Phase diagram of strong interaction in an external magnetic field

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Hot Quarks

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Motivation

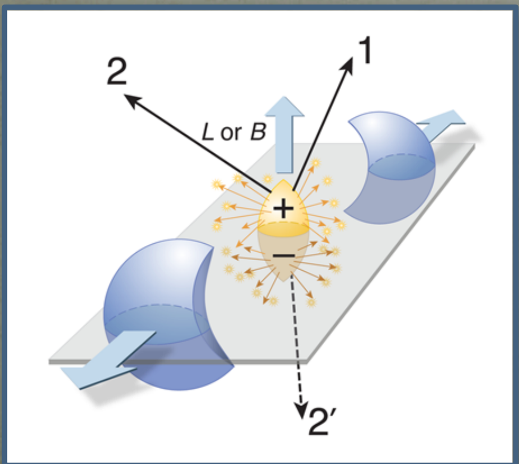
Strong interactions under strong magnetic fields can be found in nature:

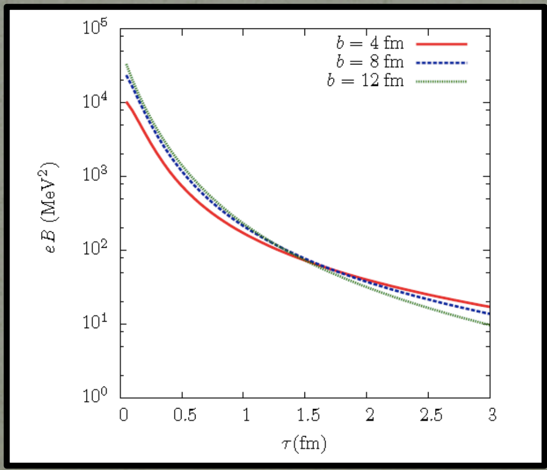
- Magnetars
- Early universe
- <u>Non-central heavy ion collisions:</u> 10^{18} Gauss

THE CHIRAL MAGNETIC EFFECT

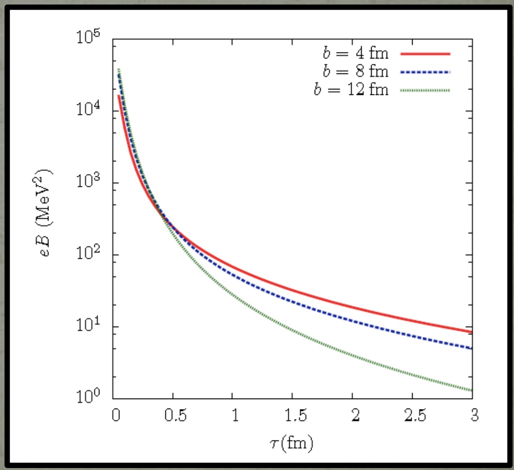
non-trivial gauge field configurations

strong magnetic field





Au-Au, 62 GeV



Au-Au, 200 GeV

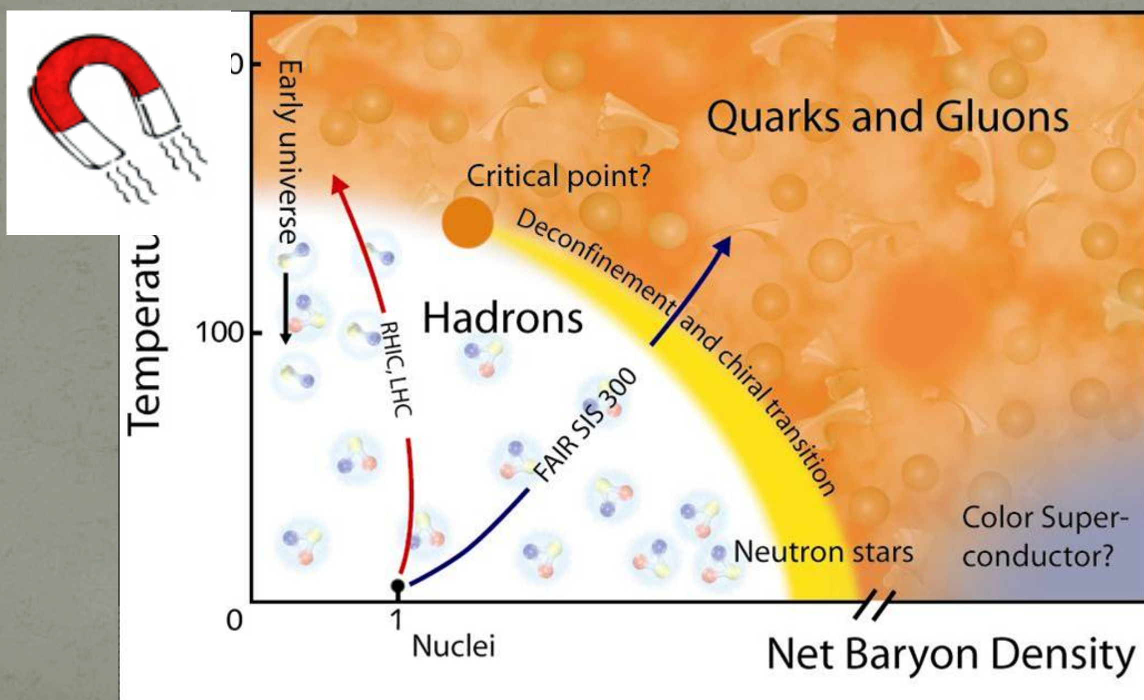
[Kharzeev, McLerran & Warringa, 2008]

- How does the QCD diagram look like including another external control parameter, the magnetic field B?

- Are there modifications in the nature of the phase transition?

