Non-linear flow response and plane correlations

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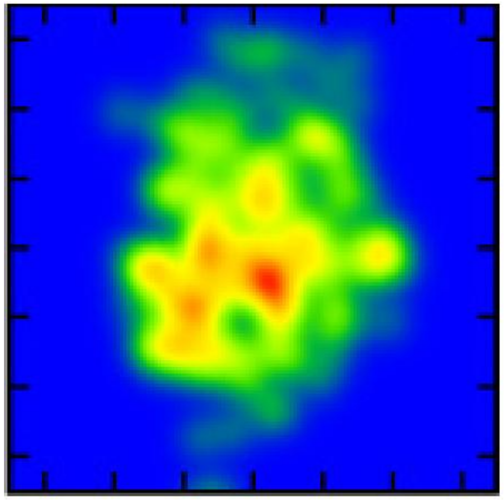
Hot Quarks 2012, Puerto Rico

In collaboration with Derek Teaney

- e-Print: arXiv:1206.1905 [nucl-th].
- Phys. Rev. C83 (2011) 064904, e-Print: arXiv:1010.1876 [nucl-th].

Motivation and outline



Initial state fluctuations(+ Event-By-Event hydro)

- Harmonic flow: v_1, v_2, v_3, etc.
- Correlations of reaction planes: Ψ_n's.

How to understand the observed correlations?

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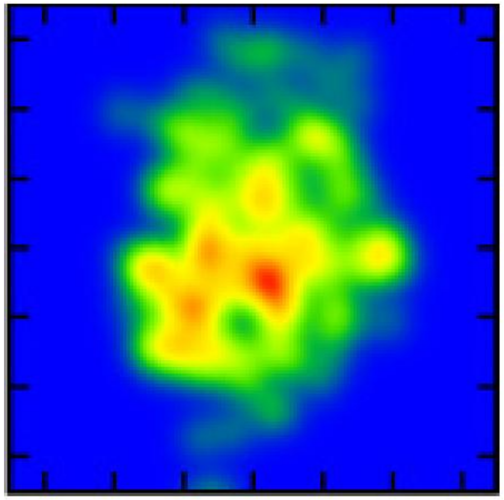

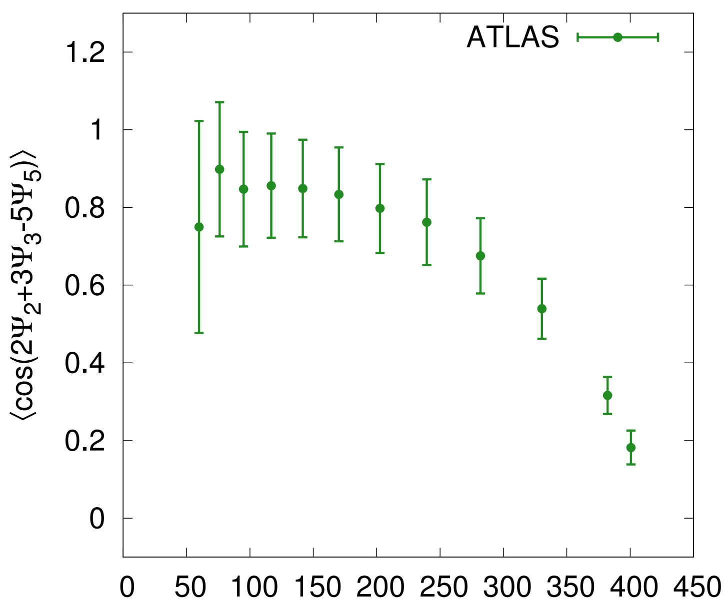

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- From medium expansion. (Non-linear resp.)
- Take into account both effects. (Total resp.)

Linear + Non-linear response.

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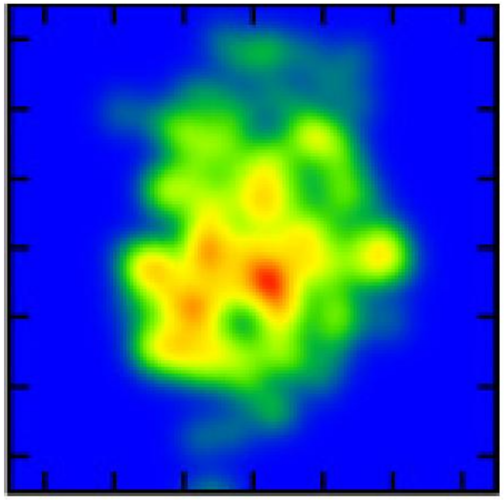

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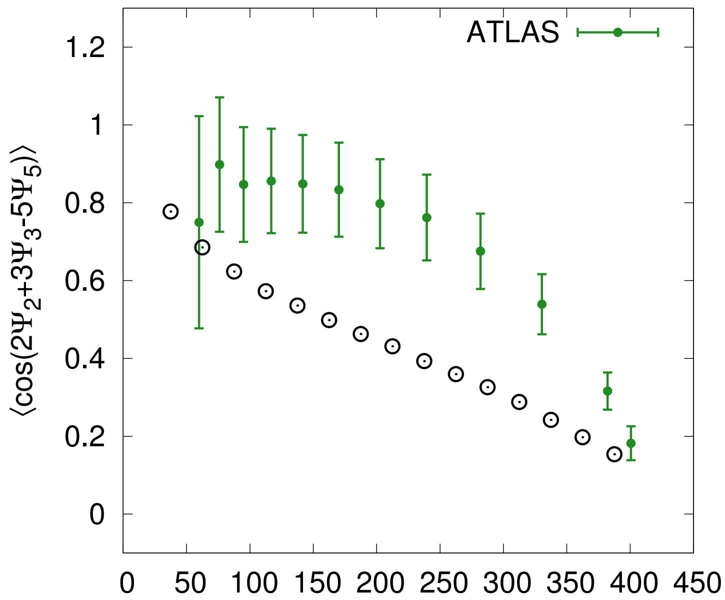

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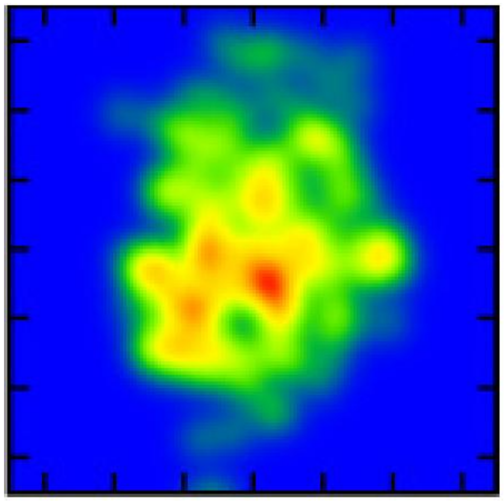

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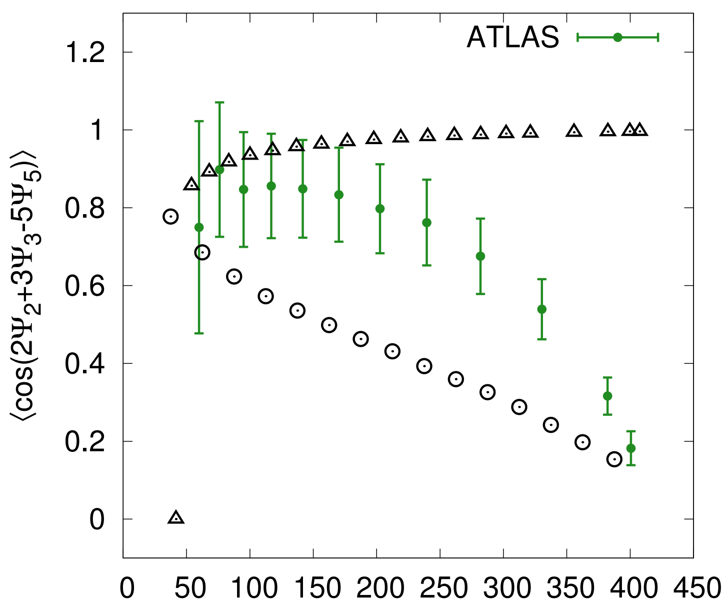

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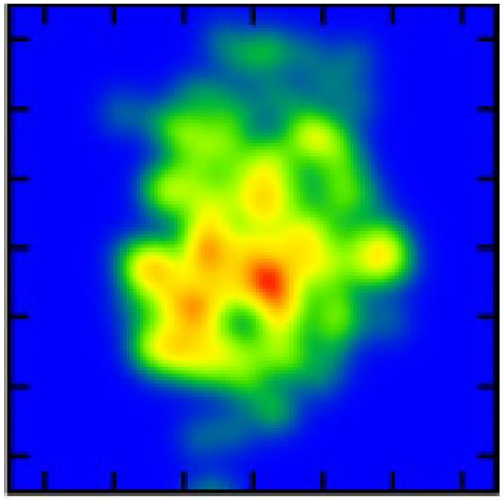

