University of Catania
INFN-LNS

"Chemical" composition of the Quark-Gluon Plasma in relativistic heavy-ion collisions

F. Scardina, V. Greco,
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Outline

- Our model: Transport theory with Quasiparticle
- Box calculation
- Results for RHIC and LHC
- Conclusion and future developments

Transport theory

$$\underbrace{p^{\mu}\partial_{\mu} f(x,p)}_{\text{Free streaming}} + \underbrace{M(X)\partial_{\mu} M(X)\partial_{p}^{\mu} f(X,p)}_{\text{Mean Field}} = \underbrace{C_{22}}_{\text{Collisions}}$$

$$\begin{aligned}\mathcal{C}_{22} \;=\; & \frac{1}{2E_1}\int \frac{d^3p_2}{(2\pi)^3 2E_2}\frac{1}{\nu}\int \frac{d^3p'_1}{(2\pi)^3 2E'_1}\frac{d^3p'_2}{(2\pi)^3 2E'_2} f'_1 f'_2 |\mathcal{M}_{1'2'\to 12}|^2 (2\pi)^4 \delta^{(4)}(p'_1+p'_2-p_1-p_2) \\ & -\frac{1}{2E_1}\int \frac{d^3p_2}{(2\pi)^3 2E_2}\frac{1}{\nu}\int \frac{d^3p'_1}{(2\pi)^3 2E'_1}\frac{d^3p'_2}{(2\pi)^3 2E'_2} f_1 f_2 |\mathcal{M}_{12\to 1'2'}|^2 (2\pi)^4 \delta^{(4)}(p_1+p_2-p'_1-p'_2)\end{aligned}$$

We consider both **elastic and inelastic collisions**

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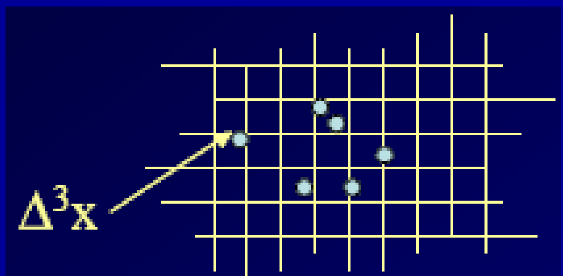
For the numerical solutions of the Boltzmann equation we use a three dimensional lattice that discretizes the space and the standard test particle methods that samples the distributions



$$\Delta t \to 0$$

$$\Delta^3 x \to 0$$

Exact solution

[Z. Xhu, et al. PRC71(04)]

Test of the model in a box

Massless case

$$p^{\mu}\partial_{\mu} f(x,p) = C_{22}$$

Equilibrium value

$$\frac{\rho_{eq}^{quark}}{\rho_{eq}^{gluon}} = \frac{\nu_{quark}}{\nu_{gluon}} = \frac{2*2*3*Nfl}{2*8} = 2.25$$

(Nfl=3)

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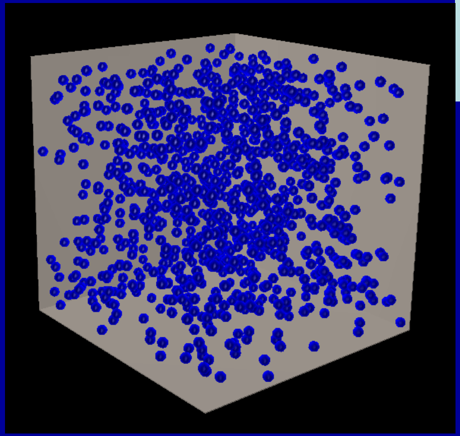
Simulations in which a particle ensemble in a **box** evolves dynamically



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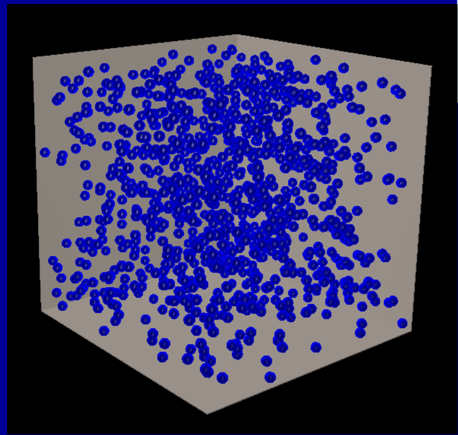
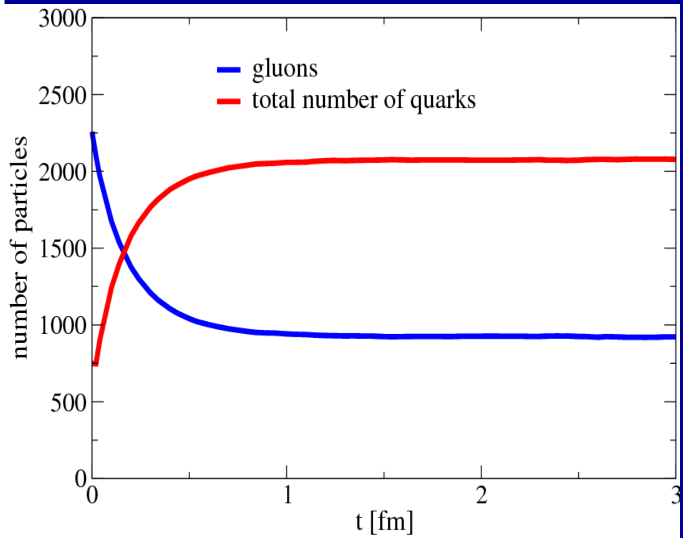
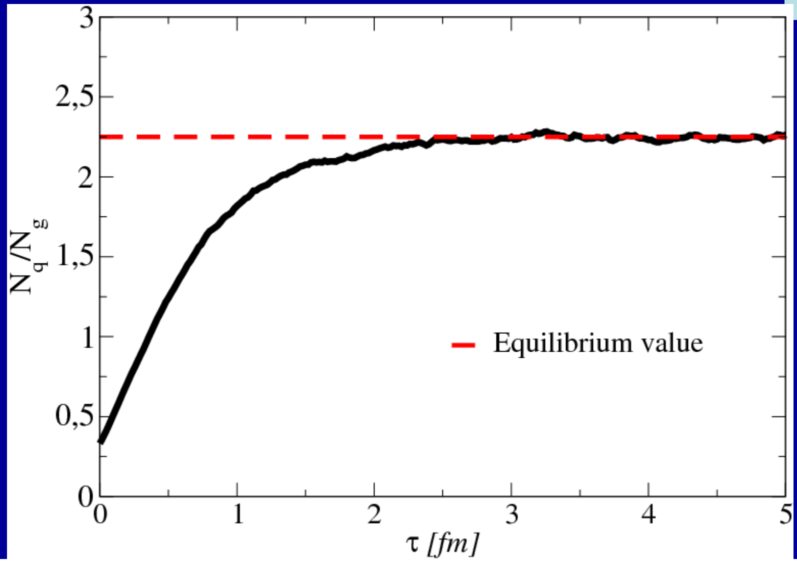


$$\frac{\rho_{Init}^{quark}}{\rho_{Init}^{gluon}} = 0{,}33$$

Inelastic collisions $gg \rightarrow q\bar{q}$ $q\bar{q} \rightarrow gg$ lead the system to **chemical equilibrium**

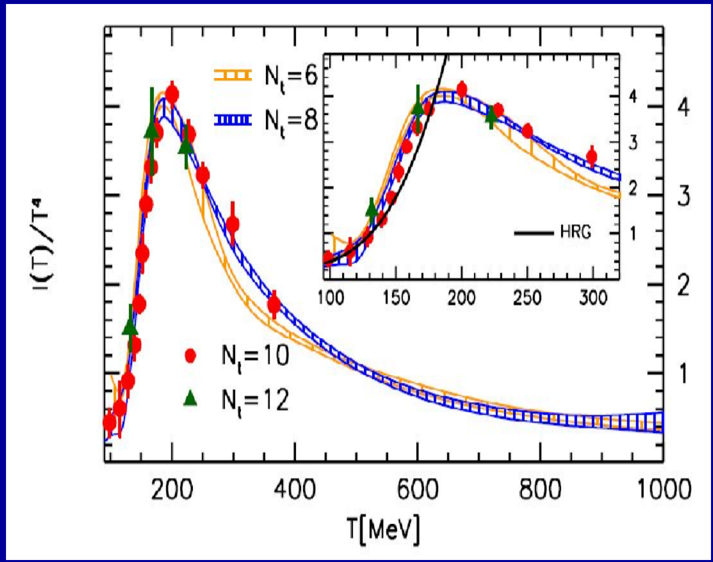
$$\frac{\rho_{eq}^{quark}}{\rho_{eq}^{gluon}} = 2.25$$





Quasiparticle model (QP)

From lQCD we know that QGP is significantly different from a massless gas showing deviation of both ε and p and exihibiting a large trace anomaly



