Meson Spectra and Thermodynamics in Soft-Wall AdS/QCD

Sean Bartz

University of Minnesota

In collaboration with

Joe Kapusta

U.S. DEPARTMENT OF ENERGY | Office of Science

AdS/CFT Correspondence

Duality between:

4D Conformal Field Theory ↔ 5D Gravity theory in AdS Space

Strongly Coupled CFT	↔	Weakly coupled Gravity
Operators	↔	Fields
Global Symmetries	↔	Gauged Symmetries

AdS/QCD

- Study non-perturbative QCD
  - Hadron structure

Strongly Coupled QCD ⟷ Weakly coupled Gravity Dual in 5 Dimensions

- QCD not scale-invariant
  - Dilaton cutoff → Soft-wall Model

Zero Temperature Action

$$\mathcal{S}_{string} = \int d^5x\sqrt{-g}\left[e^{-2\Phi}\left(R + 4\partial_M\Phi\partial^M\Phi - \frac{1}{2}\partial_M\chi\partial^M\chi - V(\Phi,\chi)\right) + e^{-\Phi}\mathcal{L}_{meson}\right]$$

$$\mathcal{L}_{meson} \equiv |DX|^2 + m_X^2|X|^2 - \kappa|X|^4 + \frac{1}{2g_5^2}(F_A^2 + F_V^2)$$

$$ds^2 = \frac{R^2}{z^2}(-dt^2 + dx^2 + dz^2)$$

Finite Temperature Action

- Black hole metric:

$$\frac{R^2}{z^2}\left(-f(z)dt^2 + d\vec{x}^2 + \frac{dz^2}{f(z)}\right)$$

- Using black hole thermodynamics:

$$T = -\frac{1}{4\pi}\frac{\partial f}{\partial z}\Big|_{z=z_h}$$

- Potential is related to *f* by background equations

Background Field Behavior

- Dilaton
  - IR: $\phi = \lambda z^2$ → Slope of Meson Trajectory
- Chiral Field
  - UV: $\chi = \sigma z^3$ → Chiral Condensate
  - IR: $\chi = \Gamma z$ → Axial-Vector Mass Splitting

2-Field Background Equations

1 $$\sqrt{6}\phi''(z) - [\chi'(z)]^2 + \frac{2\sqrt{6}\phi'(z)}{z} \quad = \quad 0$$

2 $$3e^{2\phi(z)/\sqrt{6}}\frac{z^2}{L^2}\left[\frac{1}{\sqrt{6}}\phi''(z) - \frac{1}{2}[\phi'(z)]^2 - \frac{\sqrt{6}}{z}\phi'(z) - \frac{4}{z^2}\right] \quad = \quad V(\phi(z),\chi(z))$$

3 $$e^{2\phi(z)/\sqrt{6}}\frac{z^2}{L^2}\left[\chi''(z) - 3\chi'(z)\left(\phi'(z)/\sqrt{6} + \frac{1}{z}\right)\right] \quad = \quad \left.\frac{\partial V}{\partial \chi}\right|_{\phi=\phi(z),\chi=\chi(z)}$$

