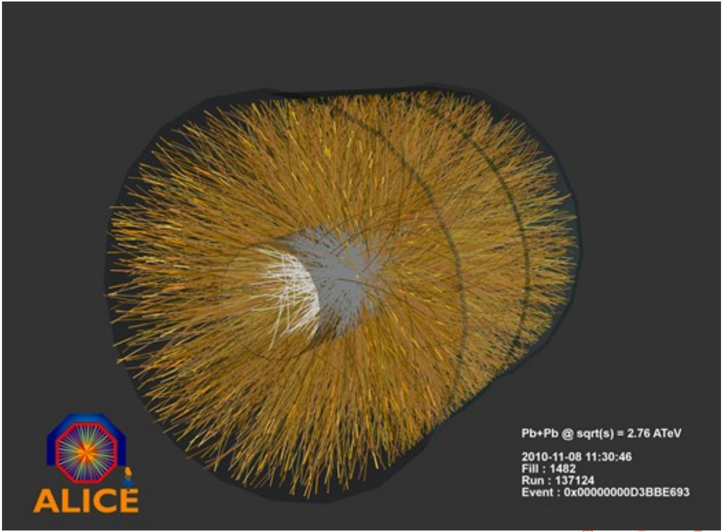
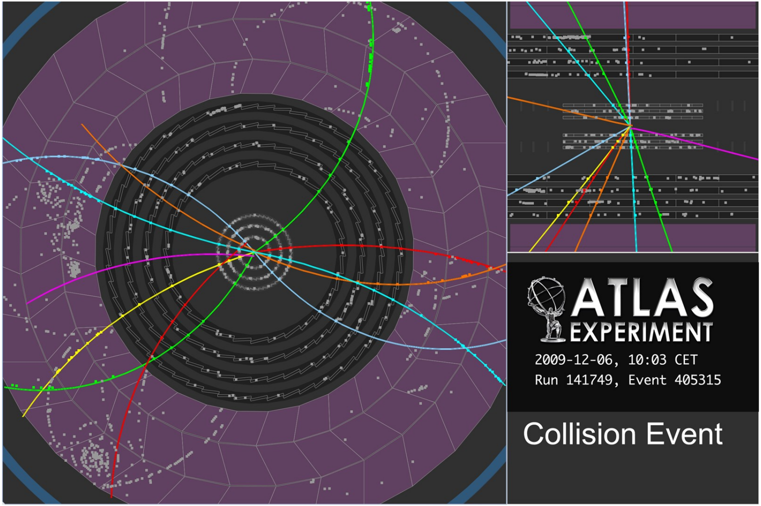
HEAVY-ION COLLISION

What we see in a collision:



PROTON-PROTON COLLISION

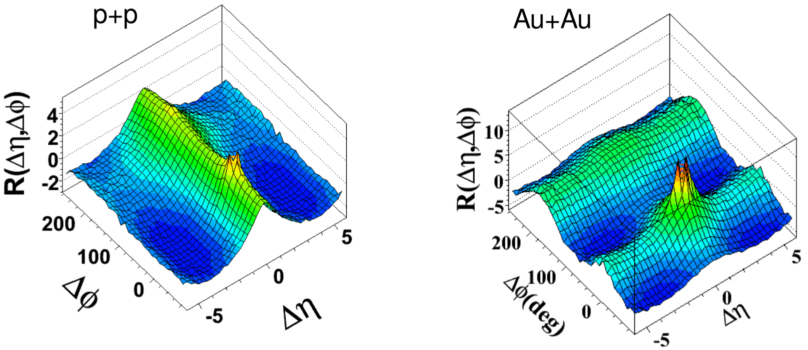
Compare to proton-proton:



http://atlas.web.cern.ch/Atlas/public/EVTDISPLAY/events.html

Two-particle correlations

Experimental motivation for hydrodynamics:



(PHOBOS, Phys.Rev. C81 (2010) 024904)

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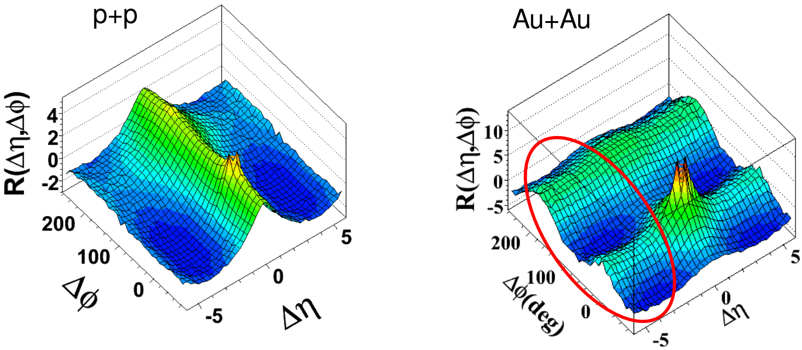