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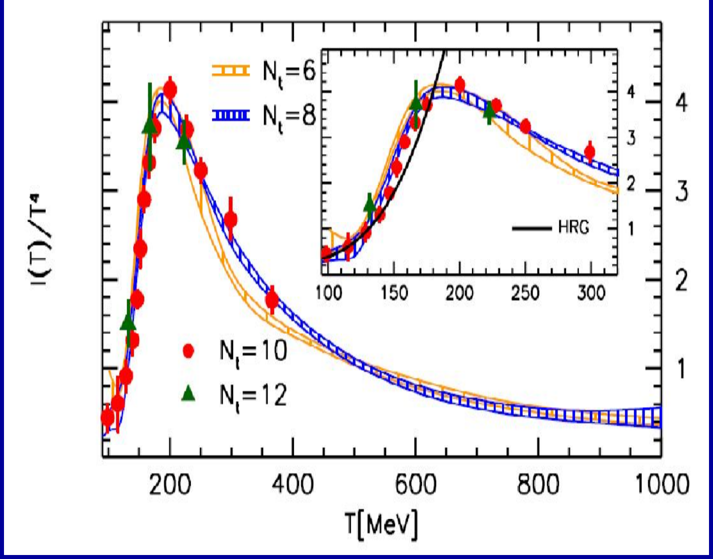

The lattice results can be described in terms of a massive quasiparticle model in which both gluons and quarks acquire thermal masses

Effective masses are generated through the interaction

In order to apply quasiparticle model it is necessary evaluate masses

They can be perturbatively evaluated

$$m_g^2 = \frac{1}{6} g^2 \left[\left(N_c + \frac{1}{2} n_f \right) T^2 \right] \qquad m_q^2 = \frac{N_c^2 - 1}{8N_c} g^2 T^2$$

We are interested in the non perturbative region, hence we use these two only to fix the mq/mg ratio and g is determined through a fit to lQCD data

Quasiparticle model (QP)

Usually it is carried out as follow

It is evaluated P_{QP} summing the contribution of all particle + Bag pressure B

$$P_{QP} = \sum_{i=u,d,s,g} d_i \int \frac{d^3p}{(2\pi)} \frac{\vec{p}^{\,2}}{3E_i} f_i(p) - B(T)$$

f(p)=equilibrium distribution functions

$$E_i = \sqrt{\vec{p}_i^{\,2} + m_i^2(T)}$$

From the pressure P_{QP} the energy density ε_{QP} can be evaluated

$$\varepsilon_{QP} = \sum_i d_i \int \frac{d^3p}{(2\pi)^3} E_i f_i(E_i) + B(m_i(T))$$

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To have **thermodynamic consistency** this relation has to be satisfied

$$\left(\frac{\partial P}{\partial m_i} \right)_{T,\mu} = 0$$

$$\frac{\partial B}{\partial m_i} + d_i \int \frac{d^3 p}{(2\pi)^3} \frac{m_i}{E_i} f(E_i) = 0$$

This equation link **B(T)** to **g(T)**, the only function that has to be determinated

g(T) is determinated imposing

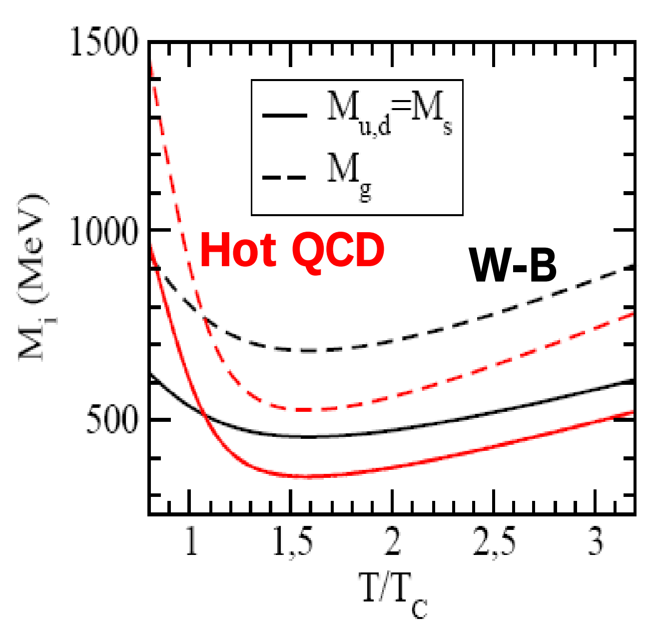
$$\varepsilon_{QP} = \varepsilon_{Lattice\_QCD}$$

Quasiparticle model (QP)

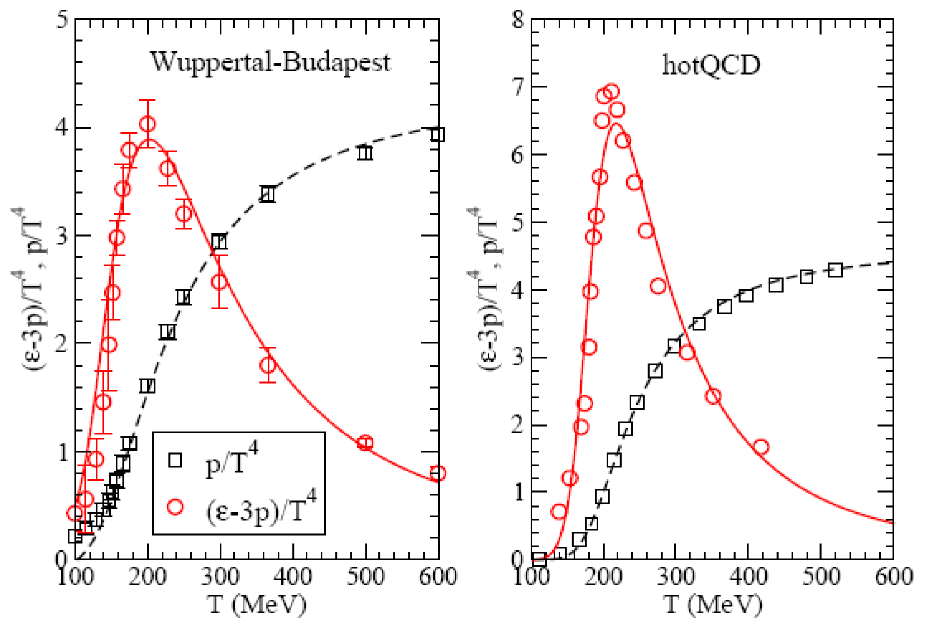
Once **g** is known it is possible to evaluate **masses** and also **B**

$$m_g^2 = \frac{1}{6} g^2 \left[\left(N_c + \frac{1}{2} n_f \right) T^2 \right]$$

$$m_q^2 = \frac{N_c^2 - 1}{8 N_c} g^2 T^2$$



Knowing masses and B the other thermodynamic quantities can be evaluated



[Plumari et. al PRD 84 094004 (2011)]

Quasiparticle model (QP)

While lattice QCD cannot be used to study the dynamical evolution of a system the quasi particle model can be used

In order to do that it is necessary to couple the Boltzamnn equation with the equation for the quasi-particle masses.

Quasiparticle model and Kinetic theory

$$p^{\mu}\partial_{\mu} f(x,p) + m_i(x)\partial_{\mu} m_i(x)\partial_p^{\mu} f(x,p) = C_{22}$$

$$\frac{\partial B}{\partial m_i} + d_i \int \frac{d^3 p}{(2\pi)^3} \frac{m_i(x)}{E_i(x)} f(x,p) = 0$$

The two equations have to be solved autoconsistently

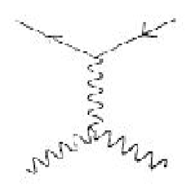
In order to solve the collision integral it is necessary to know the cross section between massive particles

$\sigma_{2\to 2}$ inelastic for quasiparticle

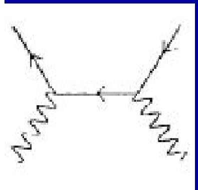
We have evaluated σ in a pQCD leading order scheme

The processes we are interested in are $gg \to q\bar{q}$ and $q\bar{q} \to gg$

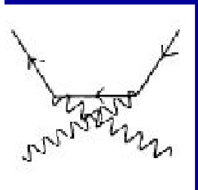
The 3 diagrams contributing to the processes correspond to the u,t,s channels, for which we have evaluated the squared matrix elements M



$$|M_s|^2 = \alpha_s^2 \pi^2 12 \, \frac{(m_q^2 + m_g^2 - t)(m_q^2 + m_g^2 - u) - 3 m_g^2 s + 2 m_q^2 m_g^2}{(s - m_g^2)^2}$$



$$|M_t|^2 = \alpha_s^2 \pi^2 \frac{8}{3} \frac{(m_q^2 + m_g^2 - t)(m_q^2 + m_g^2 - u) - 2 m_q^2 (m_q^2 + t) - m_g^2 s - 4 m_q^2 m_g^2}{(t - m^2)^2}$$



$$|M_u|^2 = \alpha_s^2 \pi^2 \frac{8}{3} \frac{(m_q^2 + m_g^2 - t)(m_q^2 + m_g^2 - u) - 2 m_q^2 (m_q^2 + u) - m_g^2 s - 4 m_q^2 m_g^2}{(u - m^2)^2}$$