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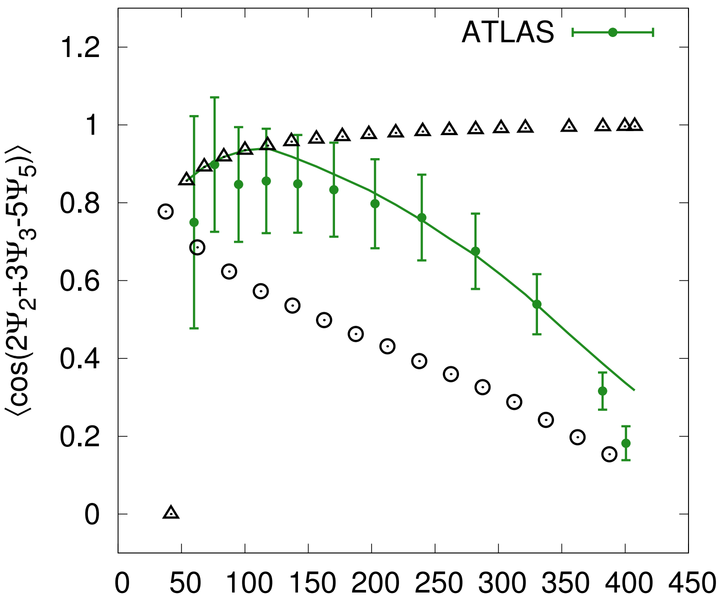

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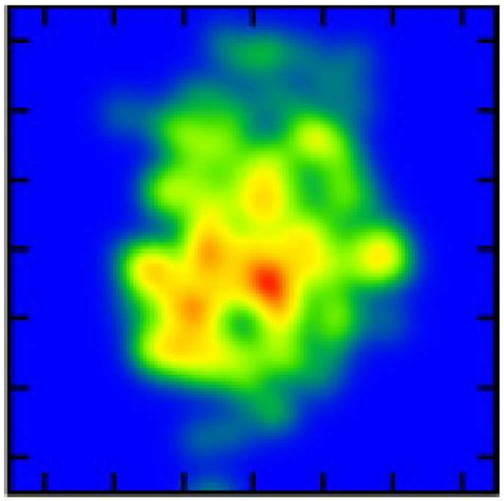

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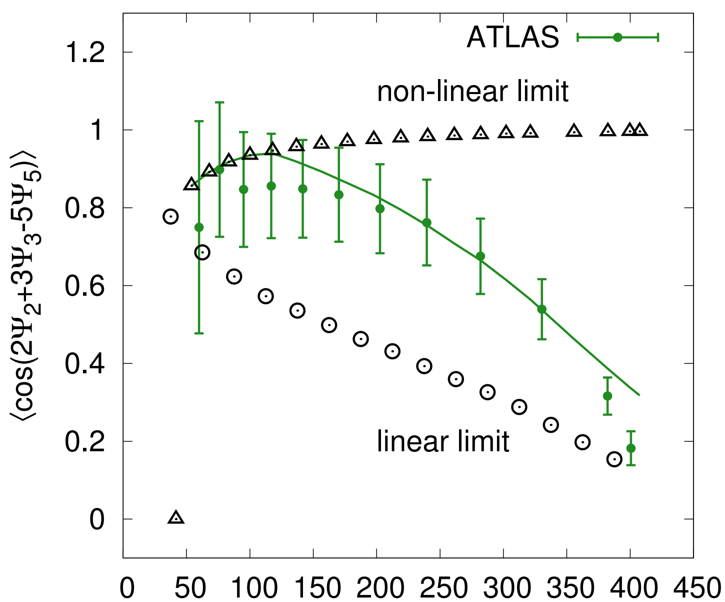

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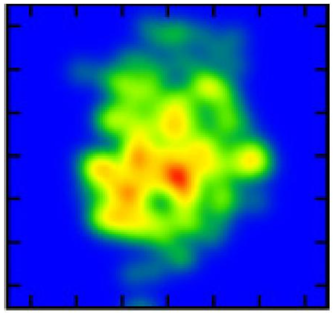
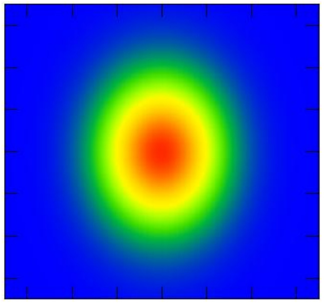
Linear + Non-linear response.

Cumulants for initial state: (not moments!)

Fluctuations in initial state as corrections:

 =  + fluctuations

- Cumulant expansion:

$$\rho(x,y) = \text{Gaussian} + \underbrace{\text{1st cumulant}}_{\varepsilon_1} + \underbrace{\text{3rd cumulant}}_{\varepsilon_3} + \underbrace{\text{4th cumulant}}_{\mathcal{C}_4} + \ldots$$

- 4th Cumulant determines **eccentricity** $\mathcal{C}_4$ and **participant angle** Φ_4.

$$\underbrace{\mathcal{C}_4 e^{4i\Phi_4}}_{\text{4th cumulant}} = -\frac{1}{\langle r^4 \rangle}\Big[\underbrace{\langle r^4 e^{i4\phi_r}\rangle}_{\varepsilon_4:\ \text{moments def.}} - \underbrace{3\langle r^2 e^{i2\phi_r}\rangle^2}_{\text{subtract } \varepsilon_2^2} \Big]$$

e.g. Gaussian with ε_2 has $\mathcal{C}_4 = 0$, but $\varepsilon_4 \propto \varepsilon_2^2 \neq 0$. So no non-linearity in def.

$\mathcal{C}_n \neq \varepsilon_n$, for $n \geq 4$.

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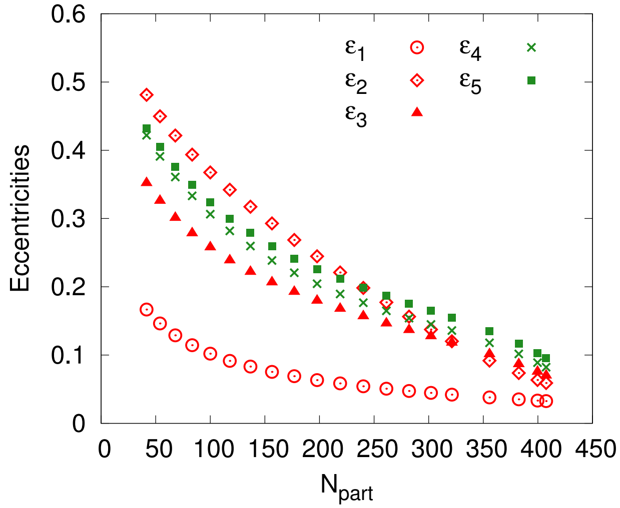
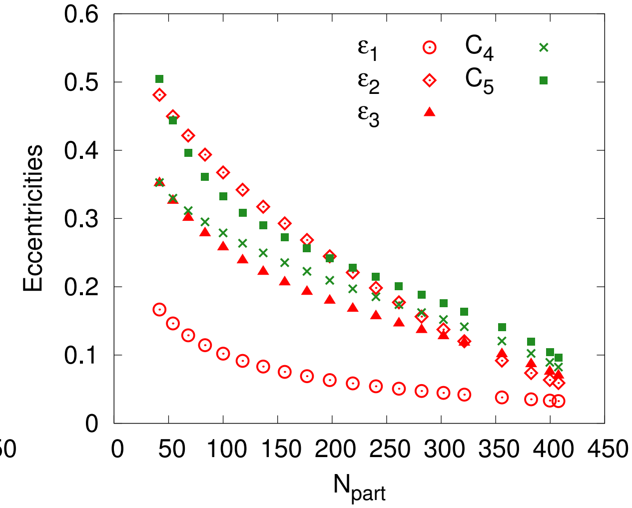
Eccentricities:





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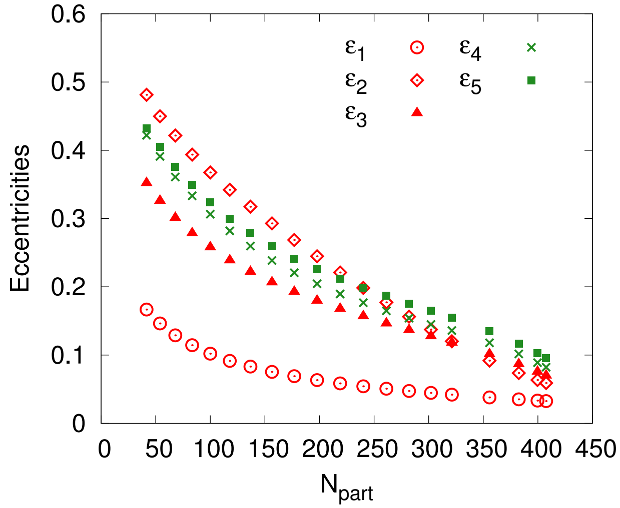
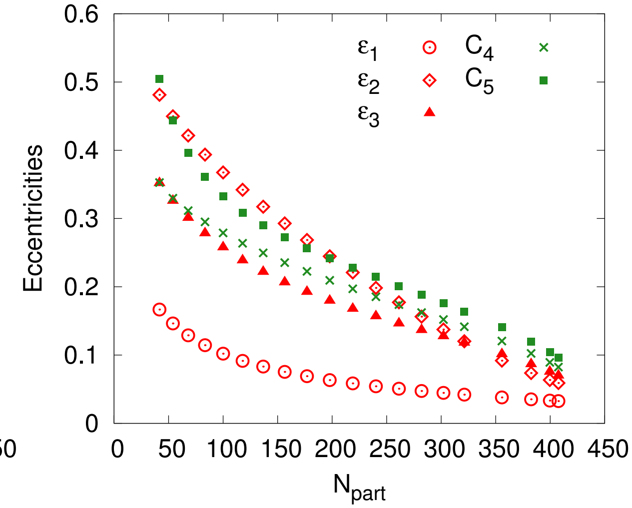
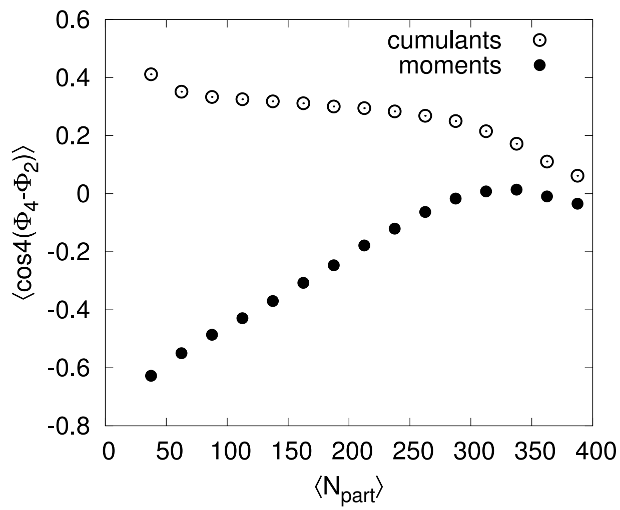
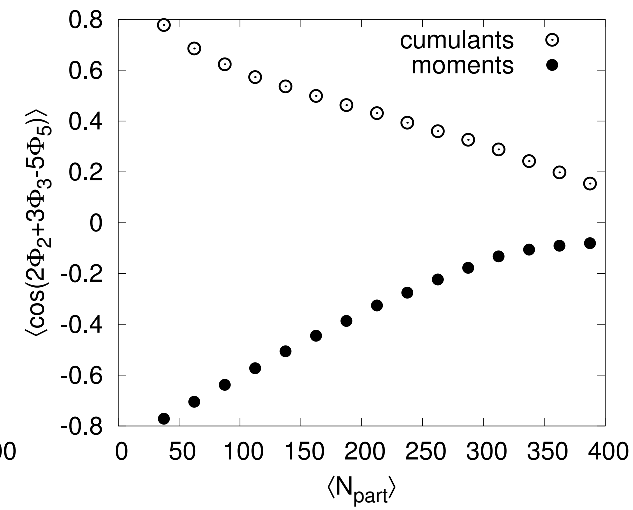