- How do the chiral and deconfinement transitions react to this magnetic field?

- How is the interplay between this two transitions?



Step by step

- Effect of a magnetic background on the chiral transition
  - Linear sigma model at finite temperature
  - How to introduce the magnetic field?

- Effective theory for the chiral and deconfining transitions:
  - The linear sigma model coupled to quarks and the Polyakov loop

- Incorporating an external magnetic field

- Free energy at one loop

- Phase structure

Effective theory

[AJM, M. Chernodub & E.S.Fraga (2010)]

A. Degrees of freedom and approximate order parameters

Chiral field: $\phi = (\sigma, \vec{\pi})\,, \qquad \vec{\pi} = (\pi^+, \pi^0, \pi^-)$

Quark spinors: $$\psi = \begin{pmatrix} u \\ d \end{pmatrix}$$

Polyakov loop: $$L(x) = \frac{1}{3}\mathrm{Tr}\,\Phi(x)\,, \quad \Phi = \mathcal{P}\exp\Big[i\int_0^{1/T} \mathrm{d}\tau A_4(\vec{x},\tau)\Big]$$

$$\text{Chiral symmetry}: \quad \begin{cases} \langle\sigma\rangle \neq 0\,, & \text{low } T \\ \langle\sigma\rangle = 0\,, & \text{high } T \end{cases}$$

$$\text{Confinement}: \quad \begin{cases} \langle L\rangle = 0\,, & \text{low } T \\ \langle L\rangle \neq 0\,, & \text{high } T \end{cases}$$

B. Chiral Lagrangian and quark interaction

$$L = \overline{\psi}\left[i\gamma^{\mu}\partial_{\mu} - g(\sigma + i\gamma^{5}\vec{\tau}\cdot\vec{\pi})\right]\psi + \frac{1}{2}\left[\partial_{\mu}\sigma\,\partial^{\mu}\sigma + \partial_{\mu}\vec{\pi}\cdot\partial^{\mu}\vec{\pi}\right] - V\left(\sigma,\vec{\pi}\right)$$

Quark kinetic term

Fermion-meson interaction

Mesons kinetic term

$$V\left(\sigma,\pi\right) = \frac{\lambda}{4}\left(\left(\pi^{2} + \sigma^{2}\right) - v^{2}\right)^{2} - h\sigma$$

Spontaneously symmetry breaking

Explicit symmetry breaking

The effective potential

[E.S.Fraga & AJM, PRD78,025016 (2008); NPA 820, 103C (2009)]

Mean field treatment with the following assumptions:

Quarks constitute a thermalized gas that acts as a thermal bath in which the chiral fields evolve.

T=0 (vacuum: broken χ symmetry; reproduces LσM and χPT)

- Quark degrees of freedom: its presence in the vacuum brings remarkable consequences.

- Heavy σ meson (M_σ~600 MeV), treated classically.

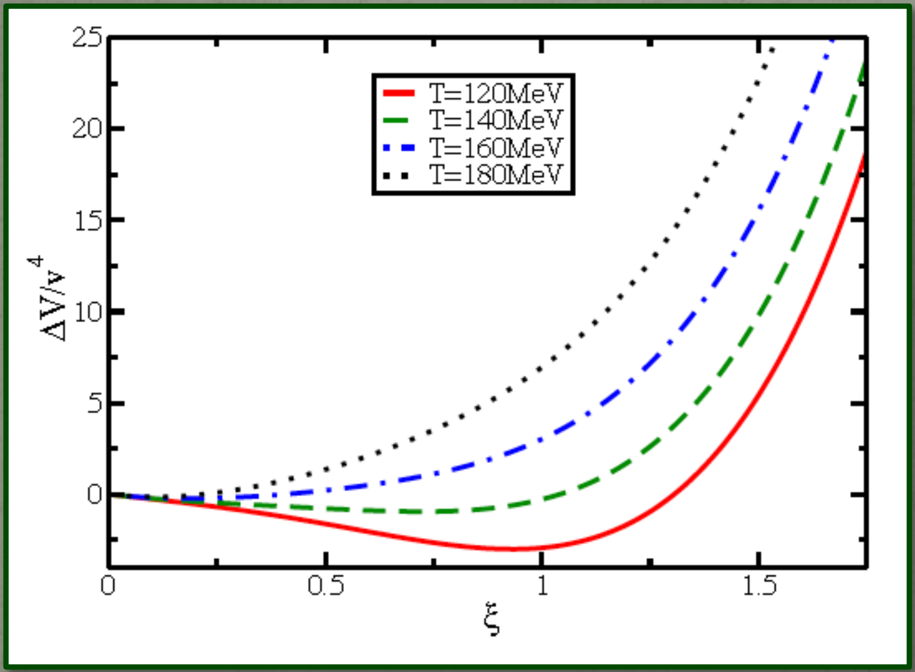
- $SU(2)_L \otimes SU(2)_R$ is spontaneously broken in the vacuum, with $\langle\sigma\rangle = f_\pi$, $\langle\pi\rangle = 0$

- h should be related to the nonzero pion mass

The fermions provide a thermal bath for the long wavelength chiral fields. Integrating over quarks:

$$\Omega(T,\mu,\phi) = V(\phi) - \frac{T}{\nu} \ln \det \left[\frac{\left(G_E^{-1} + M(\phi)\right)}{T} \right]$$

effective potential



$\xi = \sigma / v$ (v used as mass scale)

Effective potential for the chiral field σ for different values of the temperature. For low temperature the expected value of σ is non-zero. As the temperature increases it approaches to zero restoring the chiral symmetry.

[Scavenius, Mócsy, Mishustin & Rischke (2001); ...]