+ Interference terms

$\sigma_{2\text{->}2}$ inelastiche for quasiparticle

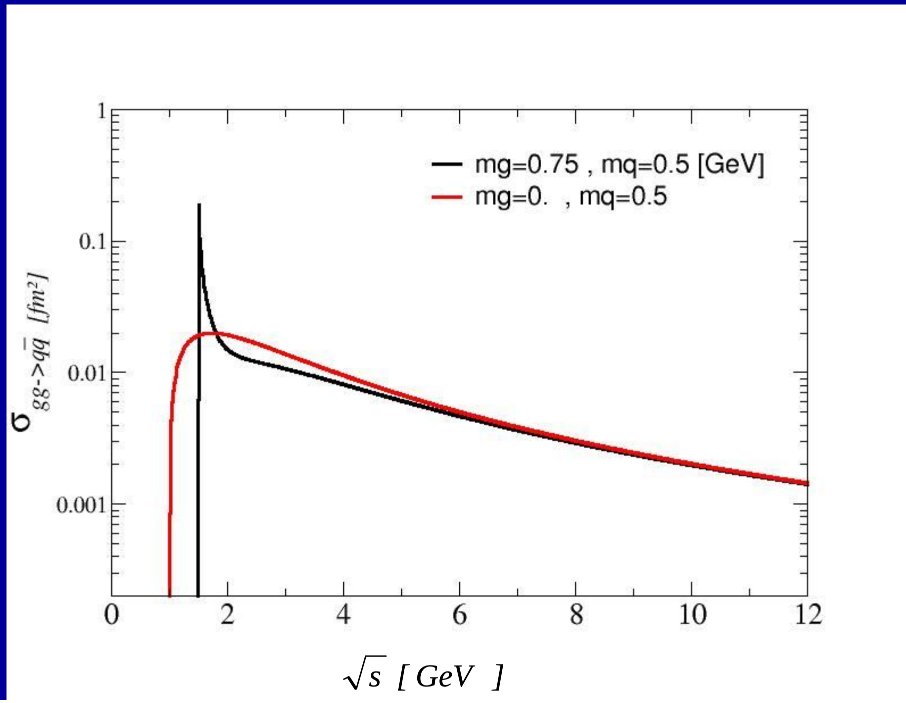
Process $gg \to q\bar{q}$

$$\sigma^{Tot}_{gg\to q\bar{q}}(s) = \frac{1}{16\pi s\left(s-4m_g^2\right)}\int_{t_-}^{t_+} dt\left(|M_s|^2 + |M_t|^2 + |M_u|^2 + Interference\, terms\right)$$

Integration limits ⟶ $t_\pm = m_q^2 + m_g^2 \mp \frac{s}{2}\left(1-\sqrt{1-4m_q^2/s-1-4m_g^2/s+16m_q^2m_g^2/s^2}\right)$

[T. S. Biro et. al PRD 42 (1990)]

Comparison with Combridge cross sections (mg=0) ⟶



[B. L. Combridge Nucl. Phys. B 151 (1979) 429]

Chemical equilibrium

Massless case

$$\rho_{eq} = \nu \frac{T^3}{\pi^2} V$$

$$\frac{\rho_{eq}^{quark}}{\rho_{eq}^{gluon}} = \frac{\nu_{quark}}{\nu_{gluon}} = \frac{2*2*3*Nfl}{2*8} = 2.25$$ (Nfl=3)

Massive case

$$\frac{\rho_{eq}^{quark}}{\rho_{eq}^{gluon}} = \frac{\nu_{quark}}{\nu_{gluon}} \frac{m_q^2(T)}{m_g^2(T)} \frac{K_2(m_q/T)}{K_2(m_g/T)}$$

Bessel function

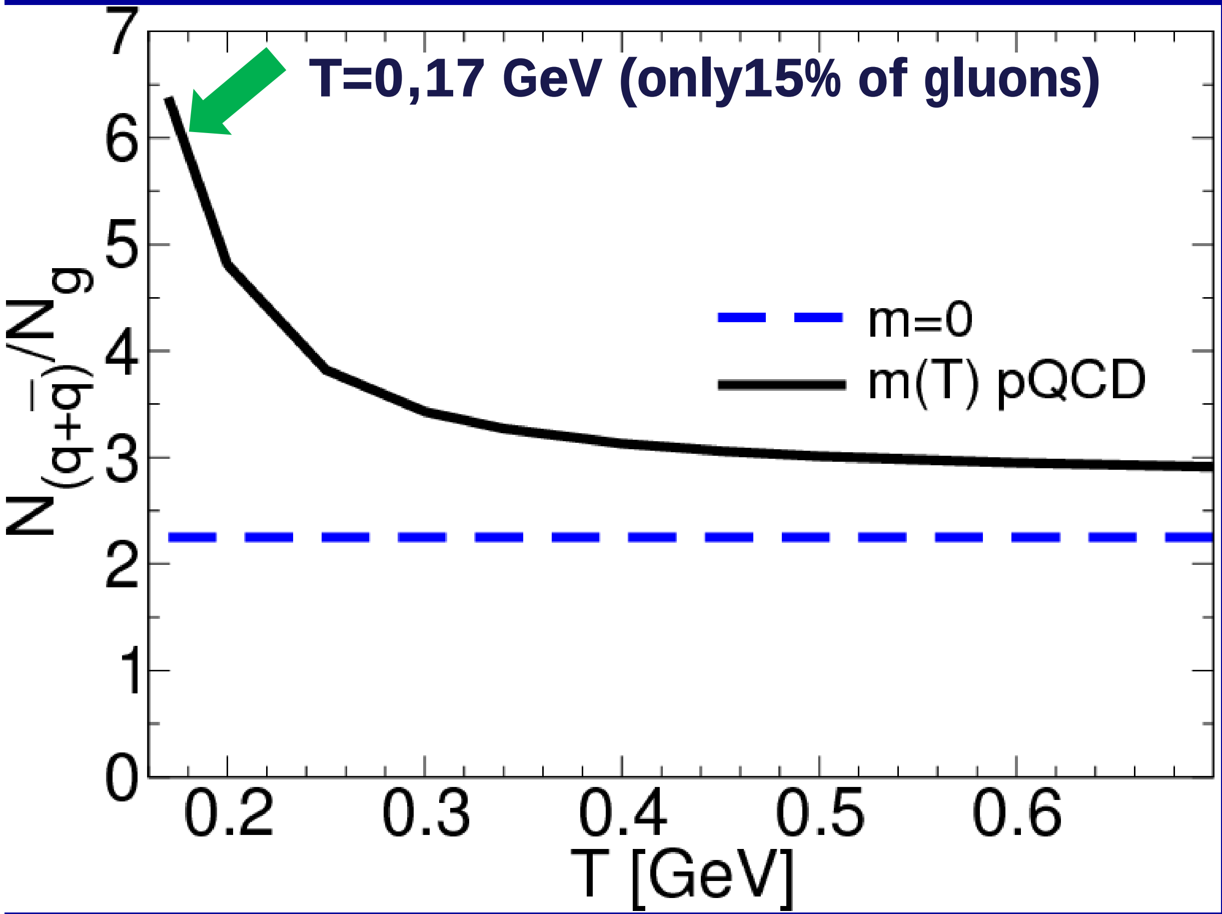
$$K_2(\frac{m}{T}) = \frac{2^n n!}{(2n!)}(\frac{m}{T})^{-n} \int_{m/T}^{\infty} d\tau(\tau^2 - (\frac{m}{t})^2)^{n-1/2} e^{-\tau}$$

n=2 , $\tau = \frac{1}{T}\sqrt{m^2 + p^2}$

The equilibrium value depends on the **ratio between the degrees of freedom** but also on **mq/mg and on m/T**

Chemical equilibrium

However the **ratio between the masses is fixed in the quasiparticle model** so **N_q/N_g** depends only on **temperature**

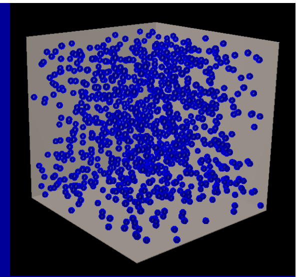


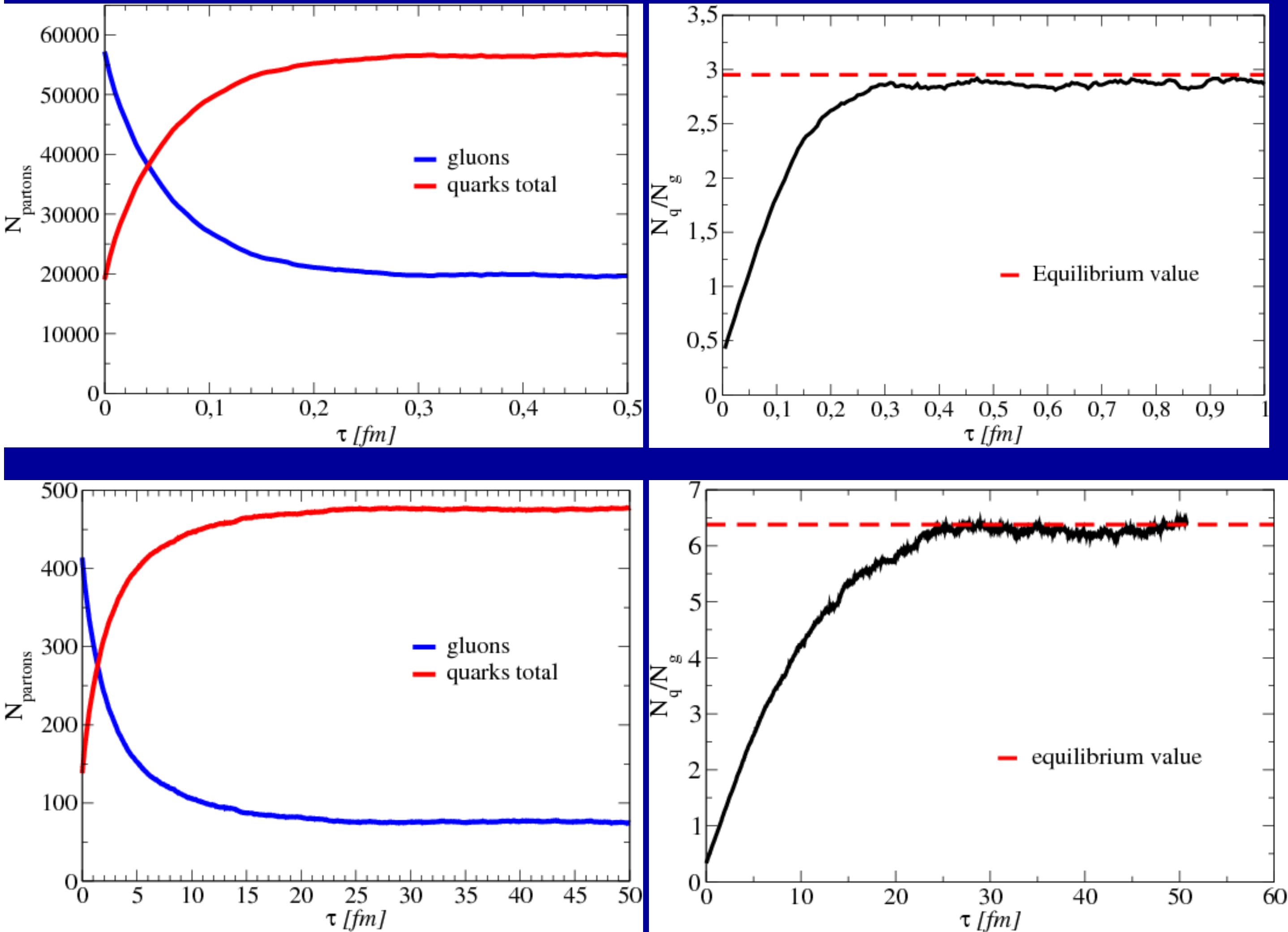
$$\frac{\rho_{eq}^{quark}}{\rho_{eq}^{gluon}} = \frac{\nu_{quark}}{\nu_{gluon}} \frac{m_q^2(T)}{m_g^2(T)} \frac{K_2(m_q/T)}{K_2(m_g/T)}$$