Initial 2-4 plane and 2-3-5 plane correlations:





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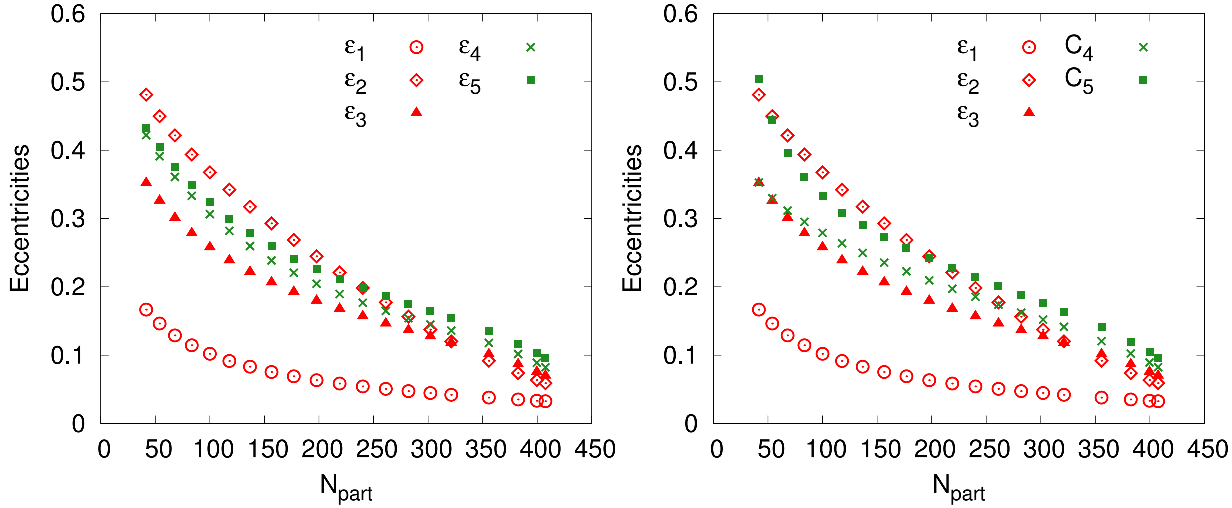

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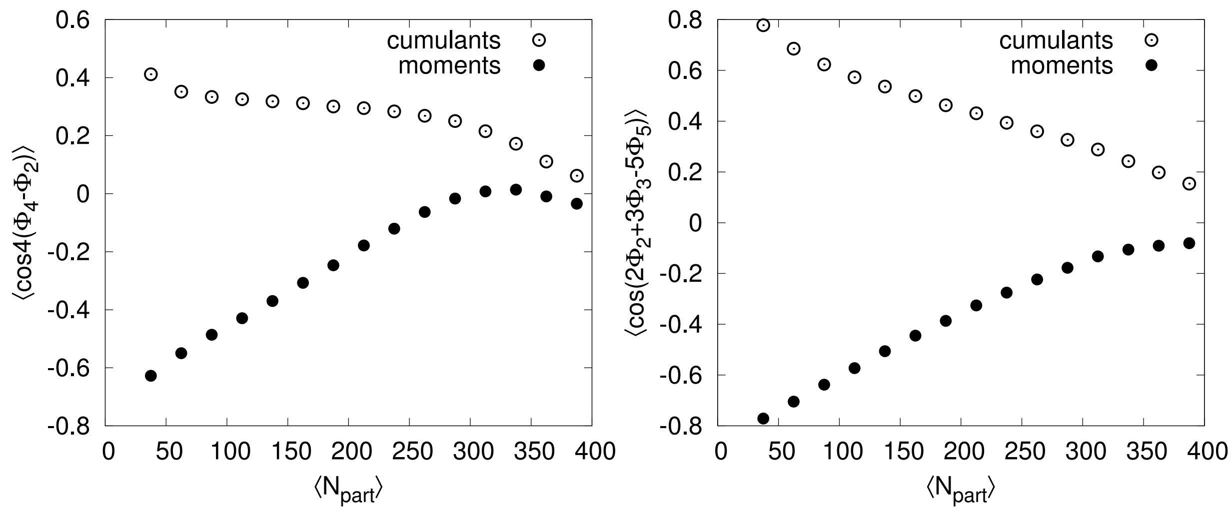

- Why we use cumulants: avoid double counting in initial conditions.

We define all geometric deformations, *i.e.* $(\mathcal{C}_n,\, \Phi_n)$, with cumulants.

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Non-linear response formalism (n=5 for example)

Flow generation in hydro:

$$\underbrace{v_5 e^{-i5\Psi_5}}_{\text{final state}} = \underbrace{\underbrace{\frac{w_5}{\mathcal{C}_5}}_{\text{linear resp.}} \times \underbrace{\mathcal{C}_5 e^{-i5\Phi_5}}_{\text{initial state}}}$$

[1] (U. Heinz and Z. Qiu, and Gardim *et al.*)

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- Assume non-linear flow response to $\underline{\varepsilon_2\varepsilon_3}$.

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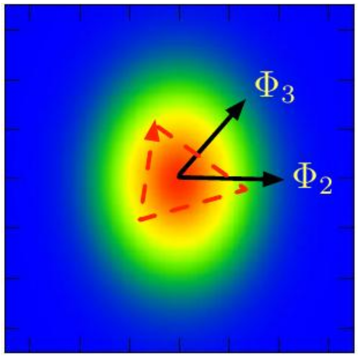

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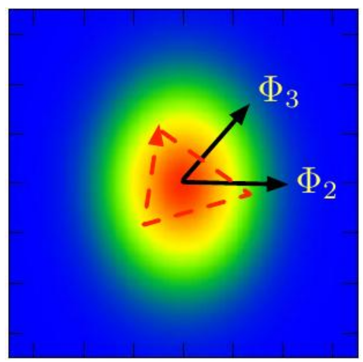

- Calculations of $v_n\{2\}$: flow from two-particle correlation.

$$v_5\{2\} = \langle\langle |w_5 e^{-i5\Phi_5} + w_{5(23)} e^{-i(3\Phi_3+2\Phi_2)}|^2 \rangle\rangle^{1/2}$$

$$v_5 \sim (\text{linear}) + (\text{non-linear}) + (\text{interference} \propto \cos(2\Phi_2 + 3\Phi_3 - 5\Phi_5)).$$

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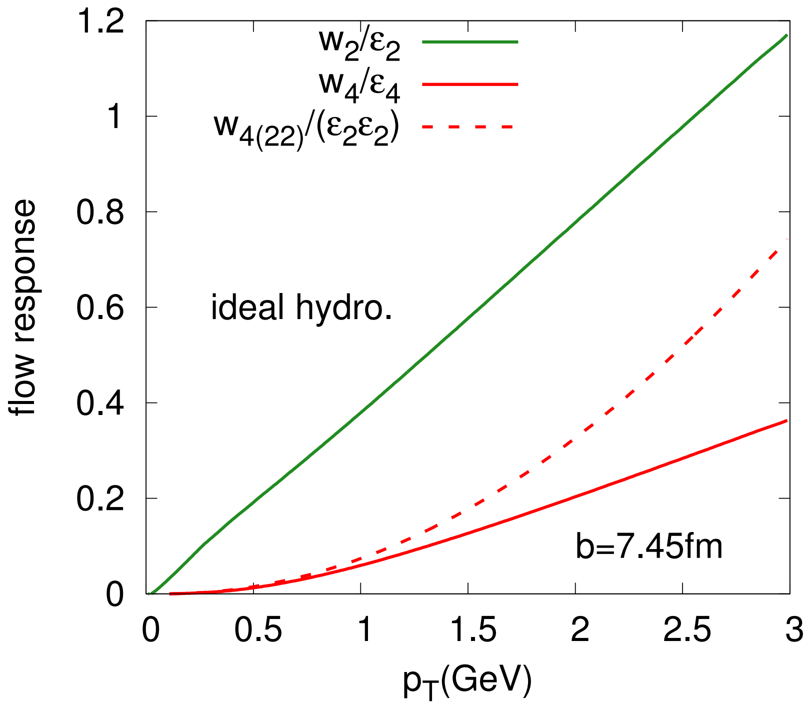
Non-linear response dependence on p_T