2-Field Power-Law Behavior

$$\chi = \chi_o z^n$$

↓

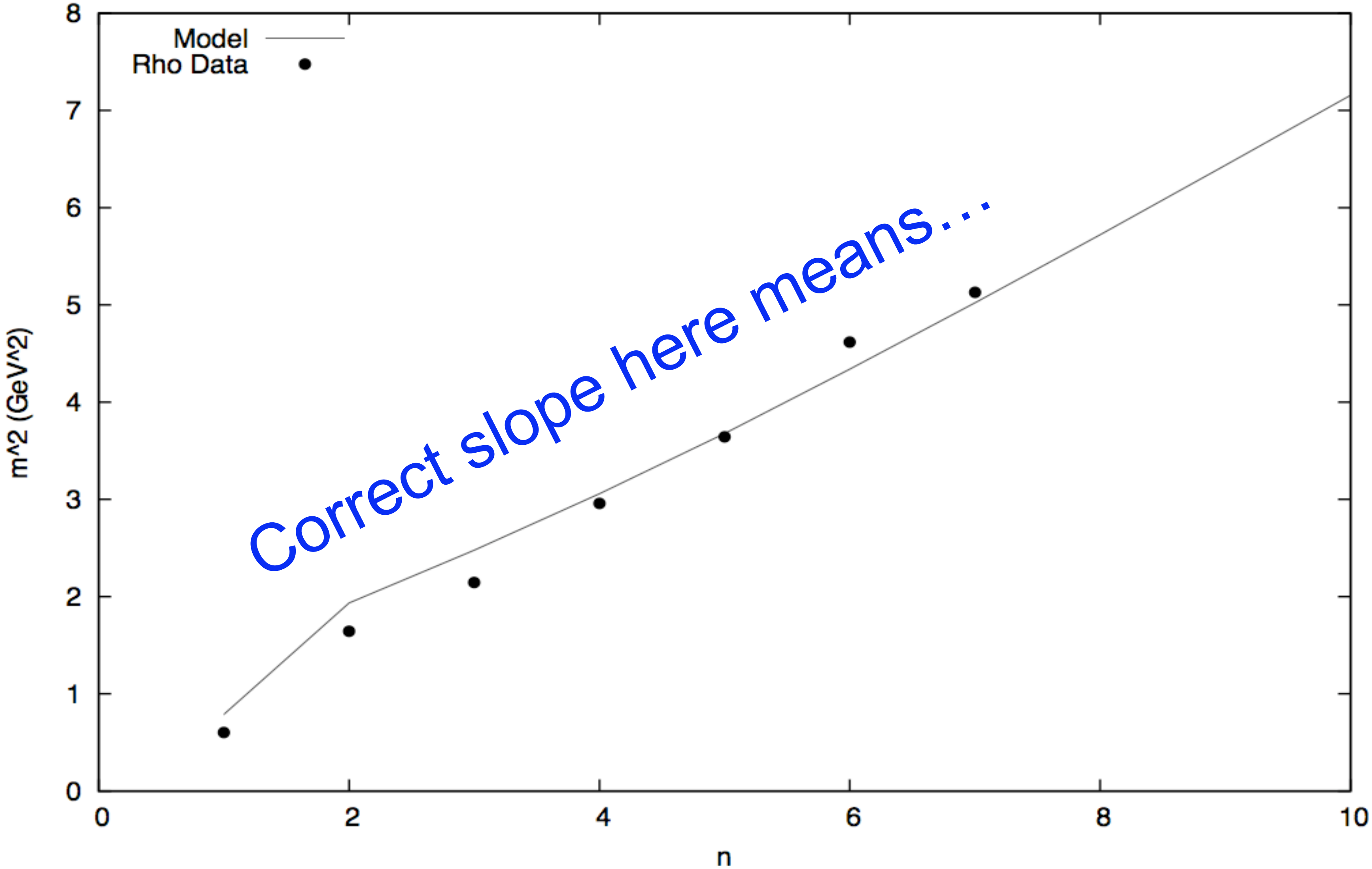
1 $$\sqrt{6}\phi''(z) - [\chi'(z)]^2 + \frac{2\sqrt{6}\phi'(z)}{z} = 0$$