In the massive case we have a **larger quark abundances** especially **at low temperature.**

Near **T_c** only the **15% of the particle should be gluons**

Test of the model in a box





T=0.6 GeV

T=0.17 GeV

The relaxations times are different because ρ it is different (ρ≈T^3) and the collision rate depends on ρ

Our code is able to reproduces the expected equilibria value

Plasma created in the heavy ion collisions during its evolution does reach chemical equilibrium ?

Why studying chemical equilibration of QGP ?

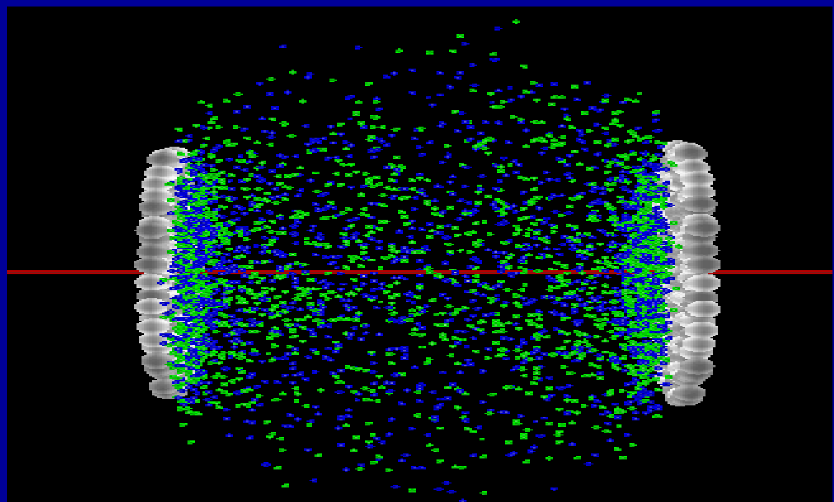
A closer look into the theoretical approaches describing the QGP probes reveals that the different models assume different chemical composition of the QGP

- In some cases the QGP is described as a gluon plasma. This is for example is assumed in the most popular jet quenching models.
- In hydrodynamics instead a chemical quark to gluon equilibration is implicit in the employment of a lattice QCD equation of state
- The coalescence model assumes a quark dominance in the plasma T_c

Simulations at RHIC and LHC

Initial conditions:

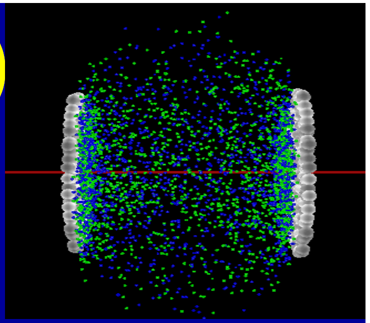
Coordinates-space	Glauber model
Momenta -space	Boltzmann-Jutter plus minijet
N_q/N_g	75% gluons 25 % quarks
Temperature	T=340 Mev (RHIC); T=600 MeV (LHC)
Initial time	τ=0.6 fm (RHIC); τ=0.3 fm (RHIC)

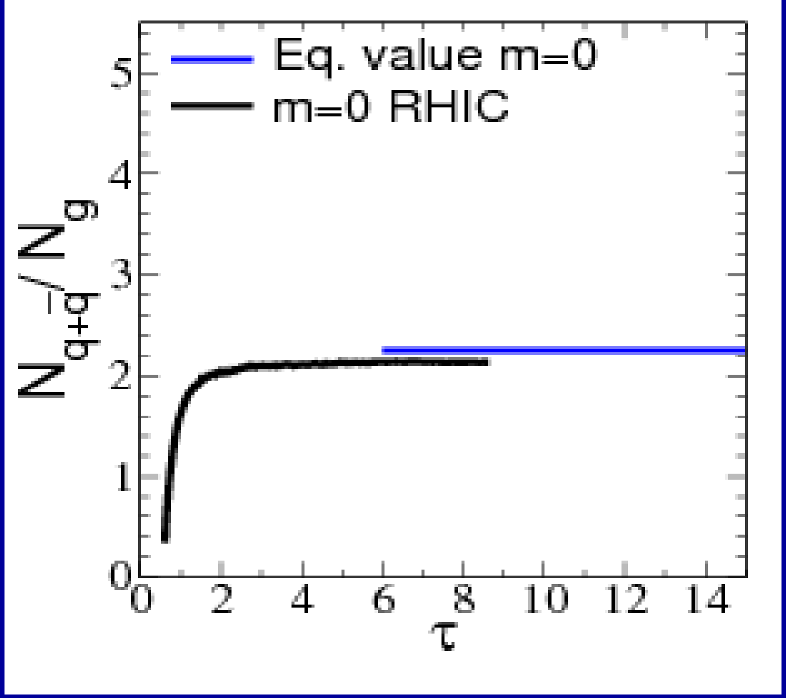


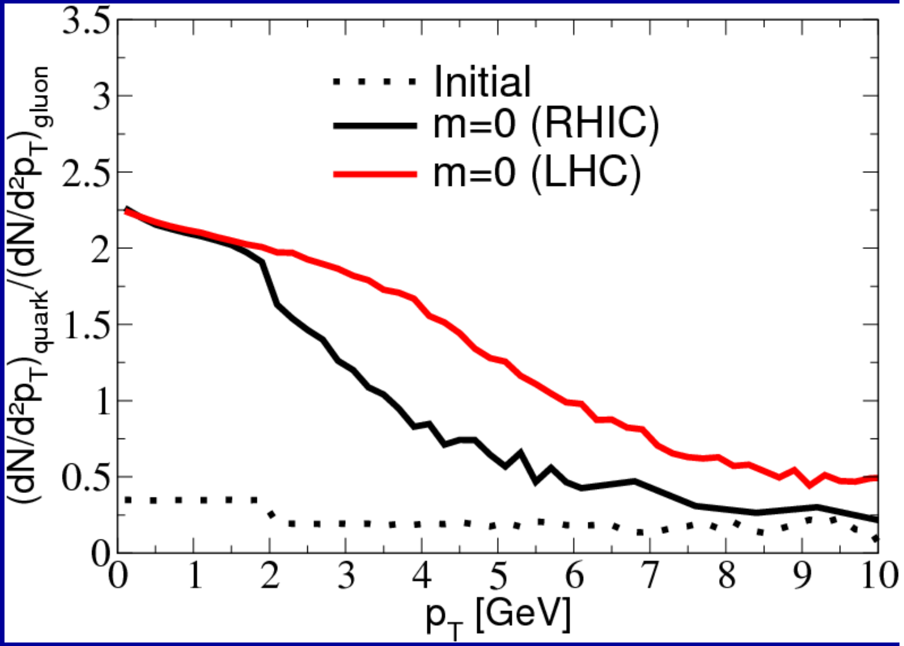
We consider the **running coupling** and we use the standard one-loop perturbative formula for the α_s

$$\alpha_s(Q^2) = \frac{4\pi}{(11 - \frac{2}{3}N_f)ln(Q^2/\Lambda^2_{QCD})}$$

Results at RHIC and LHC (massless case)







- **Nq quickly increases during the first part of the fireball evolution**
- **Open system -> it does not reach the expected equilibrium value**

- **N_q/N_g strongly decreases with p_T because the collision rate $R(\sigma,\rho)$**
- **At LHC the ratio keep an higher value in a wider region of p_T because partons have more time to equilibrate**

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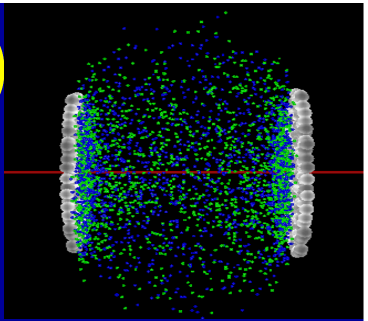