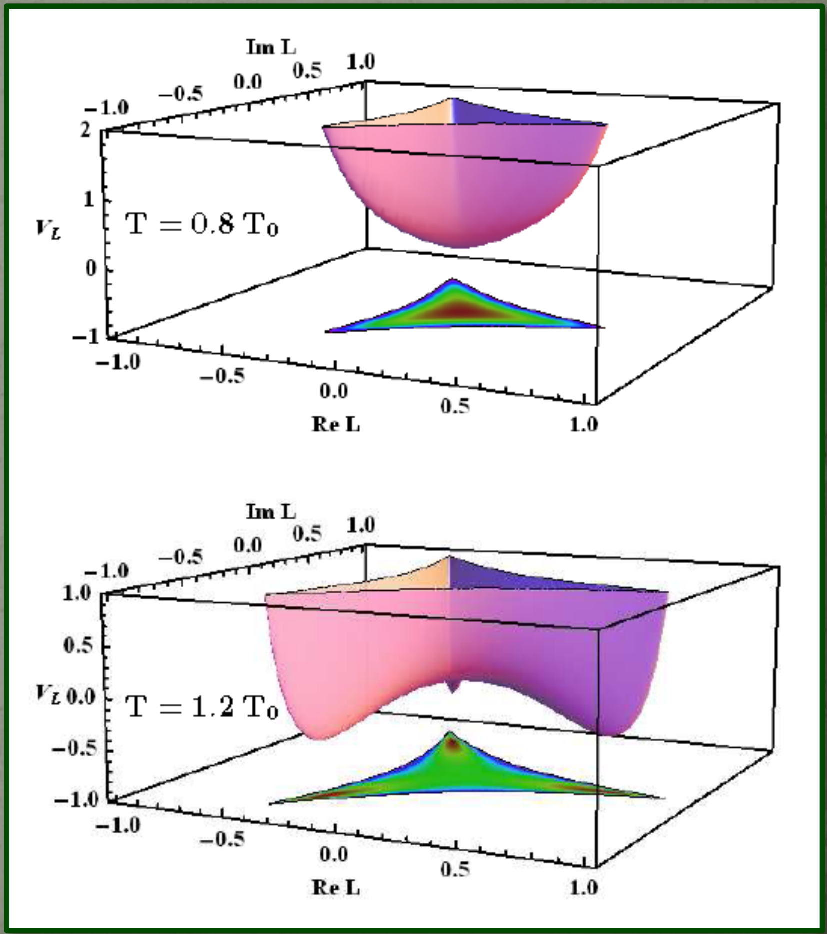
C. Confining potential

$$\frac{V_L(L,T)}{T^4} = -\frac{1}{2}a(T)\,L^*L + b(T)\,\ln\left[1 - 6\,L^*L + 4\left(L^{*3} + L^3\right) - 3\,(L^*L)^2\right]$$

$$a(T) = a_0 + a_1\left(\frac{T_0}{T}\right) + a_2\left(\frac{T_0}{T}\right)^2,$$

$$b(T) = b_3\left(\frac{T_0}{T}\right)^3$$

$$\mathcal{L}_L = -V_L(L,T)$$



Parameters fix demanding:

[Roessner et al. (2008):

• Stefan-Boltzmann limit reached at $T \to \infty$

• first order transition happens at $T=T_0$

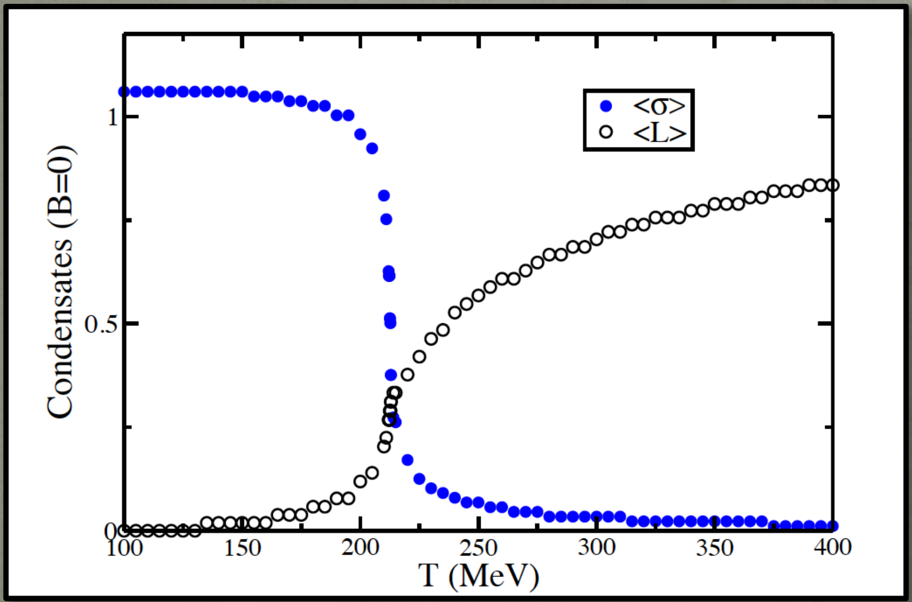
• the potential fits lattice data for thermodynamical quantities (pressure, energy density and entropy)

The interaction with the Polyakov loop is implemented via the field A_μ in the covariant derivative

$$\mathcal{L}_q = \bar{\psi}\left[i\not{D} - g(\sigma + i\gamma_5\vec{\tau}\cdot\pi)\right]\psi$$

$$\begin{cases} \not{D} = \gamma^\mu D_\mu^{(q)} \\ D_\mu^{(q)} = \partial_\mu - iA_\mu \end{cases}$$

Interaction between the mesons and the Polyakov loop only via quarks: minimal coupling



- Crossover for both transitions.
- With no magnetic field the critical temperature is the same.