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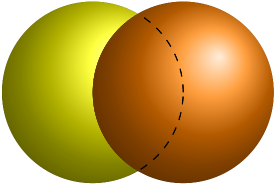
Unique long-range correlations indicate strong collective behavior

FLOW



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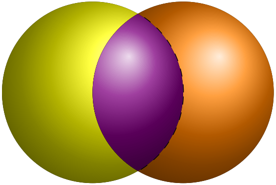

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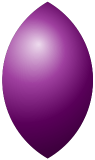

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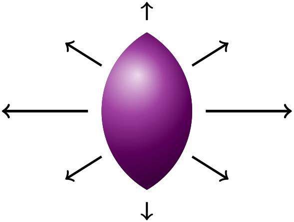

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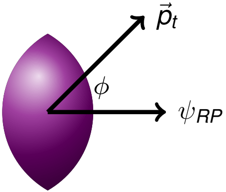
$$\frac{dN}{d\phi} \propto 1 + 2v_2 \cos 2\phi + 2v_4 \cos 4\phi + \ldots$$



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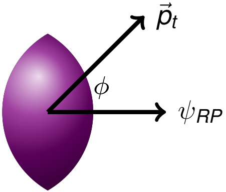

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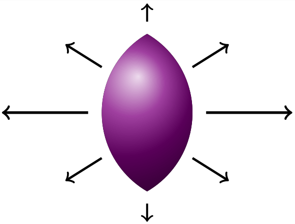

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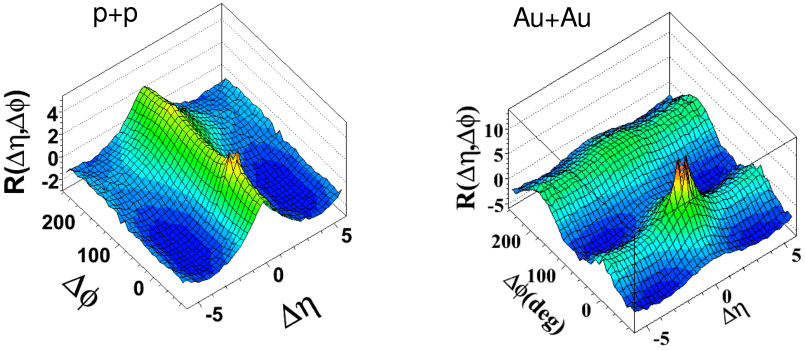

(PHOBOS, Phys.Rev. C81 (2010) 024904)

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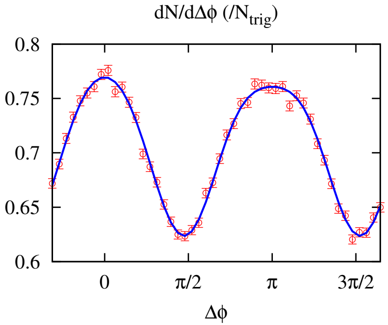

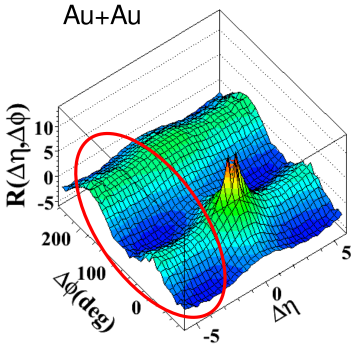

(STAR, arXiv:1010.0690)

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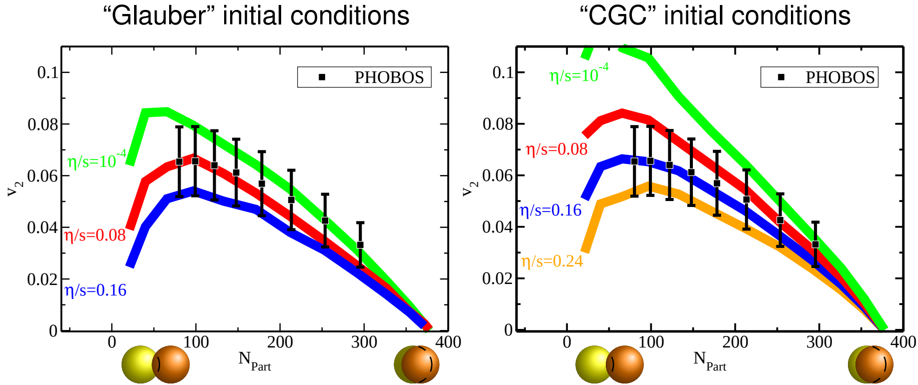
Elliptic Flow and viscosity