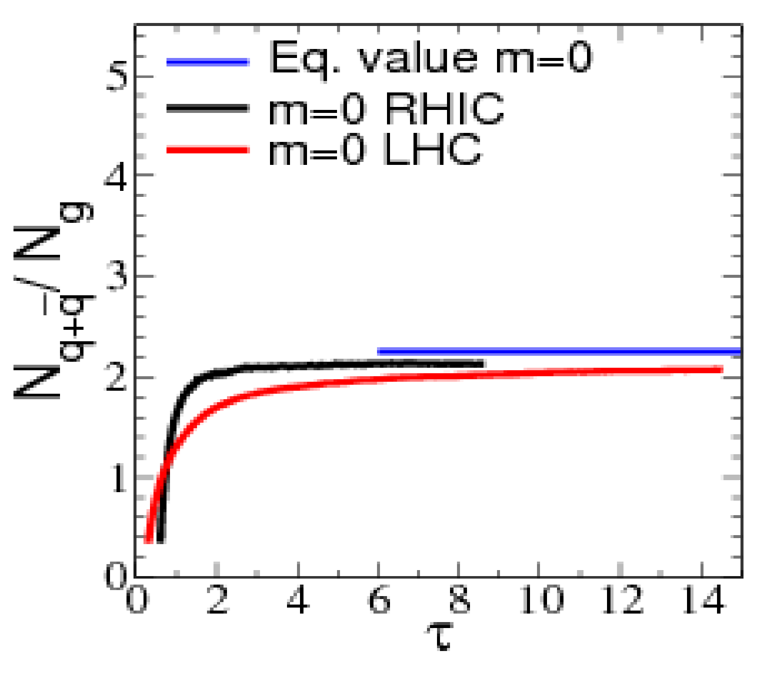

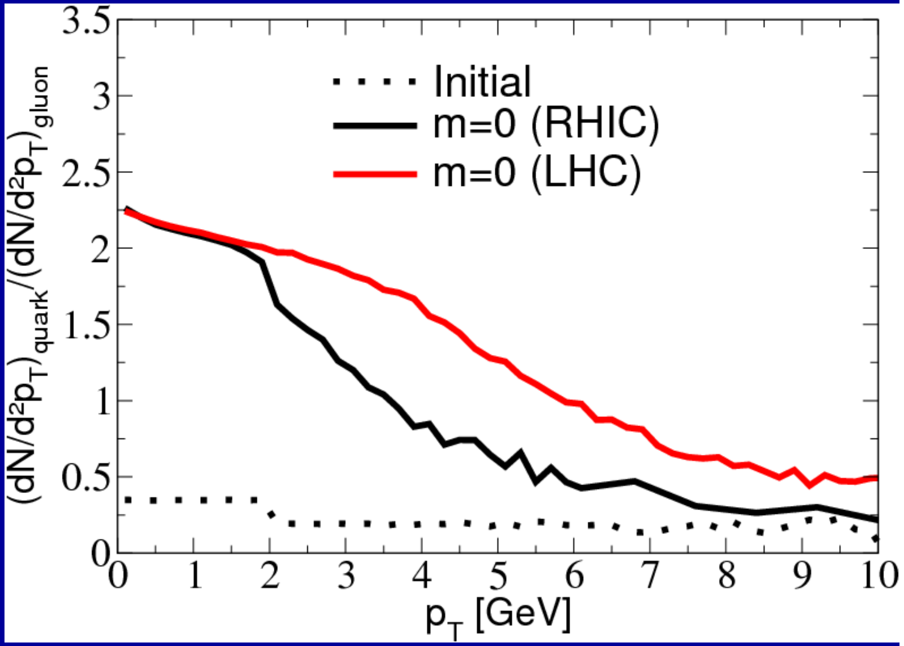

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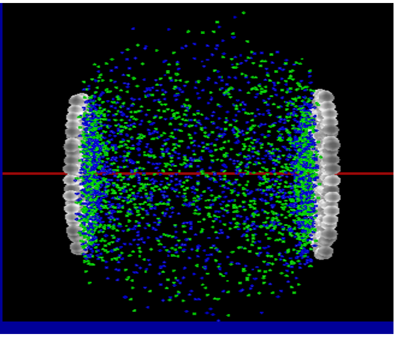
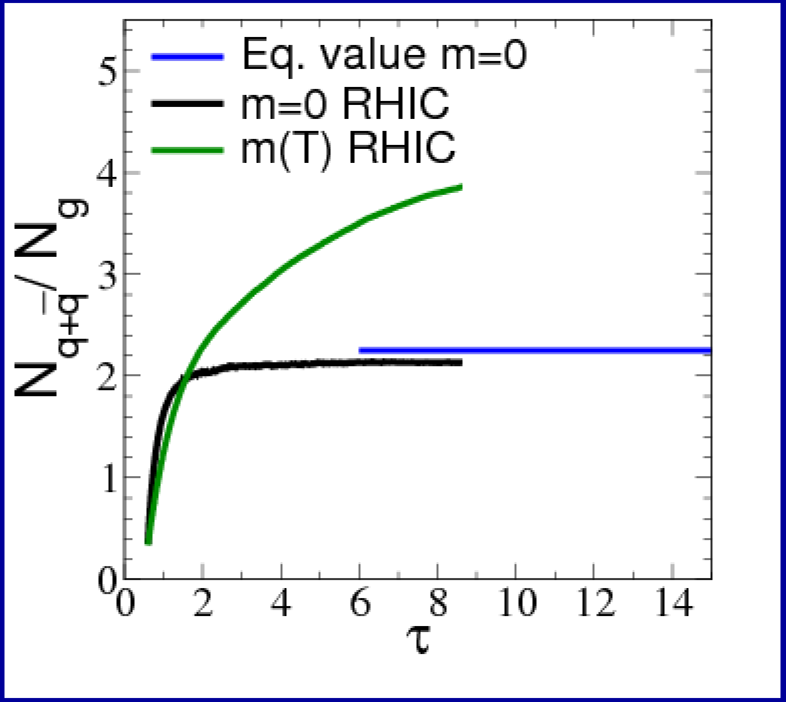
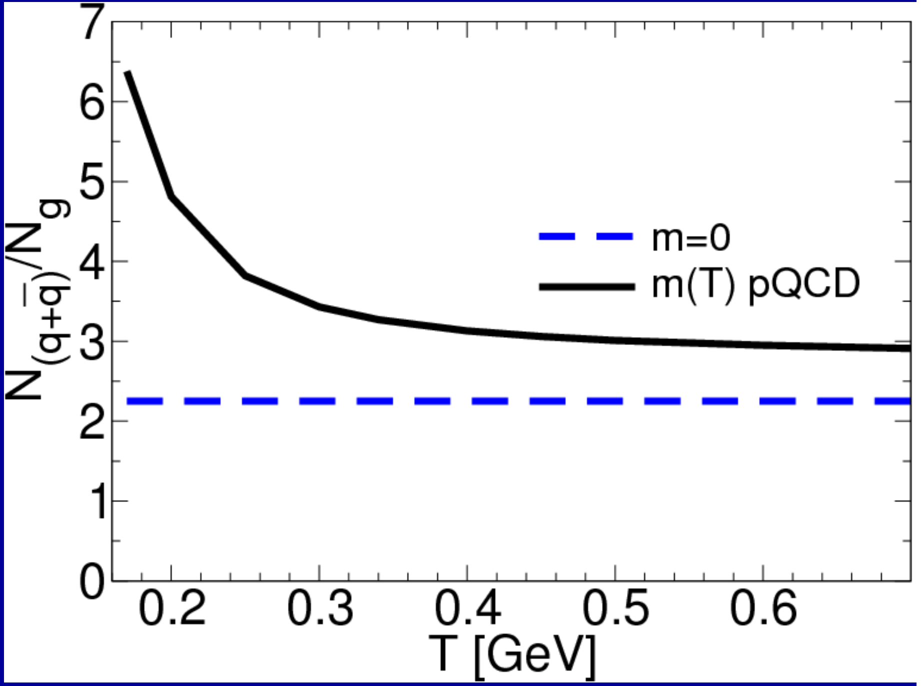
Results at RHIC (massive case)