Including an external magnetic field

[E.S.Fraga & AJM, 2008]

For simplicity we assume a magnetic field that is constant and homogeneous:

$$\vec{B} = B\hat{z} \quad \xrightarrow{\text{Gauge choice}} \quad A^\mu = (A^0, \vec{A}) = (0, -By, 0, 0)$$

- charged mesons (new dispersion relations):

$$(\partial^2 + m^2)\phi = 0, \quad \partial_\mu \to \partial_\mu + iqA_\mu \quad \Longrightarrow \quad p_{0n}^2 = p_z^2 + m^2 + (2n+1)|q|B$$

- quarks (new dispersion relations):

$$s = \pm\frac{1}{2}$$

$$(i\gamma^\mu\partial_\mu - m)\psi = 0, \quad \partial_\mu \to \partial_\mu + iqA_\mu \quad \Longrightarrow \quad p_{0n}^2 = p_z^2 + m^2 + (2n+1-2s)|q|B$$

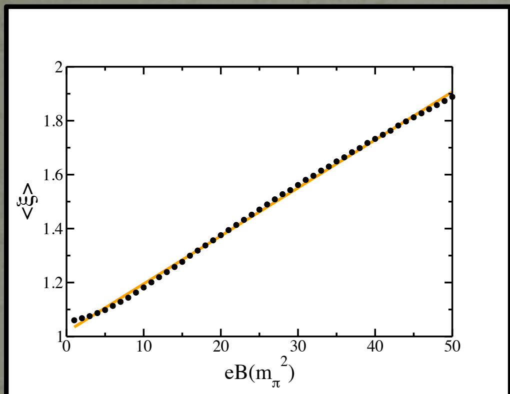
For the quarks interacting with the gauge filed:

$$D_\mu^{(q)} = \partial_\mu - iQa_\mu - iA_\mu$$

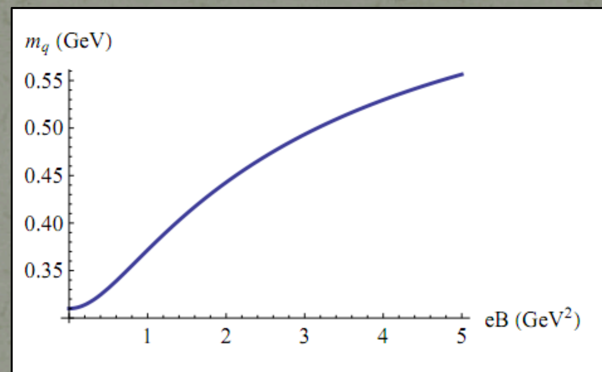
Abelian: magnetic field — Non-Abelian: Polyakov loop

Vacuum contribution: magnetic catalysis

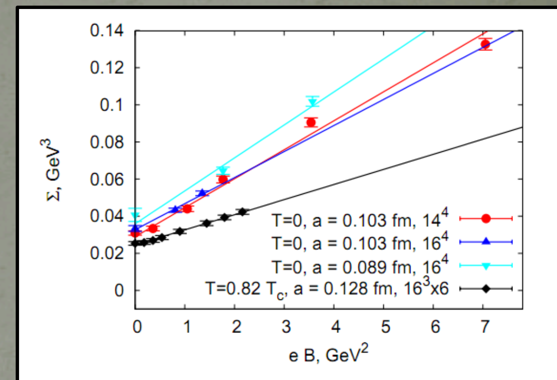
In the vacuum the value of the condensate increases as the magnitude of the magnetic field is increased: Reinforcement of the chiral symmetry breaking



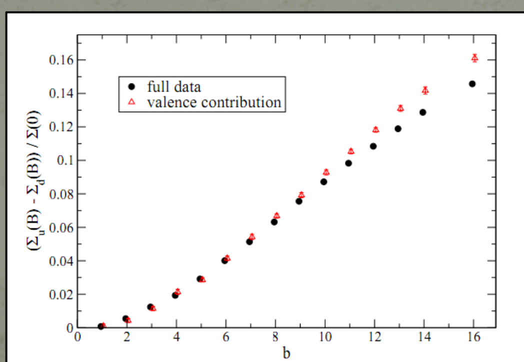
[AJM, M.Chernodub & E.S.Fraga, 2011]



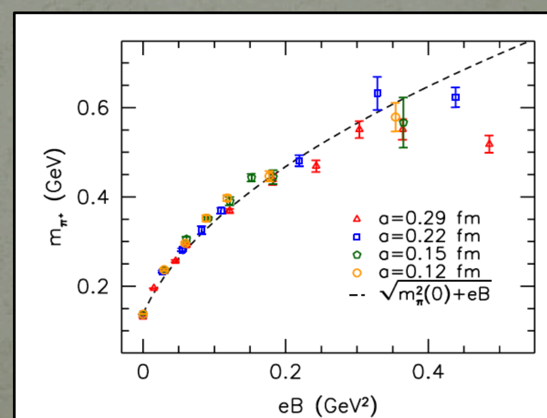
N. Callebault and D. Dudal (2011): Sakai-Sugimoto (AdS/CFT)



P. V. Buividovich, et al (2010): lattice



[M. D'Elia, F. Negro (2011): lattice]



[Bali et al (2012): lattice]

Thermal contribution: induced breaking of Z(3)