(ML & Romatschke, Phys.Rev. C78 (2008) 034915)

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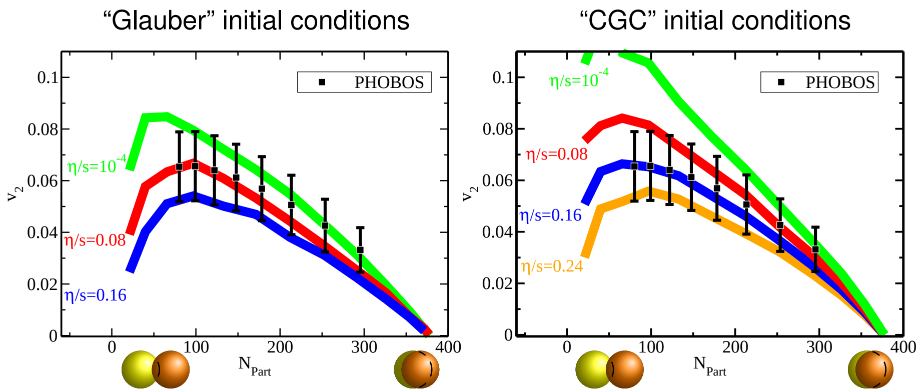

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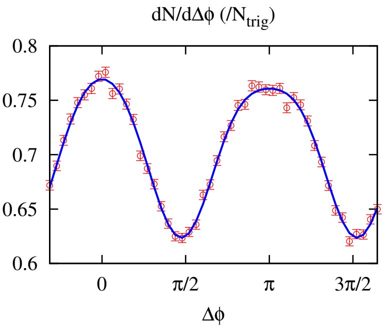

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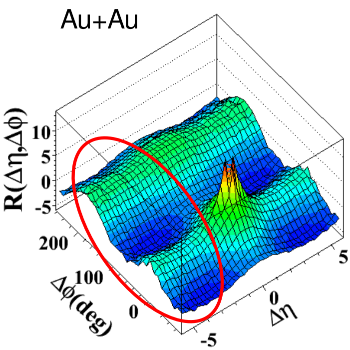

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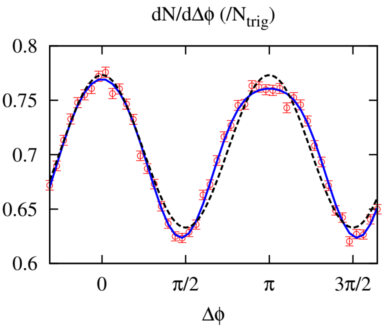

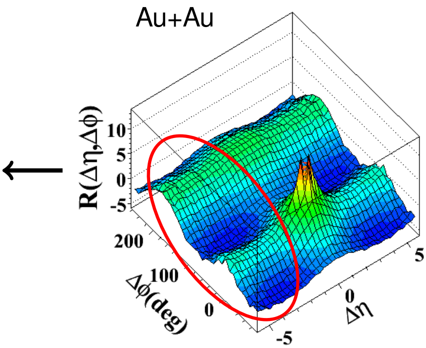

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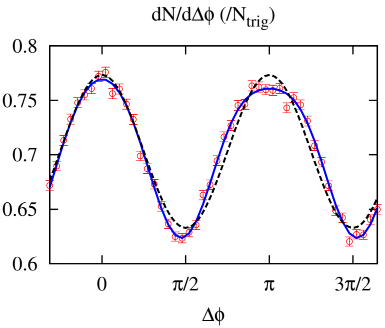

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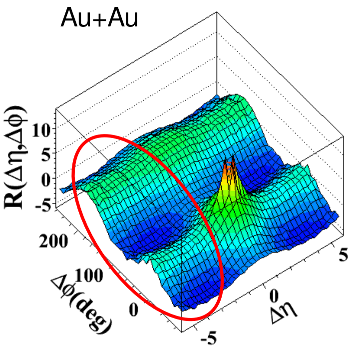

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…can be entirely explained by collective flow:

Flow fluctuations

$$\frac{dN}{d\phi} \propto 1 + 2v_2 \cos 2\phi + 2v_4 \cos 4\phi + \ldots$$

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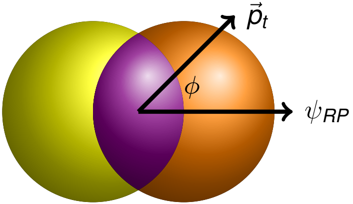

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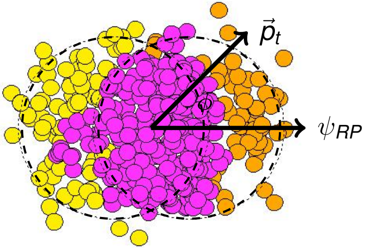

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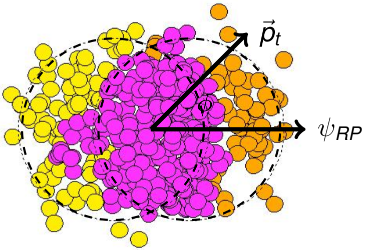

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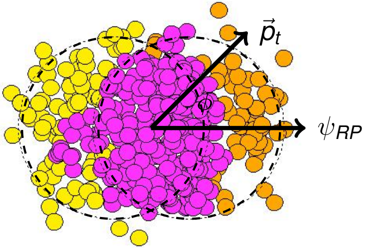

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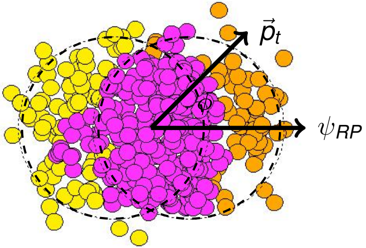

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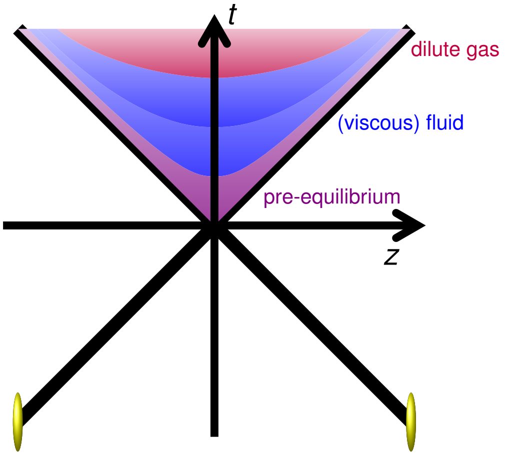
Collision Evolution



A full calculation includes:
hydro evolution, initial conditions, freeze-out

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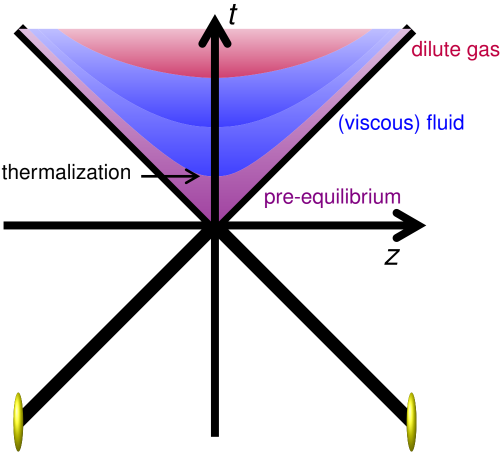

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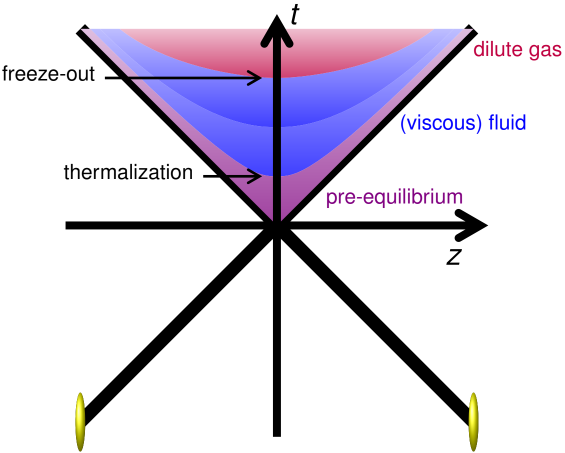

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Initial Conditions

Various models are used to generate initial conditions:

- MC-Glauber
- Color Glass Condensate-inspired pictures
  - MC-KLN
  - MCrcBK
  - IP-Glasma
- Hadronic/partonic cascades
  - UrQMD
  - NeXus
  - EPOS
  - BAMPS
  - HIJING/AMPT