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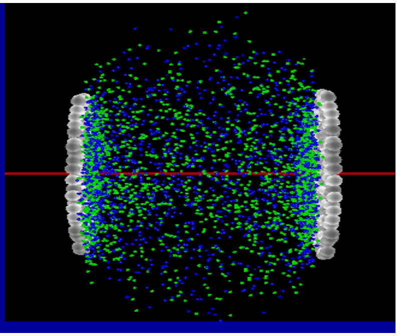
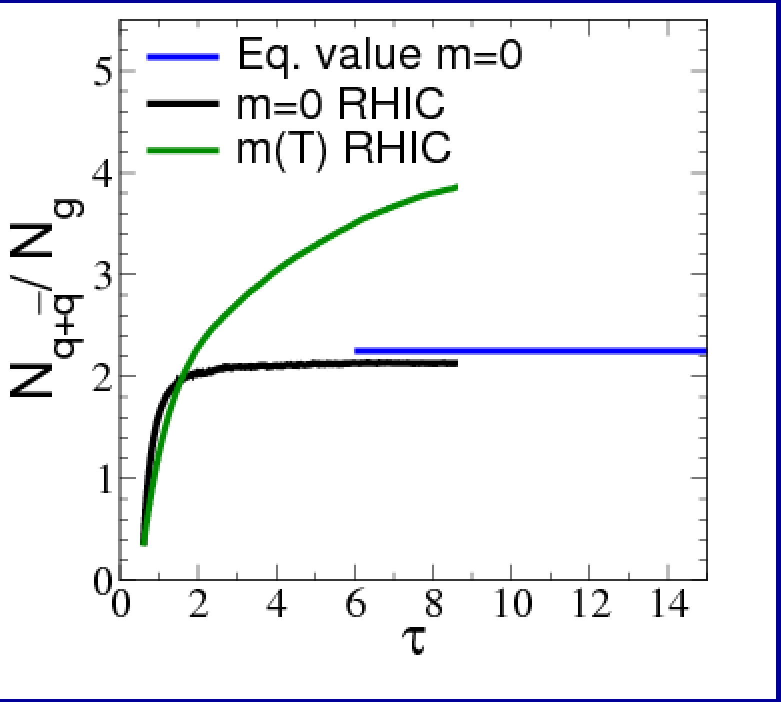
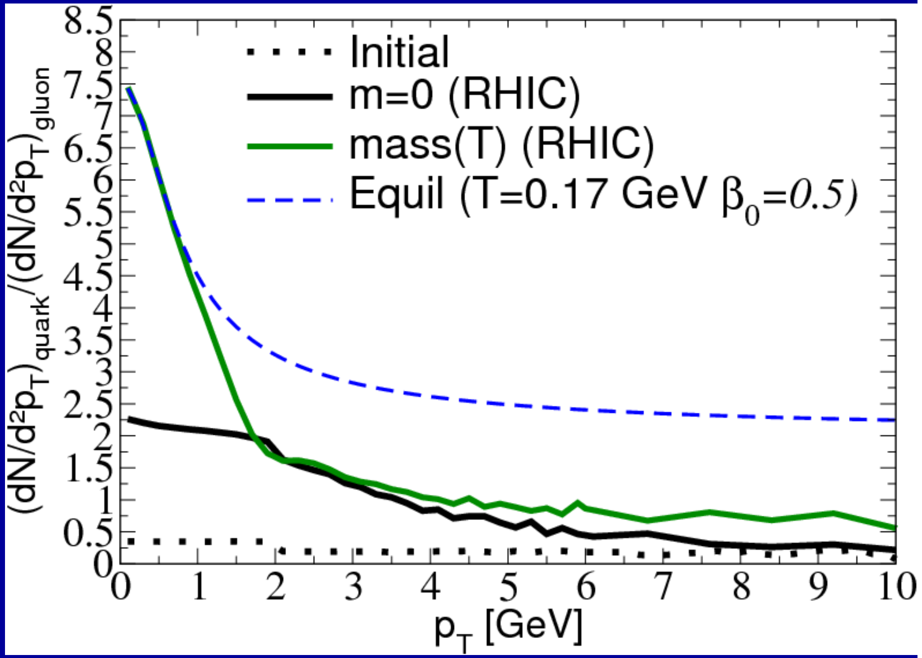
- The ratio keeps to increase and does not show the saturation behavior observed for m=0

Results at RHIC (massive case)







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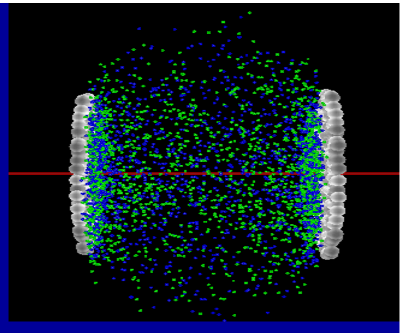
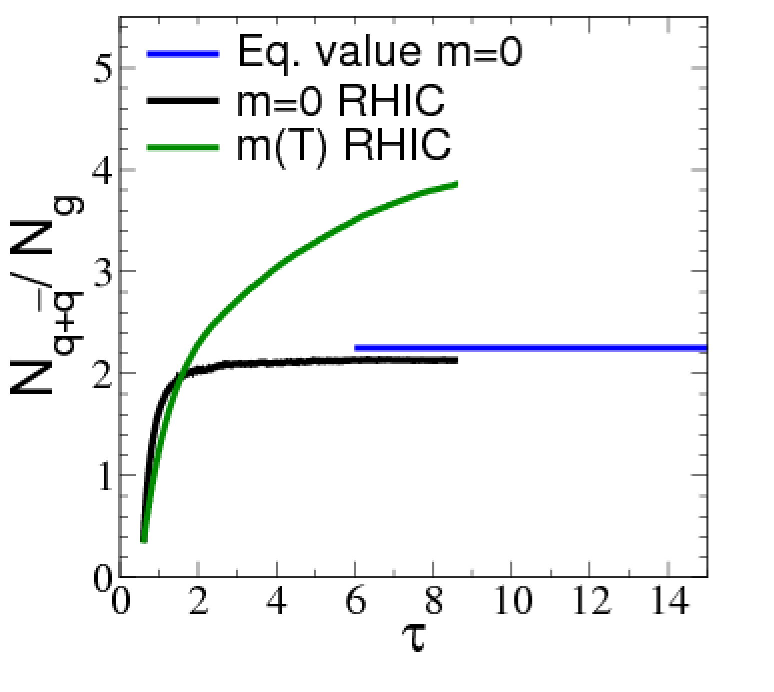
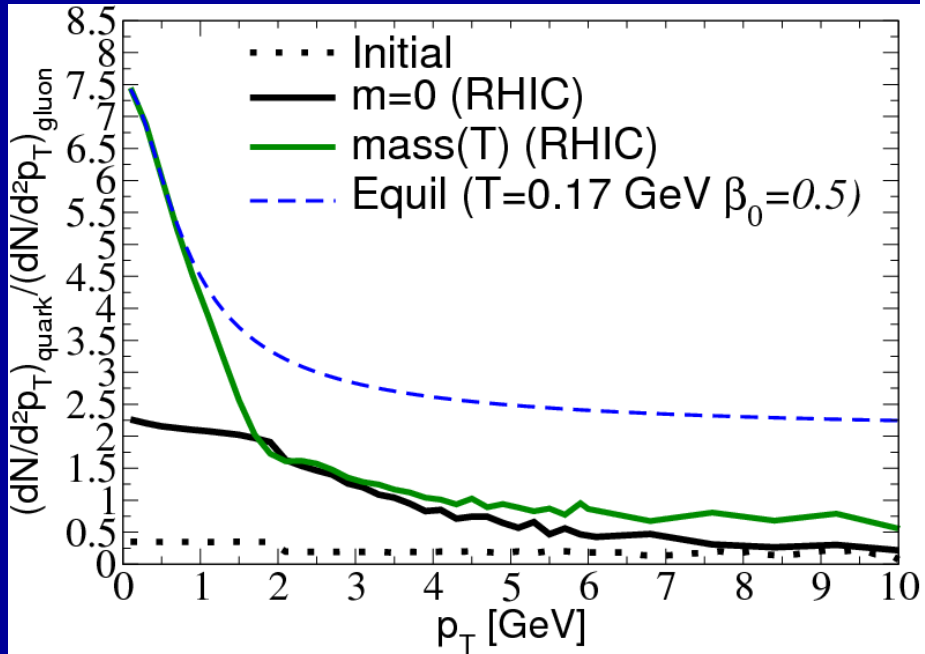
$N_q/N_g(p_T)$ shows a large difference between massless and massive case (strong momentum-dependence below pt equal to 2 GeV)

Results at RHIC (massive case)