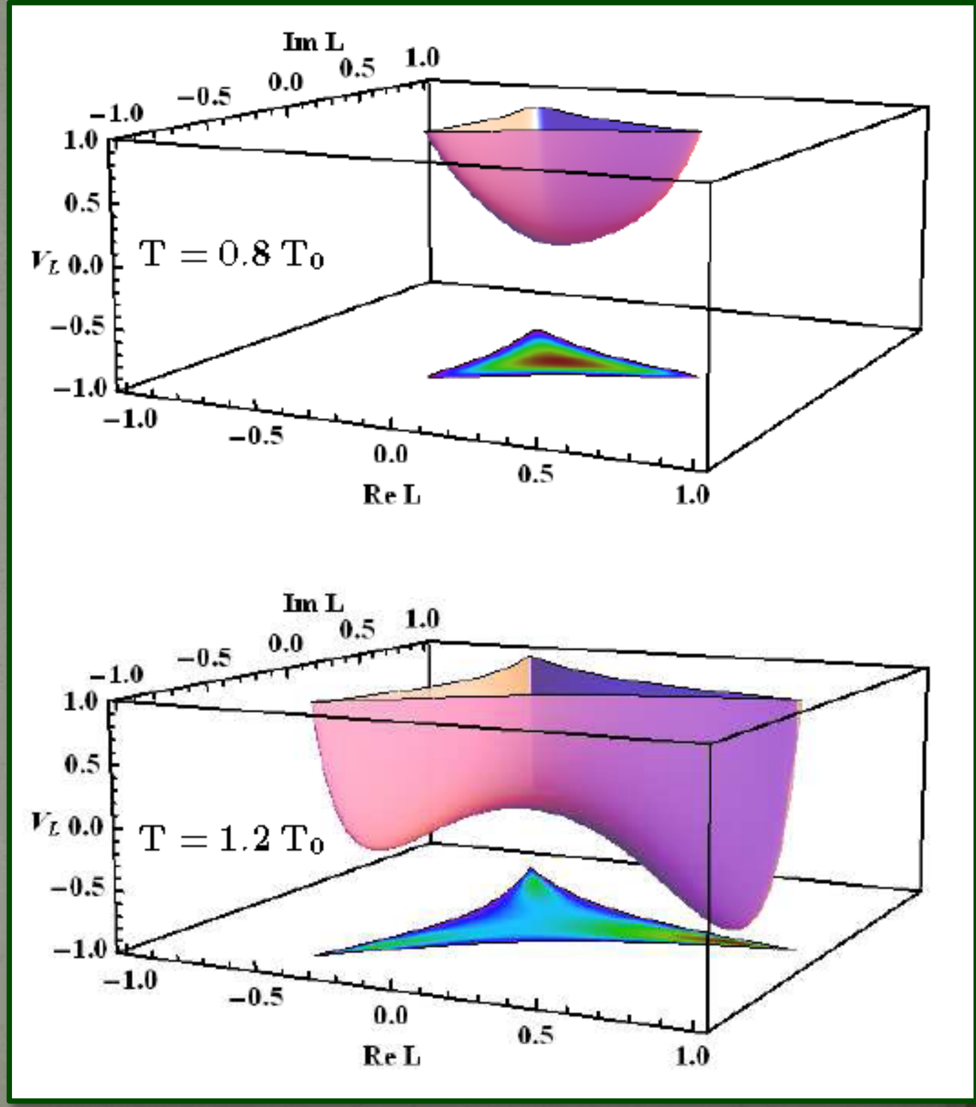
[A.J. Mizher, M. Chernodub & ESF (2010)]

The magnetic field drastically affects the potential for the Polyakov loop. For very large fields $|q|B >> m_q^2$:

$$\Omega_q^{\mathrm{para}} = -3\frac{g\sigma|q|BT}{\pi^2}K_1\left(\frac{g\sigma}{T}\right)\mathrm{Re}\,L$$

(not Z(3) invariant)

The magnetic field reinforces the breaking of Z(3) that occurs in the presence of fermions, forcing <L> to be real-valued!

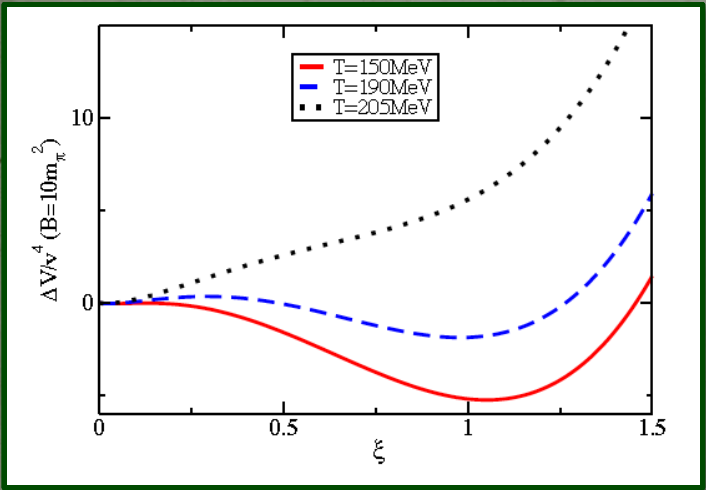


$\sqrt{eB} = 3T$

Effective potential

(i) Chiral condensate direction:

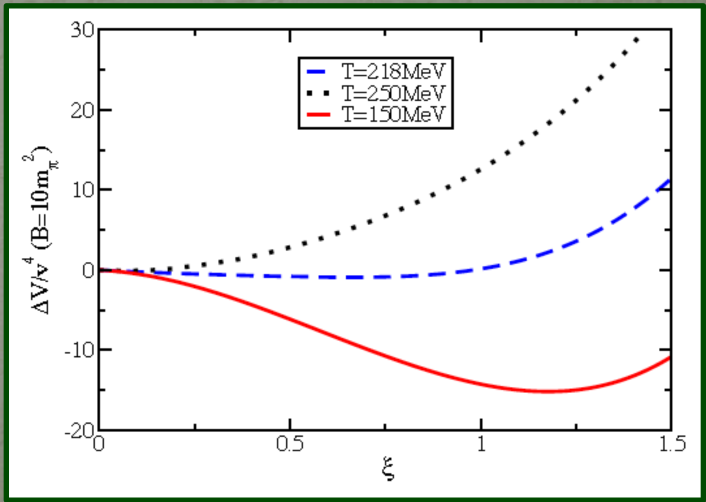
Without vacuum corrections



• A barrier appears: 1st order chiral transition.