↓

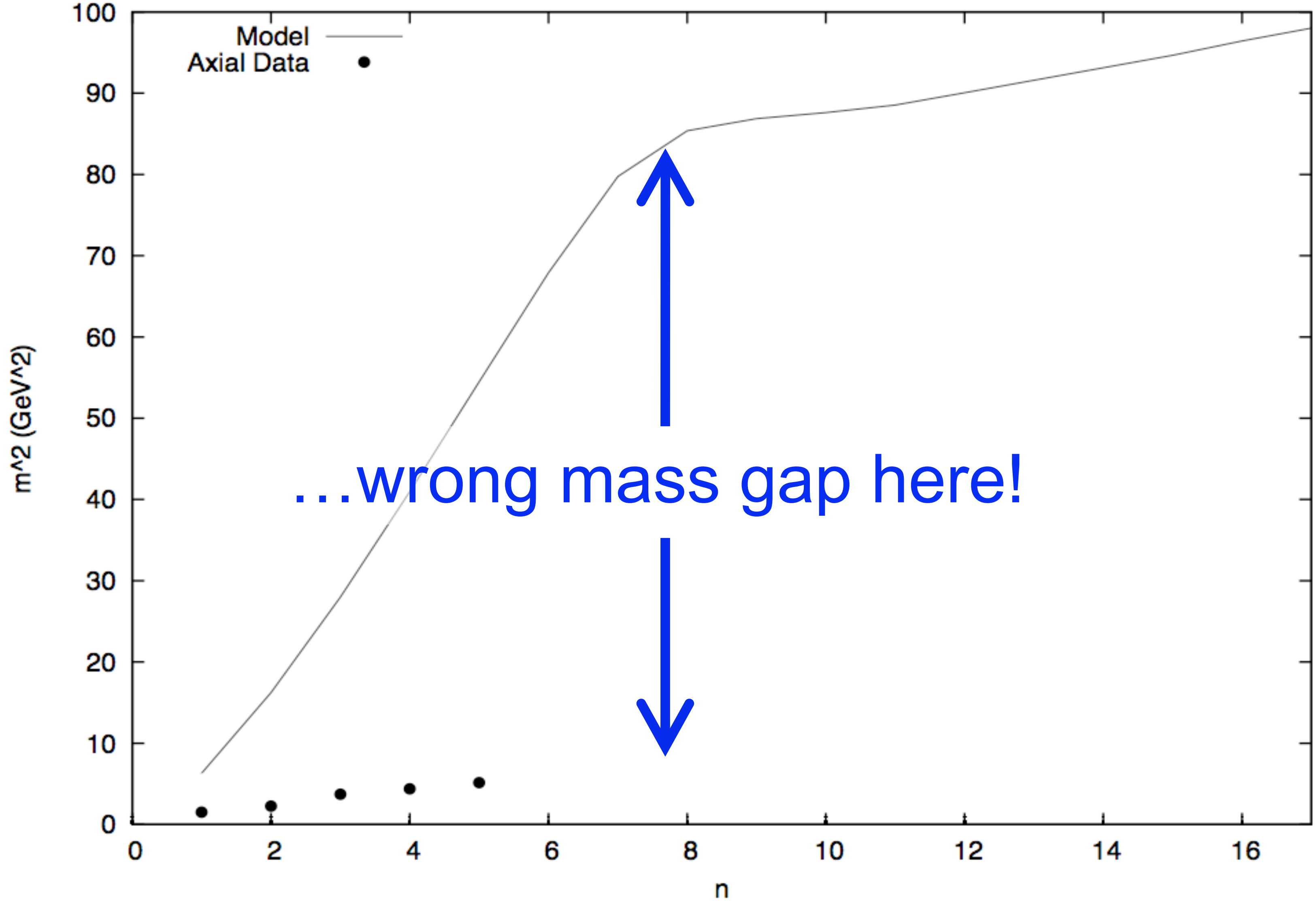
$$\phi = \frac{\sqrt{6}n}{12(2n+1)}\chi^2$$

So… λ and Γ are linked in IR! Problem?