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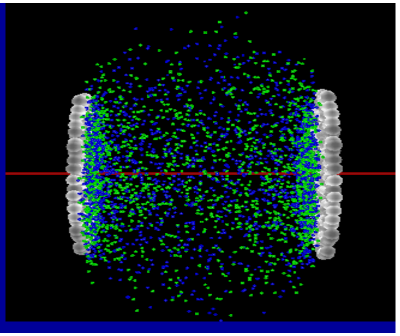
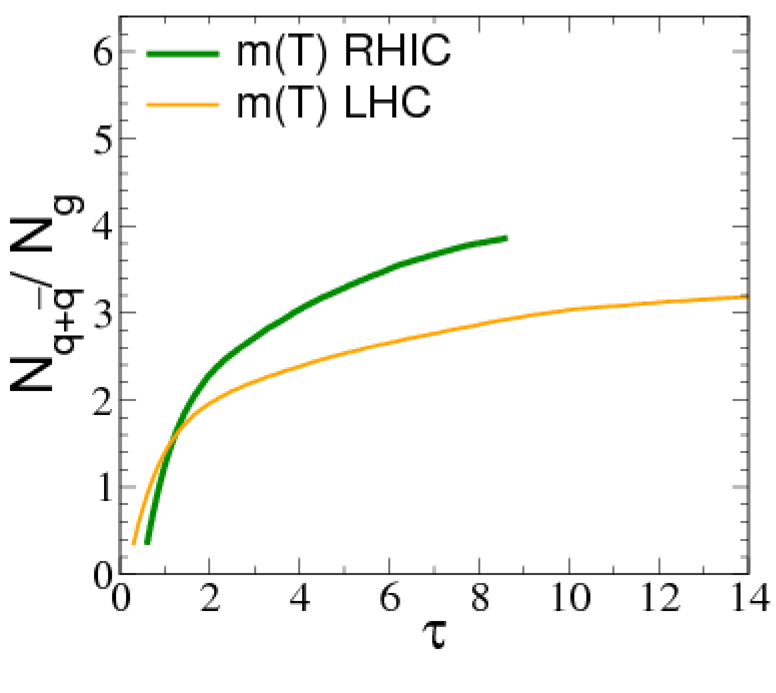
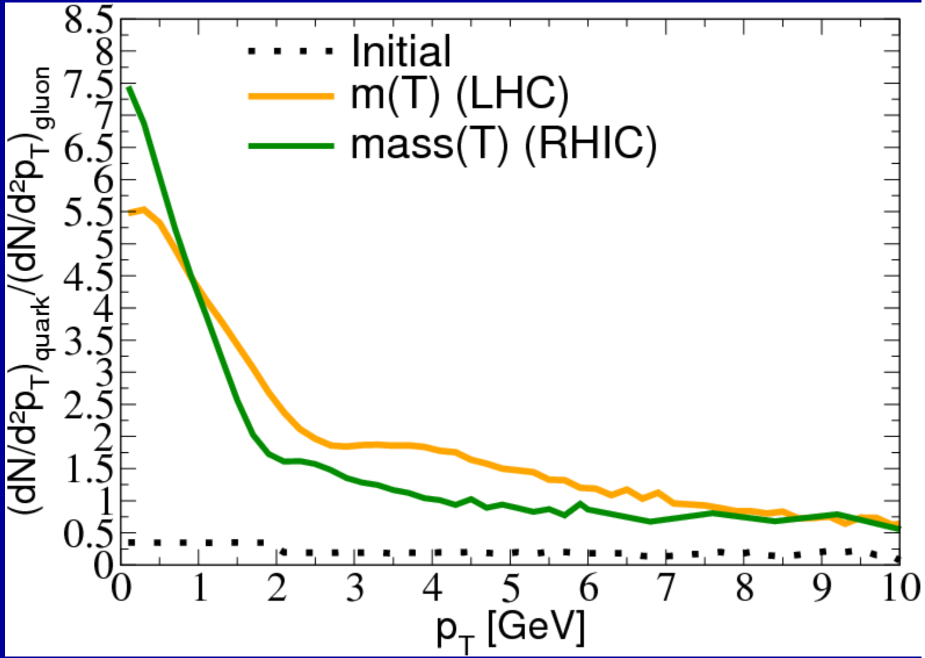
- The p_T dependence can be evaluated at equilibrium and our result scales with the expectations

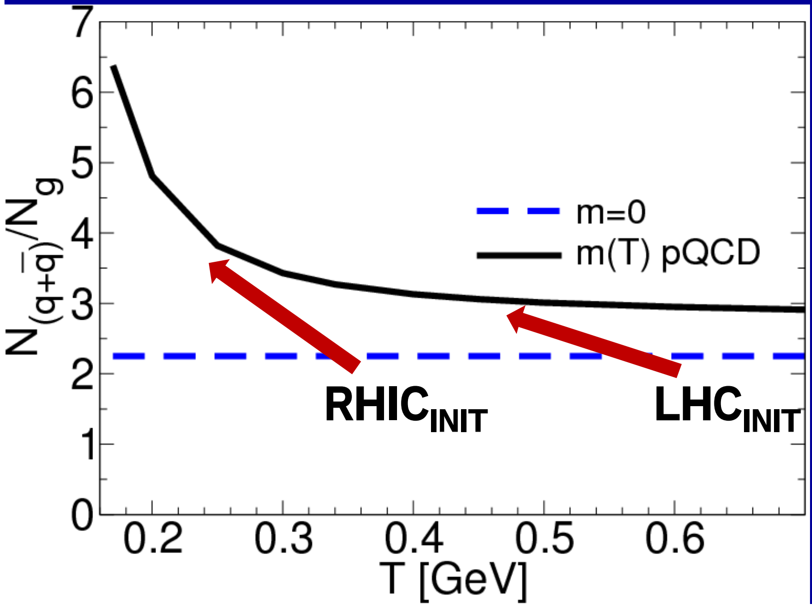
$$\frac{\left(dN/d^2p_T\right)^{quark}}{\left(dN/d^2p_T\right)^{gluoni}} = \frac{\nu_{q+\bar{q}}\, m_T^q\, e^{\gamma\left(m_T^q-\beta_0 p_T\right)/T}}{\nu_g\, m_T^g\, e^{\gamma\left(m_T^g-\beta_0 p_T\right)/T}}$$

Results ad LHC (massive case)









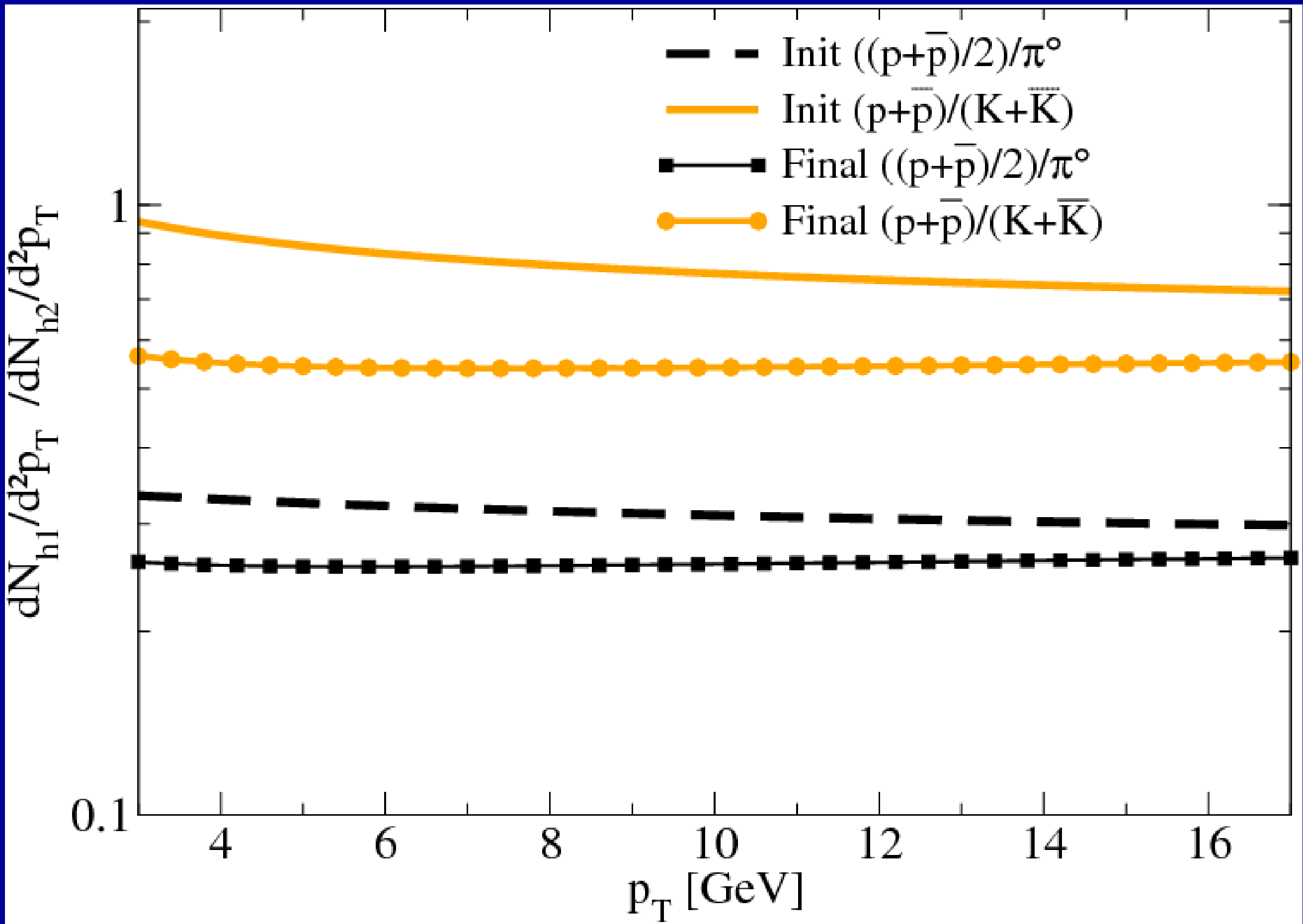
QGP has longest lifetime at LHC but N_q/N_g reaches a lower value

The difference between RHIC and LHC is due to the different temperatures experienced by the plasma during its evolution

Effects of inelastic collisions on high p_T hadrons

We have evaluated the **hadrons spectra** using the **AKK fragmentation functions**

$$\frac{dN_h}{d^2p_h} = \sum_f \int dz \frac{dN_f}{d^2p_f} D_{f\to h}(z)$$



Conclusions and Perspective

- ✓ Inelastic collision lead the initial Glasma towards a quark dominated plasma

- ✓ A realistic description of the lQCD thermodynamic using the quasi-particle model implies a ratio N_q/N_g almost two times greater than that expected in the massless case

- ✓ The inelastic collisions affect the high pT hadrons abundances

- ✓ Our results provide a support to the quark coalescence , capable to explains two main observations at RHIC: The barion/meson anomaly and the quark number scaling of the elliptic flow