• Part of the system kept in the false vacuum: some bubbles and spinodal instability, depending on the intensity of supercooling.

With vacuum corrections

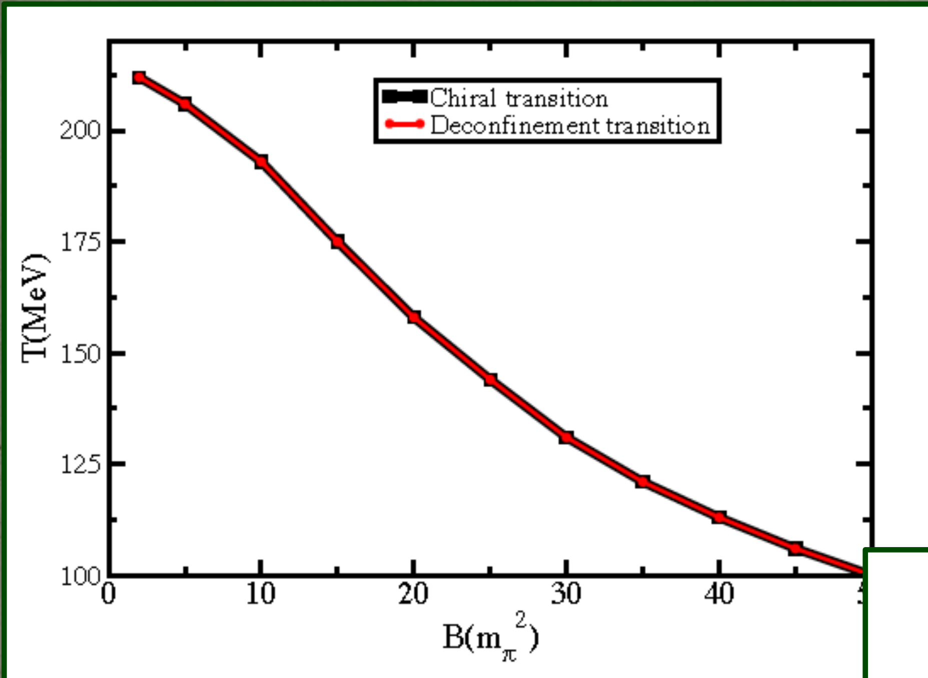


• No barrier: crossover for the chiral transition.

• System smoothly drained to the true vacuum: no bubbles or spinodal instability.

Phase diagrams

Without vacuum corrections



- Chiral and deconfinement (crossover) lines initially coincide, then split (3 phases).

- The deconfinement line flattens out for high enough B (does not go to zero).

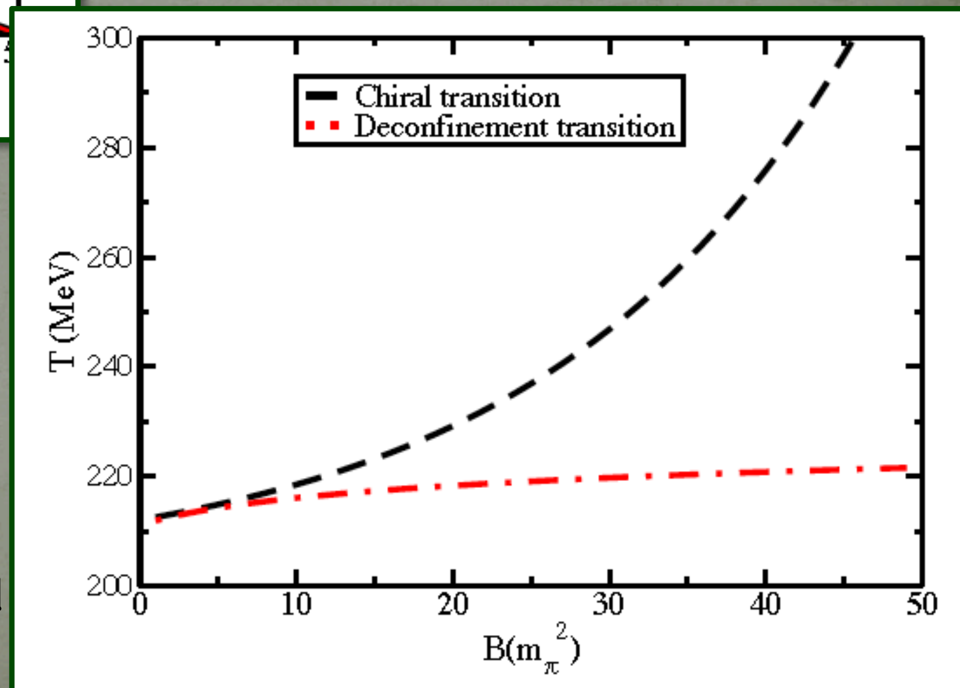
- Chiral restoration becomes more and more difficult for high B.