2-Field Results – Vector



2-Field Results – Axial



3-Field Model

Add Glueball Field

1 $$\left(\frac{d\chi}{dz}\right)^2 + \boxed{\left(\frac{dG}{dz}\right)^2} = \frac{\sqrt{6}}{z^2}\frac{d}{dz}\left(z^2\phi'(z)\right)$$

2 $$\frac{1}{2}z^2 a\phi''(z) - \frac{3}{2}a[z\phi'(z)]^2 - 3za\phi'(z) = \tilde{V}\left(\phi(z), \chi(z), \boxed{G(z)}\right) + 12$$

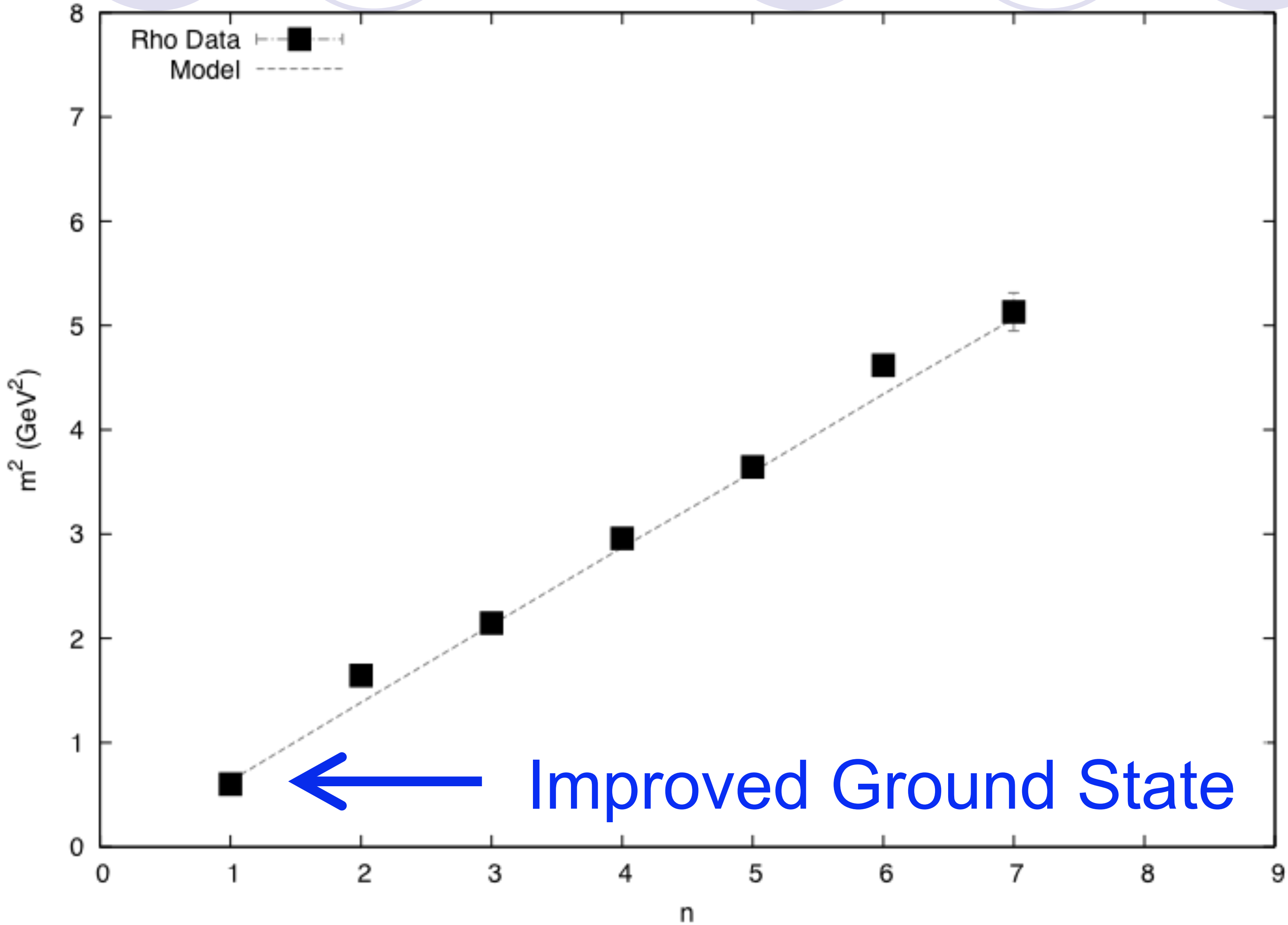
3 $$\frac{z^2}{L^2}\left[\chi''(z) - 3\chi'(z)\left(a\phi'(z) + \frac{1}{z}\right)\right] = \frac{\partial\tilde{V}}{\partial\chi}$$