w_4 and $w_{4(22)}$



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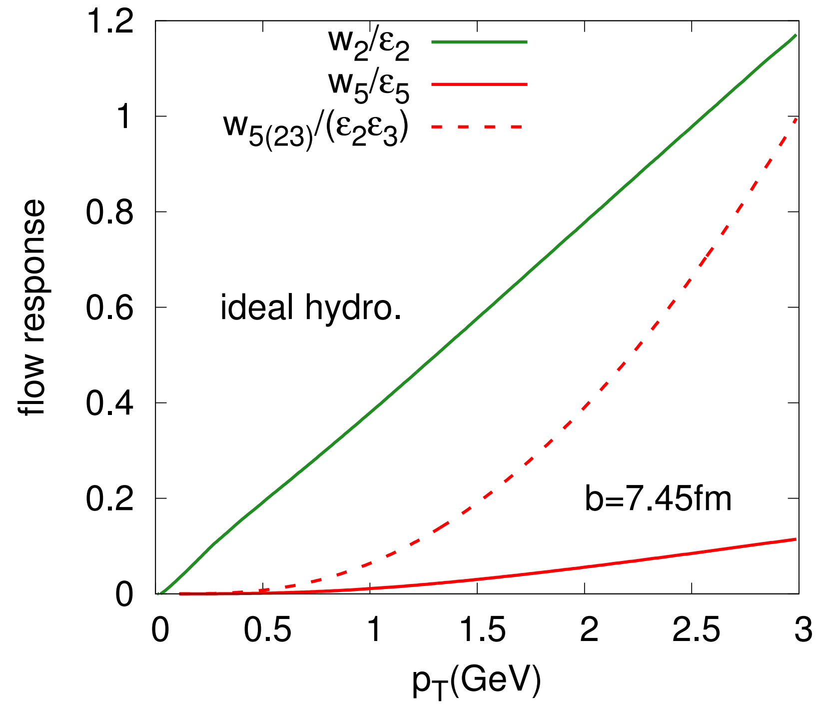


- Small p_T: non-linear response is not distinguishable from linear response.
- Large p_T: linear response $\propto p_T$, non-linear response $\propto p_T^2$.[2]

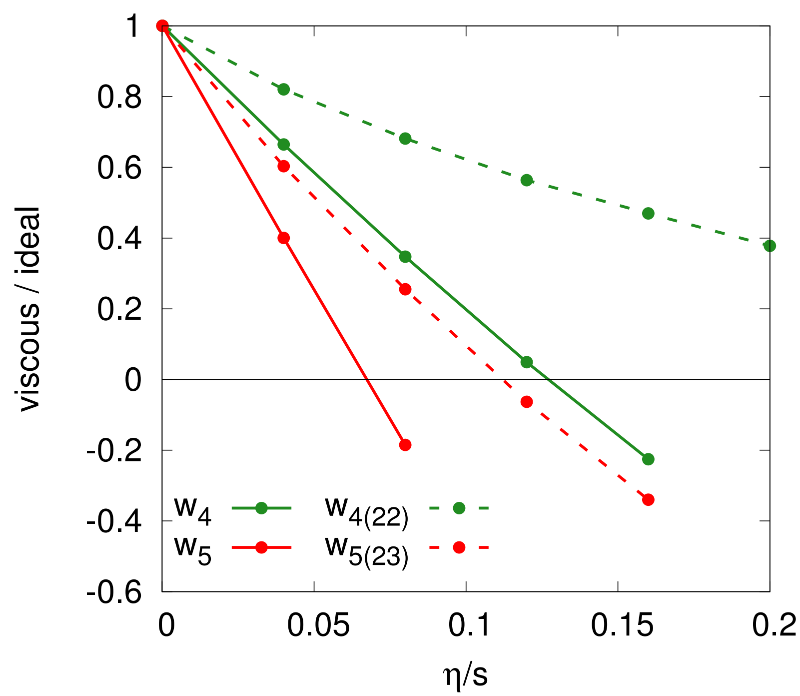
So, non-linear response becomes more significant for larger p_T.

[2] (N. Borghini and J. Ollitrault)

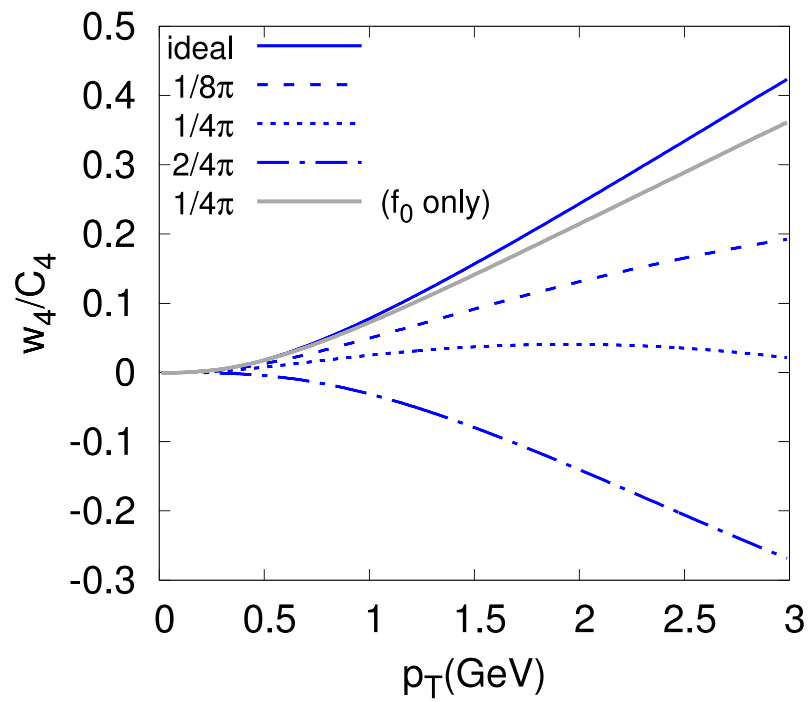
Non-linear response dependence on η/s

[3]Damping rate $\propto$ (harmonic order)$^2 \times \eta/s$:

Damping rate



linear v_4



- Damping of $w_{4(22)}$ < damping of w_4, which can be generalized to $n \geq 4$.
- δf on freeze out may be questionable for higher order flow response.

For $(n \geq 4)$, non-linear response becomes more important for larger η/s.

[3](S. Gubser and A. Yarom)

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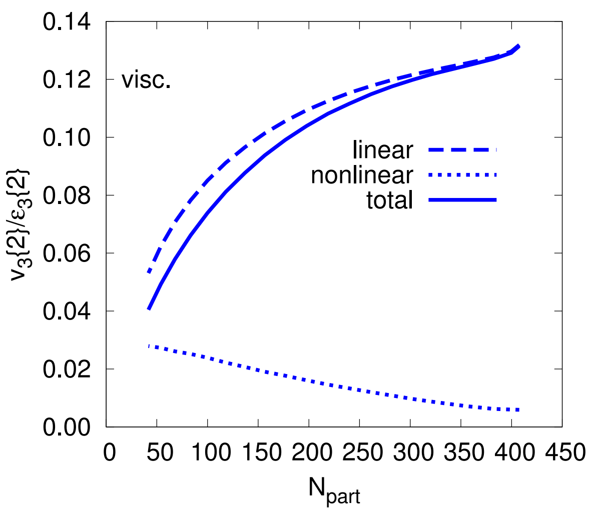
Non-linear response dependence on centrality: integrated $v_n\{2\}$

- $v_n\{2\}^2 = \underbrace{\text{linear response}}_{\langle \varepsilon^2 \rangle} + \underbrace{\text{crossing terms}}_{\langle \varepsilon^2 \cos(\ldots) \rangle} + \underbrace{\text{non-linear response}}_{\langle \varepsilon^4 \rangle}$

$v_3\{2\} : w_3, w_{3(12)}$



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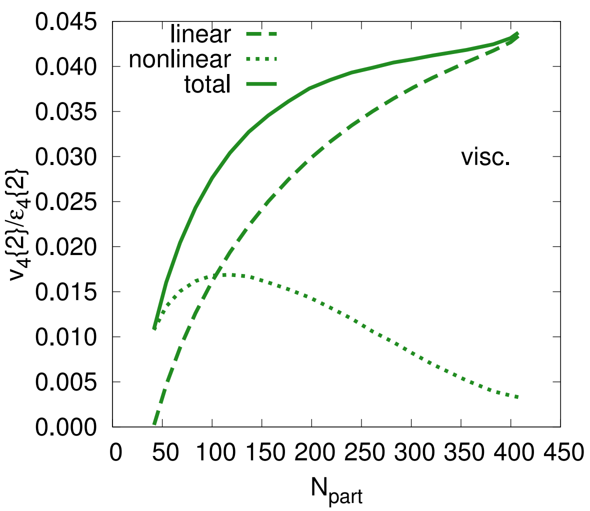
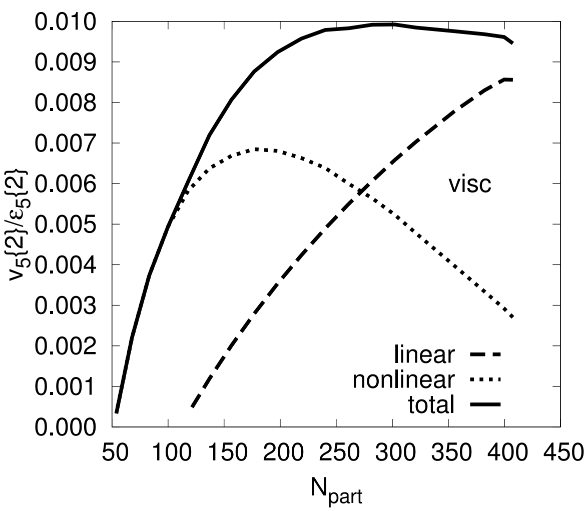


$v_5\{2\} : w_5, w_{5(23)}$



(LHC PbPb, viscous hydro, $T_{fo} = 150$MeV, PHOBOS MC-GLb.)