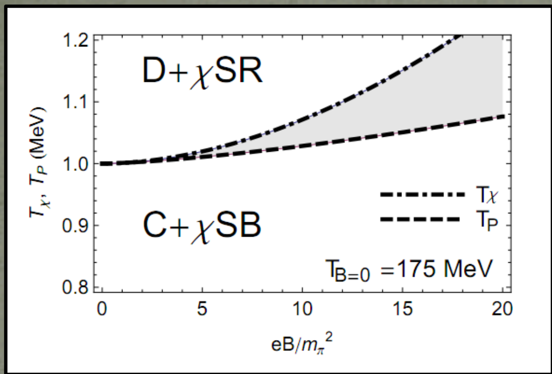
- Chiral and deconfinement lines coincide.

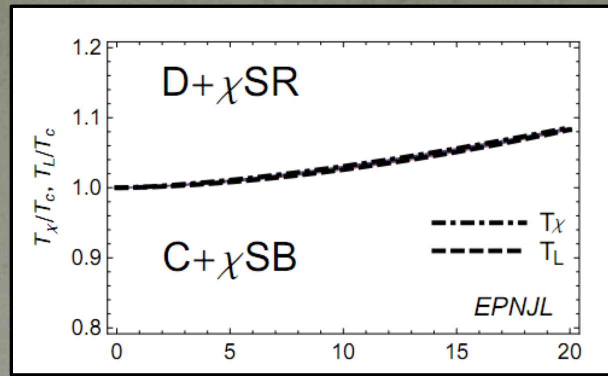
- The transitions are 1st order for a critical value of B.

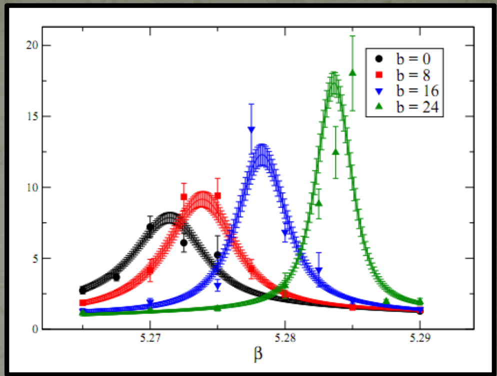
- Magnetic catalysis reproduced in the vacuum. [ESF & A.J. Mizher (2008)]

With vacuum corrections



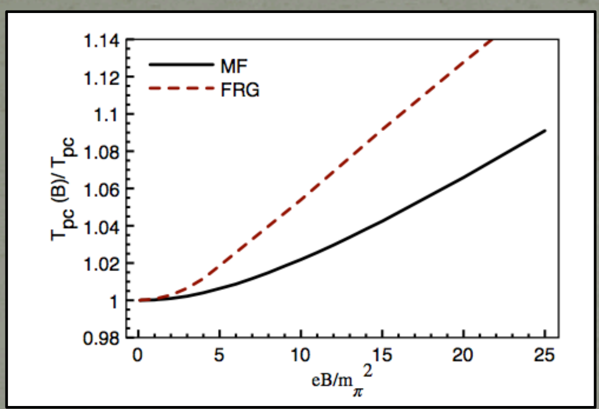


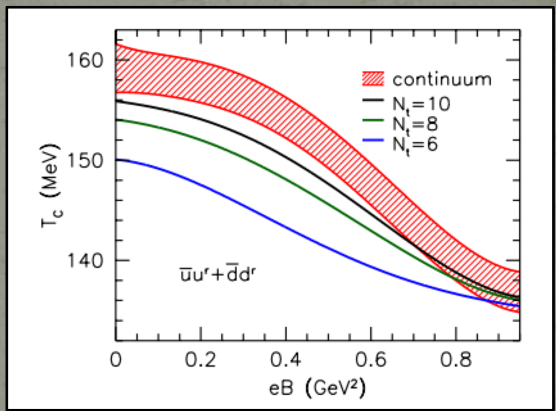




M. D'Elia, S. Mukherjee, F. Sanfilippo (2010): lattice

M. Ruggieri, R. Gatto (2010-2011): NJL and extensions





Skokov (2012): PQM

Bali et al (2012): lattice

Final remarks

- Strong magnetic fields can modify the nature and the lines of the chiral and the deconfining transitions, opening new possibilities in the study of the phase diagram of QCD.

- Break of Z(3) reinforced by the magnetic field.