4 $$\boxed{\frac{z^2}{L^2}\left[G''(z) - 3G'(z)\left(a\phi'(z) + \frac{1}{z}\right)\right] = \frac{\partial\tilde{V}}{\partial G}}$$

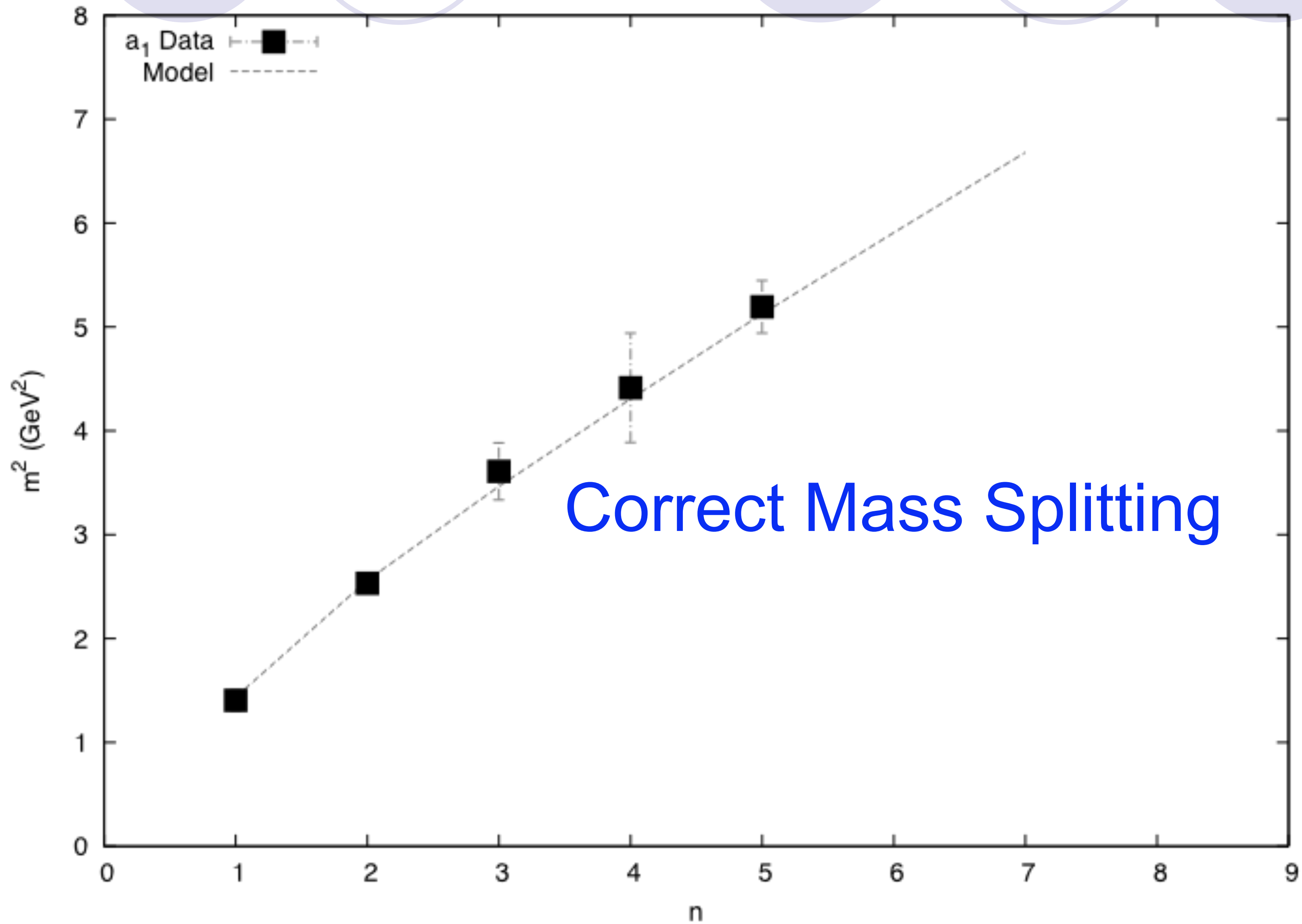
Sanity Check

- Try a simple 3-field parametrization
    - Zero quark mass
- Adjust input parameters
- Plot meson spectra