- Non-linear response is not important for v_3, but crucial for v_4 and v_5.
- Linear response dominates at central bins.
- Non-linear response dominates at peripheral bins.

Non-linear response becomes more important for larger centrality.

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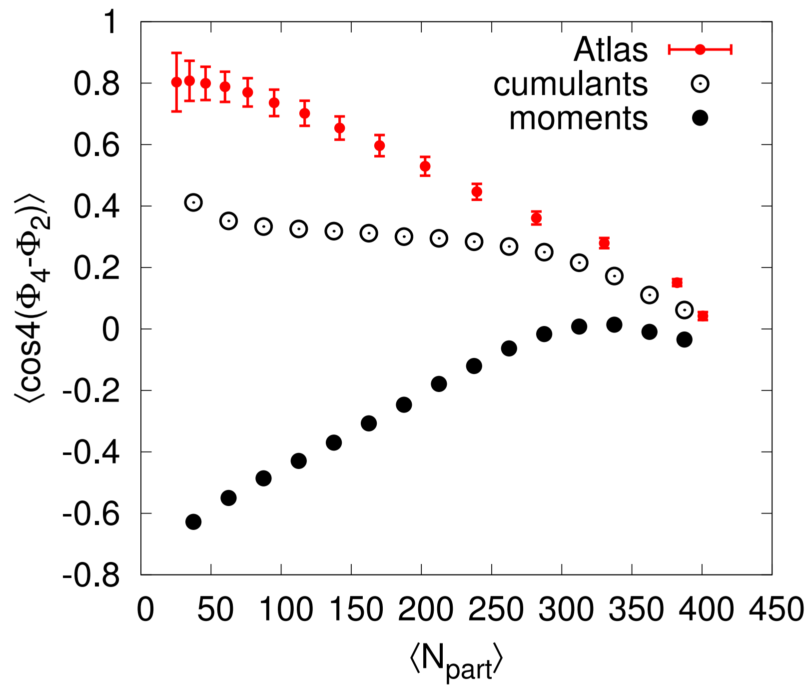
Reaction-plane correlations: linear response $(\Phi_n, \ldots) \Leftrightarrow (\Psi_n, \ldots)$

(Ψ_4, Ψ_2) correlations:



- Initial correlations from cumulants and moments are different.
- Deviations (cumulants def.) from experiment data imply NL response.
  1. At central bins(linear dominant): smaller deviations.
  2. At peripheral bins(non-linear dominant): larger deviations.

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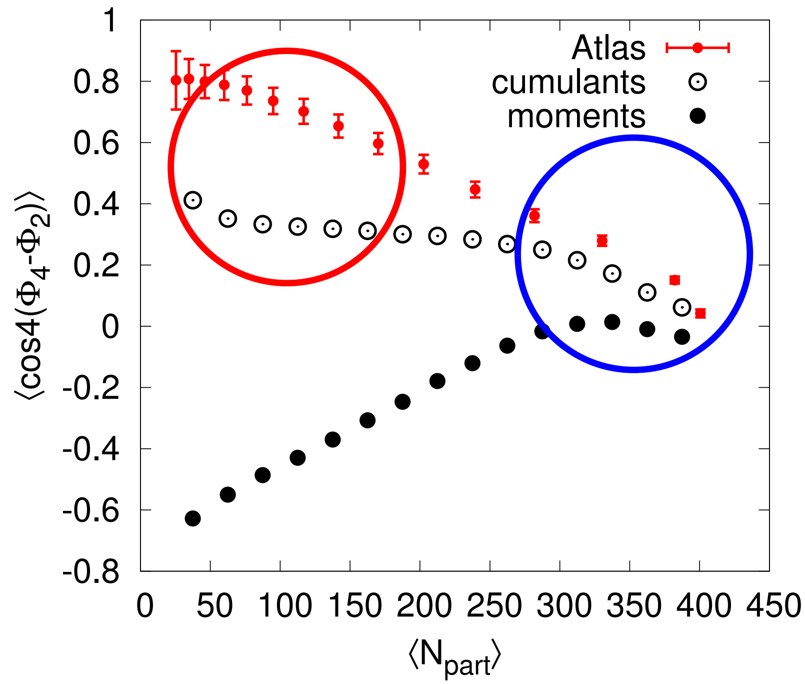

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- Understand RP plane correlation with non-linear response formalism.

$$\text{RP correlations} = \langle\langle \underbrace{\text{PP correlations}}_{\color{blue}\text{Linear response limit}} + \underbrace{\text{NL correlations}}_{\color{red}\text{NL response limit}} \rangle\rangle$$

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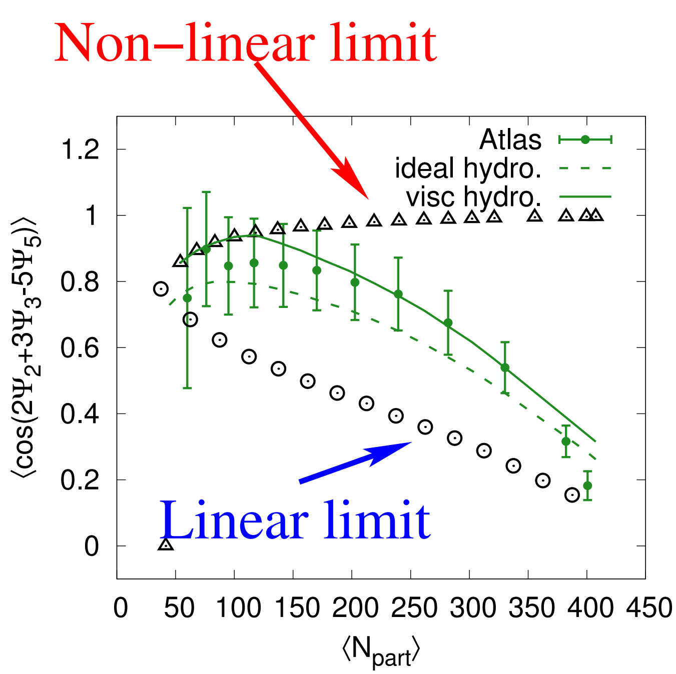

2–3–5 correlation

Reaction-plane correlations: non-linear response $(\Phi_n, \ldots) \not\Leftrightarrow (\Psi_n, \ldots)$

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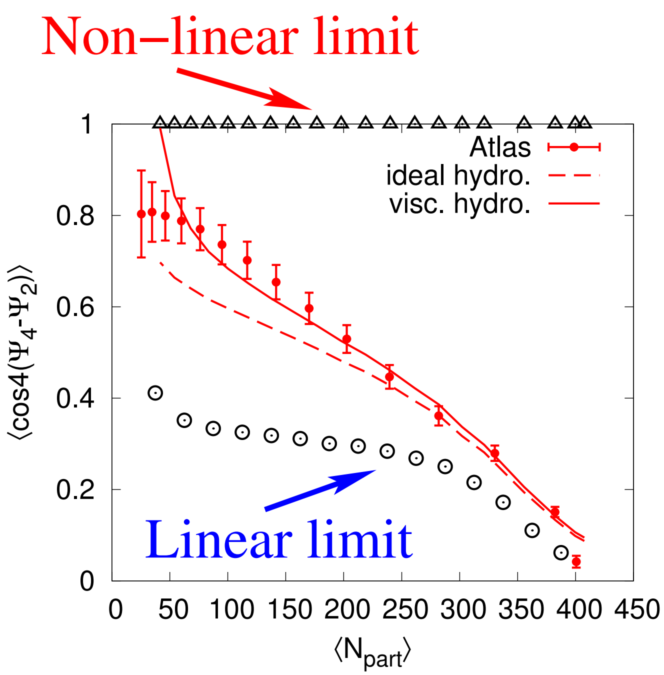
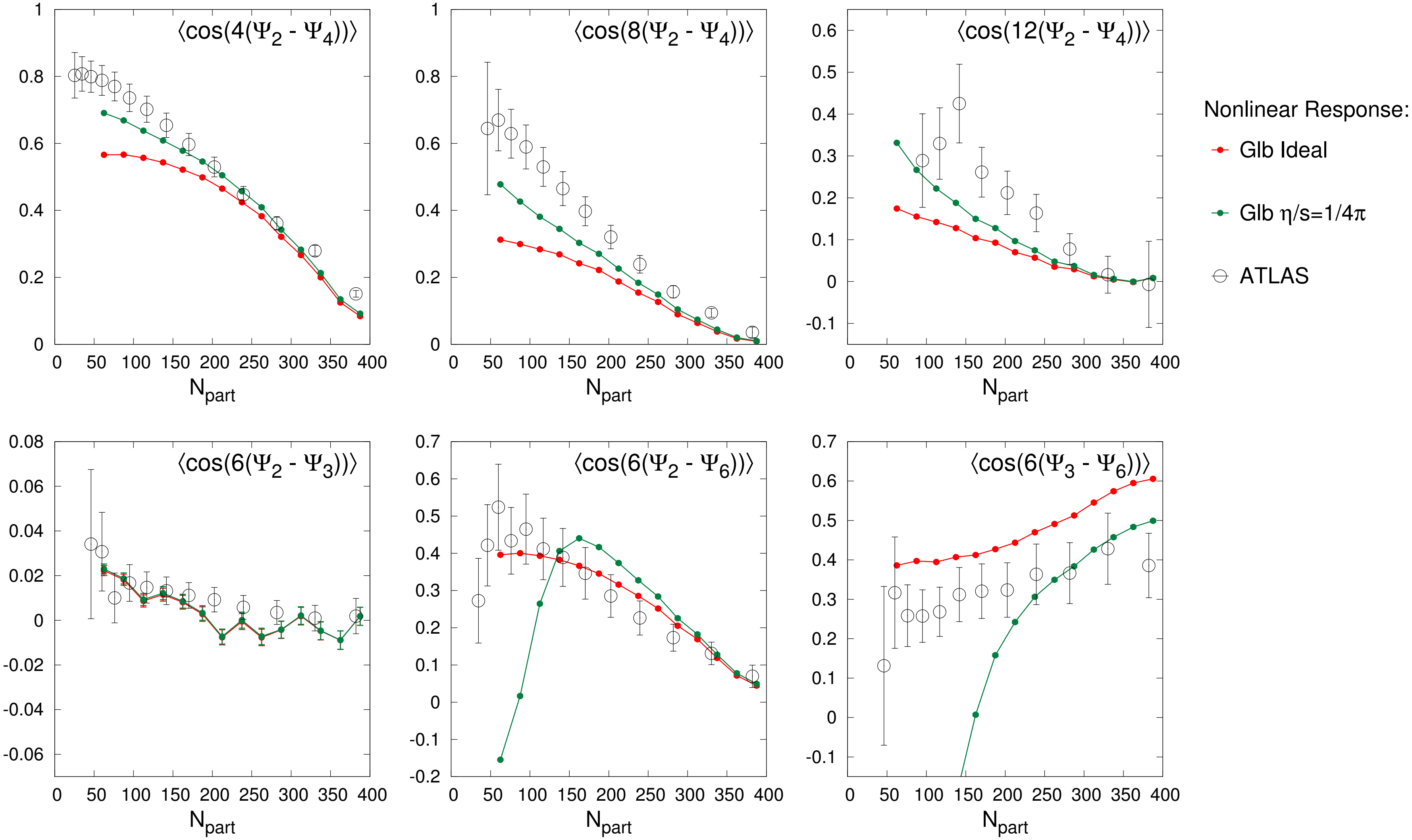