Free parameters usually include:

- "Thermalization time" τ_0
- Initial temperature T_i

HYDRO

- Equation of state $p(\epsilon)$ can be reliably obtained from Lattice QCD calculations (though not $p(\epsilon, \rho_B)$)
- Transport coefficients $\eta(T)$, $\zeta(T)$ usually taken as free parameters
- Many calculations assume approximate boost invariance near mid rapidity and solve equations in 2+1 D

FREEZE-OUT

- Must switch from fluid description to particle description
- When a fluid cell reaches a predefined criterion, particles emitted according to kinetic theory

$$f(p^\mu) = f_0(p \cdot u) + \delta f(p^\mu)$$

- Standard Cooper-Frye prescription takes this as the final distribution:

$$E\frac{d^3N}{d^3\mathbf{p}} \equiv \frac{d}{(2\pi)^3} \int p_\mu d\Sigma^\mu f\left(x^\mu, p^\mu\right),$$

(followed by the decay of unstable resonances)

- Or one can allow the particles to interact further with an "afterburner" (UrQMD, JAM, etc.)
- Can also implement separate "chemical freeze out"

SUMMARY

- Heavy-ion collisions show strong long-range correlations, well described by hydrodynamics…
- $\Longrightarrow$ medium is strongly-interacting, low-viscosity fluid
- Event-by-event fluctuations can not be neglected!
- Many new flow observables have been recently measured, with more still to come
- This information allows for precise extraction of QGP properties as well as strong constraints on theory (e.g., the geometry and fluctuations of the early stages of the collision).

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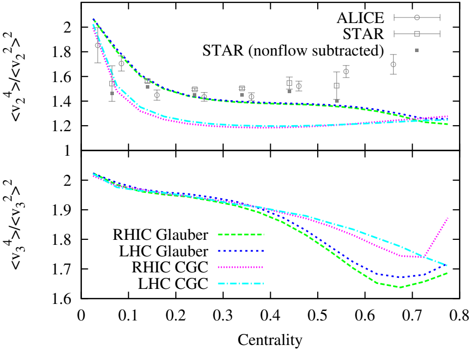
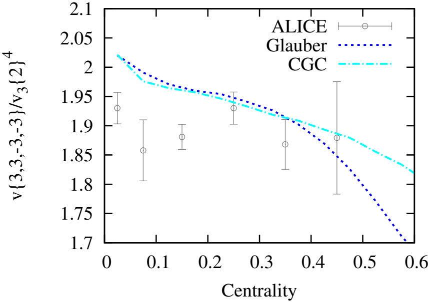
EXTRA SLIDES

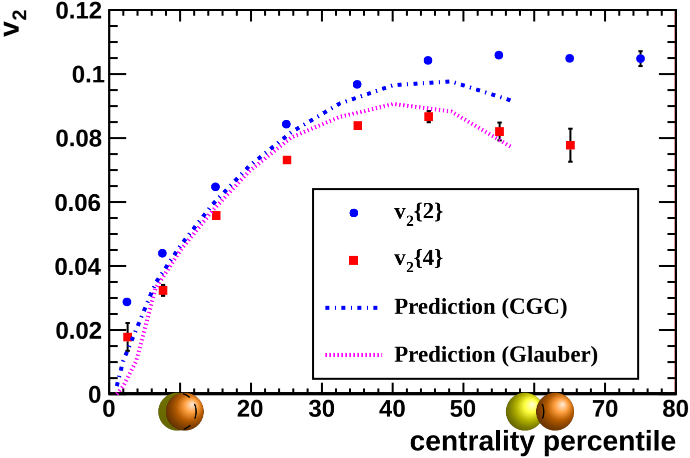




(Bhalerao, ML, Ollitrault, Phys.Rev. C84 (2011) 034910

Recent results: Elliptic flow at LHC

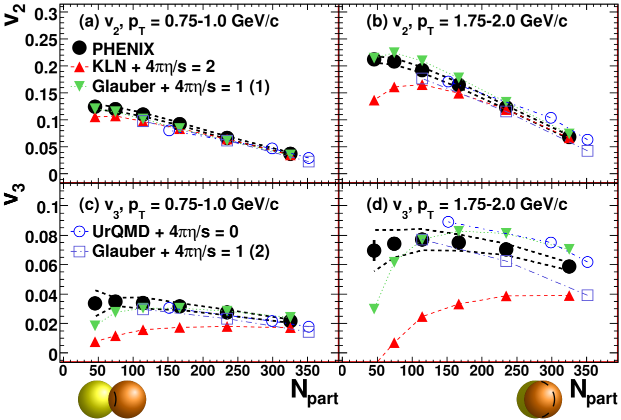
Hydro calculations correctly predicted flow at LHC:



(ML, Phys.Rev. C83 (2011) 044911)

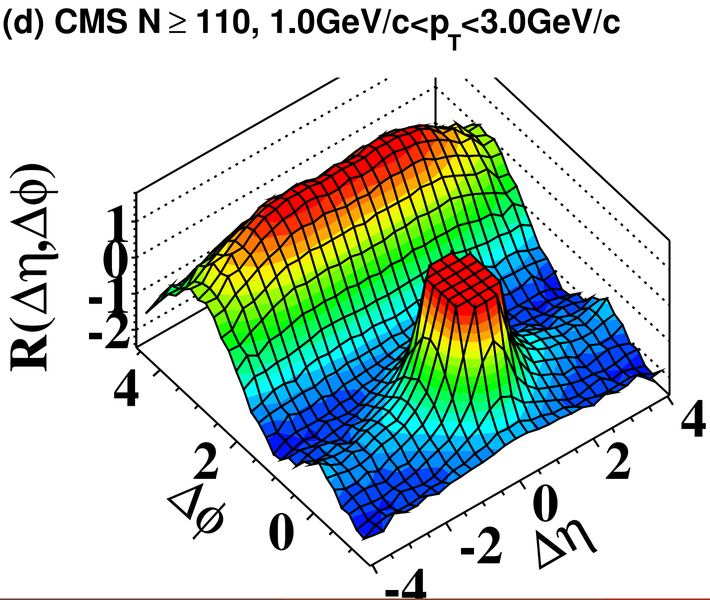
Recent results: v_n

Combining observables constrains theory



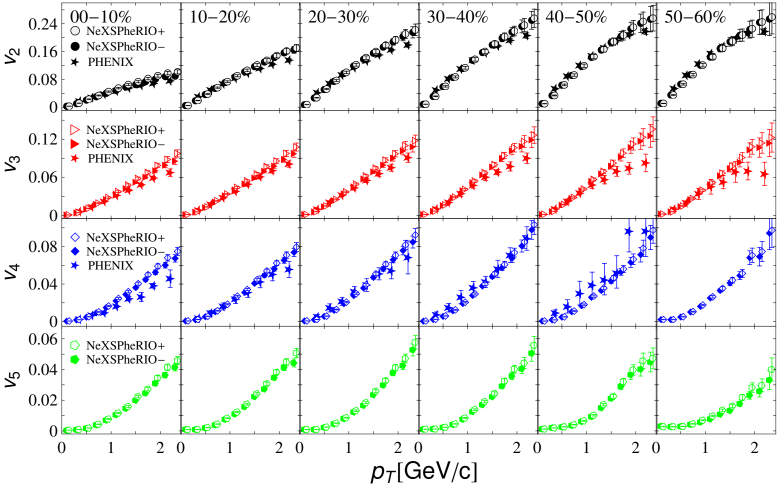
(PHENIX , Phys.Rev.Lett. 107 (2011) 252301)

CMS pp ridge



RECENT RESULTS

Hydrodynamic calculation can reproduce two-particle correlation:



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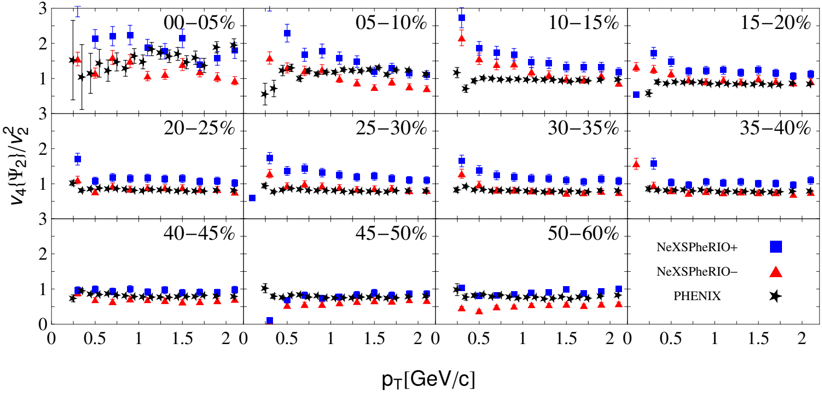

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