Parametrization – Vector

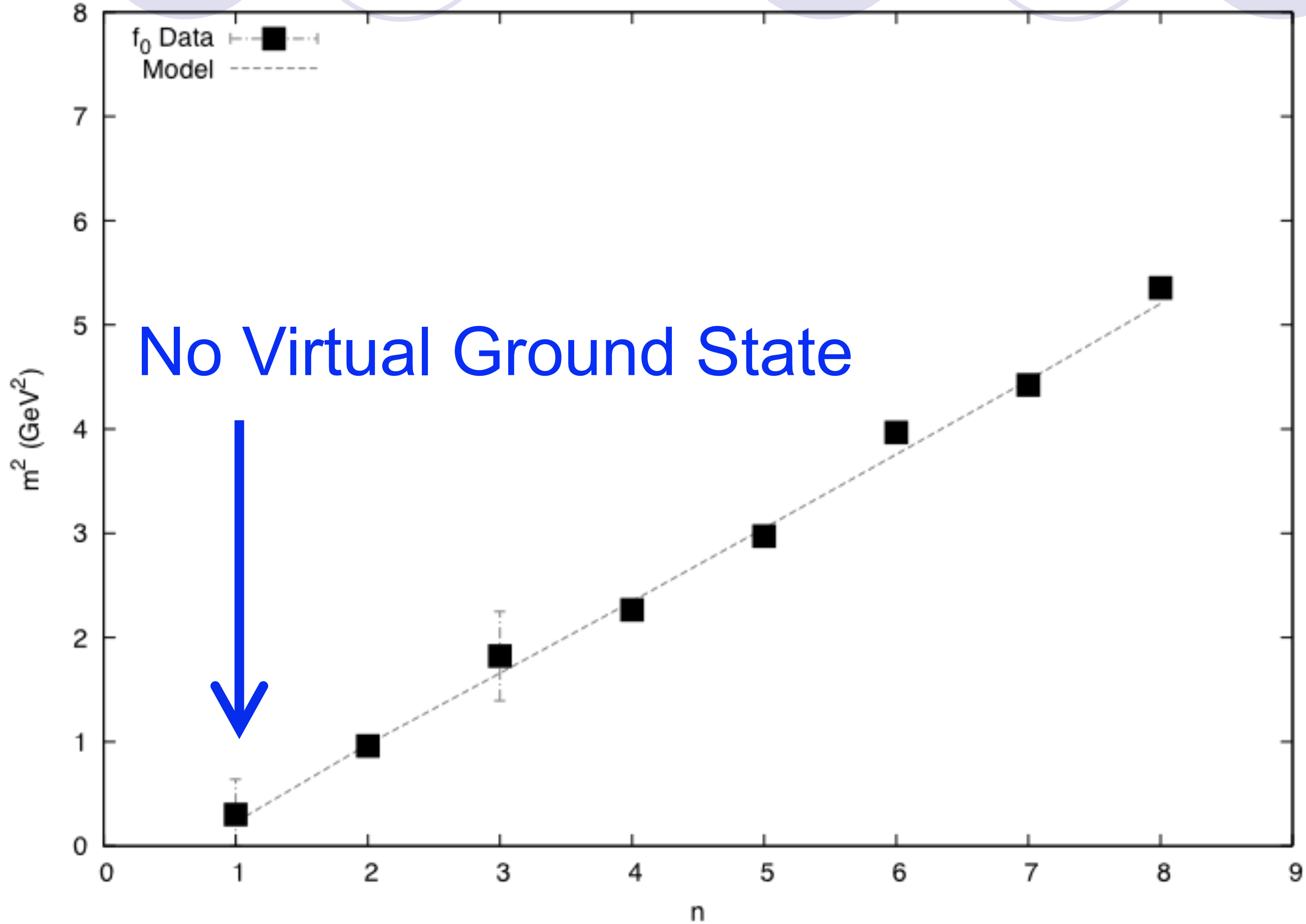


Parametrization – Axial

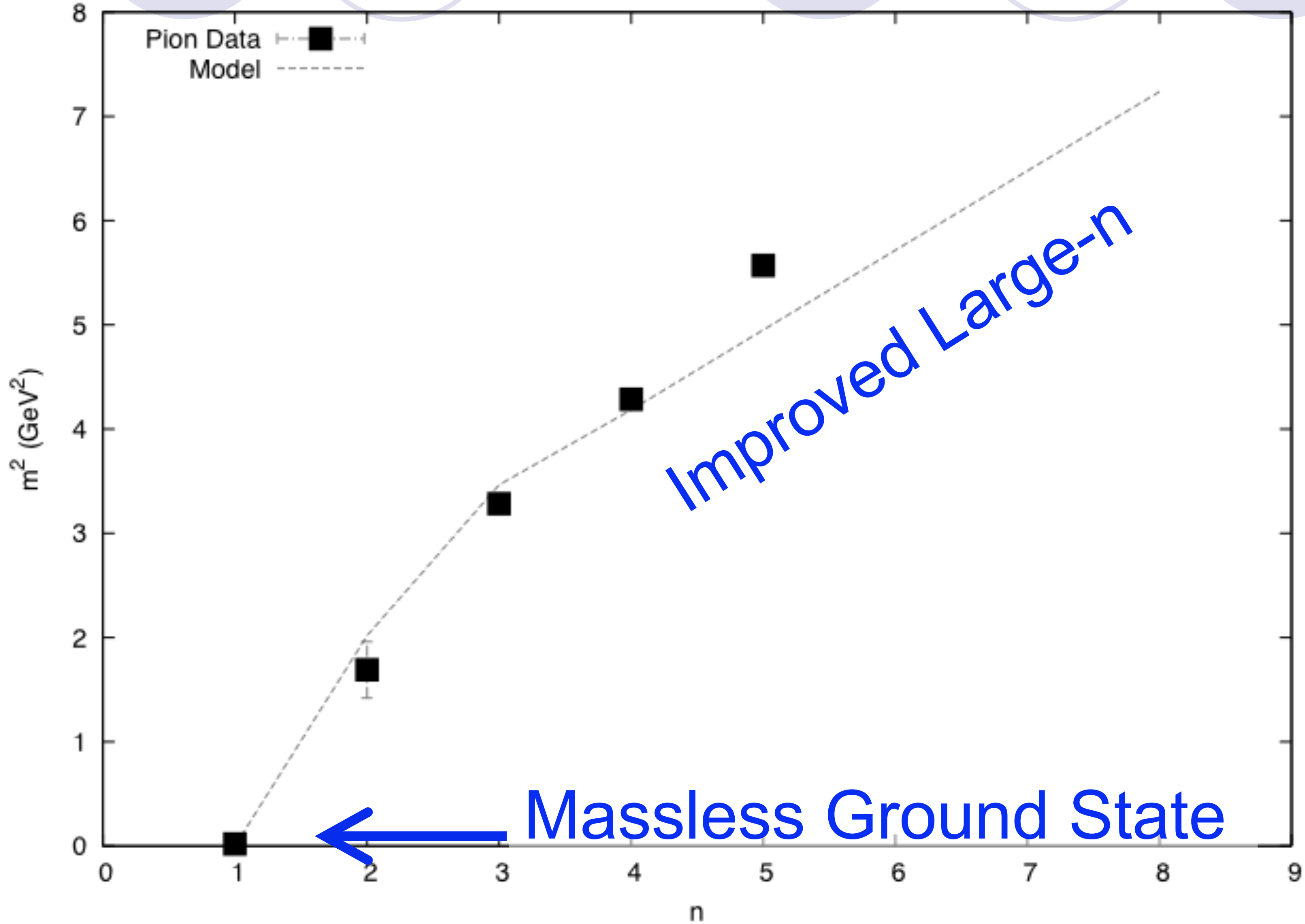


Correct Mass Splitting

Parametrization – Scalar



Parametrization – Pseudoscalar



General Potential

- Chiral and Glueball fields:

$$\chi(z) = \chi_o z^n, \quad G(z) = g_o z^m$$

  - Insert into 1 to find dilaton

- Ansatz:

$$\tilde{V} = c_o + c_1\phi + \frac{1}{2}m_\chi^2\chi^2 + c_2\phi^2 + c_3G^2 + c_4\chi^4 + c_5\phi\chi^2 + c_6G^4 + c_7\phi G^2 + c_8G^2\chi^2$$

  - Insert into 2, 3, 4. Match powers to find c_i
  - Cosmological Constant $\quad c_0 = -12$
  - Glueball mass term in IR
  - c_2 set by dilaton mass

Potential Ansatz

$$\tilde{V} = c_0 + c_1\phi + c_2\phi^2 + \frac{1}{2}m_\chi^2\chi^2$$