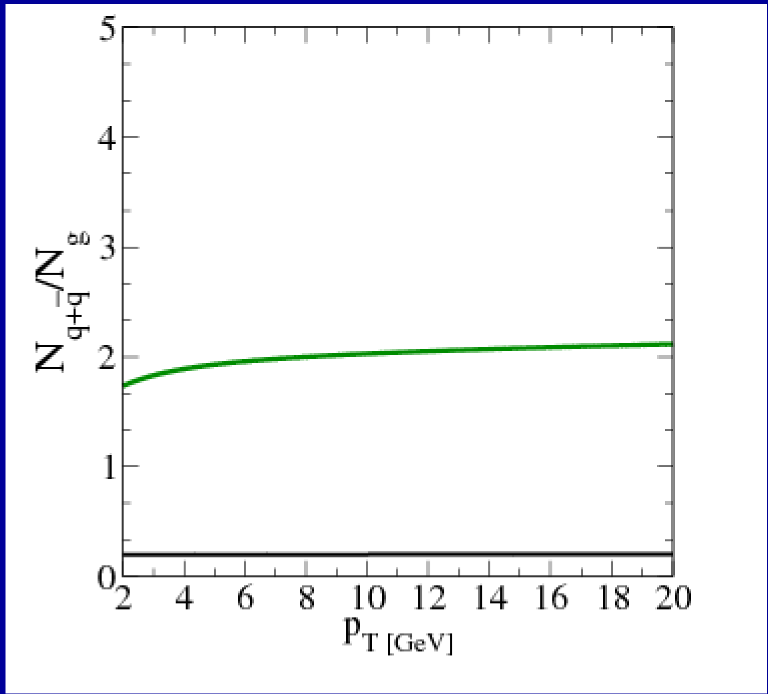
$\sigma_{2\to 2}$ inelastic for quasiparticle

- **Process** $q\bar{q} \to gg$

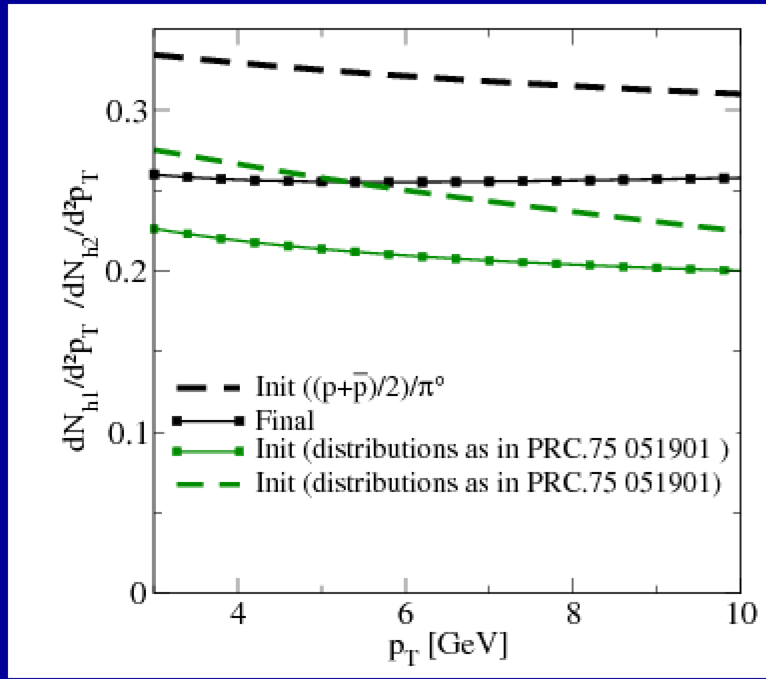
The cross section $\sigma_{q\bar{q}\to gg}$ differs from $\sigma_{gg\to q\bar{q}}$ only for the **different average over the initial color and spin states**

$$\sigma_{gg\to q\bar{q}}(s) = \frac{1}{16\pi s\left(s-4m_g^2\right)}\int_{t_-}^{t_+} dt\left(\left|M_t\right|^2 + \left|M_u\right|^2\right)$$

$$\sigma_{q\bar{q}\to gg}(s) = \frac{64}{9}\frac{1}{16\pi s\left(s-4m_q^2\right)}\int_{t_-}^{t_+} dt\left(\left|M_t\right|^2 + \left|M_u\right|^2\right)$$

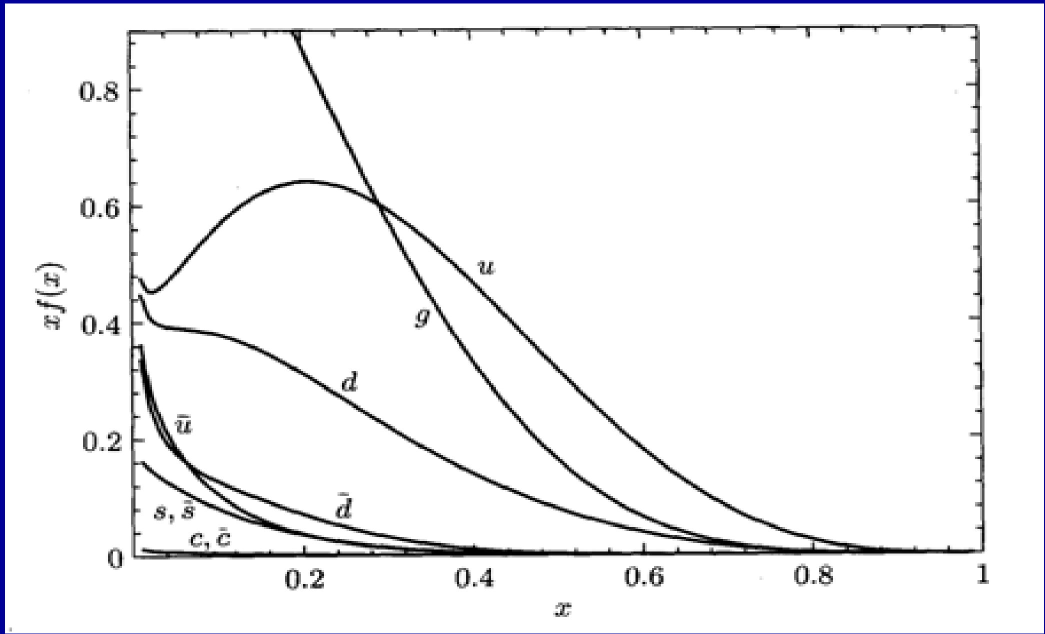


$N_{q+\bar{q}}/N_g$
$p_{T\,[GeV]}$

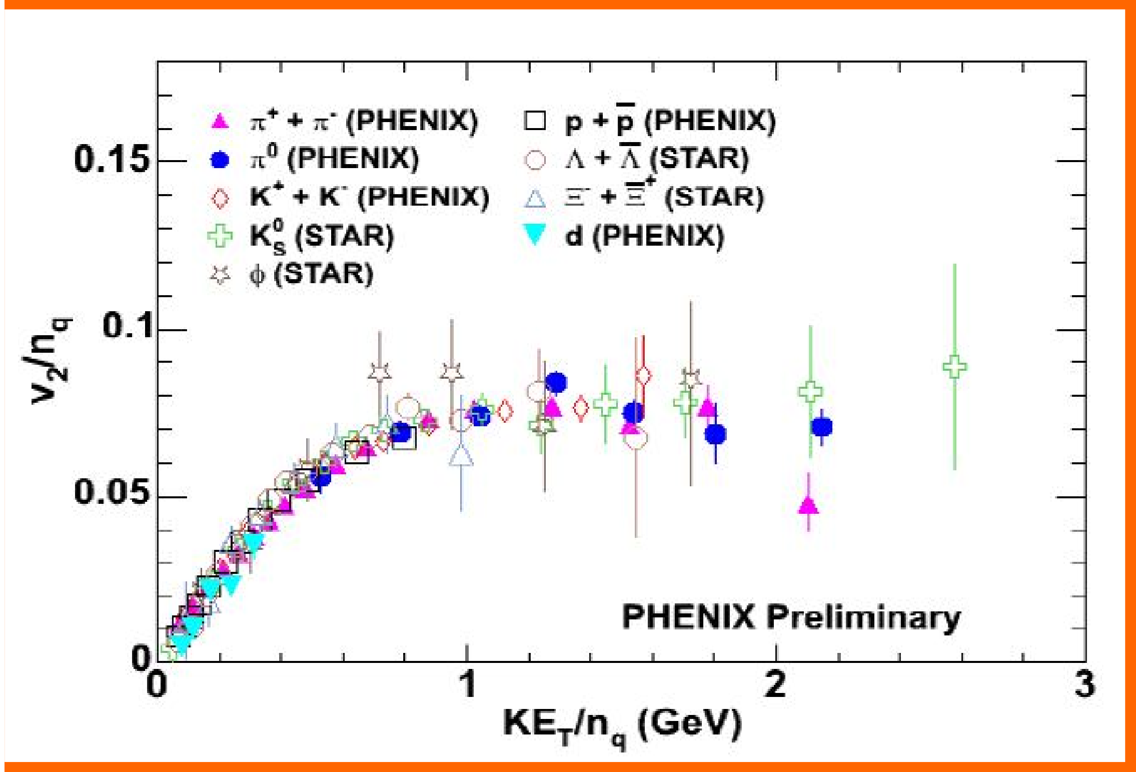


$dN_{h1}/d^2p_T\;/dN_{h2}/d^2p_T$
p_T [GeV]
Init $((p+\bar{p})/2)/\pi^o$
Final
Init (distributions as in PRC.75 051901)
Init (distributions as in PRC.75 051901)

Funzioni di distribuzioni partoniche



La coalescenza modifica il flusso ellittico



Coalescence scaling

$$\boxed{\frac{1}{n} V_2\left(\frac{p_T}{n}\right)}$$

Innalzamento del v_2

$$\boxed{\begin{aligned} v_{2,M}(p_T) &\approx 2v_{2,q}(p_T/2) \\ v_{2,B}(p_T) &\approx 3v_{2,q}(p_T/3) \end{aligned}}$$