2–4 correlation

Two-plane correlations



- $v_3(3, 12), v_4(4, 22), v_5(5, 23), v_6(33, 24, 222)$
- (LHC PbPb, $\eta/s = 1/4\pi$, $T_{fo} = 150$MeV, PHOBOS MC-GLb.)

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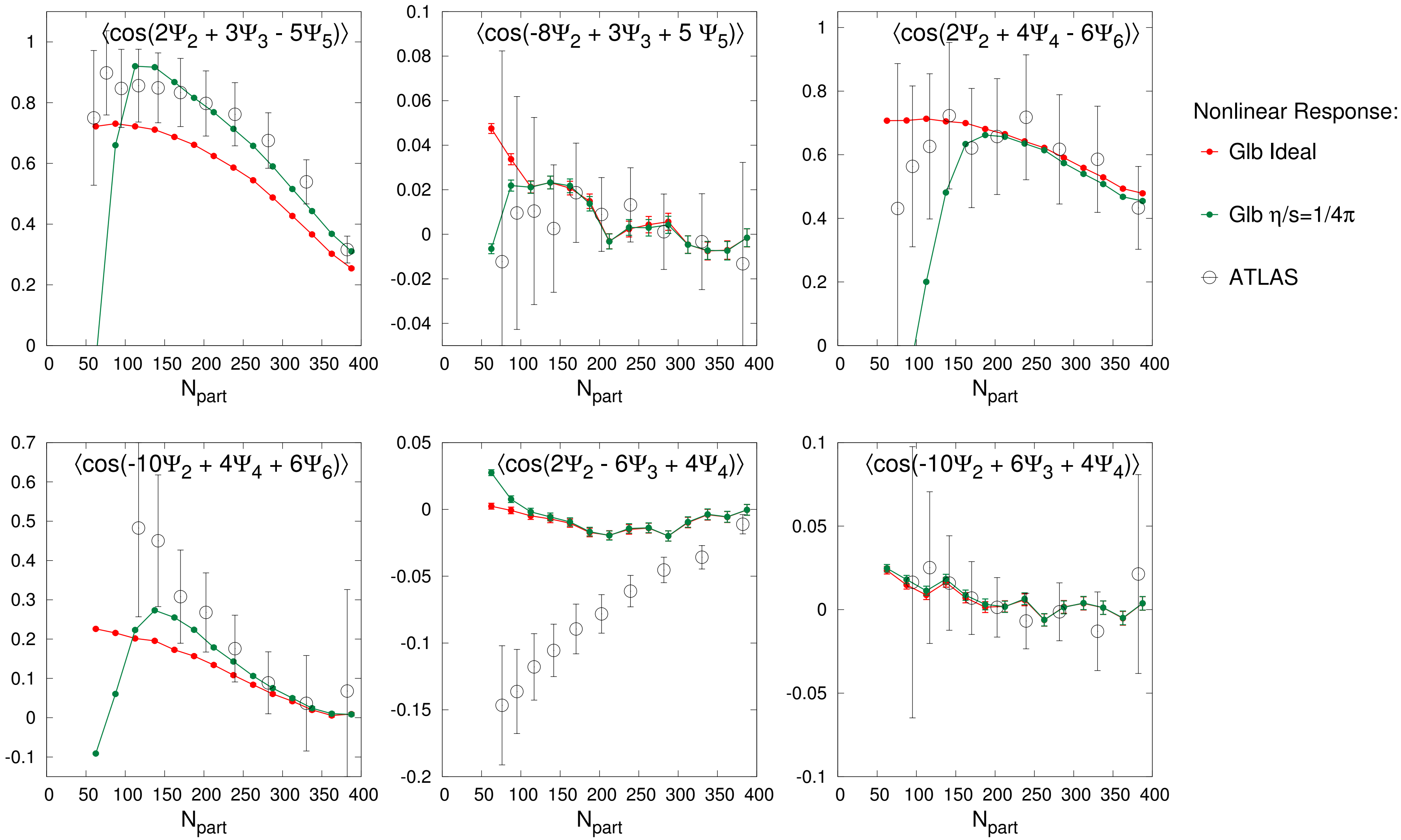
Three-plane correlations



- $v_3(3, 12), v_4(4, 22), v_5(5, 23), v_6(33, 24, 222)$
- (LHC PbPb, $\eta/s = 1/4\pi$, $T_{fo} = 150$MeV, PHOBOS MC-GLb.)

Summary and conclusions

We have developed a non-linear response formalism for (single-show) hydro:

$$\underbrace{\text{Initial correlations}}_{\text{cumulants}} \quad + \quad \underbrace{\text{Flow response}}_{\text{Linear \& \underline{Non-linear}}} \quad = \quad \underbrace{\text{Reaction plane correlations}}_{\text{Final state}}$$

- Ingredients:

  1. We use cumulant formalism to classify initial fluctuations.
  2. We take linear *and* non-linear response in hydro calculations,

     non-linear response *vs.* (p_T, η/s, centrality).
  3. Predictions are consistent to final state observables (E-B-E hydro).

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Thank you.