$$+c_3G^2 + c_4\chi^4 + c_5\phi\chi^2 + c_6G^4 + c_7\phi G^2 + c_8\chi^2G^2$$

- Chiral and Glueball fields:

$$\chi(z) = \chi_o z^n, \quad G(z) = g_o z^m$$

- Insert into background equations, match powers of *z*

Potential Ansatz

$$\tilde{V} = c_0 + c_1\phi + c_2\phi^2 + \frac{1}{2}m_\chi^2\chi^2$$

$$+c_3G^2 + c_4\chi^4 + c_5\phi\chi^2 + c_6G^4 + c_7\phi G^2 + c_8\chi^2G^2$$

$c_0 = -12$ Constant from UV to IR

$c_1 = 4\sqrt{6}$ c_2 is unconstrained

$m_x^2 = -3$ ← Matches AdS/CFT Dictionary

Potential Ansatz

$$\tilde{V} = c_0 + c_1\phi + c_2\phi^2 + \frac{1}{2}m_\chi^2\chi^2$$

$$+c_3G^2 + c_4\chi^4 + c_5\phi\chi^2 + c_6G^4 + c_7\phi G^2 + c_8\chi^2G^2$$

6 coefficients change from UV to IR

Only 3 background equations

Cannot solve unambiguously

Closer Examination…

$$\tilde{V} = c_0 + c_1\phi + c_2\phi^2 + \underline{\frac{1}{2}m_\chi^2}\chi^2$$

$$+\underline{c_3}G^2 + \underline{c_4}\chi^4 + \underline{c_5}\phi\chi^2 + \underline{c_6}G^4 + \underline{c_7}\phi G^2 + c_8\chi^2 G^2$$

Pairs of coefficients with same IR behavior

$c_3 = 0$ in UV

$c_4 = c_6 \sim c_8$ in IR

Next Steps…

- Combine terms using non-power behavior

- Ex: hyperbolic functions

$$G^2 \tanh G^2 \sim G^4$$ in UV

$$G^2 \tanh G^2 \sim G^2$$ in IR

Summary

- Two-field model gives wrong meson spectra
- Three-field model can correct this
- Thermodynamics requires a potential
- Potential derivation in progress

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