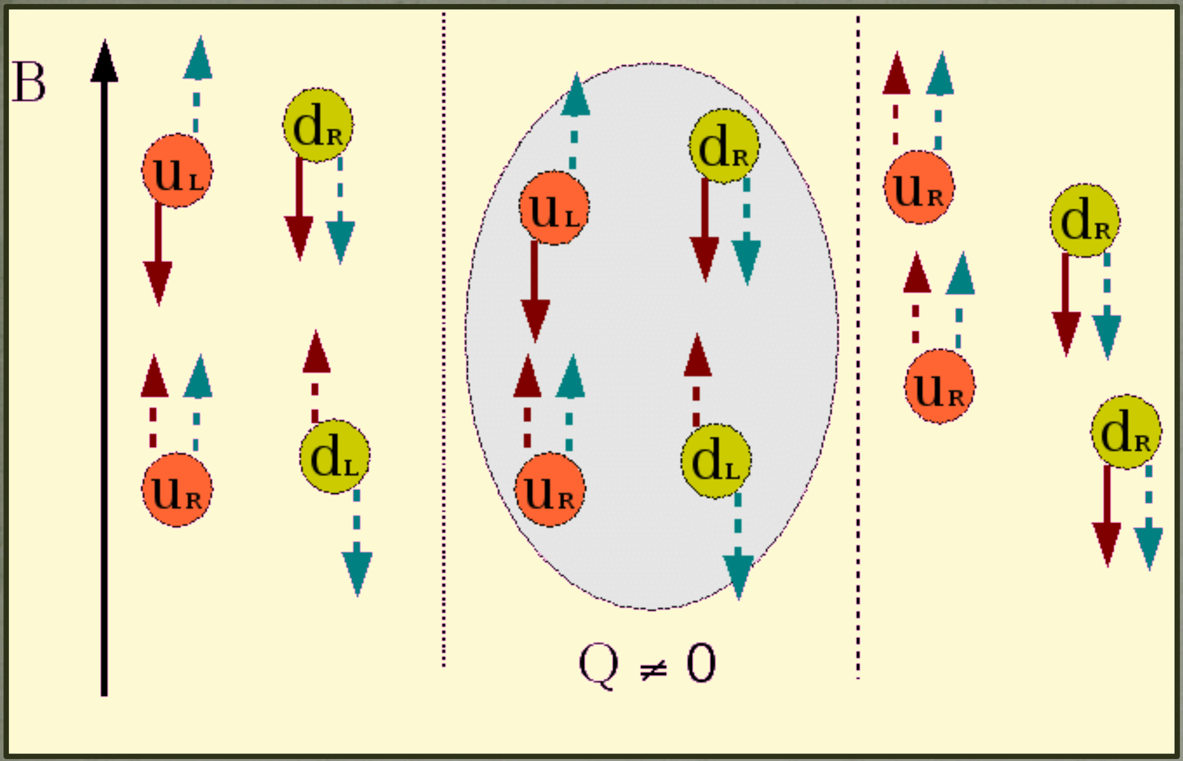
- Our approach indicates an approximately linear dependence of the chiral condensate on the magnetic field in the vacuum.

- Vacuum contributions are shown to change drastically the structure of the phase diagram. Including it keeps the transition a crossover, in accordance with lattice simulations. However in this approach the critical temperature grows with the magnitude of the magnetic field, contradicting lattice predictions.

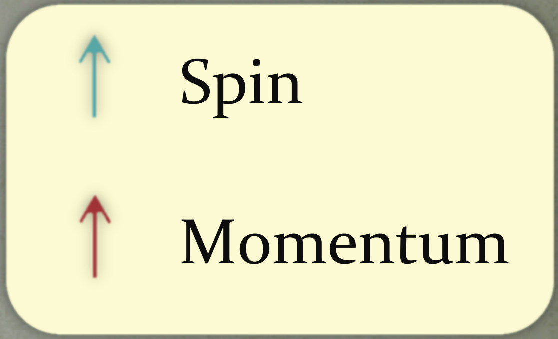
- still a lot to be done...

Thank you

Back up slides



$$h = \vec{p} \cdot \vec{s}$$



The interaction of the quarks with the non-trivial gauge fields gives rise to a difference between the number of quarks left and right. In the presence of the magnetic field it generates a current in its direction and a charge difference between the two hemisphere opposite to the reaction plane.

Effective theory for the chiral transition (LσM)

[Gell-Mann & Levy (1960); Scavenius, Mócsy, Mishustin & Rischke (2001); ...]

- Symmetry: for massless QCD, the action is invariant under $SU(N_f)_L \times SU(N_f)_R$
- "Fast" degrees of freedom: quarks
  "Slow" degrees of freedom: mesons
- Typical energy scale: hundred of MeV
- We choose $SU(N_f=2)$, for simplicity: we have pions and the sigma
- $SU(2) \otimes SU(2)$ spontaneously broken in the vacuum
- Also accommodates explicit breaking by massive quarks

Including an external magnetic field

[E.S.Fraga & AJM, 2008]

For simplicity we assume a magnetic field that is constant and homogeneous:

$$\vec{B} = B\hat{z} \quad \xrightarrow{\text{Gauge choice}} \quad A^\mu = (A^0, \vec{A}) = (0, -By, 0, 0)$$

Inserted via gauge field in the covariant derivative. For systems containing only chiral fields:

- charged mesons (new dispersion relations):

$$(\partial^2 + m^2)\phi = 0 \qquad \partial_\mu \to \partial_\mu + iqA_\mu \quad \Longrightarrow \quad p_{0n}^2 = p_z^2 + m^2 + (2n+1)|q|B$$

Landau levels:

$$\varepsilon_n \equiv \left(\frac{p_{0n}^2 - p_z^2 - m^2}{2m}\right) = \left(n + \frac{1}{2}\right)\omega_B \qquad \omega_B \equiv \frac{|q|B}{m}$$

- quarks (new dispersion relations):

$$s = \pm\frac{1}{2}$$

$$(i\gamma^\mu\partial_\mu - m)\psi = 0 \qquad \partial_\mu \to \partial_\mu + iqA_\mu \quad \Longrightarrow \quad p_{0n}^2 = p_z^2 + m^2 + (2n+1-2s)|q|B$$

For the quarks interacting with the gauge filed:

$$D_\mu^{(q)} = \partial_\mu - iQa_\mu - iA_\mu$$

Abelian: magnetic field

Non-Abelian: Polyakov loop

Integration length:

$$\int \frac{d^4k}{(2\pi)^4} \longrightarrow \sum_l \int \frac{d^3k}{(2\pi)^3}$$

Finite temperature

$$\int \frac{d^4k}{(2\pi)^4} \mapsto \frac{|q|B}{2\pi} \sum_{n=0}^{\infty} \int \frac{dk_0}{2\pi} \frac{dk_z}{2\pi}$$

External magnetic field

$$T \sum_\ell \int \frac{d^3k}{(2\pi)^3} \mapsto \frac{|q|BT}{2\pi} \sum_\ell \sum_{n=0}^{\infty} \int \frac{dk_z}{2\pi}$$

l: Matsubara index

n: Landau level index