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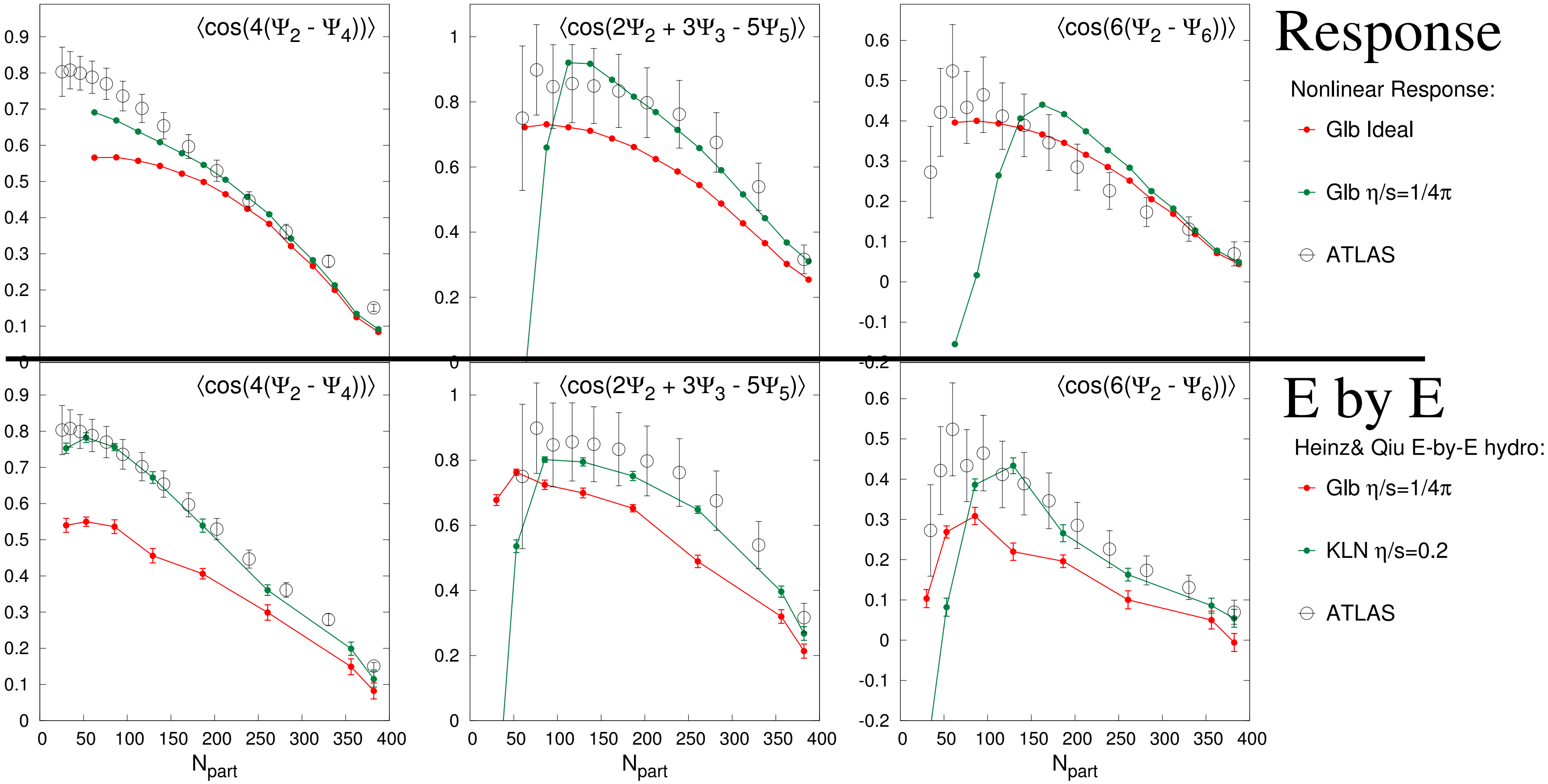
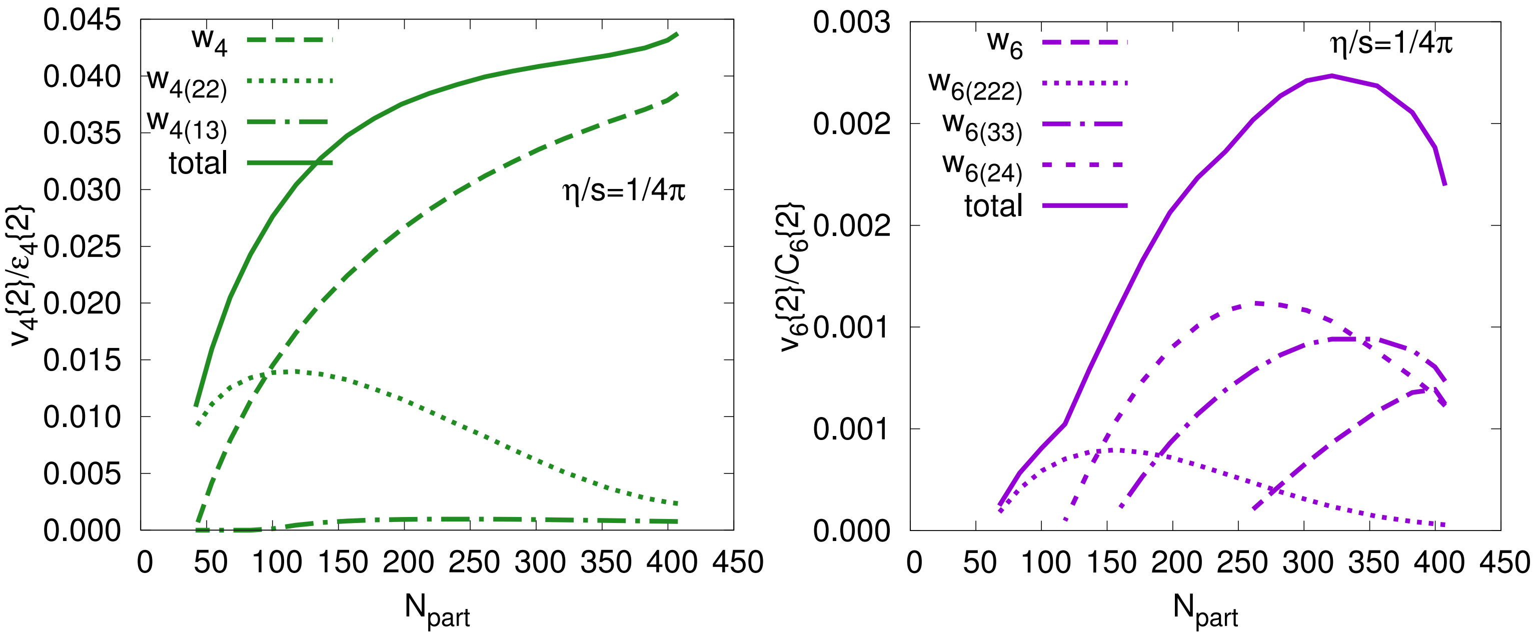
Backup slides

Not fair comparison with E-By-E hydro



Problem related to those ”. . .”

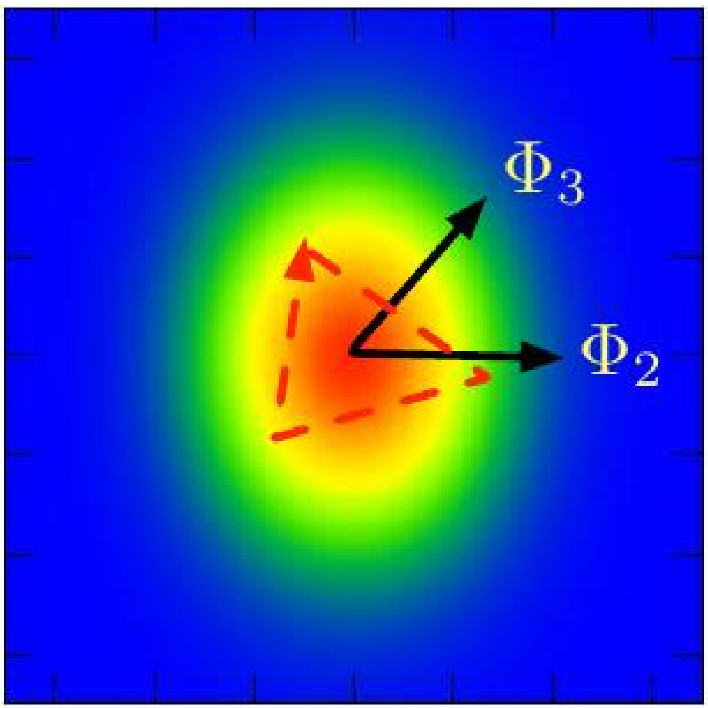


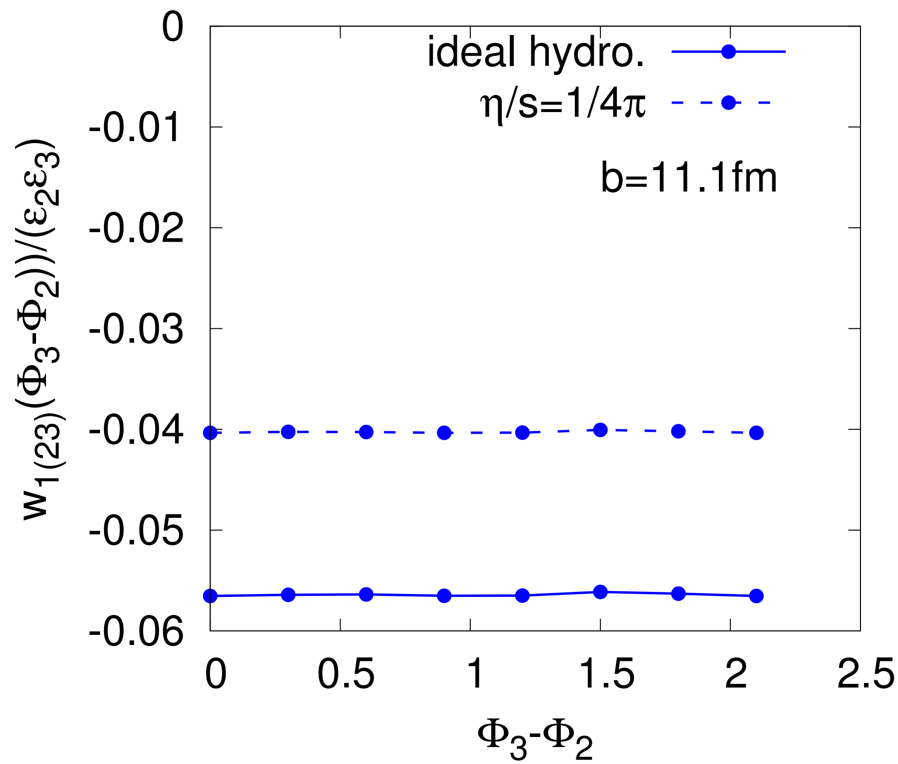
- There are infinite number of non-linear resp. terms.
- Only some of them dominate non-linear flow generations.
- It is more complex for higher order harmonic flow, e.g. v_6.

Non-linear response dependence on angle

- For $w_{4(22)}$ the angle dependence is trivial.

- $\Phi_2 = \Phi_R$ fixed, while Φ_3 rotates $\rightarrow w_{1(23)}$, ($w_{5(23)}$ similar!).





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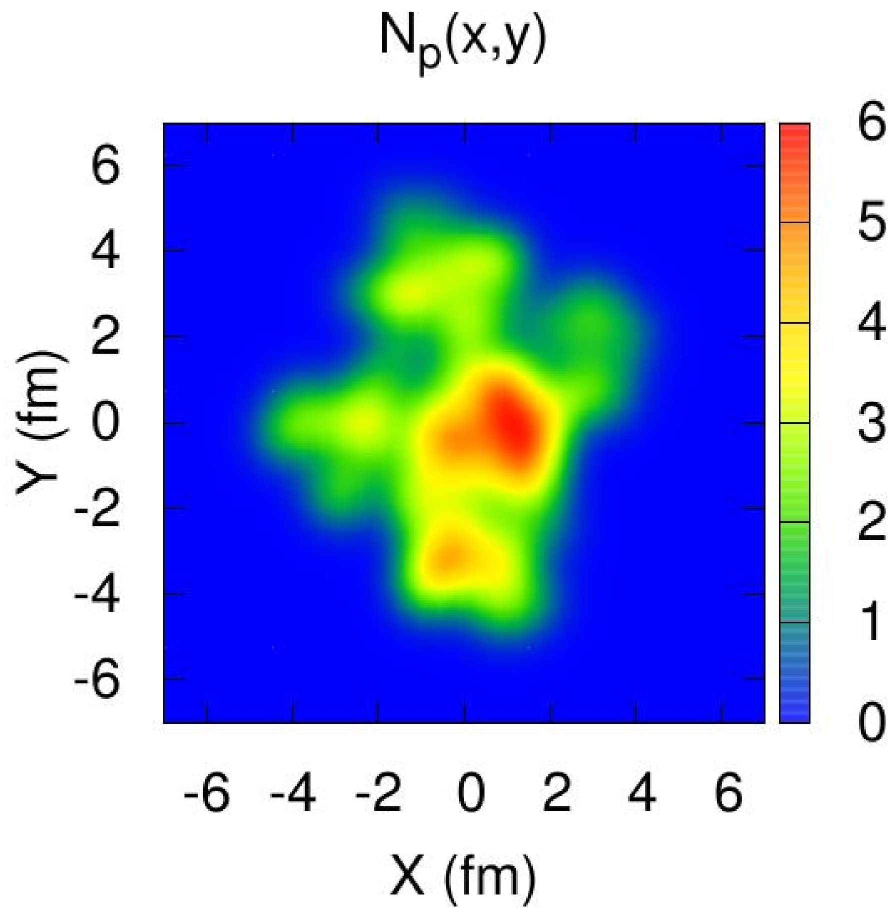
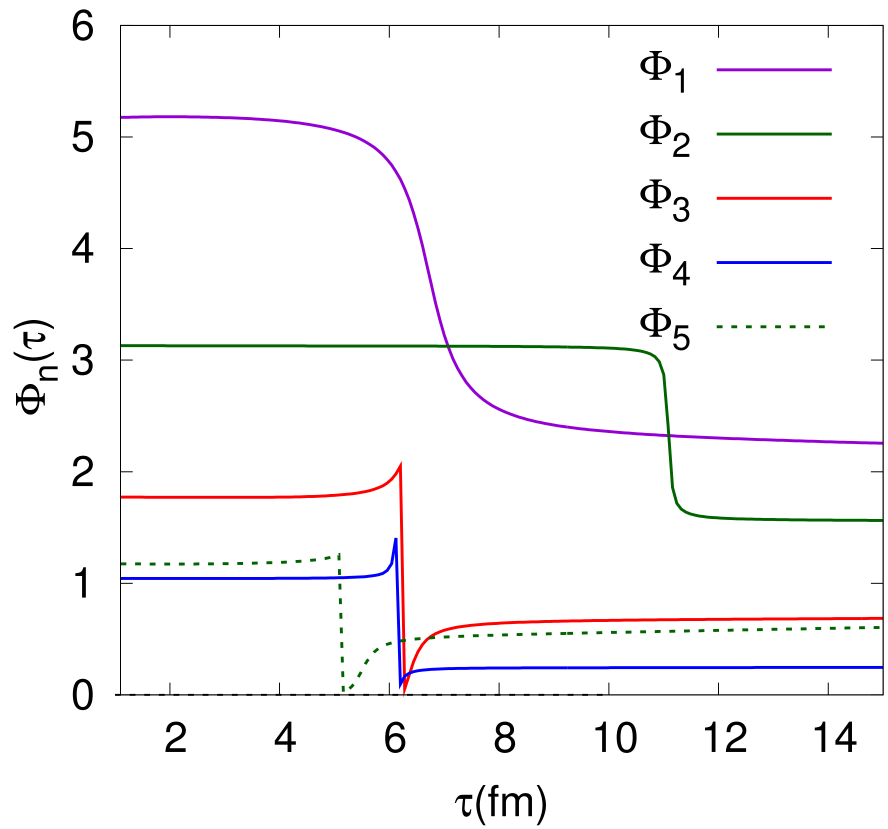
Origins of non-linearity – medium expansion

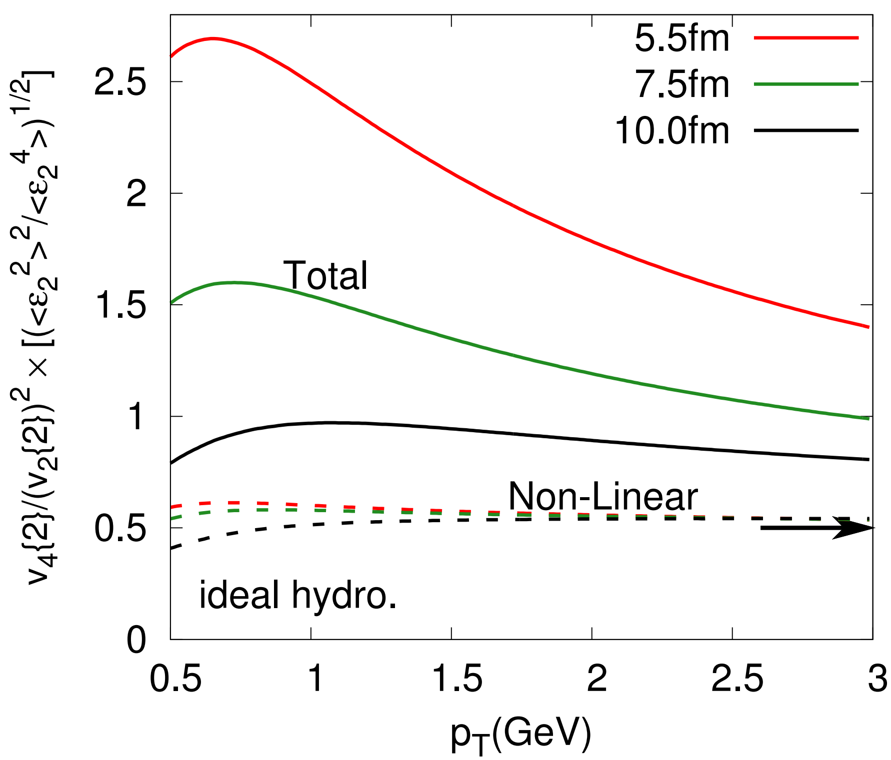


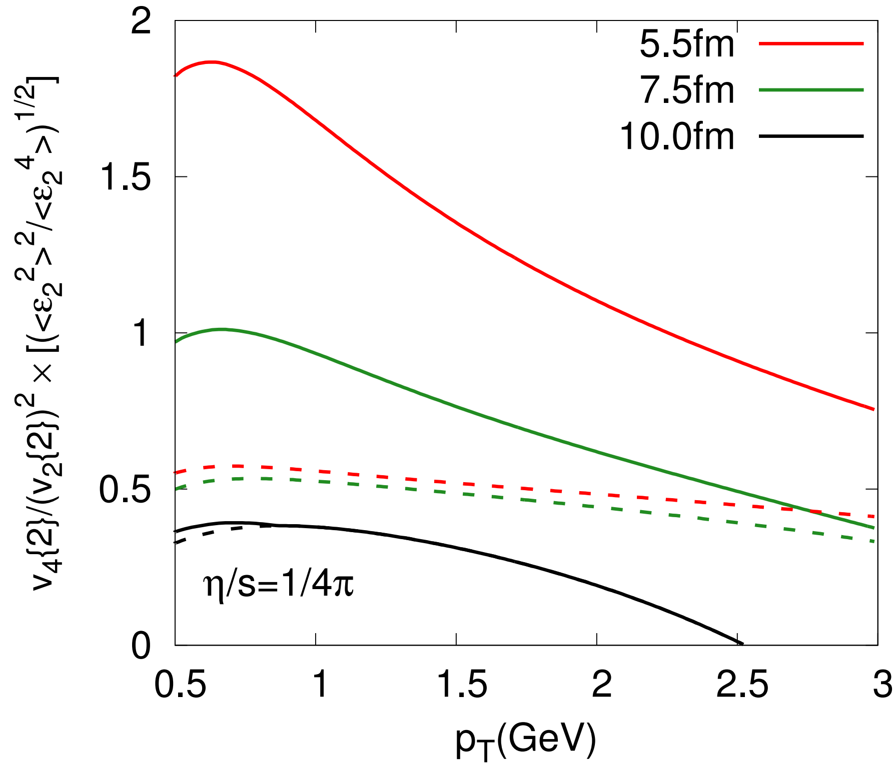


- Shape switch at certain time – $\Delta\Phi_n = |\pi/n|$
- Each of the deformations evolves independently.

Different angular deformations do NOT interact during expansion.

$v_4\{2\}(p_T)/v_2\{2\}(p_T)^2$





- Scaling behavior reproduced for ideal hydro. large p_T limit.
- Linear contribution affects the scaling.
- The real observables are dressed with quantities from events average.

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